The negative energy density in the Natario warp drive spacetime for a "realistic" manned interstellar spaceflight

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November 23, 2026

Abstract

In 1994 Alcubierre developed the first warp drive work using a function that distorts the spacetime geometry generating the warp drive effect. This function is known as the Alcubierre shape function. Seven years later in 2001 the second warp drive work appeared developed by Natario. The Natario warp drive also uses a shape function and the Natario shape function can be obtained using the Alcubierre shape function. However the energy density requirements to sustain a warp drive with superluminal velocities are tremendously high. In this work we introduce a Natario shape function able to reduce these energy density requirements. We also discuss briefly the environment of interstellar space travels. We adopted in this work a pedagogical language and a presentation style designated to novices,newcomers or enthusiasts.We use extensively pictures for pedagogical purposes in order to better illustrate our points of view.All the pictures were taken from Public Domain websites.

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1 Introduction:

The Natario warp drive appeared for the first time in 2001.([1]).Although the idea of the warp dive as a spacetime distortion that allows a spaceship to travel faster than light predated the Natario work by 7 years Natario introduced in 2001 the new concept of a propulsion vector to define or to generate a warp drive spacetime.

This propulsion vector nX uses the form $nX = X^i e_i$ where X^i are the shift vectors responsible for the spaceship propulsion or speed and e_i are the Canonical Basis of the Coordinates System where the shift vectors are based or placed.

Natario (See pg 5 in [1]) defined a warp drive vector with constant speeds over the x-axis of motion in Polar Coordinates(See pg 4 in [1]).However Polar Coordinates are not real tridimensional $3D$ coordinates since it uses only the two Canonical Basis $\mathbf{e}_r$ and $\mathbf{e}_\theta$.

Natario used Polar Coordinates and constant speeds.(See pg 4 in [1]) but for a real $3D$ Spherical Coordinates another warp drive vector must be calculated.Remember that a real spaceship is a tridimensional $3D$ object inserted inside a tridimensional $3D$ warp bubble that must be defined in real $3D$ Spherical Coordinates with all the tridimensional $3D$ Canonical Basis $\mathbf{e}_r$,$\mathbf{e}_\theta$ and $\mathbf{e}_\phi$.Remember also that a real warp drive must accelerate or de-accelerate in order to be accepted as a physical valid model.The Natario warp drive in real $3D$ Spherical Coordinates with constant and variable velocities is presented in [15] and [26].

Natario used the original ADM formalism in Polar Coordinates and constant speeds but even in Polar Coordinates it is possible to have a Natario warp drive with variable speeds as in [20].

Also it is possible to assemble Natario warp drives in Polar Coordinates with constant and variable speeds in different ADM formalisms as demonstrated in [18] and [19].

Natario used the original ADM formalism with the so-called lapse function always equal to 1 but it is possible to have a Natario warp drive in Polar Coordinates with constant and variable speeds using a lapse function different than 1 as demonstrated in [17].

In General Relativity unfortunately the problem of the null-like geodesics $ds^2 = 0$ Horizon in which the warp bubble is causally disconnected from the ship wether in original or modified ADM formalisms exists.

In Polar Coordinates with constant or variable speeds using both the original or modified ADM formalisms the problem of the null-like geodesics $ds^2 = 0$ Horizon for the Natario warp drive is presented in [21],[22] and [23].

In real $3D$ Spherical Coordinates with constant or variable speeds using both the original or modified ADM formalisms the problem of the null-like geodesics $ds^2 = 0$ Horizon for the Natario warp drive is presented in [24],[25],[26],[27] and [28].

In General Relativity unfortunately the problem of the relativistic geometrical Horizon $g_{00} = 0$ in which the warp drive metric completely collapses resembling the Event Horizon of a black hole also exists.

The problem of the relativistic geometrical Horizon for the Natario warp drive is presented in [29] and [30].

The "ex-libris" of the original Natario warp drive work is the so-called "Zero Expansion" that occurs in Polar Coordinates but do not occurs in real $3D$ Spherical Coordinates as demonstrated in [16].

The so-called "Zero Expansion" behavior is described in pg 5 in [1] and section 2 in [16].It can be described as in front of the bubble a contraction of spacetime exists in the radial direction; this is however exactly balanced by an expansion of spacetime in the perpendicular directions so the net final result is ZERO however this do not work in real $3D$ Spherical Coordinates.Hence the so-called "Zero Expansion" behavior cannot be applied to a real spaceship immersed in $3D$ Spherical Coordinates.

In this work we concentrate ourselves in the otiginal Natario warp drive in Polar Coordinates.([1]):

The negative energy density according to Natario in Polar Coordinates is given by(see pg 5 in [1]:

Equations written in the Geometrized System of Units $c = G = 1$:(see Appendix A)

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(l'(rs))^2\cos^2\theta + \left(l'(rs) + \frac{r}{2}l''(rs)\right)^2\sin^2\theta\right] \tag{1}$$

Rewriting the Natario negative energy density in cartezian coordinates in the equatorial plane we should expect for:(see Appendix D)

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(l'(rs))^2\right] \tag{2}$$

This is a very important result:The negative energy density in the equatorial plane of the Natario warp drive in front of the ship IS NOT ZERO and this changes the whole picture or changes the rules of the game!!!

This Natario warp drive negative energy density in front of the ship with repulsive gravitational behavior(see pg 116 in [31]) deflects all the incoming hazardous particles of the Interstellar Medium(see Appendices E and F in this work) and offers an excellent protection to the crew members as depicted in the works [11],[12],[13] and mainly [14].

However the negative energy density is directly proportional to the square of the ship velocity vs and also proportional to the square of the first and second order derivatives of the Natario shape function $l(rs)$.In this work we discuss the reduction of the negative energy density requirements to low and affordable levels.

In this work we present the new Natario shape function $l(rs)$ defined using the Alcubierre shape function $f(rs)$:

$$l(rs) = \frac{1}{2}[1 - (f(rs)^A)]^B \tag{3}$$

In the new Natario shape function $l(rs)$ above the factors A and B are both equal to the modulus of the bubble diameter D.A is the inner factor IF while B is the outer factor OF.

This shape function with the factors $A = B$ both equal to the modulus of the bubble diameter D can reduce the negative energy density to low and affordable levels.

We discovered this function in 10 November 2026:In the original form A was equal to the modulus of the bubble radius R and B was equal to the modulus of the bubble thickness parameter @ however such a function did not reduced the negative energy density requirements in our numerical simulations model to a satisfactory level.

In 18 November 2026 we made a second attempt with new numerical simulations in our model making $A = B$ both equal to the modulus of the bubble diameter D.Then $A = B = D = 2R$ and we got exactly the satisfactory results obtained for the negative energy density requirements by our previous work in [14].(see the Remarks Section in this work).

The Natario shape function presented in [14] is given by:(see eq 17 in [14]):

$$N(rs) = [\frac{1}{2}][1 - f(rs)^{WF}]^{WF} \tag{4}$$

The derivative square of the Natario shape function in [14] is(see eq 18 in [14]) :

$$N'(rs)^2 = [\frac{1}{4}]WF^4[1 - f(rs)^{WF}]^{2(WF-1)}[f(rs)^{2(WF-1)}]f'(rs)^2 \tag{5}$$

This Natario shape function in [14] appeared first in eqs 10 and 11 in [35].

The first order derivative of the Natario shape function in [14] is given by(see eq 48 in [14]):

$$N'(rs) = -[\frac{1}{2}]WF^2[1 - f(rs)^{WF}]^{(WF-1)}[f(rs)^{(WF-1)}]f'(rs) \tag{6}$$

The second order derivative of the Natario shape function in [14]is given by the lengthly expression(see eqs 50,51 and 52 in [14]):

$$N''(rs) = [\frac{1}{2}]WF^3(WF-1)[1 - f(rs)^{WF}]^{(WF-2)}[f(rs)^{2(WF-1)}]f'(rs)^2 \tag{7}$$

$$-[\frac{1}{2}]WF^2[1 - f(rs)^{WF}]^{(WF-1)}(WF-1)[f(rs)^{(WF-2)}]f'(rs)^2 \tag{8}$$

$$-[\frac{1}{2}]WF^2[1 - f(rs)^{WF}]^{(WF-1)}[f(rs)^{(WF-1)}]f''(rs) \tag{9}$$

Note that the Alcubierre shape function $f(rs)$ and its first and second order derivatives appears in all the equations above.

We can see that the square of the second order derivative of the Natario shape function in [14] $N''(rs)^2$ would result in a very algebraic complicated expression.In order to simplify our study we decompose the second order derivative of the Natario shape function in separated algebraic expressions as shown below:(see eqs 54 to 57 in [14])

$$N''(rs) = P + Q + R \tag{10}$$

With the expressions for A B and C given respectively by:

$$P = [\frac{1}{2}]WF^3(WF-1)[1-f(rs)^{WF}]^{(WF-2)}[f(rs)^{2(WF-1)}]f'(rs)^2 \tag{11}$$

$$Q = -[\frac{1}{2}]WF^2[1-f(rs)^{WF}]^{(WF-1)}(WF-1)[f(rs)^{(WF-2)}]f'(rs)^2 \tag{12}$$

$$R = -[\frac{1}{2}]WF^2[1-f(rs)^{WF}]^{(WF-1)}[f(rs)^{(WF-1)}]f''(rs) \tag{13}$$

The works in [14] and [35] uses the so-called "warp factor" WF to reduce the levels of negative energy density requirements to sustain a Natario warp bubble without giving a clue on the origin of WF simply defining it as an "arbitrary parameter".

Since 18 November 2026 WF can now finally be defined as being the diameter of the warp bubble in modulus and this line of reason finally gives a physical meaning to WF giving also a physical meaning to the works in [14] and [35].

Then $WF = A = B$ and we keep both Natario shape functions $N(rs)$ and $l(rs)$.

The advantage of having different inner and outer factors separated being A the inner factor IF and B as the outer factor OF is the fact that it is possible to test other possibilities with A and B possessing different values in a generic mathematical formalism using different numerical simulations in our model.

For example it is possible to try another combination in which A is the modulus of the thickness parameter @ while B is the modulus of the radius R.

We adopted in this work a pedagogical language and a presentation style that perhaps will be considered as tedious,monotonous, or exhaustive by experienced readers and we designated this work for novices,newcomers or enthusiasts.We use extensively pictures for pedagogical purposes in order to better illustrate our points of view.All the pictures were taken from Public Domain websites.

2 The Alcubierre warp drive

The warp drive work that predates Natario by 7 years was written by Alcubierre in 1994.(see [2])

The Alcubierre warp drive works using an expansion of spacetime behind a spaceship leaving away the departure point and a contraction of spacetime in front of the spaceship bringing the destination point close to the ship.(abs in [2]).

This expansion of the spacetime behind a spaceship mimics the expansion of our Universe as pointed out by Alcubierre(pg 2 in [2]).We know that the expansion of the Universe is accelerating and if the distance between us and a galaxy far and far away is extremely large the speed of the recession may well exceed the light speed limit.(see pg 98 in [32] and pg 377 in [33]).

For the experimental verification of the acceleration of the Universe see for example the bottom of pg 355 and top of pg 356 eq 8.155 in [34].

But the most important thing:Alcubierre introduced an analytical function to distort the flat Minkowsky spacetime metric generating the warp drive bubble and hence the geometry that allows faster than light space travel.This analytical function is known as the Alcubierre shape function given by the following equation:(see eq 6 pg 4 in [2],eq 3 pg 3 in [3],eq 135 pg 27 in [4],eq 2 pg 4 in [5] and eq 6 pg 3 in [36])

$$f\left(r_s\right) = \frac{\tanh\left(\sigma\left(r_s + R\right)\right) - \tanh\left(\sigma\left(r_s - R\right)\right)}{2\tanh\left(\sigma R\right)} , \tag{14}$$

$$f\left(r_s\right) = \frac{\tanh\left(@\left(r_s + R\right)\right) - \tanh\left(@\left(r_s - R\right)\right)}{2\tanh\left(@ R\right)} , \tag{15}$$

Alcubierre worked in the so-called "remote frame" in which an observer at the rest observes the spaceship passing by him.[1].The Alcubierre shape function $f(rs)$ gives the value of 1 inside the bubble and zero outside the bubble.The region where $1 > f(rs) > 0$ is known as the Alcubierre warped region.(see eq 7 pg 4 in [2],eq 136 pg 27 in [4],eq 7 pg 3 in [36]).

$$\lim_{\sigma\to\infty} f\left(r_s\right) = \begin{cases} 1 & \text{for} \quad r_s \in [-R,\, R] \\ 0 & \text{otherwise} \end{cases} , \tag{16}$$

In the Alcubierre shape function $f(rs)$,R is the radius of the warp bubble,@(or σ) is the "thickness parameter" related to the thickness of the warp bubble which means to say the region where $1 > f(rs) > 0$.A large @(or σ) concentrates the Alcubierre warped region in a thin shell.According to Alcubierre pg 4 in [2] @(or σ) can have arbitrary values and rs is the distance between the center of the bubble and a point P that can lies inside or outside the bubble.In the center of the bubble $rs = 0$.

[1]the difference between remote and ship frames is explained in [37]

A large @ = 50000 makes the term $tanh[@(rs+R)] \cong 1$ so we can use the following approach for the Alcubierre shape function:

$$f(rs) = \frac{1}{2}([1 - tanh[@(rs-R)]) \tag{17}$$

However the energy needed to sustain a warp bubble is negative violating all the known energy conditions.(see paragraph 4.6 pgs 174 and 175 in [34],see eq 9.2.19 pg 219,eq 9.2.20 pg 220 in [32]).Alcubierre also pointed out this violation in pg 8 in [2].

This negative energy density is directly proportional to the factor $(\frac{c^2}{G})vs^2$ in which $c = 3 \times 10^8 \frac{meters}{second}$ is the light speed $G = 6,67 \times 10^{-11} \frac{Newton \times meters^2}{kilograms^2}$ is the Newton gravitational constant $c^2 = 9 \times 10^{16}(\frac{meters}{second})^2$ is the square of the light speed and vs^2 is the square of the spaceship velocity all in the International System of Units $SI - MKS$.

Considering an Alcubierre warp drive moving with $vs = 200$ which means to say 200 times light speed in order to make a round trip from Earth to a nearby star at 20 light-years away in a reasonable amount of time(in months not in years) we would get in the expression of the negative energy the factor $c^2 = (3 \times 10^8)^2 = 9 \times 10^{16}$ being divided by $6,67 \times 10^{-11}$ giving $1,35 \times 10^{27}$ and this is multiplied by $(6 \times 10^{10})^2 = 36 \times 10^{20}$ coming from the term $vs = 200$ giving $1,35 \times 10^{27} \times 36 \times 10^{20} = 1,35 \times 10^{27} \times 3,6 \times 10^{21} = 4,86 \times 10^{48}$!!!

A number with 48 zeros!!!The planet Earth have a mass[2] of about $6 \times 10^{24} kg$

This term is 1.000.000.000.000.000.000.000.000 times bigger in magnitude than the mass of the planet Earth!!!or better:The amount of negative energy density needed to sustain a warp bubble at a speed of 200 times faster than light requires the magnitude of the masses of 1.000.000.000.000.000.000.000.000 planet Earths!!!-Impossible to be achieved!!!

Lets illustrate this:the Alcubierre warp drive needs an amount of negative energy density of about 10^{48} in order to travel at 200 times light speed while the mass of the Earth is of about 10^{24}.Note the proportion between 10^{48} and 10^{24} the magnitude of the masses of 1.000.000.000.000.000.000.000.000 planet Earths.

Imagine that we can reduce the mass of the Earth from 10^{24} to 1 kilogram.Shrinking 24 degrees of magnitude from the factor 10^{48} by the same proportion we would get again 10^{24}.The proportion between a body of mass 1 kilogram and the Earth mass 10^{24} is the same proportion between the mass of the Earth 10^{24} and the factor 10^{48} in order to travel at 200 times light speed.

[2]see Wikipedia:The free Encyclopedia

Alcubierre provided the expression to compute the negative energy density in the Alcubierre warp drive in eq 19 pg 8 in [2] .See also the bottom of pg 5 in [2] but we prefer to quote the negative energy density in the Alcubierre warp drive directly from the works of Pfenning and Ford,Lobo,Lobo and Alcubierre and finally Lobo and Visser.The equations below are being given in the Geometrized System of Units in which $c = G = 1$(See Appendix A in this work).

Quoting directly from Pfenning and Ford in [3] eq 8 pg 6 see fig 3 pg 7 "We now turn our attention to the energy density distribution of the warp drive metric. Using the first integral of the geodesic equations, it is easily shown that

$$\langle T^{\mu\nu}u_\mu u_\nu\rangle = \langle T^{00}\rangle = \frac{1}{8\pi}G^{00} = -\frac{1}{8\pi}\frac{v_s^2(t)\rho^2}{4r_s^2(t)}\left(\frac{df(r_s)}{dr_s}\right)^2, \tag{18}$$

where $\rho = [y^2 + z^2]^{1/2}$, is the radial distance perpendicular the x-axis as was defined above. We immediately see that the energy density measured by any geodesic observer is always negative, as was shown in Alcubierre's original paper[2]. In Figure 3, we see that the distribution of negative energy density is concentrated in a toroidal region perpendicular to the direction of travel".

$$\langle T^{\mu\nu}u_\mu u_\nu\rangle = \langle T^{00}\rangle = \frac{1}{8\pi}G^{00} = -\frac{1}{8\pi}\frac{v_s^2(t)[y^2 + z^2]}{4r_s^2(t)}\left(\frac{df(r_s)}{dr_s}\right)^2, \tag{19}$$

Quoting directly from Lobo in [4] eq 140 pg 28 fig 3

$$T_{\mu\nu}\,U^\mu\,U^\nu = -\frac{1}{32\pi}\frac{v^2(x^2+y^2)}{r^2}\left(\frac{df}{dr}\right)^2 < 0\,. \tag{20}$$

"It is easy to verify that the energy density is distributed in a toroidal region around the z-axis, in the direction of travel of the warp bubble".

Quoting directly from Lobo and Alcubierre in [5] eq 8 pg 5 se fig 2 pg 6

$$T_{\mu\nu}\,U^\mu U^\nu = -\frac{1}{32\pi}\frac{v^2(x^2+y^2)}{r^2}\left(\frac{df}{dr}\right)^2 < 0\,. \tag{21}$$

"It is easy to verify that the energy density is distributed in a toroidal region around the z-axis, in the direction of travel of the warp bubble"

Quoting directly from Lobo and Visser in [36] eq 11 pg 4

$$T_{\hat{\mu}\hat{\nu}}\,U^{\hat{\mu}}\,U^{\hat{\nu}} = -\frac{1}{32\pi}\frac{v^2(x^2+y^2)}{r^2}\left[\frac{\mathrm{f}}{\mathrm{r}}\right]^2 < 0\,. \tag{22}$$

In all the quotes above we see that the distribution of negative energy is concentrated in a toroidal region perpendicular to the direction of travel or the direction of motion of the warp bubble and there is nothing in front of the ship in the equatorial plane so the ship dont have a protection.See Appendices H,I and J in this work about the negative energy density distribution in the Alcubierre warp drive.

If the motion is over the x-axis the negative energy density concentrates over the y and z axis while on a motion over the z-axis the negative energy density concentrates over the x and y axis.

In an equatorial plane over the x-axis $y^2 + z^2 = 0$ the negative energy density is zero according to the equation of Pfenning and Ford in [3] eq 8 pg 6 while in an equatorial plane over the z-axis $x^2 + y^2 = 0$ the negative energy density is also zero according to the equations of Lobo in [4] eq 140 pg 28,Lobo and Alcubierre in [5] eq 8 pg 5 and finally Lobo and Visser in [36] eq 11 pg 4.

Unfortunately the interstellar space is not empty and a spaceship at 200 times faster than light would suffer collisions against the interstellar medium in the equatorial plane and this would be catastrophic since there is nothing in front of the spaceship to protect it.See Appendices E and F in this work about the composition of the interstellar medium.

Also the interstellar space is fulfilled with photons of cosmic background radiation that would suffer highly energetic Doppler blueshifts in a head-on collision with the spaceship also in the equatorial plane since the negative energy density that could deflect these photons lies above and below the spaceship but not in the front.((See Appendices G,H,I,J and L in this work))

At 200 times faster than light the energy of these cosmic background blueshifted photons is equivalent to the energy of a solar photosphere and think in a solar photosphere entering in the bubble threatening the ship integrity and the crew member lives !!!.(pg 11 in [9]) in Clark,Hiscock and Larson.

Also Barcelo,Finazzi and Liberati pg 6 and 7 in [10] mentions the fact that these Doppler blueshifted photons will raise the temperature inside the bubble to the Hawking temperature generating a harsh environment that would instantaneously imply in the destruction of the ship and the death of the crew members!!!.

So at 200 times faster than light an Alcubierre warp drive would be completely destroyed!!!.

Even worst !!! The equations of the negative energy density are directly proportional to the square of the first order derivative of the Alcubierre shape function multiplied by 10^{48} !!! and unfortunately the first order derivative of the Alcubierre shape function possesses high values worsening the negative energy density requirements

The Alcubierre warp drive although impossible to be achieved it provided us with a very important scientific tool essential to the study of the Natario warp drive.This tool is the Alcubierre shape function!!!.

The Alcubierre shape function can be modified to fulfill the requirements of the negative energy density for the Natario warp drive.We will explore this is the next sections.

3 The Natario warp drive continuous shape function $l(rs)$.

The Natario warp drive likes its Alcubierre counterpart needs a shape function to generate the spacetime distortion.According to Natario(pg 5 in [1]) any function that gives 0 inside the bubble and $\frac{1}{2}$ outside the bubble while being $0 < l(rs) < \frac{1}{2}$ in the Natario warped region is a valid shape function for the Natario warp drive.The Natario warped region resembles the Alcubierre warped region although is radically different.

We can obtain the Natario shape function $l(rs)$ using the Alcubierre shape function $f(rs)$ as follows:

$$f(rs) = \frac{1}{2}([1 - tanh[@(rs - R)]) \tag{23}$$

The Alcubierre shape function gives the values of 1 inside the bubble and zero outside the bubble while the Natario shape function gives the values of 0 inside the bubble and $\frac{1}{2}$ outside the bubble.While Alcubierre worked in the so-called "remote frame" in which an external observer sees the spaceship passing by him,Natario worked in the so called "ship frame" in which the observer is placed in the ship inside the bubble.[3]

Natario deals with the spacetime as a "strain" tensor of Fluid Mechanics(pg 5 in [1]).Imagine the observer being a fish inside an aquarium and the aquarium is floating in the surface of a river but carried out by the river stream.The warp bubble in this case is the aquarium.An observer in the margin of the river (remote frame)would see the aquarium passing by him at a large speed but inside the aquarium the fish is at the rest with respect to his local neighborhoods.The fish as being the spaceship inside the bubble is the ship frame observer.

The Natario shape function $l(rs)$ is given by:

$$l(rs) = \frac{1}{2}[1 - f(rs)] \tag{24}$$

$$l(rs) = \frac{1}{2}[1 - [\frac{1}{2}[1 - tanh[@(rs - R)]]] \tag{25}$$

This shape function gives the result of $l(rs) = 0$ inside the warp bubble and $l(rs) = \frac{1}{2}$ outside the warp bubble while being $0 < l(rs) < \frac{1}{2}$ in the Natario warped region.

For the Natario shape function introduced above it is easy to figure out when $f(rs) = 1$(interior of the Alcubierre bubble) then $l(rs) = 0$(interior of the Natario bubble) and when $f(rs) = 0$(exterior of the Alcubierre bubble)then $l(rs) = \frac{1}{2}$(exterior of the Natario bubble) and this function satisfies the Natario requirements for a shape function.(pg 5 in [1])

[3]the difference between remote and ship frames is explained in [37]

The negative energy density according to Natario in Polar Coordinates is given by(see pg 5 in [1]:

Equations written in the Geometrized System of Units $c = G = 1$:(see Appendix A)

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(l'(rs))^2\cos^2\theta + \left(l'(rs) + \frac{r}{2}l''(rs)\right)^2\sin^2\theta\right] \tag{26}$$

In the bottom of pg 4 in [1] Natario defined the x-axis as the polar axis.In the top of page 5 we can see that $x = r\cos(\theta)$ implying in $\cos(\theta) = \frac{x}{r}$ and in $\sin(\theta) = \frac{y}{r}$

Rewriting the Natario negative energy density in cartezian coordinates we should expect for:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(l'(rs))^2(\frac{x}{rs})^2 + \left(l'(rs) + \frac{r}{2}l''(rs)\right)^2(\frac{y}{rs})^2\right] \tag{27}$$

Considering motion in the equatorial plane of the Natario warp bubble (x-axis only) then $[y^2 + z^2] = 0$ and $r^2 = [(x - xs)^2]$ and making $xs = 0$ the center of the bubble as the origin of the coordinate frame for the motion of the Eulerian observer then $r^2 = x^2$ because in the equatorial plane $y = z = 0$.

Rewriting the Natario negative energy density in cartezian coordinates in the equatorial plane we should expect for:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(l'(rs))^2\right] \tag{28}$$

This is a very important result:The negative energy density in the equatorial plane of the Natario warp drive IS NOT ZERO and this changes the whole picture or changes the rules of the game!!!

Compare with the Alcubierre negative energy density in the equatorial plane according to the equation of Pfenning and Ford in [3] eq 8 pg 6:(See Appendices G,H,I,J and L in this work).The negative energy density in the Alcubierre equatorial plane IS ZERO!!!

$$\langle T^{\mu\nu}u_{\mu}u_{\nu}\rangle = \langle T^{00}\rangle = \frac{1}{8\pi}G^{00} = -\frac{1}{8\pi}\frac{v_s^2(t)[y^2+z^2]}{4r_s^2(t)}\left(\frac{df(r_s)}{dr_s}\right)^2, \dashrightarrow [y^2+z^2] = 0 \dashrightarrow \frac{1}{8\pi}G^{00} = 0 \tag{29}$$

The negative energy density have repulsive gravitational behavior and is distributed along all the bubble volume even in the equatorial plane in front of the ship so any hazardous incoming objects in front of the bubble (Doppler blueshifted photons or space dust or debris etc) would then be deflected by the repulsive behavior of the negative energy in front of the bubble never reaching the bubble walls(see pg 116 in [31]).(See Appendices B,C,D and K in this work).

The Natario warp drive have negative energy density in the equatorial plane in front of the ship protecting the ship and the crew members while the Alcubierre warp drive dont have negative energy density in the equatorial plane in front of the ship.Natario warp drive offers protection while Alcubierre warp drive dont.This is the reason why Natario warp drive is better than Alcubierre warp drive.

Considering the interstellar navigation in a hostile environment the Natario warp drive is better than Alcubierre warp drive.However another major serious problem exists and this problem must be solved.

Rewriting both the Alcubierre and Natario negative energy density equations in the International System of Units $SI-MKS$(see Appendix A in this work) we get:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[3(l'(rs))^2\cos^2\theta + \left(l'(rs)+\frac{r}{2}l''(rs)\right)^2\sin^2\theta\right] \tag{30}$$

$$\langle T^{\mu\nu}u_{\mu}u_{\nu}\rangle = \langle T^{00}\rangle = \frac{1}{8\pi}G^{00} = -\frac{c^2}{G}\frac{1}{8\pi}\frac{v_s^2(t)[y^2+z^2]}{4r_s^2(t)}\left(\frac{df(r_s)}{dr_s}\right)^2, \tag{31}$$

Note that for a spaceship at 200 times light speed the infamous factor $(\frac{c^2}{G})vs^2$ appears again in both the Natario and Alcubierre equations of negative energy density distribution with the order of magnitude of 10^{48} which is 1.000.000.000.000.000.000.000.000 times bigger in magnitude than the mass of the planet Earth!!!.

Also looking for both equations we can see that the Alcubierre equation of negative energy density is directly proportional to the square of the first order derivative of the Alcubierre shape function $f(rs)$ and this derivative have very high values worsening the negative energy density requirements.

Since the Natario shape function $l(rs)$ is defined using the Alcubierre shape function $f(rs)$ the Natario negative energy density is also directly proportional to the square of the first order derivative of the Natario shape function $l(rs)$ plus also directly proportional to the square of the second order derivative of the Natario shape function $l(rs)$ and these derivatives of first and second order possesses high values also worsening the negative energy density requirements.

Using the Natario shape function presented in the eqs 11 and 12 of this work the Natario negative energy density requirements are even worst when compared to the Alcubierre negative energy density requirements due to the presence of the second order derivative of the Natario shape function.

We need another Natario shape function $l(rs)$ that satisfies the Natario requirements for a warp drive spacetime with very low derivatives and hence very low negative energy density requirements able to complete obliterate the factor 10^{48} allowing a spaceship to travel with 200 times light speed.

Such a function can also be defines using the Alcubierre shape function $f(rs)$ and is given by:

$$l(rs) = \frac{1}{2}[1 - (f(rs)^A)]^B \tag{32}$$

$$l(rs) = \frac{1}{2}[1 - ([\frac{1}{2}[1 - tanh[@(rs - R)]])^A]^B \tag{33}$$

$$f(rs) = \frac{1}{2}([1 - tanh[@(rs - R)]) \tag{34}$$

In the Natario shape function $l(rs)$ above the factor A and the factor B are both equal to the modulus of the bubble diameter $D = 2R.A$ is the inner factor IF while B is the outer factor $OF.A = B = D = 2R$-

For the Natario shape function introduced above it is easy to figure out when $f(rs) = 1$(interior of the Alcubierre bubble) then $l(rs) = 0$(interior of the Natario bubble) and when $f(rs) = 0$(exterior of the Alcubierre bubble)then $l(rs) = \frac{1}{2}$(exterior of the Natario bubble) and this function satisfies the Natario requirements for a shape function.(pg 5 in [1])

This shape function with the factors $A = B = IF = OF$ both equal to the modulus of the bubble diameter $D = 2R$ can reduce the negative energy density to low and affordable levels.

We discovered this function in 10 November 2026:In the original form A was equal to the modulus of the bubble radius R and B was equal to the modulus of the bubble thickness parameter @ however such a function did not reduced the negative energy density requirements in our numerical simulations model to a satisfactory level.

In 18 November 2026 we made a second attempt with new numerical simulations in our model making $A = B$ both equal to the modulus of the bubble diameter D.Then $A = B = D = 2R$ and we got exactly the satisfactory results obtained for the negative energy density requirements by our previous work in [14].(see the Remarks Section in this work)

The Natario shape function presented in [14] is given by:(see eq 17 in [14]):

$$N(rs) = [\frac{1}{2}][1 - f(rs)^{WF}]^{WF} \tag{35}$$

The works in [14] and [35] uses the so-called "warp factor" WF to reduce the levels of negative energy density requirements to sustain a Natario warp bubble without giving a clue on the origin of WF simply defining it as an "arbitrary parameter".

Since 18 November 2026 WF can now finally be defined as being the diameter of the warp bubble in modulus and this line of reason finally gives a physical meaning to WF giving also a physical meaning to the works in [14] and [35].

Then $WF = A = B$ and we keep both Natario shape functions $N(rs)$ and $l(rs)$.Note that if $A = B$ $l(rs)$ and $N(rs)$ becomes the same function.$l(rs) = \frac{1}{2}[1 - (f(rs)^A)]^B = N(rs) = [\frac{1}{2}][1 - f(rs)^{WF}]^{WF}$.

In the next sections we will compute the first and second order derivatives of this new Natario shape function $l(rs)$ with the numerical plots needed to demonstrate that the factor 10^{48} due to the terms $(f(rs)^{(A)}$ and $[1-(f(rs)^A)]^B)$ or $(f(rs)^{(WF)}$ and $[1-(f(rs)^{WF})]^{WF})$ can be completely obliterated allowing affordable Natario negative energy density requirements even with speeds of 200 times faster than light.

The advantage of having different inner and outer factors separated being A the inner factor IF and B as the outer factor OF is the fact that it is possible to test other possibilities with A and B possessing different values in a generic mathematical formalism using different numerical simulations in our model.

4 The first and second order derivatives $l'(rs)$ and $l''(rs)$ of the Natario shape function and its squares.

$$f(rs) = \frac{1}{2}([1 - tanh[@(rs - R)]) \tag{36}$$

$$f'(rs) = -\frac{1}{2}[\frac{@}{cosh^2[@(rs - R)]}] \tag{37}$$

$$f'(rs)^2 = \frac{1}{4}[\frac{@^2}{cosh^4[@(rs - R)]}] \tag{38}$$

$$f''(rs) = [\frac{(@^2)sinh[@(rs - R)]}{cosh^3[@(rs - R)]}] \tag{39}$$

$$f''(rs)^2 = [\frac{(@^2)sinh[@(rs - R)]}{cosh^3[@(rs - R)]}]^2 = [\frac{(@^4)sinh[@(rs - R)]^2}{cosh^6[@(rs - R)]}] \tag{40}$$

Above are being presented the Alcubierre shape function $f(rs)$ and its first and second order derivatives.The Alcubierre shape function $f(rs)$ gives the value of 1 inside the bubble and zero outside the bubble.(see eq 7 pg 4 in [2],eq 136 pg 27 in [4],eq 7 pg 3 in [36]).The region where $1 > f(rs) > 0$ is known as the Alcubierre warped region.In the Alcubierre shape function $f(rs)$,R is the radius of the warp bubble,@ is the "thickness parameter" related to the thickness of the warp bubble which means to say the region where $1 > f(rs) > 0$.A large @ concentrates the Alcubierre warped region in a thin shell.According to Alcubierre pg 4 in [2] @ can have arbitrary values and rs is the distance between the center of the bubble and a point P that can lies inside or outside the bubble.In the center of the bubble $rs = 0$.

In a bubble of large radius R with size enough to contains a spaceship the spaceship lies in the center of the bubble in the innermost regions of the bubble where $f(rs) = 1$.The spaceship position PS is well inside the bubble and far from the radius $PS << R$.An external observer lies at a faraway distance outside the bubble.The external observer position PO with respect to the bubble radius is $PO >> R$.In the external regions $f(rs) = 0$.

Inside and outside the bubble the derivatives of the Alcubierre shape function are null $f'(rs) = 0$.The region where $1 > f(rs) > 0$ the Alcubierre warped region is the region where $f(rs)$ starts to decrease from 1(inner warped region) ending the decrease when $f(rs)$ approaches zero but not zero.(outer warped region)

Then the geometric position where the Alcubierre warped regions begins with respect to R the bubble radius is $rs = R - \epsilon < R$(inner warped region) and the geometric position where the Alcubierre warped regions ends with respect to R the bubble radius is $rs = R + \epsilon > R$.(outer warped region).The thickness of the warp bubble is then $2 \times \epsilon$.As large as @ becomes as smaller ϵ becomes too.

As large as @ becomes as small the distance between the inner warped region and the outer warped region placing the inner region close to the outer region in a thin shell.

An example;For a bubble with 100 meters radius $R = 100$ and a thickness parameter $@ = 50000$ from 0 to $99,996$ meters,$f(rs) = 1$(inside the bubble).At $99,997$ meters $f(rs)$ starts to decrease(inner warped region) and at $100,003$ meters $f(rs)$ approaches zero but not zero.(outer warped region) and at $100,004$ meters finally $f(rs) = 0$.(outside the bubble)

However a large thickness parameter is a very serious problem because this means large derivatives of the Alcubierre shape function that will not obliterate the factor 10^{48}.

Perhaps using a different and modified Alcubierre shape function the factor 10^{48} may be circumvented however the interstellar matter and the Doppler blueshifted photons would always remains in front of the Alcubierre bubble that have no protections against interstellar impacts.(See Appendices G,H,I,J and L in this work).

And since the Natario bubble offers protection against interstellar impacts is more suitable for interstellar space travel.(see Appendices B,C,D and K in this work).

However the Natario shape function($l(rs)$ or $N(rs)$) uses the Alcubierre shape function $f(rss)$ in its definition.

$$l(rs) = \frac{1}{2}[1 - (f(rs)^A)]^B \tag{41}$$

The Natario shape function $N(rs)$ was presented in [14] given by:(see eq 17 in [14]).(This shape function appeared first in eq 10 in [35]).:

$$N(rs) = [\frac{1}{2}][1 - f(rs)^{WF}]^{WF} \tag{42}$$

The first order derivatives and its squares are given by:

$$l'(rs) = (-\frac{1}{2})(AB)[1 - (f(rs)^A)]^{B-1}[(f(rs)^{A-1}]f'(rs) \tag{43}$$

$$N'(rs) = -[\frac{1}{2}]WF^2[1 - f(rs)^{WF}]^{(WF-1)}[f(rs)^{(WF-1)}]f'(rs) \tag{44}$$

$$l'(rs)^2 = (\frac{1}{4})[(AB)^2][1 - (f(rs)^A)]^{(2(B-1)}[(f(rs)^{2(A-1)}]f'(rs)^2 \tag{45}$$

$$N'(rs)^2 = [\frac{1}{4}]WF^4[1 - f(rs)^{WF}]^{2(WF-1)}[f(rs)^{2(WF-1)}]f'(rs)^2 \tag{46}$$

Making $A = B$ both equal to the modulus of the bubble diameter D.Then $A = B = D = 2R$ and we got exactly the numerical simulations satisfactory results obtained for the negative energy density requirements by our previous work in [14].(see the Remarks Section in this work).

The works in [14] and [35] uses the so-called "warp factor" WF to reduce the levels of negative energy density requirements to sustain a Natario warp bubble without giving a clue on the origin of WF simply defining it as an "arbitrary parameter".

WF can now finally be defined as being the diameter of the warp bubble in modulus $D = 2R$ and this line of reason finally gives a physical meaning to WF giving also a physical meaning to the works in [14] and [35].

The advantage of having different inner and outer factors separated being A the inner factor IF and B as the outer factor OF is the fact that it is possible to test other possibilities with A and B possessing different values in a generic mathematical formalism using different numerical simulations in our model.

Then $A = B = D = 2R = WF$ in modulus.

The second order derivative of the Natario shape function $l''(rs)$ would result in a very algebraic complicated expression.In order to simplify our study we decompose the second order derivative of the Natario shape function in separated algebraic expressions as shown below:

$$l''(rs) = (-\frac{1}{2})(AB)[U + V + W] \tag{47}$$

$$l''(rs) = (-\frac{1}{2})(WF^2)[U + V + W] \tag{48}$$

$$U = -A(B-1)[1 - (f(rs)^A)]^{B-2}[(f(rs)^{2(A-1)}]f'(rs)^2 \tag{49}$$

$$U = -WF(WF-1)[1 - (f(rs)^{WF})]^{WF-2}[(f(rs)^{2(WF-1)}]f'(rs)^2 \tag{50}$$

$$V = (A-1)[1 - (f(rs)^A)]^{B-1}[(f(rs)^{A-2}]f'(rs)^2 \tag{51}$$

$$V = (WF-1)[1 - (f(rs)^{WF})]^{WF-1}[(f(rs)^{WF-2}]f'(rs)^2 \tag{52}$$

$$W = [1 - (f(rs)^A)]^{B-1}[(f(rs)^{A-1}]f''(rs) \tag{53}$$

$$W = [1 - (f(rs)^{WF})]^{WF-1}[(f(rs)^{WF-1}]f''(rs) \tag{54}$$

$$l''(rs) = (-\frac{1}{2})(AB)[U + V + W] \tag{55}$$

$$l''(rs) = (-\frac{1}{2})(WF^2)[U + V + W] \tag{56}$$

$$(-\frac{1}{2})(WF^2)[U] = (-\frac{1}{2})(WF^2)[-WF(WF-1)[1-(f(rs)^{WF})]^{WF-2}[(f(rs)^{2(WF-1)}]f'(rs)^2] \tag{57}$$

$$(-\frac{1}{2})(WF^2)[U] = [\frac{1}{2}]WF^3(WF-1)[1-f(rs)^{WF}]^{(WF-2)}[f(rs)^{2(WF-1)}]f'(rs)^2 = P \tag{58}$$

$$(-\frac{1}{2})(WF^2)[V] = (-\frac{1}{2})(WF^2)[(WF-1)[1-(f(rs)^{WF})]^{WF-1}[(f(rs)^{WF-2}]f'(rs)^2] = Q \tag{59}$$

$$(-\frac{1}{2})(WF^2)[W] = (-\frac{1}{2})(WF^2)[[1-(f(rs)^{WF})]^{WF-1}[(f(rs)^{WF-1}]f''(rs)] = R \tag{60}$$

$$l''(rs) = (-\frac{1}{2})(WF^2)[U + V + W] \tag{61}$$

$$(-\frac{1}{2})(WF^2)[U] = [\frac{1}{2}]WF^3(WF-1)[1-f(rs)^{WF}]^{(WF-2)}[f(rs)^{2(WF-1)}]f'(rs)^2 = P \tag{62}$$

$$(-\frac{1}{2})(WF^2)[V] = (-\frac{1}{2})(WF^2)[(WF-1)[1-(f(rs)^{WF})]^{WF-1}[(f(rs)^{WF-2}]f'(rs)^2] = Q \tag{63}$$

$$(-\frac{1}{2})(WF^2)[W] = (-\frac{1}{2})(WF^2)[[1-(f(rs)^{WF})]^{WF-1}[(f(rs)^{WF-1}]f''(rs)] = R \tag{64}$$

$$l''(rs) = [\frac{1}{2}]WF^3(WF-1)[1-f(rs)^{WF}]^{(WF-2)}[f(rs)^{2(WF-1)}]f'(rs)^2 \tag{65}$$

$$-[\frac{1}{2}]WF^2[1-f(rs)^{WF}]^{(WF-1)}(WF-1)[f(rs)^{(WF-2)}]f'(rs)^2 \tag{66}$$

$$-[\frac{1}{2}]WF^2[1-f(rs)^{WF}]^{(WF-1)}[f(rs)^{(WF-1)}]f''(rs) \tag{67}$$

We can see that the square of the second order derivative of the Natario shape function in [14] $N''(rs)^2$ would result in a very algebraic complicated expression.In order to simplify our study we decompose the second order derivative of the Natario shape function in separated algebraic expressions as shown below:(see eqs 54 to 57 in [14])

$$N''(rs) = P + Q + R \tag{68}$$

$$P = [\frac{1}{2}]WF^3(WF-1)[1-f(rs)^{WF}]^{(WF-2)}[f(rs)^{2(WF-1)}]f'(rs)^2 \tag{69}$$

$$Q = -[\frac{1}{2}]WF^2[1-f(rs)^{WF}]^{(WF-1)}(WF-1)[f(rs)^{(WF-2)}]f'(rs)^2 \tag{70}$$

$$R = -[\frac{1}{2}]WF^2[1-f(rs)^{WF}]^{(WF-1)}[f(rs)^{(WF-1)}]f''(rs) \tag{71}$$

The second order derivative of the Natario shape function $l''(rs)$ is then given by:

$$l''(rs) = P + Q + R \tag{72}$$

$$P = [\frac{1}{2}]WF^3(WF-1)[1-f(rs)^{WF}]^{(WF-2)}[f(rs)^{2(WF-1)}]f'(rs)^2 \tag{73}$$

$$Q = -[\frac{1}{2}]WF^2[1-f(rs)^{WF}]^{(WF-1)}(WF-1)[f(rs)^{(WF-2)}]f'(rs)^2 \tag{74}$$

$$R = -[\frac{1}{2}]WF^2[1-f(rs)^{WF}]^{(WF-1)}[f(rs)^{(WF-1)}]f''(rs) \tag{75}$$

Since 18 November 2026 WF can now finally be defined as being the diameter of the warp bubble in modulus and this line of reason finally gives a physical meaning to $WF = 2R = D$ giving also a physical meaning to the works in [14] and [35].

In order to avoid a lengthly work we will not repeat here the numerical simulations already given in the sections 4 to 10 in [14].From these simulations anyone can see that the factor 10^{48} can be completely circumvented allowing a Natario warp drive bubble to travel 200 times faster than light.

5 Conclusion

In this work we concentrated ourselves in the otiginal Natario warp drive in Polar Coordinates.([1]). The negative energy density according to Natario in Polar Coordinates is given by(see pg 5 in [1]:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(l'(rs))^2\cos^2\theta + \left(l'(rs) + \frac{r}{2}l''(rs)\right)^2\sin^2\theta\right] \tag{76}$$

Rewriting the Natario negative energy density in cartezian coordinates we should expect for:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(l'(rs))^2(\frac{x}{rs})^2 + \left(l'(rs) + \frac{r}{2}l''(rs)\right)^2(\frac{y}{rs})^2\right] \tag{77}$$

Considering motion in the equatorial plane of the Natario warp bubble (x-axis only) then $[y^2 + z^2] = 0$ and $r^2 = [(x - xs)^2]$ and making $xs = 0$ the center of the bubble as the origin of the coordinate frame for the motion of the Eulerian observer then $r^2 = x^2$ because in the equatorial plane $y = z = 0$.

Rewriting the Natario negative energy density in cartezian coordinates in the equatorial plane we should expect for:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(l'(rs))^2\right] \tag{78}$$

This is a very important result:The negative energy density in the equatorial plane of the Natario warp drive in front of the ship IS NOT ZERO and this changes the whole picture or changes the rules of the game!!!

This Natario warp drive negative energy density in front of the ship with repulsive gravitational behavior(see pg 116 in [31]) deflects all the incoming hazardous particles of the Interstellar Medium and offers an excellent protection to the crew members as depicted in the works [11],[12],[13] and mainly [14].

Compare with the Alcubierre negative energy density in the equatorial plane according to the equation of Pfenning and Ford in [3] eq 8 pg 6:The negative energy density in the Alcubierre equatorial plane IS ZERO!!!

$$\langle T^{\mu\nu}u_{\mu}u_{\nu}\rangle = \langle T^{00}\rangle = \frac{1}{8\pi}G^{00} = -\frac{1}{8\pi}\frac{v_s^2(t)[y^2+z^2]}{4r_s^2(t)}\left(\frac{df(r_s)}{dr_s}\right)^2, \dashrightarrow [y^2+z^2] = 0 \dashrightarrow \frac{1}{8\pi}G^{00} = 0 \tag{79}$$

The Alcubierre warp drive dont have negative energy density in front of the ship to deflect the incoming hazardous particles of the Interstellar Medium.

The distribution of energy for a Natario warp drive in $3D$ Spherical coordinates will appear in a future work.Since a spacetime in $3D$ Spherical coordinates can be dimensionally reduced to a one in $2D$ Polar Coordinates it is reasonable to suppose that the negative energy density will remain in front of the ship deflecting hazardous interstellar matter even in $3D$ Spherical coordinates.See sections 5 and 9 in [30].

In this work we presented the new Natario shape function $l(rs)$ defined using the Alcubierre shape function $f(rs)$:

$$l(rs) = \frac{1}{2}[1 - (f(rs)^A)]^B \tag{80}$$

In the new Natario shape function $l(rs)$ above the factors A and B are both equal to the modulus of the bubble diameter D.A is the inner factor IF while B is the outer factor OF.

This shape function with the factors $A = B$ both equal to the modulus of the bubble diameter D can reduce the negative energy density to low and affordable levels.

In the original form A was equal to the modulus of the bubble radius R and B was equal to the modulus of the bubble thickness parameter @ however such a function did not reduced the negative energy density requirements in our numerical simulations model to a satisfactory level.

However we made a second attempt with new numerical simulations in our model making $A = B$ both equal to the modulus of the bubble diameter D.Then $A = B = D = 2R$ and we got exactly the satisfactory results obtained for the negative energy density requirements by our previous work in [14].

The Natario shape function presented in [14] is given by:(see eq 17 in [14]):

$$N(rs) = [\frac{1}{2}][1 - f(rs)^{WF}]^{WF} \tag{81}$$

The works in [12],[13] and [14] uses the so-called "warp factor" WF to reduce the levels of negative energy density requirements to sustain a warp bubble without giving a clue on the origin of WF simply defining it as an "arbitrary parameter".

WF can now be defined as being the diameter of the warp bubble in modulus and this line of reason finally gives a physical meaning to $WF = 2R = D$ giving also a physical meaning to the works in [12],[13] and [14].

This negative energy density is directly proportional to the factor $(\frac{c^2}{G})vs^2$ in which $c = 3 \times 10^8 \frac{meters}{second}$ is the light speed $G = 6,67 \times 10^{-11} \frac{Newton \times meters^2}{kilograms^2}$ is the Newton gravitational constant $c^2 = 9 \times 10^{16}(\frac{meters}{second})^2$ is the square of the light speed and vs^2 is the square of the spaceship velocity all in the International System of Units $SI - MKS$.

Considering a warp drive moving with $vs = 200$ which means to say 200 times light speed in order to make a round trip from Earth to a nearby star at 20 light-years away in a reasonable amount of time(in months not in years) we would get in the expression of the negative energy the factor $c^2 = (3 \times 10^8)^2 = 9 \times 10^{16}$ being divided by $6,67 \times 10^{-11}$ giving $1,35 \times 10^{27}$ and this is multiplied by $(6 \times 10^{10})^2 = 36 \times 10^{20}$ coming from the term $vs = 200$ giving $1,35 \times 10^{27} \times 36 \times 10^{20} = 1,35 \times 10^{27} \times 3,6 \times 10^{21} = 4,86 \times 10^{48}$!!!

A number with 48 zeros!!!The planet Earth have a mass[4] of about $6 \times 10^{24} kg$

The numerical simulations presented in the sections 4 to 10 in [14] shows that the factor 10^{48} can be completely circumvented allowing a Natario warp drive bubble to travel 200 times faster than light.

The warp drive as an artificial superluminal geometric tool that allows to travel faster than light may well have an equivalent in the Nature.According to the modern Astronomy the Universe is expanding and as farther a galaxy is from us as faster the same galaxy recedes from us.The expansion of the Universe is accelerating and if the distance between us and a galaxy far and far away is extremely large the speed of the recession may well exceed the light speed limit.(see pg 98 in [32] and pg 377 in [33]).

For the experimental verification of the acceleration of the Universe see for example the bottom of pg 355 and top of pg 356 eq 8.155 in [34].

[4]see Wikipedia:The free Encyclopedia

6 Appendix A:Dimensional Reduction from $\frac{c^4}{G}$ to $\frac{c^2}{G}$

The Alcubierre expressions for the Negative Energy Density in Geometrized Units $c = G = 1$ are given by(pg 4 in [1])(pg 8 in [2]):[5]:

$$\rho = -\frac{1}{32\pi} vs^2 \left[f'(rs)\right]^2 [\frac{y^2+z^2}{rs^2}] \tag{82}$$

$$\rho = -\frac{1}{32\pi} vs^2 \left[\frac{df(rs)}{drs}\right]^2 [\frac{y^2+z^2}{rs^2}] \tag{83}$$

In this system all physical quantities are identified with geometrical entities such as lengths,areas or dimensionless factors.Even time is interpreted as the distance travelled by a pulse of light during that time interval,so even time is given in lengths.Energy,Momentum and Mass also have the dimensions of lengths.We can multiply a mass in kilograms by the conversion factor $\frac{G}{c^2}$ to obtain the mass equivalent in meters.On the other hand we can multiply meters by $\frac{c^2}{G}$ to obtain kilograms.The Energy Density($\frac{Joules}{meters^3}$)in Geometrized Units have a dimension of $\frac{1}{length^2}$ and the conversion factor for Energy Density is $\frac{G}{c^4}$.Again on the other hand by multiplying $\frac{1}{length^2}$ by $\frac{c^4}{G}$ we retrieve again ($\frac{Joules}{meters^3}$). [6].

This is the reason why in Geometrized Units the Einstein Tensor have the same dimension of the Stress Energy Momentum Tensor(in this case the Negative Energy Density)and since the Einstein Tensor is associated to the Curvature of Spacetime both have the dimension of $\frac{1}{length^2}$.

$$G_{00} = 8\pi T_{00} \tag{84}$$

Passing to normal units and computing the Negative Energy Density we multiply the Einstein Tensor (dimension $\frac{1}{length^2}$) by the conversion factor $\frac{c^4}{G}$ in order to retrieve the normal unit for the Negative Energy Density ($\frac{Joules}{meters^3}$).

$$T_{00} = \frac{c^4}{8\pi G} G_{00} \tag{85}$$

Examine now the Alcubierre equations:

$vs = \frac{dxs}{dt}$ is dimensionless since time is also in lengths.$\frac{y^2+z^2}{rs^2}$ is dimensionless since both are given also in lengths. $f(rs)$ is dimensionless but its derivative $\frac{df(rs)}{drs}$ is not because rs is in meters. So the dimensional factor in Geometrized Units for the Alcubierre Energy Density comes from the square of the derivative and is also $\frac{1}{length^2}$.Remember that the speed of the Warp Bubble vs is dimensionless in Geometrized Units and when we multiply directly $\frac{1}{length^2}$ from the Negative Energy Density in Geometrized Units by $\frac{c^4}{G}$ to obtain the Negative Energy Density in normal units $\frac{Joules}{meters^3}$ the first attempt would be to make the following:

$$\rho = -\frac{c^4}{G}\frac{1}{32\pi} vs^2 \left[f'(rs)\right]^2 [\frac{y^2+z^2}{rs^2}] \tag{86}$$

$$\rho = -\frac{c^4}{G}\frac{1}{32\pi} vs^2 \left[\frac{df(rs)}{drs}\right]^2 [\frac{y^2+z^2}{rs^2}] \tag{87}$$

[5]See Geometrized Units in Wikipedia

[6]See Conversion Factors for Geometrized Units in Wikipedia

But note that in normal units vs is not dimensionless and the equations above do not lead to the correct dimensionality of the Negative Energy Density because the equations above in normal units are being affected by the dimensionality of vs.

In order to make vs dimensionless again,the Negative Energy Density is written as follows:

$$\rho = -\frac{c^4}{G}\frac{1}{32\pi}(\frac{vs}{c})^2\left[f'(rs)\right]^2[\frac{y^2+z^2}{rs^2}] \tag{88}$$

$$\rho = -\frac{c^4}{G}\frac{1}{32\pi}(\frac{vs}{c})^2\left[\frac{df(rs)}{drs}\right]^2[\frac{y^2+z^2}{rs^2}] \tag{89}$$

Giving:

$$\rho = -\frac{c^2}{G}\frac{1}{32\pi}vs^2\left[f'(rs)\right]^2[\frac{y^2+z^2}{rs^2}] \tag{90}$$

$$\rho = -\frac{c^2}{G}\frac{1}{32\pi}vs^2\left[\frac{df(rs)}{drs}\right]^2[\frac{y^2+z^2}{rs^2}] \tag{91}$$

As already seen.The same results are valid for the Natario Energy Density

Note that from

$$\rho = -\frac{c^4}{G}\frac{1}{32\pi}(\frac{vs}{c})^2\left[f'(rs)\right]^2[\frac{y^2+z^2}{rs^2}] \tag{92}$$

$$\rho = -\frac{c^4}{G}\frac{1}{32\pi}(\frac{vs}{c})^2\left[\frac{df(rs)}{drs}\right]^2[\frac{y^2+z^2}{rs^2}] \tag{93}$$

Making $c = G = 1$ we retrieve again

$$\rho = -\frac{1}{32\pi}vs^2\left[f'(rs)\right]^2[\frac{y^2+z^2}{rs^2}] \tag{94}$$

$$\rho = -\frac{1}{32\pi}vs^2\left[\frac{df(rs)}{drs}\right]^2[\frac{y^2+z^2}{rs^2}] \tag{95}$$

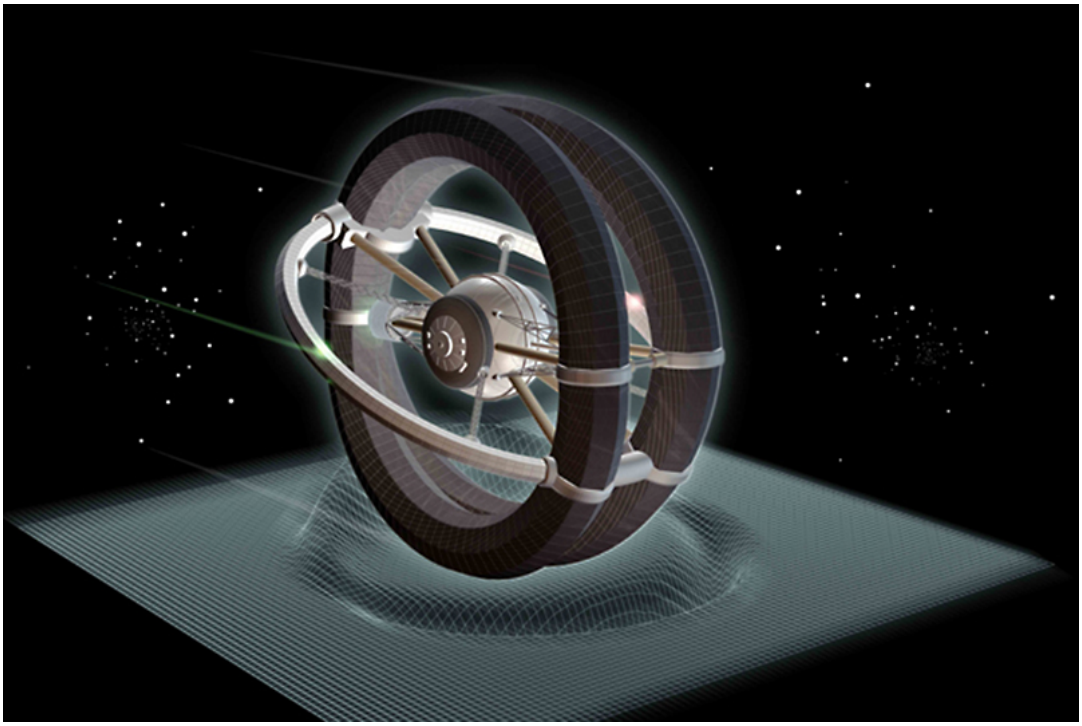

Figure 1: Artistic representation of the negative energy density in the Natario warp drive .Note in the bottom of the figure the Alcubierre expansion of the normal volume elements .(Source:Internet)

7 Appendix B:Artistic Presentation of the negative energy density in the Natario warp drive-Polar Coordinates

According to the geometry of the Natario warp drive the spacetime contraction in one direction(radial) is balanced by the spacetime expansion in the remaining direction(perpendicular).(pg 5 in [1]).

The expansion of the normal volume elements in the Natario warp drive is given by the following expressions(pg 5 in [1]).

$$K_{rr} = \frac{\partial X^r}{\partial r} = -2v_s l'(r)\cos\theta \tag{96}$$

$$K_{\theta\theta} = \frac{1}{r}\frac{\partial X^\theta}{\partial \theta} + \frac{X^r}{r} = v_s l'(r)\cos\theta; \tag{97}$$

$$K_{\varphi\varphi} = \frac{1}{r\sin\theta}\frac{\partial X^\varphi}{\partial \varphi} + \frac{X^r}{r} + \frac{X^\theta \cot\theta}{r} = v_s l'(r)\cos\theta \tag{98}$$

$$\theta = K_{rr} + K_{\theta\theta} + K_{\varphi\varphi} = 0 \tag{99}$$

If we expand the radial direction the perpendicular direction contracts to keep the expansion of the normal volume elements equal to zero.This figure is a pedagogical example of the graphical presentarion of the Natario warp drive.

The "rings" in the figure were included to illustrate how the expansion in one direction can be counterbalanced by the contraction in the other directions.These "rings" keeps the expansion of the normal volume elements in the Natario warp drive equal to zero.

Note also that the graphical presentation of the Alcubierre warp drive expansion of the normal volume elements according to fig 1 pg 10 in [2] is also included

Note also that the energy density in the Natario warp drive $3+1$ spacetime being given by the following expressions(pg 5 in [1]):

$$\rho = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(l'(r))^2\cos^2\theta + \left(l'(r)+\frac{r}{2}l''(r)\right)^2\sin^2\theta\right]. \tag{100}$$

$$\rho = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(\frac{dl(r)}{dr})^2\cos^2\theta + \left(\frac{dl(r)}{dr}+\frac{r}{2}\frac{d^2l(r)}{dr^2}\right)^2\sin^2\theta\right]. \tag{101}$$

Is being distributed around all the space involving the ship(above the ship $\sin\theta = 1$ and $\cos\theta = 0$ while in front of the ship $\sin\theta = 0$ and $\cos\theta = 1$).The negative energy in front of the ship "deflect" photons or other particles so these will not reach the ship inside the bubble.The illustrated "rings" are the obstacles that deflects photons or incoming particles from outside the bubble never allowing these to reach the interior of the bubble.

The negative energy density have repulsive gravitational behavior and is distributed along all the bubble volume even in the equatorial plane so any hazardous incoming objects in front of the bubble (Doppler blueshifted photons or space dust or debris) would then be deflected by the repulsive behavior of the negative energy in front of the bubble never reaching the bubble walls(see pg 116 in [31])

-)-Energy directly above the ship$(y-axis)$

$$\rho = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[\left(\frac{dl(r)}{dr}+\frac{r}{2}\frac{d^2l(r)}{dr^2}\right)^2\sin^2\theta\right]. \tag{102}$$

-)-Energy directly in front of the ship$(x-axis)$

$$\rho = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(\frac{dl(r)}{dr})^2\cos^2\theta\right]. \tag{103}$$

This is a very important result:The negative energy density in the equatorial plane of the Natario warp drive in front of the ship IS NOT ZERO and this changes the whole picture or changes the rules of the game!!!

The distribution of energy presented in this Appendix is valid only for the Natario warp drive vector in Polar Coordinates without the lapse function.For the case of the lapse function see Section 3 and Appendix I in [17].

Also the Zero-Expansion behavior is valid only in Polar Coordinates and do not occurs in $3D$ Spherical Coordinates.See [16].

The distribution of energy for a Natario warp drive in $3D$ Spherical coordinates will appear in a future work.Since a spacetime in $3D$ Spherical coordinates can be dimensionally reduced to a one in $2D$ Polar Coordinates it is reasonable to suppose that the negative energy density will remain in front of the ship deflecting hazardous interstellar matter even in $3D$ Spherical coordinates.See sections 5 and 9 in [30].

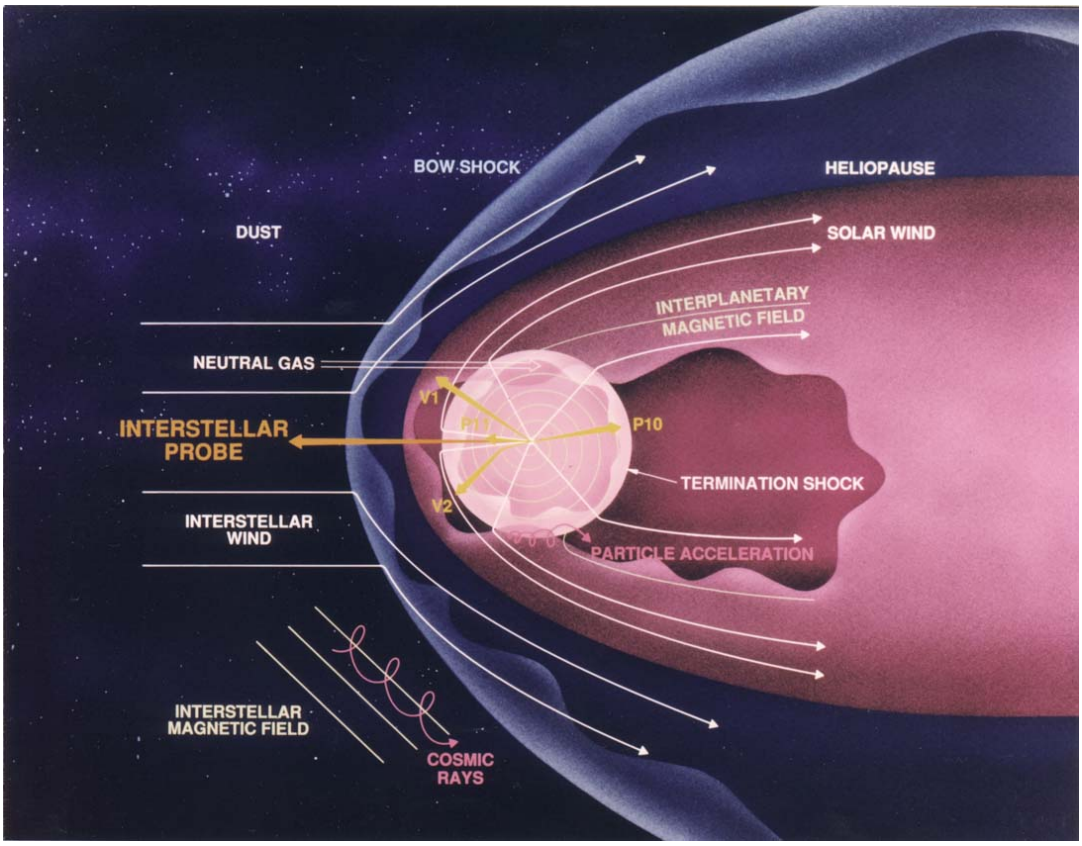

Figure 2: Artistic representation of a Natario warp drive in a real superluminal space travel .Note the negative energy in front of the ship deflecting incoming hazardous interstellar matter(brown arrows).(Source:Internet)

8 Appendix C:Artistic Presentation of a Natario warp drive in a real faster than light interstellar spaceflight

Above is being presented the artistic presentation of a Natario warp drive in a real interstellar superluminal travel.The "ball" or the spherical shape is the Natario warp bubble with the negative energy surrounding the ship in all directions and mainly protecting the front of the bubble.See Appendices B and K

The brown arrows in the front of the Natario bubble are a graphical presentation of the negative energy in front of the ship deflecting interstellar dust,neutral gases,hydrogen atoms,interstellar wind photons etc.See Appendices E and F for the composition of the Interstellar Medium IM.

The spaceship is at the rest and in complete safety inside the Natario bubble.

In order to allow to the negative energy density of the Natario warp drive the deflection of incoming hazardous particles from the Interstellar Medium(IM) the Natario warp drive energy density must be heavier or denser when compared to the IM density.

The negative energy density have repulsive gravitational behavior and is distributed along all the bubble volume even in the equatorial plane so any hazardous incoming objects in front of the bubble (Doppler blueshifted photons or space dust or debris) would then be deflected by the repulsive behavior of the negative energy in front of the bubble never reaching the bubble walls(see pg 116 in [31]).

9 Appendix D:The Natario warp drive negative energy density in Cartezian coordinates

The negative energy density according to Natario in Polar Coordinates is given by(see pg 5 in [1])[7]:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(l'(rs))^2\cos^2\theta + \left(l'(rs) + \frac{r}{2}l''(rs)\right)^2\sin^2\theta\right] \tag{104}$$

In the bottom of pg 4 in [1] Natario defined the x-axis as the polar axis.In the top of page 5 we can see that $x = r\cos(\theta)$ implying in $\cos(\theta) = \frac{x}{r}$ and in $\sin(\theta) = \frac{y}{r}$

Rewriting the Natario negative energy density in cartezian coordinates we should expect for:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(l'(rs))^2(\frac{x}{rs})^2 + \left(l'(rs) + \frac{r}{2}l''(rs)\right)^2(\frac{y}{rs})^2\right] \tag{105}$$

Considering motion in the equatorial plane of the Natario warp bubble (x-axis only) then $[y^2 + z^2] = 0$ and $r^2 = [(x - xs)^2]$ and making $xs = 0$ the center of the bubble as the origin of the coordinate frame for the motion of the Eulerian observer then $r^2 = x^2$ because in the equatorial plane $y = z = 0$.

Rewriting the Natario negative energy density in cartezian coordinates in the equatorial plane we should expect for:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(l'(rs))^2\right] \tag{106}$$

This is a very important result:The negative energy density in the equatorial plane of the Natario warp drive in front of the ship IS NOT ZERO and this changes the whole picture or changes the rules of the game!!!

The negative energy density have repulsive gravitational behavior and is distributed along all the bubble volume even in the equatorial plane in front of the ship so any hazardous incoming objects in front of the bubble (Doppler blueshifted photons or space dust or debris etc) would then be deflected by the repulsive behavior of the negative energy in front of the bubble never reaching the bubble walls(see pg 116 in [31]).

The distribution of energy presented in this Appendix is valid only for the Natario warp drive vector in Polar Coordinates without the lapse function.For the case of the lapse function see Section 3 and Appendix I in [17].

The distribution of energy for a Natario warp drive in $3D$ Spherical coordinates will appear in a future work.Since a spacetime in $3D$ Spherical coordinates can be dimensionally reduced to a one in $2D$ Polar Coordinates it is reasonable to suppose that the negative energy density will remain in front of the ship deflecting hazardous interstellar matter even in $3D$ Spherical coordinates.See sections 5 and 9 in [30].

[7] $l(r)$ is the Natario shape function.Equation written in the Geometrized System of Units $c = G = 1$

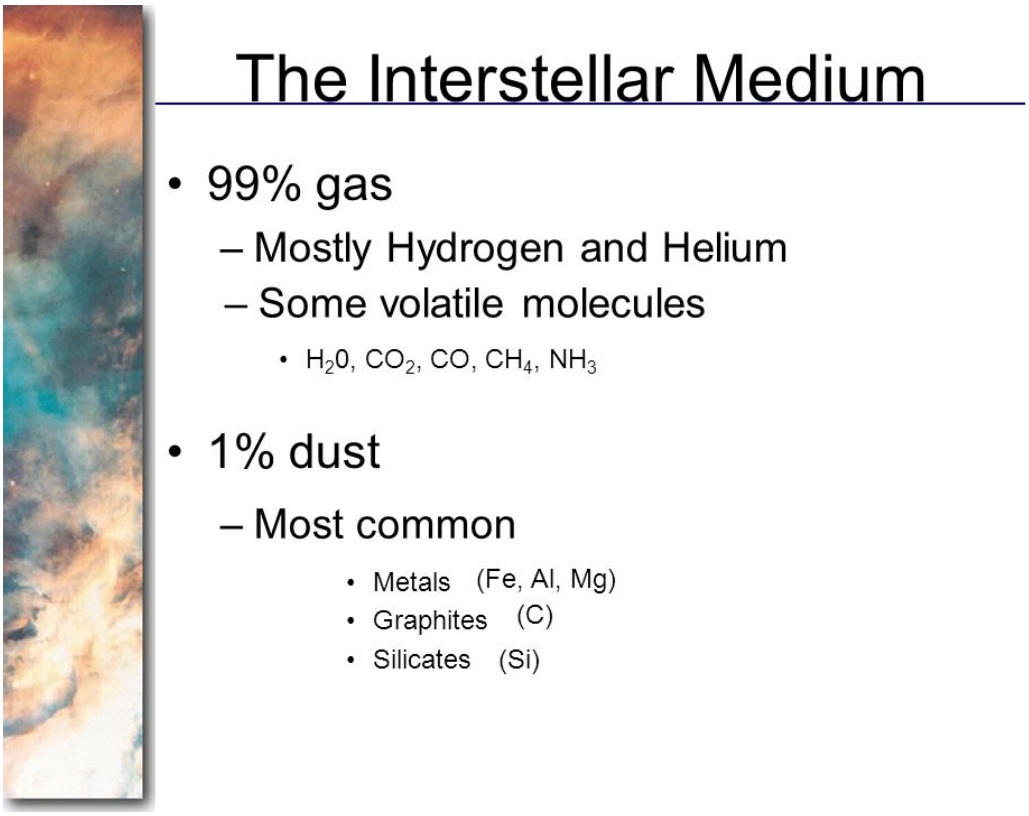

Figure 3: Composition of the Interstellar Medium IM(Source:Internet)

10 Appendix E:Composition of the Interstellar Medium IM

The problem of collisions between a warp drive spaceship moving at superluminal velocity and the potentially dangerous particles from the Interstellar Medium IM is not new.

It was first noticed in 1999 in the work of Chad Clark,Will Hiscock and Shane Larson(see [9]).Later on in 2010 it appeared again in the work of Carlos Barcelo,Stefano Finazzi and Stefano Liberatti(see [10]).In 2012 the same problem of collisions against hazardous IM particles appeared in the work of Brendan McMonigal,Geraint Lewis and Philip O'Byrne(see [6]).

The last work addressing interstellar collisions was the work in ([7]) in 2022.It covers the analysis of Siyu Bian ,Yi Wang,Zun Wang and Mian Zhu.

All these works uses the geometry of the original Alcubierre warp drive 1994 paper in [2] and the results outlined in these works are completely correct.

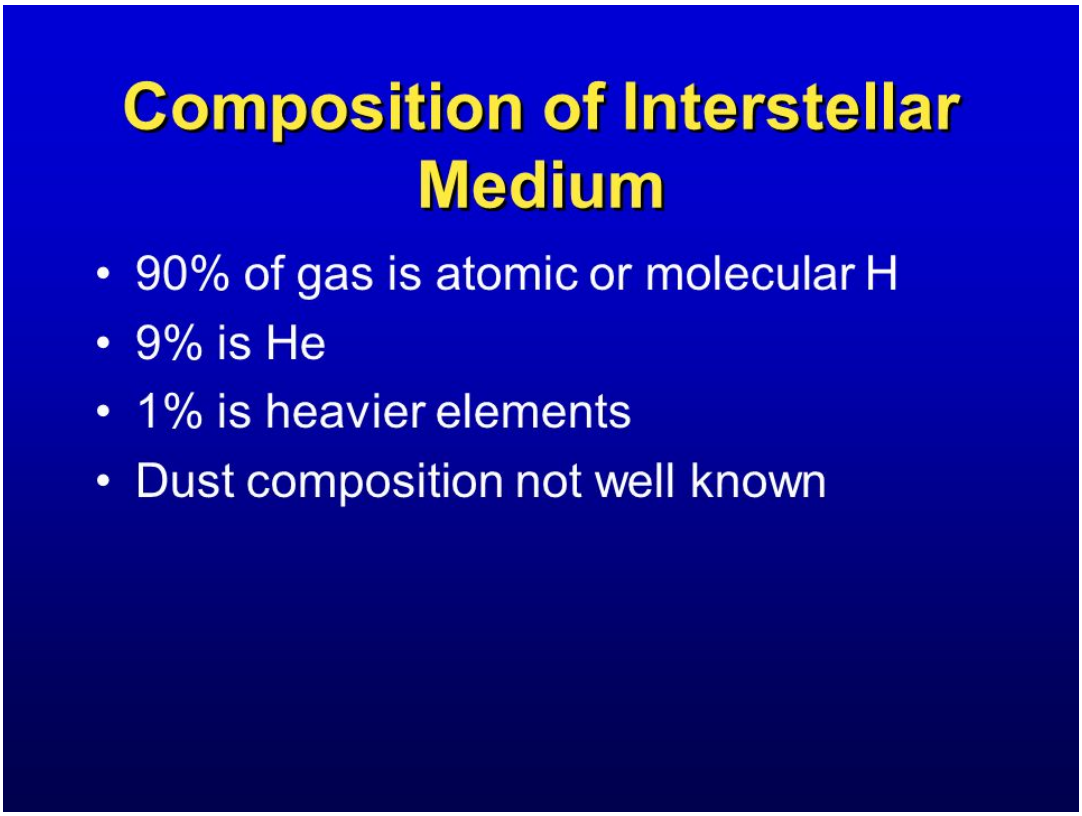

Figure 4: Composition of the Interstellar Medium IM(Source:Internet)

11 Appendix F:Composition of the Interstellar Medium IM

The Natario warp drive is probably the best candidate(known until now) for an interstellar space travel considering the fact that a spaceship in a real superluminal interstellar spaceflight will encounter(or collide against) hazardous objects(Doppler blueshifted photons,asteroids,comets,interstellar dust and debris etc) and due to a different distribution of the negative energy density in front of the ship with repulsive gravitational behavior(see pg 116 in [31]) deflecting all the incoming hazardous particles of the Interstellar Medium the Natario spacetime and its negative energy density in front of the ship(see Appendices B,D and K in this work) offers an excellent protection to the crew members as depicted in the works [11],[12],[13] and mainly [14].The work in [14] borrowed the idea from [8].

The works in [12],[13] and [14] uses the so-called "warp factor" WF to reduce the levels of negative energy density requirements to sustain a warp bubble without giving a clue on the origin of WF simply defining it as an "arbitrary parameter".

WF can be defined as being the diameter of the warp bubble in modulus and this line of reason finally gives a physical meaning to WF and finally a physical meaning to the works [12],[13] and [14] .

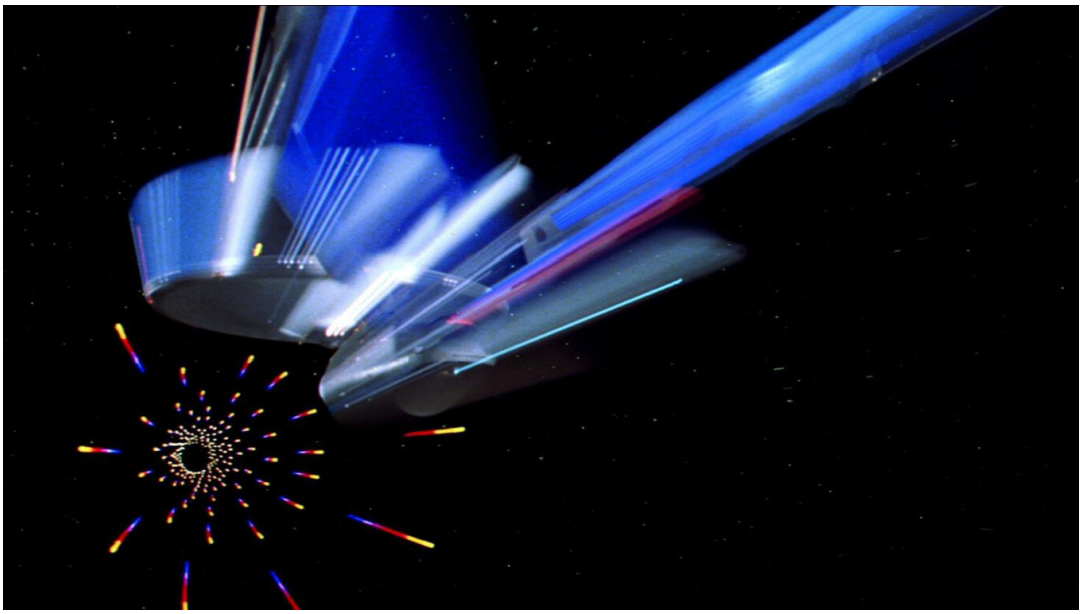

Figure 5: Artistic Presentation of a warp drive spaceship in interstellar space colliding with highly Doppler-Blueshifted photons in the Alcubierre warp drive(Source:Internet)

12 Appendix G:Artistic Presentation of a warp drive spaceship in interstellar space colliding with highly Doppler-Blueshifted Photons in the Alcubierre warp drive

The picture above borrowed from science-fiction depicts one of the most serious(and dangerous) problems a spaceship would confront in a realistic interstellar travel:Collisions with highly Doppler-Blueshifted photons.This problem was first pointed out in 1999 in the work of Chad Clark,Will Hiscock and Shane Larson [9].In 2010 it appeared again in the work of Carlos Barcelo,Stefano Finazzi and Stefano Liberatti [10].In 2012 the same problem of collisions against hazardous IM photons appeared in the work of Brendan McMonigal,Geraint Lewis and Philip O'Byrne [6].

All these works uses the geometry of the original Alcubierre warp drive 1994 paper in [2] and the results outlined in these works are completely correct.

Look to fig 3 pg 7 Pfenning and Ford in [3],fig 3 pg 28 Lobo in [4],fig 2 pg 6 in Lobo and Alcubierre in [5]

In all these figures we see that the distribution of negative energy is concentrated in a toroidal region perpendicular to the direction of travel or the direction of motion of the warp bubble and there is nothing in front of the ship to avoid the arrival of these blueshifted photons so the ship dont have a protection.There is nothing in front of the ship to deflect these blueshifted photons.

This is better illustrated looking to fig 2 pg 4 in Barcelo,Finazzi and Liberati in [10] or the Appendix H in this work:

The "blue arrow" in front of the ship represents the blueshifted photons entering in the bubble following the direction of motion.The negative energy density lies above and below the ship but not in front so there is nothing to deflect the blueshifted photons.See also Appendix I,J and L in this work where the negative energy density above and below the ship but not in front can be seen.

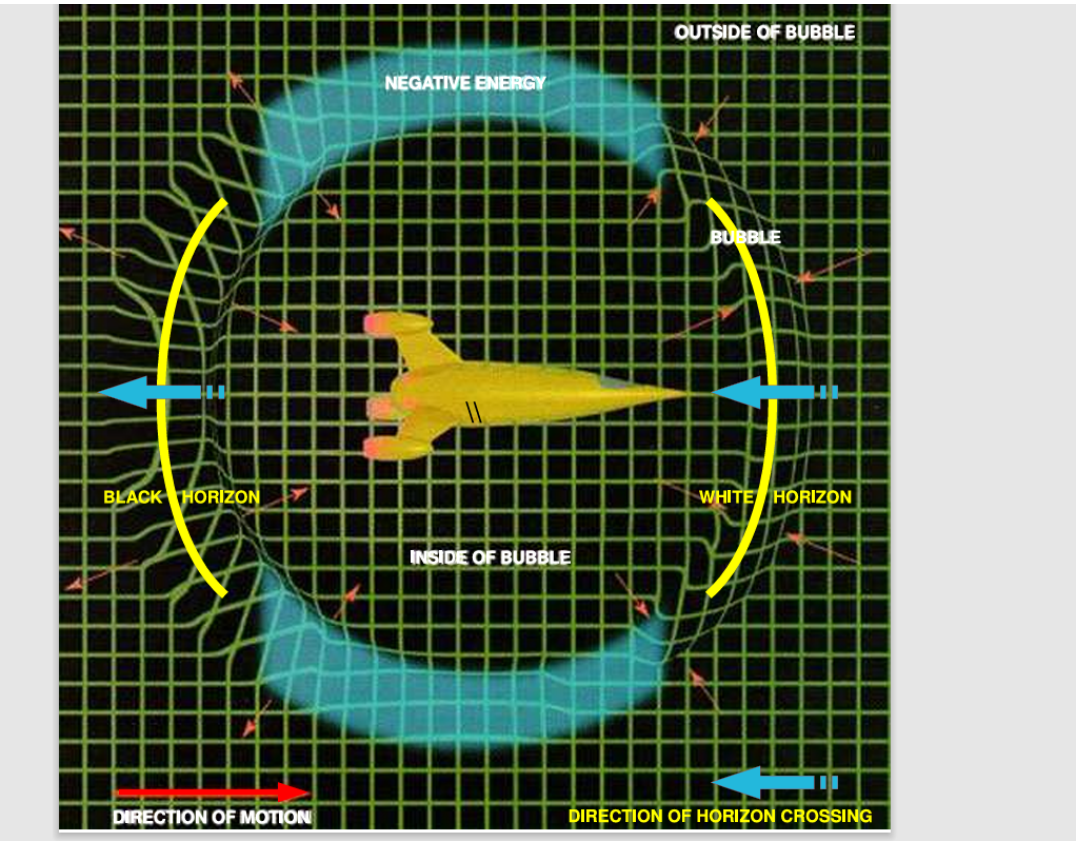

Figure 6: Artistic Presentation of the negative energy density in the Alcubierre warp drive(Source:Fig 2 pg 4 in Barcelo,Finazzi and Liberati work in [10]

13 Appendix H:Artistic Presentation of the negative energy density in the Alcubierre warp drive

The picture above depicts the collisions of the Alcubierre warp drive with highly Doppler-Blueshifted photons or the interstellar matter.(See Appendices E and F in this work about the interstellar medium).

The "blue arrow" represents the Doppler-Blueshifted photons or the interstellar matter coming towards the spaceship inside the bubble.The negative energy density lies in a toroidal region perpendicular to the direction of travel or the direction of motion(equatorial plane) of the warp bubble and there is nothing in front of the ship to protect it.

Clark,Hiscock and Larson pg 11 in [9] mention how hazardous for the ship and the crew members these collisions with Doppler-Blueshifted photons can be.

At 200 times faster than light the energy of these cosmic background blueshifted photons is equivalent to the energy of a solar photosphere and think in a solar photosphere entering in the bubble threatening the ship integrity and the crew member lives.(pg 11 in [9])

Also Barcelo,Finazzi and Liberati pg 6 and 7 in [10] mentions the fact that these Doppler blueshifted photons will raise the temperature inside the bubble to the Hawking temperature generating a harsh environment that would instantaneously imply in the destruction of the ship and the death of the crew members.

Examining the work of Siyu Bian ,Yi Wang,Zun Wang and Mian Zhu in [7].

According to pg 10 in [7] the major part of the incoming IM particles or photons will move towards the spaceship in a trajectory over or very close to the x-axis.Et Voila!!!The equatorial plane!!!.

The "blue arrow" in the figure represents the incoming IM particles or photons

These photons or particles applies over the surface of the warp bubble a pressure directly proportional to the bubble speed vs and as fast the ship or the bubble moves then the pressure becomes more and more stronger and this is a very serious problem that exists in the Alcubierre warp drive spacetime.

This is due to the fact that the warp bubble when moving across interstellar space possesses Cross-Sections and the area of each Cross-Section in the Alcubierre geometry is exposed to impacts against the hazardous IM particles and these impacts generates the mentioned pressure.As large the areas of each Cross-Section are then the surfaces exposed to impacts becomes larger too.

The pressure applied by interstellar photons over the Cross-Section of a warp bubble in the Alcubierre spacetime is given by;(see eq 5.2 pg 11 in [7])

$$dp_p = \frac{E_{av}}{4}\rho_p(\Delta A)\frac{vs}{c}dt \tag{107}$$

In the eq above dp_p is the total momentum of the photons travelling towards the Cross-Section area ΔA of the warp bubble in the Alcubierre spacetime in an infinitesimal amount of time dt,E_{av} is the average energy of each interstellar photon,ρ_p is the "surface density" or the number of impacted photons scattered across the area ΔA of the Cross-Section and vs is the speed of the bubble.

The "force" F exerted by the photons over the area ΔA of the Cross-Section would be given by:

$$F = \frac{dp_p}{dt} = \frac{E_{av}}{4}\rho_p(\Delta A)\frac{vs}{c} \tag{108}$$

Note that the force is directly proportional to the area ΔA of the Cross-Section and also directly proportional to the speed of the bubble vs.As large the bubble is or as fast the bubble moves then the force exerted by the interstellar photons against the Cross-Section becomes stronger and stronger.

The pressure P applied by the interstellar photons over the area ΔA of the Cross-Section would be given by;

$$P = \frac{1}{\Delta A}\frac{dp_p}{dt} = \frac{E_{av}}{4}\rho_p\frac{vs}{c} \tag{109}$$

Note that the pressure is directly proportional to the speed of the bubble vs.As fast the bubble moves then the pressure exerted by the interstellar photons against the Cross-Section becomes stronger and stronger.

The pressure applied by interstellar dust over the Cross-Section of a warp bubble in the Alcubierre spacetime is given by;(see eq 5.4 pg 12 in [7])

$$dp_d = P_{av}\rho_d(\pi\Delta A)vsdt \tag{110}$$

In the eq above dp_d is the total momentum of the dust particles travelling towards the Cross-Section area ΔA of the warp bubble in the Alcubierre spacetime in an infinitesimal amount of time dt,P_{av} is the average momentum of each interstellar dust particle,ρ_d is the "surface density" or the number of impacted dust particles scattered across the area ΔA of the Cross-Section and vs is the speed of the bubble.

The "force" F exerted by the dust particles over the area ΔA of the Cross-Section would be given by;

$$F = \frac{dp_d}{dt} = P_{av}\rho_d(\pi\Delta A)vs \tag{111}$$

Like the previous case the force is directly proportional to the area ΔA of the Cross-Section and also directly proportional to the speed of the bubble vs.

The part of the Alcubierre bubble that suffers the pressure from the interstellar matter is graphically represented by the yellow circular lines in the figure.

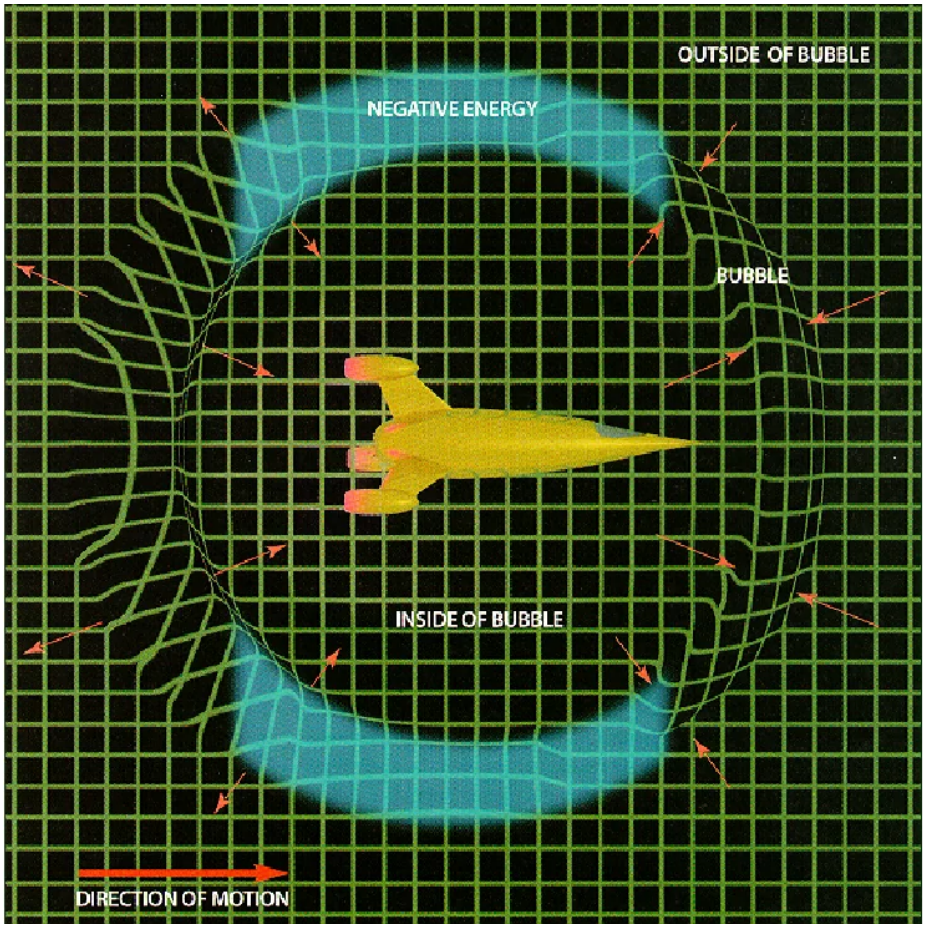

Figure 7: Another Artistic Presentation of the negative energy density in the Alcubierre warp drive(Source:Internet)

14 Appendix I:Another Artistic Presentation of the negative energy density in the Alcubierre warp drive

The negative energy density lies in a toroidal region perpendicular to the direction of travel or the direction of motion(equatorial plane) of the warp bubble above and below the spaceship and there is nothing in front of the spaceship to protect it.

The "red arrows" in the right side of the bubble graphically represents the part of the Alcubierre bubble that suffers the pressure of the interstellar matter in the work of Siyu Bian ,Yi Wang,Zun Wang and Mian Zhu in [7].

Although the negative energy lies above and below the ship,the part of the bubble in front of the ship graphically represented by the "red arrows" is the region in the spacetime distortion where the spacetime contracts in front of the ship bringing the destination point close to the ship but also bringing to the ship neighborhoods all the hazardous interstellar matter between the ship and the destination point.See Appendices E and F in this work about the interstellar medium.

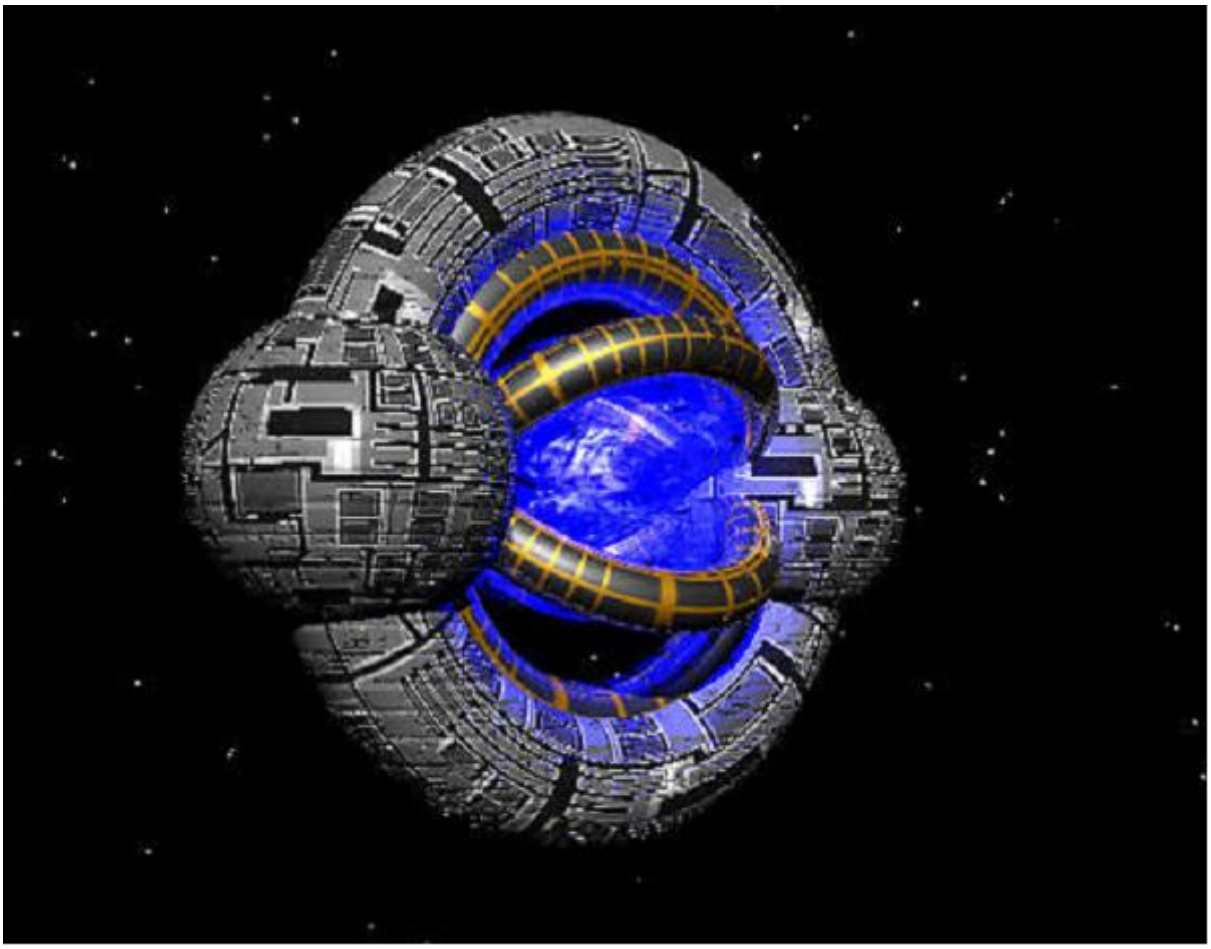

Figure 8: Another Artistic Presentation of the negative energy density in the Natario warp drive(Source:Internet)

15 Appendix K:Another Artistic Presentation of the negative energy density in the Natario warp drive-Polar Coordinates

Negative energy density according to Natario in Polar Coordinates (see pg 5 in [1])[8]:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(l'(rs))^2\cos^2\theta + \left(l'(rs) + \frac{r}{2}l''(rs)\right)^2\sin^2\theta\right] \tag{112}$$

Natario negative energy density in cartezian coordinates:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(l'(rs))^2(\frac{x}{rs})^2 + \left(l'(rs) + \frac{r}{2}l''(rs)\right)^2(\frac{y}{rs})^2\right] \tag{113}$$

[8] $l(r)$ is the Natario shape function.Equation written in the Geometrized System of Units $c = G = 1$

Considering motion in the equatorial plane of the Natario warp bubble (x-axis only) then $[y^2+z^2]=0$ and $y=z=0$.

Natario negative energy density in cartezian coordinates in the equatorial plane($x-axis$):

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(l'(rs))^2\right] \tag{114}$$

This is a very important result:The negative energy density in the equatorial plane of the Natario warp drive in front of the spaceship !!!IS NOT ZERO!!!.The Natario warp drive offers protection to the spaceship and this changes the whole picture or changes the rules of the game!!!

Energy directly above the ship($y-axis$):

$$\rho = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[\left(\frac{dl(r)}{dr}+\frac{r}{2}\frac{d^2l(r)}{dr^2}\right)^2\sin^2\theta\right]. \tag{115}$$

The negative energy density have repulsive gravitational behavior and is distributed along all the bubble volume even in the equatorial plane in front of the ship so any hazardous incoming objects in front of the bubble (Doppler blueshifted photons or space dust or debris etc) would then be deflected by the repulsive behavior of the negative energy in front of the bubble never reaching the bubble walls(see pg 116 in [31]).

The negative energy density is graphically represented by the "brown small rings" in the sides of the figure and mainly by the "large grey ring".



Figure 9: Another Artistic Presentation of a warp drive spaceship in interstellar space colliding with highly Doppler-Blueshifted photons in the Alcubierre warp drive(Source:Internet)

16 Appendix L:Another Artistic Presentation of a warp drive spaceship in interstellar space colliding with highly Doppler-Blueshifted Photons in the Alcubierre warp drive

The picture above also borrowed from science-fiction also depicts the collisions with highly Doppler-Blueshifted photons.

Claek,Hiscock and Larson pg 11 in [9] mention how hazardous for the ship and the crew members these collisions can be.

At 200 times faster than light the energy of these cosmic background blueshifted photons is equivalent to the energy of a solar photosphere and think in a solar photosphere entering in the bubble threatening the ship integrity and the crew member lives.(pg 11 in [9])

Also Barcelo,Finazzi and Liberati pg 6 and 7 in [10] mentions the fact that these Doppler blueshifted photons will raise the temperature inside the bubble to the Hawking temperature generating a harsh environment that would instantaneously imply in the destruction of the ship and the death of the crew members.

The Alcubierre warp drive dont have negative energy density in front of the ship able to deflect these incoming blueshifted photons graphically represented by the "blue arrow" fig 2 pg 4 in Barcelo,Finazzi and Liberati in [10] or the Appendix H in this work:This concludes the fact that the Alcubierre warp drive is impossible for realistic interstellar space travel.

17 Remarks.

The Natario shape function $l(rs)$ that satisfies the Natario requirements for a warp drive spacetime with very low derivatives and hence very low negative energy density requirements able to complete obliterate the factor 10^{48} allowing a spaceship to travel with 200 times light speed is given by:

$$l(rs) = \frac{1}{2}[1 - (f(rs)^A)]^B \tag{116}$$

We discovered this function in 10 November 2026:In the original form A was equal to the modulus of the bubble radius R and B was equal to the modulus of the bubble thickness parameter @ however such a function did not reduced the negative energy density requirements in our numerical simulations model to a satisfactory level.

In 18 November 2026 we made a second attempt with new numerical simulations in our model making $A = B$ both equal to the modulus of the bubble diameter D.Then $A = B = D = 2R = WF$ and we got exactly the satisfactory results obtained for the negative energy density requirements by our previous work in [14].

$$N(rs) = [\frac{1}{2}][1 - f(rs)^{WF}]^{WF} \tag{117}$$

We can provide the numerical simulations of $l(rs)$ and $N(rs)$ for those interested.

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