Uniformly Accelerated Motion Observed from a Perpendicularly Moving Inertial Frame: Kinematics, Proper-Time Parametrization, Tetrad Construction, and Boosted Rindler Coordinates

Sangwha Yi

Department of Mathematics, Daejeon University,
Daejeon, Republic of Korea
leeshwa@edu.dju.ac.kr

November 18, 2026

Abstract

We analyze the motion of a point particle executing uniformly accelerated motion with constant proper acceleration in an inertial frame S', as observed from a second inertial frame S whose velocity relative to S' is perpendicular to the acceleration axis. The boost velocity is kept in a general direction $\mathbf{v}_0 = (0, v_{0y}, v_{0z})$ inside the plane perpendicular to the acceleration. Starting from the Lorentz transformation for a perpendicular boost, we derive the complete kinematics in S: the x-coordinate obeys a hyperbolic law with reduced acceleration parameter $a_0 = \alpha/\gamma$, the transverse coordinates grow linearly in time, the velocity parallel to the acceleration is suppressed by the Lorentz factor, and the coordinate acceleration at the instant of rest in S' is reduced by γ^2. We then extend the analysis to dynamics and derive the transformation of the four-momentum, as well as the three-force in S; the latter acquires transverse components even though the coordinate acceleration is purely longitudinal. In the perpendicular configuration the momentum transformation reduces to $E = \gamma E'$, $p_x = p'_x$, $p_y = \gamma v_{0y} E'/c^2$, and $p_z = \gamma v_{0z} E'/c^2$, each of which admits a transparent physical reading. We construct the orthonormal tetrad adapted to the accelerated observer, verify its orthonormality under $\eta = \mathrm{diag}(+,-,-,-)$, and give the explicit construction of boosted Rindler coordinates for a general perpendicular boost direction. The inverse transformation $\xi^a(t,x,y,z)$, the dual one-forms, the v_0-independent Rindler metric, and the horizon structure are given explicitly. The rest mass and proper acceleration are shown to be invariant, and all results reduce to the Galilean limit for $v_0 \ll c$. The construction applies to straight hyperbolic motion with vanishing rotation coefficient $\kappa = 0$.

Contents

1 Introduction **2**

2 Setup and notation **3**

3 Hyperbolic motion in the rest frame S' **3**

4 Perpendicular Lorentz boost to S **4**
4.1 Boost in a general perpendicular direction 4
4.2 Worldline in S in proper time 4

5 Kinematics in S: trajectory, velocity, and acceleration **4**
5.1 Trajectory . . . 4
5.2 Velocity . . . 5
5.3 Acceleration . . . 5

6 Energy–momentum transformation **5**
6.1 General transformation . . . 5
6.2 Perpendicular configuration . . . 5
6.3 Explicit time dependence . . . 5
6.4 Consistency and mass-shell invariance . . . 6
6.5 Three-force in S . . . 6

7 Proper-time parametrization in S **6**

8 Orthonormal tetrad **6**
8.1 Orthonormality . . . 7

9 Boosted Rindler coordinates: explicit construction **7**
9.1 Forward transformation . . . 7
9.2 Inverse transformation . . . 8
9.3 Horizon structure . . . 8
9.4 Dual one-forms and metric . . . 8

10 Consistency checks and limiting cases **9**
10.1 Acceleration transformation . . . 9
10.2 Invariance of proper acceleration . . . 9
10.3 Galilean limit . . . 9
10.4 Special case $v_{0z} = 0$ . . . 9

11 Discussion **9**

12 Conclusion **9**

1 Introduction

Uniformly accelerated motion in special relativity—often called *hyperbolic motion*—is the motion of a particle subject to a constant proper acceleration. It was studied by Born [1] and by Rindler [2, 3], and plays a central role in relativistic kinematics, in the analysis of the Unruh effect, and in the theory of rigid motion. The Rindler coordinates, which describe the perception of Minkowski space by a uniformly accelerated observer, have become a standard tool in relativistic quantum information and in the study of horizons.

We consider a particle that executes hyperbolic motion along the x'-axis in its instantaneous rest frame S', viewed from another inertial frame S that moves with constant velocity perpendicular to the acceleration axis. Because the boost is perpendicular to the acceleration, transverse spatial coordinates are unaffected by Lorentz contraction, while the time coordinate mixes with the boost direction. The resulting motion in S is a superposition of the hyperbolic law along x and a uniform drift along the boost direction. Unlike treatments in which the boost is taken along a single transverse axis, we keep the boost direction $\mathbf{v}_0 = (0, v_{0y}, v_{0z})$ general inside the plane perpendicular to the acceleration; the one-component case is recovered by setting $v_{0z} = 0$.

The purpose of this paper is threefold. First, we derive the complete kinematics of the particle in S: trajectory, velocity, acceleration, proper-time parametrization, and the three-force. Second, we derive the transformation of the four-momentum in the same perpendicular

configuration and show that the transformation laws take a remarkably simple form whose physical content can be read off directly. Third, we construct the orthonormal tetrad adapted to the accelerated observer, and we give the explicit construction of boosted Rindler coordinates for a general perpendicular boost direction, including the inverse transformation, the dual one-forms, and the horizon structure.

2 Setup and notation

Let S and S' be inertial frames related by a Lorentz boost, with the boost velocity $\mathbf{v}_0$ of S' relative to S lying in the yz-plane:

$$\mathbf{v}_0 = (0, v_{0y}, v_{0z}), \qquad v_0^2 = v_{0y}^2 + v_{0z}^2. \tag{1}$$

Define the boost Lorentz factor

$$\gamma = \frac{1}{\sqrt{1-\beta_0^2}}, \qquad \beta_0 = \frac{v_0}{c}. \tag{2}$$

Throughout, γ denotes the boost Lorentz factor, consistent with the time-dilation relation $dt = \gamma\, dt'$ for a clock at rest in S'. The particle's own Lorentz factor will be denoted γ_p and γ_p' in S and S', respectively, to avoid confusion.

It is convenient to introduce the orthonormal basis vectors of the yz-plane aligned with, and perpendicular to, the boost direction:

$$\hat{n} = \frac{(v_{0y}, v_{0z})}{v_0}, \qquad \hat{m} = \frac{(-v_{0z}, v_{0y})}{v_0}. \tag{3}$$

A point particle of rest mass m moves in S' along the x'-axis with constant proper acceleration $\alpha > 0$, starting from rest at the origin at $t' = 0$:

$$x'(0) = 0, \qquad \dot{x}'(0) = 0, \qquad y'(t') = z'(t') = 0. \tag{4}$$

The proper acceleration α is a Lorentz scalar; its magnitude is the same in every inertial frame. We denote

$$L := \frac{c^2}{\alpha}, \qquad \theta := \frac{\alpha\tau}{c}, \tag{5}$$

where τ is the proper time of the accelerated particle. The reduced acceleration parameter is

$$a_0 := \frac{\alpha}{\gamma} = \alpha\sqrt{1-\beta_0^2}. \tag{6}$$

3 Hyperbolic motion in the rest frame S'

The worldline of a particle with constant proper acceleration α along the x'-axis, starting from rest at the origin, is

$$x'(t') = \frac{c^2}{\alpha}\left(\sqrt{1+\frac{\alpha^2 t'^2}{c^2}} - 1\right), \qquad y'(t') = z'(t') = 0. \tag{7}$$

The velocity and coordinate acceleration are

$$V_x'(t') = \frac{\alpha t'}{\sqrt{1+\alpha^2 t'^2/c^2}}, \qquad a_x'(t') = \frac{\alpha}{(1+\alpha^2 t'^2/c^2)^{3/2}}. \tag{8}$$

At $t' = 0$, $V'_x = 0$ and $a'_x = \alpha$; as $t' \to \infty$, $V'_x \to c$. The particle's Lorentz factor in S' and its momentum–energy are

$$\gamma'_p(t') = \sqrt{1 + \frac{\alpha^2 t'^2}{c^2}}, \qquad p'_x(t') = \gamma'_p m V'_x = m\alpha t', \qquad E'(t') = \gamma'_p mc^2. \tag{9}$$

The linear form $p'_x = m\alpha t'$ is a special feature of one-dimensional hyperbolic motion: $dp'_x/dt' = m\alpha$ is constant because the coordinate force equals the proper force in the instantaneous rest frame.

The proper time satisfies $d\tau = dt'/\gamma'_p$, hence

$$\tau = \frac{c}{\alpha}\,\mathrm{arsinh}\!\left(\frac{\alpha t'}{c}\right), \qquad \text{or} \qquad ct'(\tau) = L\sinh\theta, \quad x'(\tau) = L\left(\cosh\theta - 1\right). \tag{10}$$

4 Perpendicular Lorentz boost to S

4.1 Boost in a general perpendicular direction

Decompose spatial vectors into components parallel and perpendicular to $\mathbf{v}_0$: $\mathbf{r}'_\parallel = (\mathbf{r}'\cdot\mathbf{v}_0)\mathbf{v}_0/v_0^2$, $\mathbf{r}'_\perp = \mathbf{r}' - \mathbf{r}'_\parallel$. Since $\mathbf{v}_0$ lies in the yz-plane and $\mathbf{r}' = (x', 0, 0)$, we have $\mathbf{r}'_\parallel = 0$ and $\mathbf{r}'_\perp = (x', 0, 0)$. The Lorentz transformation from S' to S reads

$$t = \gamma\left(t' + \frac{\mathbf{v}_0\cdot\mathbf{r}'}{c^2}\right), \qquad \mathbf{r}_\parallel = \gamma\left(\mathbf{r}'_\parallel + \mathbf{v}_0 t'\right), \qquad \mathbf{r}_\perp = \mathbf{r}'_\perp. \tag{11}$$

For the particle, $\mathbf{v}_0\cdot\mathbf{r}' = 0$, hence

$$t = \gamma t', \qquad x = x'(t'), \qquad y = \gamma v_{0y} t' = v_{0y} t, \qquad z = \gamma v_{0z} t' = v_{0z} t. \tag{12}$$

4.2 Worldline in S in proper time

Inserting (10) into (12) and using $\theta = \alpha\tau/c$ and $L = c^2/\alpha$,

$$\boxed{\begin{aligned} ct(\tau) &= \gamma L\sinh\theta, \\ x(\tau) &= L\left(\cosh\theta - 1\right), \\ y(\tau) &= \gamma L\frac{v_{0y}}{c}\sinh\theta, \\ z(\tau) &= \gamma L\frac{v_{0z}}{c}\sinh\theta. \end{aligned}} \tag{13}$$

Along the boost direction $\hat{n}$, $r_\parallel(\tau) = \gamma L\beta_0\sinh\theta$ and $r_\perp(\tau) = 0$.

5 Kinematics in S: trajectory, velocity, and acceleration

5.1 Trajectory

Substituting $t' = t/\gamma$ into (7) and using $x = x'$,

$$x(t) = \frac{c^2}{\alpha}\left[\sqrt{1 + \frac{\alpha^2 t^2}{\gamma^2 c^2}} - 1\right] = \frac{c^2}{\gamma a_0}\left[\sqrt{1 + \frac{a_0^2 t^2}{c^2}} - 1\right], \tag{14}$$

$$y(t) = v_{0y} t, \qquad z(t) = v_{0z} t. \tag{15}$$

5.2 Velocity

Differentiating (14),

$$V_x(t) = \frac{a_0 t/\gamma}{\sqrt{1 + a_0^2 t^2/c^2}} = \frac{V_x'(t')}{\gamma}, \qquad V_y = v_{0y}, \qquad V_z = v_{0z}. \tag{16}$$

The particle's Lorentz factor in S is

$$\gamma_p = \frac{1}{\sqrt{1 - V^2/c^2}} = \gamma\gamma_p'. \tag{17}$$

5.3 Acceleration

Differentiating (16),

$$a_x(t) = \frac{a_0/\gamma}{\left(1 + a_0^2 t^2/c^2\right)^{3/2}}, \qquad a_x(0) = \frac{\alpha}{\gamma^2}, \qquad a_y = a_z = 0. \tag{18}$$

The coordinate acceleration in S at the instant of rest in S' is reduced by γ^2 relative to the proper acceleration.

6 Energy–momentum transformation

6.1 General transformation

The four-momentum of a particle of rest mass m is

$$P^\mu = \left(\frac{E}{c}, \mathbf{p}\right), \quad E = \gamma_p mc^2, \quad \mathbf{p} = \gamma_p m\mathbf{V}, \quad P^\mu P_\mu = m^2c^2. \tag{19}$$

Under a boost with velocity $\mathbf{v}_0$,

$$E = \gamma\left(E' + \mathbf{v}_0 \cdot \mathbf{p}'\right), \quad p_\parallel = \gamma\left(p_\parallel' + v_0\frac{E'}{c^2}\right), \quad p_\perp = p_\perp'. \tag{20}$$

6.2 Perpendicular configuration

In the present problem $\mathbf{p}' = (p_x', 0, 0)$ and $\mathbf{v}_0$ lies in the yz-plane, so $\mathbf{v}_0 \cdot \mathbf{p}' = 0$. Equations (20) reduce to

$$\boxed{E = \gamma E', \quad p_x = p_x', \quad p_y = \gamma v_{0y}\frac{E'}{c^2}, \quad p_z = \gamma v_{0z}\frac{E'}{c^2}.} \tag{21}$$

The physical content is: (i) $E = \gamma E'$ involves only time dilation; (ii) $p_x = p_x'$ is invariant because x is perpendicular to the boost; (iii) p_y and p_z acquire a term proportional to E', reflecting the relativistic inertia of energy.

6.3 Explicit time dependence

Combining (9) with (21), and using $t' = t/\gamma$ and $a_0 = \alpha/\gamma$,

$$E(t) = \gamma mc^2\sqrt{1 + \frac{a_0^2 t^2}{c^2}} = \gamma_p(t)mc^2, \qquad p_x(t) = ma_0 t, \tag{22}$$

$$p_y(t) = \gamma v_{0y} m\sqrt{1 + \frac{a_0^2 t^2}{c^2}}, \qquad p_z(t) = \gamma v_{0z} m\sqrt{1 + \frac{a_0^2 t^2}{c^2}}. \tag{23}$$

6.4 Consistency and mass-shell invariance

Dividing (22)–(23) by $\gamma_p m$ reproduces (16). As a consistency check, we verify that the mass-shell condition is preserved:

$$E^2 - \mathbf{p}^2c^2 = \gamma^2 E'^2 - p_x'^2c^2 - \gamma^2\beta_0^2 E'^2 = E'^2 - p_x'^2c^2 = m^2c^4. \tag{24}$$

This confirms that the three features of (21) are mutually consistent with, and jointly ensure, the invariance of the rest mass.

6.5 Three-force in S

The three-force in S is $\mathbf{F} = d\mathbf{p}/dt$. From (22)–(23),

$$F_x = \frac{dp_x}{dt} = ma_0 = \frac{m\alpha}{\gamma}, \tag{25}$$

$$F_y = \frac{dp_y}{dt} = \frac{\gamma m v_{0y} a_0^2 t}{c^2\sqrt{1 + a_0^2t^2/c^2}}, \qquad F_z = \frac{dp_z}{dt} = \frac{\gamma m v_{0z} a_0^2 t}{c^2\sqrt{1 + a_0^2t^2/c^2}}. \tag{26}$$

Although the coordinate acceleration is purely along x, the three-force acquires transverse components because the particle's relativistic mass $\gamma_p m$ grows with time while the transverse velocities v_{0y}, v_{0z} remain constant. At $t = 0$, $F_y = F_z = 0$; as $t \to \infty$, $F_y \to \gamma m v_{0y} a_0/c$ and $F_z \to \gamma m v_{0z} a_0/c$.

The power delivered by the three-force satisfies

$$\mathbf{F}\cdot\mathbf{V} = F_xV_x + F_yV_y + F_zV_z = \frac{\gamma m a_0^2 t}{\sqrt{1 + a_0^2t^2/c^2}} = \frac{dE}{dt}, \tag{27}$$

which is a consistency check of the energy–momentum transformation (21).

7 Proper-time parametrization in S

Using $a_0 = \alpha/\gamma$, we can write (13) in terms of τ and a_0:

$$\boxed{\begin{aligned} t(\tau) &= \frac{c}{a_0}\sinh\!\left(\frac{\gamma a_0\tau}{c}\right), \\ x(\tau) &= \frac{c^2}{\gamma a_0}\left[\cosh\!\left(\frac{\gamma a_0\tau}{c}\right) - 1\right], \\ y(\tau) &= v_{0y}\frac{c}{a_0}\sinh\!\left(\frac{\gamma a_0\tau}{c}\right), \\ z(\tau) &= v_{0z}\frac{c}{a_0}\sinh\!\left(\frac{\gamma a_0\tau}{c}\right). \end{aligned}} \tag{28}$$

8 Orthonormal tetrad

We construct the orthonormal tetrad $e_{(a)}$ carried by the accelerated observer, working with signature $\eta = \mathrm{diag}(+,-,-,-)$ and components ordered as (ct, x, y, z). Evaluating the coordinate

derivatives at $\xi^1 = \xi^2 = \xi^3 = 0$,

$$\boxed{\begin{aligned} e_{(0)} &= \frac{1}{c}\frac{\partial X}{\partial \tau} = \left(\gamma\cosh\theta,\ \sinh\theta,\ \gamma\frac{v_{0y}}{c}\cosh\theta,\ \gamma\frac{v_{0z}}{c}\cosh\theta\right), \\ e_{(1)} &= \frac{\partial X}{\partial \xi^1} = \left(\gamma\sinh\theta,\ \cosh\theta,\ \gamma\frac{v_{0y}}{c}\sinh\theta,\ \gamma\frac{v_{0z}}{c}\sinh\theta\right), \\ e_{(2)} &= \left(\gamma\beta_0,\ 0,\ \gamma\frac{v_{0y}}{v_0},\ \gamma\frac{v_{0z}}{v_0}\right), \\ e_{(3)} &= \left(0,\ 0,\ -\frac{v_{0z}}{v_0},\ \frac{v_{0y}}{v_0}\right). \end{aligned}} \tag{29}$$

Here $e_{(0)} = U/c$, $e_{(1)}$ points along x', $e_{(2)}$ along $\hat{n}$, and $e_{(3)}$ along $\hat{m}$.

Remark 1. *Equations (29) give the tetrad at $\xi^1 = \xi^2 = \xi^3 = 0$. At general ξ, the unit timelike normal to the τ =const hypersurface is $e_{(0)}(\xi) = \partial_\tau X/|\partial_\tau X|$. Since $\partial_\tau X = c(1+\xi^1/L)e_{(0)}(0)$ and $|\partial_\tau X| = c(1+\xi^1/L)$, we have $e_{(0)}(\xi) = e_{(0)}(0)$. The tetrad components (29) are therefore independent of ξ.*

8.1 Orthonormality

Using $\gamma^2(1-\beta_0^2) = 1$ and $v_{0y}^2 + v_{0z}^2 = v_0^2$,

$$\begin{aligned} e_{(0)}^2 = +1, \quad e_{(1)}^2 = -1, \quad e_{(2)}^2 = -1, \quad e_{(3)}^2 = -1, \\ e_{(a)}\cdot e_{(b)} = 0 \quad (a \neq b). \end{aligned} \tag{30}$$

Thus $e_{(a)}\cdot e_{(b)} = \eta_{ab}$.

9 Boosted Rindler coordinates: explicit construction

We give the explicit construction of Rindler coordinates for a general perpendicular boost direction. Let $(\tau, \xi^1, \xi^2, \xi^3)$ denote the accelerated-frame coordinates, with ξ^1 along x, ξ^2 along $\hat{n}$, and ξ^3 along $\hat{m}$.

9.1 Forward transformation

$$\boxed{\begin{aligned} ct &= \gamma\left[(L+\xi^1)\sinh\theta + \beta_0\,\xi^2\right], \\ x &= (L+\xi^1)\cosh\theta - L, \\ r_\parallel &= \gamma\left[\xi^2 + \beta_0(L+\xi^1)\sinh\theta\right], \\ r_\perp &= \xi^3, \end{aligned}} \tag{31}$$

with $r_\parallel, r_\perp$ the components along $\hat{n}, \hat{m}$. In terms of y, z,

$$\boxed{\begin{aligned} y &= \gamma\left[\xi^2 + \beta_0(L+\xi^1)\sinh\theta\right]\frac{v_{0y}}{v_0} - \xi^3\frac{v_{0z}}{v_0}, \\ z &= \gamma\left[\xi^2 + \beta_0(L+\xi^1)\sinh\theta\right]\frac{v_{0z}}{v_0} + \xi^3\frac{v_{0y}}{v_0}. \end{aligned}} \tag{32}$$

Setting $\xi^1 = \xi^2 = \xi^3 = 0$ recovers (13).

9.2 Inverse transformation

Given (t, x, y, z),

$$r_\parallel = \frac{v_{0y}y + v_{0z}z}{v_0}, \qquad r_\perp = \frac{-v_{0z}y + v_{0y}z}{v_0}, \tag{33}$$

$$ct' = \gamma(ct - \beta_0 r_\parallel), \qquad r'_\parallel = \gamma(r_\parallel - \beta_0 ct), \qquad r'_\perp = r_\perp. \tag{34}$$

Then

$$\boxed{\begin{aligned} &R = \sqrt{(x+L)^2 - (ct')^2}, \\ &\xi^1 = R - L, \\ &\theta = \operatorname{artanh}\left(\frac{ct'}{x+L}\right) = \operatorname{arsinh}\left(\frac{ct'}{R}\right), \\ &\tau = \frac{c}{\alpha}\theta, \\ &\xi^2 = r'_\parallel, \qquad \xi^3 = r'_\perp. \end{aligned}} \tag{35}$$

The domain of validity is the Rindler wedge $x + L > |ct'|$, $R > 0$.

9.3 Horizon structure

The Rindler horizon consists of the null planes

$$x + L = \pm ct', \tag{36}$$

which in S take the form

$$x + L = \pm\gamma\left(ct - \beta_0 r_\parallel\right). \tag{37}$$

The surface $x = -L$ is not itself the horizon; it is the *bifurcation surface* where the two null planes (36) intersect at $ct' = 0$.

9.4 Dual one-forms and metric

From (31), the coordinate basis vectors satisfy

$$\partial_\tau X = c\left(1 + \frac{\xi^1}{L}\right) e_{(0)}, \qquad \partial_{\xi^1} X = e_{(1)}, \qquad \partial_{\xi^2} X = e_{(2)}, \qquad \partial_{\xi^3} X = e_{(3)}. \tag{38}$$

The resulting Rindler metric is independent of v_0:

$$\boxed{ds^2 = c^2\left(1 + \frac{\alpha\xi^1}{c^2}\right)^2 d\tau^2 - (d\xi^1)^2 - (d\xi^2)^2 - (d\xi^3)^2.} \tag{39}$$

Equivalently, in terms of $\theta = \alpha\tau/c$ and $L = c^2/\alpha$,

$$ds^2 = (L + \xi^1)^2\, d\theta^2 - (d\xi^1)^2 - (d\xi^2)^2 - (d\xi^3)^2. \tag{40}$$

The dual one-form basis reads

$$e^{(0)} = c\left(1 + \frac{\alpha\xi^1}{c^2}\right) d\tau = (L + \xi^1)\, d\theta, \quad e^{(1)} = d\xi^1, \quad e^{(2)} = d\xi^2, \quad e^{(3)} = d\xi^3, \tag{41}$$

so that $ds^2 = (e^{(0)})^2 - (e^{(1)})^2 - (e^{(2)})^2 - (e^{(3)})^2$.

Remark 2. *Although (39) does not depend explicitly on v_0, the embedding of the accelerated frame in S does: the boost direction $\hat{n}$ and the orthogonal axis $\hat{m}$ determine the orientation of ξ^2, ξ^3 in S.*

10 Consistency checks and limiting cases

10.1 Acceleration transformation

For a perpendicular boost, at the instant of rest in S', $a_\perp = a'_\perp/\gamma^2$, in agreement with (18).

10.2 Invariance of proper acceleration

The proper acceleration is $\alpha = \sqrt{-A^\mu A_\mu}$. In S' at $t' = 0$, $U'^\mu = (c, 0, 0, 0)$ and $A'^\mu = (0, \alpha, 0, 0)$, so α is invariant. Under the perpendicular boost, $A^\mu = (0, \alpha, 0, 0)$ and $-A^\mu A_\mu = \alpha^2$.

10.3 Galilean limit

For $v_0 \ll c$, $\gamma \to 1$, $a_0 \to \alpha$, and (14) gives $x \simeq \frac{1}{2}\alpha t^2$, $y = v_{0y}t$, $z = v_{0z}t$.

10.4 Special case $v_{0z} = 0$

For a boost purely in y, $\hat{n} = (0, 1, 0)$, $\hat{m} = (0, 0, 1)$, and (31) reduces to the one-component treatment.

11 Discussion

The motion in S is a superposition of a reduced hyperbolic motion along x (with parameter $a_0 = \alpha/\gamma$) and a uniform drift (v_{0y}, v_{0z}) along the boost direction. The two decouple because the boost is perpendicular to the acceleration.

The relation $V_x = V'_x/\gamma$ follows from the relativity of simultaneity: the same $dx = dx'$ occurs over $dt = \gamma dt'$. Each time derivative brings a factor $1/\gamma$, giving $a_x = a'_x/\gamma^2$.

The energy–momentum transformation (21) and the three-force (25)–(26) complete the kinematic picture. The transverse momenta grow as $\gamma_p m v_{0y,z}$, reflecting the relativistic inertia of energy. The three-force acquires transverse components, and the power identity (27) confirms consistency.

The tetrad (29) and the boosted Rindler coordinates (31) give the explicit accelerated-frame description for a general perpendicular boost direction. The metric (39) is independent of v_0, while the embedding in S depends on v_0 through $\hat{n}, \hat{m}$. Because $e_{(2)}, e_{(3)}$ are independent of θ, the spatial triad does not rotate along the worldline; the rotation coefficient κ vanishes.

Scope. The present construction applies to straight hyperbolic motion with vanishing rotation coefficient $\kappa = 0$. It does not directly apply to the constant-laboratory-force model with nonzero transverse momentum, for which $\kappa = gp_\perp/(mc) \neq 0$; that case requires a separate treatment.

12 Conclusion

The main results are:

1. Trajectory in S: $x(t) = \frac{c^2}{\gamma a_0}\left[\sqrt{1 + a_0^2 t^2/c^2} - 1\right]$, $y = v_{0y}t$, $z = v_{0z}t$.
2. Velocity: $V_x = V'_x/\gamma$, $V_y = v_{0y}$, $V_z = v_{0z}$.
3. Coordinate acceleration at $t = 0$: $a_x(0) = \alpha/\gamma^2$.
4. Energy–momentum transformation: $E = \gamma E'$, $p_x = p'_x$, $p_{y,z} = \gamma v_{0y,z} E'/c^2$.
5. Three-force: $F_x = m\alpha/\gamma$, $F_{y,z} = \gamma m v_{0y,z} a_0^2 t/(c^2\sqrt{1 + a_0^2 t^2/c^2})$, with $\mathbf{F} \cdot \mathbf{V} = dE/dt$.

6. Proper-time parametrization (28).

7. Orthonormal tetrad (29) with $e_{(a)} \cdot e_{(b)} = \eta_{ab}$.

8. Boosted Rindler coordinates (31) with inverse (35), dual forms (41), and metric (39).

9. Horizon structure: null planes $x + L = \pm ct'$, bifurcation surface $x = -L$.

10. Invariance: $E^2 - \mathbf{p}^2c^2 = m^2c^4$ and $-A^\mu A_\mu = \alpha^2$.

These results provide a complete kinematical and dynamical description of uniformly accelerated motion as viewed from a perpendicularly moving frame, together with the associated tetrad and boosted Rindler coordinates. They illustrate the interplay between time dilation, the relativity of simultaneity, the relativistic inertia of energy, and the geometry of perpendicular boosts.

References

[1] M. Born, "Die Theorie des starren Elektrons in der Kinematik des Relativitätsprinzips," *Annalen der Physik* **30**, 1–56 (1909).

[2] W. Rindler, "Hyperbolic Motion in Curved Space Time," *Physical Review* **119**, 2082–2089 (1960).

[3] W. Rindler, *Relativity: Special, General, and Cosmological*, 2nd ed. (Oxford University Press, Oxford, 2006).

[4] J. D. Jackson, *Classical Electrodynamics*, 3rd ed. (Wiley, New York, 1999), Ch. 11.

[5] C. Møller, *The Theory of Relativity*, 2nd ed. (Oxford University Press, Oxford, 1972).

[6] H. Goldstein, C. P. Poole, and J. L. Safko, *Classical Mechanics*, 3rd ed. (Addison Wesley, San Francisco, 2002), Ch. 7.