The Mathematical Verification Challenge of Feynman Diagram IIc

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Abstract: Quantum electrodynamics predicts the electron anomalous magnetic moment with extraordinary precision. However, our review of the principal historical and modern calculations has not identified a fully documented, independently auditable analytical derivation of the contribution assigned to diagram IIc, originally evaluated by Karplus and Kroll (1950) and corrected by Petermann (1957). We propose an open mathematical challenge to derive this contribution directly from the QED amplitude without assuming its accepted value, documenting every essential analytical or computational step. In light of recent breakthroughs demonstrating superhuman mathematical capabilities in specific domains, we particularly encourage the use of artificial intelligence to address this challenge. A conclusive solution may confirm the reference result or establish a demonstrable discrepancy or obstruction under explicitly matched conventions. The challenge presupposes neither an error in the accepted result nor the absence of an existing complete derivation.

1 Introduction

Quantum electrodynamics (QED) is one of the most precisely tested theories in physics, with the electron anomalous magnetic moment (AMM), defined as $a_e = (g-2)/2$, providing one of its most remarkable tests. The electron magnetic moment has been measured with a relative precision of 0.13 parts per trillion (ppt), making it the most precisely measured property of an elementary particle [1]. The comparison with QED provides an exceptionally stringent test, whose interpretation depends on the independently measured fine-structure constant.

In the perturbative formulation of QED, the electron AMM is expressed as an infinite series in powers of the fine-structure constant [6]:

$$a_e = C_1\left(\frac{\alpha}{\pi}\right) + C_2\left(\frac{\alpha}{\pi}\right)^2 + C_3\left(\frac{\alpha}{\pi}\right)^3 + C_4\left(\frac{\alpha}{\pi}\right)^4 + \cdots$$

The leading coefficient is Schwinger's $C_1 = 1/2$ [5]. Throughout this paper, C_2 denotes the mass-independent fourth-order QED coefficient, conventionally written as $A_1^{(4)}$, excluding the small mass-dependent contributions from virtual muon and tau loops. Its accepted value is [22]:

$$C_2 = -0.328478965579193\ldots$$

Karplus and Kroll's first calculation of C_2 (1950) [7] comprised seven contributions, labeled I, IIa, IIb, IIc, IId, IIe, and IIf. Contributions IIb and IIf vanish, leaving five contributions with a reported sum of $C_2 = -2.973$.

In 1957, Petermann [8] corrected contributions I and IIc (Table 1). The dominant correction was to IIc, from -3.178 to −0.564, reducing the magnitude of C_2 by approximately a factor of nine, to 0.328. The magnitude of this historical correction, together with the incomplete documentation of its derivation, makes IIc a particularly relevant case for independent mathematical verification.

Barbieri, Mignaco, and Remiddi (1972) [14] presented the seven two-loop one-particle-irreducible (1PI) vertex graphs, grouped into five categories: crossed (I), ladder (IIa), corner (IIc, two symmetric graphs), self-mass (IId, two symmetric graphs), and vacuum polarization (IIe). Unlike the historical classification of Karplus and Kroll, this set does not include separate external-leg or external-photon correction diagrams.

Diagram	K & K (1950)	Petermann (1957)
I	−0.499	−0.467
IIa	+0.778	+0.778
IIc	−3.178	−0.564
IId	−0.090	−0.090
IIe	+0.016	+0.016
Total (C_2)	**−2.973**	**−0.328**

Table 1: Contributions to C_2 reported by Karplus and Kroll (1950) and Petermann (1957). Historical numerical values are reproduced as reported; discrepancies with evaluations of the displayed analytical expressions are retained.

Recent advances in artificial intelligence have demonstrated superhuman mathematical capabilities in specific

domains, extending beyond competition-level problem solving to original mathematical research [25]. In 2026, AI-driven research led to the disproof of Erdős's unit distance conjecture [26] and produced proposed solutions to two Millennium Prize Problems: the Navier–Stokes existence and smoothness problem and the Riemann hypothesis, the latter in its generalized form [27, 28]. These remarkable developments provide a timely opportunity to revisit long-standing mathematical problems in theoretical physics. In this context, we propose an open challenge to independently derive and verify the contribution of Feynman diagram IIc, particularly encouraging researchers to exploit the emerging mathematical capabilities of artificial intelligence.

2 Diagram IIc: The Missing Steps

Diagram IIc contains a second-order vertex correction inserted into the Schwinger diagram, as illustrated below.

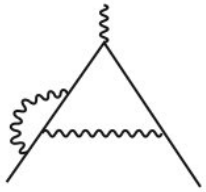

Figure 1: Diagram IIc

Equation numbers in this section refer to Karplus and Kroll [7]; the auxiliary quantities retain the definitions given in that work. Karplus and Kroll express its contribution through equation (42), reproduced here in their original notation:

$$\begin{aligned}\bar{M}^{IIc} = {} & \frac{ie\alpha^2}{4\pi^4\hbar c}(2\pi)^4 \int d^4p_1\, d^4p_2\, A^e_\mu(p_1-p_2) \\ & \times \int \frac{d^4k}{k^2+\lambda^2} \int_0^1 u\,du \int_0^1 dv\, \bar{\psi}(p_1)\gamma_\nu \\ & \times \frac{i\gamma(p_1-k)-\kappa}{(p_1-k)^2+\kappa^2}\gamma_\mu \frac{i\gamma(p_2-k)-\kappa}{(p_2-k)^2+\kappa^2} \\ & \times \left\{ \frac{K_\nu(p_2-k,p_2;u,v)}{\Lambda'^2(p_2-k,p_2;u,v)} \right. \\ & \quad +\gamma_\nu(\Lambda'^2-\Lambda_0^2)\left[\int_0^1 \frac{dz}{\Lambda_0^2+(\Lambda'^2-\Lambda_0^2)z}\right. \\ & \quad \left.\left. -\frac{2\kappa^2(1-u-\frac{1}{2}u^2)}{\Lambda'^2\Lambda_0^2}\right]\right\}\psi(p_2)\end{aligned}$$

(Karplus and Kroll, Eq. 42)

The original calculation of diagram IIc does not provide the complete sequence of intermediate steps connecting the initial amplitude in Eq. (42) to the final result in Eq. (61). Karplus and Kroll explicitly acknowledge the omission of essential computational details on page 547: "The expressions for IIa, IIc and I become successively more complicated and very much more tedious to evaluate and cannot be given in detail here" [7]. Footnote 23 further indicates that details of two independent calculations were available from the authors but were not included in the publication. Consequently, the published derivation remains incomplete: Eq. (42) provides the starting amplitude and Eq. (61) states the final result, without documenting the complete intermediate calculations. One representative intermediate integral is provided in Eq. (60), reproduced below:

$$\begin{aligned}&\int_0^1 du \int_0^1 dv \int_0^1 dt \int_0^1 dw \int_0^1 dz \\ &\times \frac{2wt(1-t)(1-uv)}{\{vw[1-uv+ut(1-v)]^2+ut(1-uv)\}^2}\end{aligned}$$

(Karplus and Kroll, Eq. 60)

The combined expression is stated in equation (61):

$$\begin{aligned}\mu^{IIc} = \frac{\alpha^2}{\pi^2}\Bigg(&-\frac{1}{2}\ln\frac{\lambda^2}{\kappa^2} - \frac{323}{24} + \frac{31}{9}\pi^2 \\ &- \frac{49}{6}\pi^2\ln 2 + \frac{49}{3}L_2 + \frac{34}{3}L_1\Bigg)\end{aligned}$$

(Karplus and Kroll, Eq. 61)

where,

$$L_1 = \int_0^1 \frac{\ln^2(1-x)}{x}\,dx = 2\zeta(3)$$

$$L_2 = \int_0^1 \frac{\ln^2(1+x)}{x}\,dx = \frac{1}{4}\zeta(3)$$

(Karplus and Kroll, Eq. 62a and 62b)

Reconstructing Karplus and Kroll's original calculation would require deriving, evaluating, and combining the integrals connecting equations (42) and (61), including renormalization and extraction of $F_2(0)$. The present challenge also permits independent analytical routes.

Equation (61) includes the infrared-divergent term $-\frac{1}{2}\ln(\lambda^2/\kappa^2)$, which cancels against the corresponding contribution from IId. Hereafter, IIc denotes the finite coefficient remaining after this term has been separated under the stated conventions. For Karplus and Kroll's original evaluation, this coefficient is:

$$II_c = -\frac{323}{24} + \frac{31}{9}\pi^2 - \frac{49}{6}\pi^2\ln(2) + \frac{107}{4}\zeta(3),$$

$$II_c = -3.17689624711729\ldots$$

Petermann (1957) subsequently corrected this contribution, obtaining the finite value:

$$II_c = -0.564020941344683\ldots$$

3 Prior Work and Verification Gap

The published calculations of Karplus and Kroll (1950) and Petermann (1957) do not provide a complete derivation of diagram IIc [7, 8], as discussed in [2, 3, 4].

Barbieri, Mignaco, and Remiddi (1972) [14][15] independently confirmed the accepted fourth-order coefficient using dispersion relations and Cutkosky rules, with extensive symbolic calculations in SCHOONSCHIP.

They also criticized Terentiev's earlier dispersive calculation [13] for overlooking infrared divergences and containing compensating intermediate errors. This criticism, published a decade later, illustrates that agreement with the final result alone does not establish the validity of a calculation.

Bonciani, Mastrolia, and Remiddi (2004) [18] provide analytical expressions for the two-loop QED vertex form factors, including the Corner diagrams corresponding to historical IIc. However, several intermediate steps relevant to the present verification criteria are not explicitly documented. In Section 3 (p. 6), the authors present Eq. (16) as an example of reduction to master integrals for diagram (g), explicitly acknowledging that the corresponding expressions for other diagrams are omitted: "Similar formulas hold for the other form factors and graphs, but are too lengthy to be reported here". For the Corner diagram (c), Eqs. (32)–(33) provide the initial amplitude and Dirac numerator, while Eq. (35) presents the final analytical form factor. The intermediate Dirac projections, diagram-specific integration-by-parts (IBP) reduction coefficients, and substitution of master integrals connecting these expressions are not provided. The preceding work [17] supplies the master integrals but not these Corner-specific reductions. Furthermore, although the renormalization counterterms are given in Section 4 and the complete two-loop magnetic form factor in Eq. (117), no explicit derivation connects the Corner contributions to Petermann's finite historical IIc result under matched subtraction and infrared conventions. These omissions do not imply that the published calculations are incorrect or that the missing transformations cannot be independently reconstructed.

Our review focused on the works listed in Table 2 and relevant references therein, examining amplitudes, analytical reductions, renormalization, infrared treatment, and computational procedures.

Using GPT-6 Astra and the publications listed in Table 2, we attempted an independent reconstruction of diagram IIc but obtained no complete, independently verifiable derivation. Preliminary attempts with diagrams IIe and IId encountered fewer difficulties. These exploratory exercises are presented neither as validated mathematical results nor as evidence that IIc cannot be reconstructed from published information. Despite the independent confirmations and methodological advances reviewed above, we have not identified a derivation satisfying the present auditability criteria. Whether the documented omissions can be overcome through indepen-

Authors	Calculation and documentation
Karplus & Kroll (1950)	Original incorrect result; authors explicitly acknowledge omitted calculations (p. 547). Eq. (42) gives the amplitude, Eq. (60) a representative integral, and Eq. (61) the final result without the complete intermediate derivation. [7]
Petermann (1957)	Corrected analytical expression without the complete derivation. [8]
Petermann (1958)	Bounds for IIc (-0.60 ± 0.11), rather than an exact analytical evaluation. [9]
Sommerfield (1957–1958)	Independent confirmation of the fourth-order coefficient using Green's functions. [10][11]
Smrz & Ulehla (1960)	Independent confirmation without published supporting calculations. [12]
Terentiev (1962)	Dispersive calculation criticized by Barbieri, Mignaco, and Remiddi for its infrared treatment and compensating intermediate errors. [13][14]
Barbieri, Mignaco & Remiddi (1972)	Analytical form factors and individual diagram contributions using SCHOONSCHIP; our review did not identify a fully documented reconstruction of historical IIc satisfying the present criteria. [14][15]
Knecht (2002)	Review of fourth-order analytical results and two-loop topologies, without a new IIc derivation. [16]
Bonciani et al. (2003)	Analytical master integrals using differential equations and FORM; complete documentation of the reduction needed to reconstruct historical IIc is not provided. [17]
Mastrolia & Remiddi (2003)	Analytical form factors via dispersion relations and low-momentum expansions; no complete auditable derivation of historical IIc. [19]
Bonciani et al. (2004)	Corner amplitudes and form factors (Eqs. 32–41), renormalization (Sec. 4), and low-momentum limits (Sec. 7). Corner-specific IBP reduction coefficients are not provided; no complete derivation connecting the initial amplitude to historical IIc under matched conventions. [18]
Aoyama et al. (2012)	Numerical QED methodology using Feynman parametrization and automated renormalization. [20]
Volkov (2018)	Independent numerical evaluation of individual diagrams (Table V). Agreement for the combined IIc + IId contribution (Table XIII), but different individual finite values under an alternative subtraction scheme. No explicit derivation of the historical IIc value. [21]
Aoyama et al. (2019)	Fourth-order numerical methodology and renormalization. [22]
Kitano et al. (2021)	Independent stochastic calculation of the total fourth-order coefficient without Feynman diagrams or a separate IIc evaluation. [23]
Volkov (2024)	Divergence-elimination and renormalization framework for individual diagrams; no complete analytical evaluation of historical IIc. [24]

Table 2: Scope and documentation of historical and modern calculations relevant to diagram IIc, as assessed in our review against the requirements of Section 4.

dent reconstruction is the question addressed by the following mathematical challenge.

4 The Mathematical Challenge

We propose an open challenge to determine analytically the finite contribution associated with diagram IIc. The use of AI-assisted methods is encouraged. Petermann's expression is the reference result to be tested, not a required conclusion:

$$\mathrm{IIc} = -\frac{67}{24} + \frac{\pi^2}{18} + \frac{\pi^2}{3}\ln 2 - \frac{1}{2}\zeta(3),$$

$$\mathrm{IIc} = -0.564020941344683\ldots$$

A proposed solution must satisfy the following requirements:

1. **Diagrammatic starting point:** Derive the amplitude of the original diagram IIc from the QED Feynman rules. Establish its equivalence to equation (42) of Karplus and Kroll or demonstrate a substantive incompatibility.

2. **Derivation and conventions:** Document all essential steps, mathematical and computational techniques, and conventions used in the calculation, from the starting amplitude to the extraction of $F_2(0)$, including analytical reduction, integration, regularization, and renormalization.

   Specify the gauge, normalization, UV/IR prescriptions, and treatment of external-leg and symmetry-related contributions. Identify the conventions governing the separation and allocation of terms between IIc and IId, and establish their correspondence with the historical prescriptions of Karplus and Kroll, as corrected by Petermann. Clearly distinguish documented conventions from reconstructed assumptions, independently justifying the latter. Agreement of the combined contribution alone does not verify the individual historical IIc value.

3. **Analytical result:** Determine analytically the finite IIc contribution or establish that it is undefined or non-unique under the specified prescriptions. Numerical agreement or reproduction of the total fourth-order coefficient alone is insufficient. Any unique result must follow from these prescriptions without using the reference value to fix residual freedom.

4. **Infrared cancellation and combined contribution:** Determine whether the infrared-divergent terms cancel between IIc and IId under the stated prescriptions and, if they do, independently derive the resulting finite combined contribution. The reference expression is:

$$IIc + IId = -\frac{7}{3} + \frac{\pi^2}{3}\ln 2 - \frac{1}{2}\zeta(3).$$

$$IIc + IId = -0.653998963627425\ldots$$

   The derivation must establish the relationship between the individual finite contributions and their combined result under matched conventions.

5. **Independence:** Published reference values of IIc, IIc + IId, and C_2 must not be used to determine intermediate results or calculation parameters, nor may IIc be inferred by subtraction from a known total. Published methods and intermediate results may be used if independently verifiable without assuming the reference values. Reproducing the historical calculation sequence is not required.

6. **Computational reproducibility:** Document all essential computer-assisted transformations sufficiently to allow independent reproduction and verification, including the necessary code or procedures, inputs, intermediate results, and software information. Routine algebraic operations and auxiliary tools need not be documented separately when their validity can be independently verified.

An optional historical objective is to reconstruct Karplus and Kroll's original evaluation, identify any demonstrable mathematical errors, and explain the relationship between that evaluation, Petermann's expression, and the independently established outcome.

5 Outcomes and Implications

The challenge admits three broad categories of outcomes:

1. **Confirmation.** An independent and reproducible analytical derivation confirms Petermann's finite IIc contribution and the combined contribution IIc + IId under matched conventions. This would strengthen confidence in the mathematical consistency and reproducibility of the fourth-order QED calculations.

2. **Negative resolution.** An independent mathematical analysis establishes a discrepancy, inconsistency, or non-uniqueness in the historical IIc contribution under the stated prescriptions. Its validity and implications would be assessed by the wider scientific community, particularly if it affects IIc + IId or the fourth-order coefficient C_2.

3. **Inconclusive outcome.** Neither an independent confirmation nor a demonstrable discrepancy, inconsistency, or non-uniqueness has been established. In this case, the challenge would remain unresolved, and no conclusion could be drawn regarding the mathematical validity of the currently accepted result.

Beyond diagram IIc, the challenge addresses the broader issue of mathematical reproducibility in high-order QED calculations. The tenth-order electron anomalous magnetic moment involves 12,672 Feynman vertex diagrams [22], requiring extensive analytical and computational work. Independent verification need not require identical methods or closed-form solutions for every diagram, but essential transformations must remain auditable. The broader purpose is to promote mathematical transparency and reproducibility in QED calculations.

At a time when artificial intelligence is addressing some of the most challenging mathematical problems in history, diagram IIc offers a concrete opportunity to apply these emerging superhuman capabilities to the independent verification of calculations in theoretical physics. We invite physicists, mathematicians, and AI researchers to undertake this challenge through conventional methods, human–AI collaboration, or autonomous AI reasoning, and to submit analytical derivations, technical reports, or reproducible computational implementations. Proposed solutions should be made publicly available to facilitate independent scrutiny and assessment against the stated verification criteria.

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