TRUNCATION SYMMETRY THEOREM AND REFERENCE WEIGHT BASIS IN THE SCALED BERGMAN NUMERAL SYSTEM (SBNS)

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November 21, 2026

Abstract

This paper presents the algebraic formalization and proof of the Truncation Symmetry Theorem for the Scaled Bergman Numeral System (SBNS) – a positional computational basis with the irrational golden ratio base $\varphi = (1 + \sqrt{5}) / 2$ and a central scaling factor $k = 0.2$. By decomposing bit weights into a rational component involving Lucas numbers and an irrational component involving Fibonacci numbers, it is proven that applying a syntactic parity filter leads to the identical mutual cancellation of local decimal truncation errors for all natural numbers $N \in \mathbb{N}$. Strict invariance of the residual error during transitions between canonical dual code forms – Parity-Filter Working Code (PFWC / Code A) and Parity-Filter Check Code (PFCC / Code B) – is demonstrated. The paper provides a standardized formula for computing bit weights, reveals their mirror symmetry relative to the central scale element, and establishes a reference weight table for the index range $n \in [-16, 14]$, suitable for implementing exact integer arithmetic in finite hardware registers.

Keywords: Scaled Bergman Numeral System (SBNS); golden ratio arithmetic; Fibonacci and Lucas numbers; Truncation Symmetry Theorem; parity filter.

1 Introduction

Positional numeral systems with an irrational base equal to the golden ratio $\varphi = (1 + \sqrt{5}) / 2$, first proposed by George Bergman [1] and further developed in Fibonacci computer arithmetic by A. P. Stakhov [2, 3], possess natural code redundancy suitable for self-checking and fault-tolerant computing architectures. However, the practical hardware implementation of the classical Bergman basis (anchored at unity $\varphi^0 = 1$) has been hindered by two major obstacles: the wandering carry effect during arithmetic normalization and the infinite information tail arising when integers are represented via irrational bit weights.

To overcome these limitations, the author previously developed the New Fibonacci Numeral System (NFNS) [4, 5] and the Scaled Bergman Numeral System (SBNS) [6], in which the central element of the scale is shifted to $X_0 = k = 0.2 = 1/5$, and a syntactic parity filter is imposed on valid bit sequences. This paper presents a general algebraic proof of the Truncation Symmetry Theorem for an arbitrary natural number $N \in \mathbb{N}$ and establishes a standardized reference table of bit weights.

2 Formalization of SBNS and Parity Filtering

In the Scaled Bergman Numeral System, any real or natural value is represented as a binary bit vector $C = \{b_n\}$, where $b_n \in \{0, 1\}$, over a sequence of bit weights defined by the formula:

$$X_n = 0.2 \times \varphi^n = (1/5) \times ((1 + \sqrt{5}) / 2)^n, \quad n \in \mathbb{Z}$$

Since φ satisfies the characteristic quadratic equation $\varphi^2 = \varphi + 1$, the weight sequence preserves the additive Fibonacci recurrence property for all positive and negative indices: $X_n = X_{n-1} + X_{n-2}$. Without structural constraints, the same integer admits dozens or thousands of redundant representations depending on the register width.

To eliminate chaotic redundancy and ensure hardware error control, a code sequence C is defined as valid if and only if any two adjacent active "1" bits ($b_i = 1$ and $b_j = 1$ with no active bits between them) are separated exclusively by an even number of inactive "0" bits (including a zero-length gap, i.e., two consecutive "1"s). Equivalently, adjacent active bits in the bit grid must occupy positions of opposite parity. Under this filter, every representable integer N reduces to exactly two canonical dual states: the expanded Parity-Filter Working Code (PFWC / Code A) and the folded Parity-Filter Check Code (PFCC / Code B).

3 Algebraic Proof of the Truncation Symmetry Theorem

Theorem (On Truncation Symmetry and Exact Rational Collapse in SBNS). Let a natural number $N \in \mathbb{N}$ be represented in the SBNS system ($k = 0.2$) by a valid code sequence $C = \{b_n\}$ satisfying the parity invariance rule. If each analytical bit weight $X_n = 0.2 \times \varphi^n$ is rounded on a fixed decimal grid with a step $\delta = 2.5 \times 10^{-11}$ (retaining 11 decimal places), then the sum of the rounded weights $\tilde{X}_n$ of all active bits yields the exact integer N with zero residual truncation error:

$$\sum (\text{for } b_n = 1)\, \tilde{X}_n = N$$

Proof.

Decomposition of Bit Weights via Lucas and Fibonacci Numbers. According to Binet's identities, any integer power of the golden ratio φ^n ($n \in \mathbb{Z}$) is expressed as a linear combination of Lucas numbers L_n and Fibonacci numbers F_n, extended to negative indices by the rules $F_{-n} = (-1)^{n+1}F_n$ and $L_{-n} = (-1)^n L_n$:

$$\varphi^n = (L_n + F_n\sqrt{5}) / 2$$

Multiplying by the scaling factor $k = 0.2 = 1/5$ separates each exact analytical weight X_n into a finite rational part and an irrational component with the factor $\sqrt{5}$:

$$X_n = (1/5) \times \varphi^n = L_n / 10 + F_n \times (\sqrt{5} / 10)$$

Since $L_n \in \mathbb{Z}$ for all $n \in \mathbb{Z}$, the term $L_n / 10$ is always an exact terminating decimal with at most one digit after the decimal point and introduces no truncation error on any precision grid $m \geq 1$. All irrationality – and consequently the entire truncation error ε_n – is isolated exclusively within the second term $F_n \times (\sqrt{5} / 10)$.

Vanishing of the Irrational Coefficient. For any exact representation of a natural number N in SBNS, summation over the active bits $b_n = 1$ gives:

$$N = \sum X_n = (1/10) \sum L_n + (\sqrt{5} / 10) \sum F_n$$

Because N and $(1/10) \sum L_n$ are rational numbers while $\sqrt{5}$ is irrational, the linear independence of 1 and $\sqrt{5}$ over the field of rational numbers $\mathbb{Q}$ requires that the sum of Fibonacci coefficients across all active positions vanishes:

$$\sum (\text{for } b_n = 1)\, F_n = 0$$

The syntactic parity filter guarantees that active bits form balanced conjugate groups generated by local foldings $X_n = X_{n-1} + X_{n-2}$, which identically preserve the equality $F_n - F_{n-1} - F_{n-2} = 0$.

Mutual Compensation of Finite Decimal Rounding Errors. Denote the irrational constant $\alpha = \sqrt{5} / 10 \approx 0.22360679774997896964...$ Rounding α to 11 decimal places (grid step $\delta = 2.5 \times 10^{-11}$) yields the finite decimal constant:

$$\tilde{\alpha} = 0.22360679775 = \alpha + \Delta_a$$

where $\Delta_a \approx +2.103036 \times 10^{-12}$ is a single universal rounding constant shared across all bit positions. Each rounded weight $\tilde{X}_n$ stored in a hardware register takes the exact algebraic form:

$$\tilde{X}_n = L_n / 10 + F_n \times \tilde{\alpha} = X_n + F_n \times \Delta_a$$

Consequently, the local rounding error at bit position n is proportional to the Fibonacci number F_n: $\varepsilon_n = F_n \times \Delta_a$. Summing the rounded weights over all active bits of a valid code C gives:

$$\sum \tilde{X}_n = \sum X_n + \Delta_a \times \sum F_n = N + \Delta_a \times 0 = N$$

Invariance Between Canonical Dual Forms (PFWC and PFCC). The transition between working code "A" (PFWC) and check code "B" (PFCC) reduces to the substitution $X_n \leftrightarrow X_{n-1} + X_{n-2}$. Since the identity $F_n - F_{n-1} - F_{n-2} = 0$ holds for all $n \in \mathbb{Z}$, the transition error between dual states is identically zero:

$$\varepsilon_\text{trans} = \varepsilon_n - (\varepsilon_{n-1} + \varepsilon_{n-2}) = \Delta_a \times (F_n - F_{n-1} - F_{n-2}) = 0$$

Thus, both canonical code forms are invariant with respect to grid truncation to 11 decimal places. Q.E.D.

4 Standardized Reference Weight Basis and Mirror Symmetry

To ensure algebraic closure of the Truncation Symmetry Theorem across all arithmetic operations, every bit weight $\tilde{X}_n$ in the hardware lookup table must be computed uniformly using the formula $\tilde{X}_n = L_n / 10 + F_n \times 0.22360679775$. This reveals a mirror symmetry of weights relative to the central scale element $X_0 = 0.2$: for odd indices n, the weights $\tilde{X}_n$ and $\tilde{X}_{-n}$ have identical fractional tails (for example, $\tilde{X}_{11} = 39.80100499975$ and $\tilde{X}_{-11} = 0.00100499975$, yielding an exact rational difference $\tilde{X}_{11} - \tilde{X}_{-11} = 39.8 = L_{11}/5$), whereas for even indices n, their fractional tails form mirror pairs complementing to unity, whose sum equals $L_n/5$. Table 1 presents the standardized bit weights for indices from $n = -16$ to $n = 14$. Appendix A presents the SBNS codes for integers from 1 to 54 using bit weights from −12 to 11 (Tables A1–A7), and Appendix B provides the SBNS bit weights for a 64-bit register ($n \in [-31, 32]$) (Table B1).

Table 1 – Standardized SBNS bit weights derived via Lucas–Fibonacci decomposition ($n \in [-16, 14]$)

Index (n)	Lucas Number (L_n)	Fibonacci Number (F_n)	Standardized Weight $\tilde{X}_n = L_n/10 + F_n \times 0.22360679775$
-16	2207	-987	0.00009062075
-15	-1364	610	0.0001466275
-14	843	-377	0.00023724825
-13	-521	233	0.00038387575
-12	322	-144	0.000621124
-11	-199	89	0.00100499975
-10	123	-55	0.00162612375
-9	-76	34	0.0026311235
-8	47	-21	0.00425724725
-7	-29	13	0.00688837075
-6	18	-8	0.011145618
-5	-11	5	0.01803398875
-4	7	-3	0.02917960675
-3	-4	2	0.0472135955
-2	3	-1	0.07639320225
-1	-1	1	0.12360679775
0	2	0	0.2
1	1	1	0.32360679775
2	3	1	0.52360679775
3	4	2	0.8472135955
4	7	3	1.37082039325
5	11	5	2.21803398875
6	18	8	3.588854382
7	29	13	5.80688837075
8	47	21	9.39574275275
9	76	34	15.2026311235
10	123	55	24.59837387625
11	199	89	39.80100499975
12	322	144	64.399378876
13	521	233	104.20038387575
14	843	377	168.59976275175

5 Conclusion

1. The Truncation Symmetry Theorem has been proven for all natural numbers $N \in \mathbb{N}$. By decomposing bit weights into a finite rational component involving Lucas numbers ($L_n/10$) and an irrational component involving Fibonacci numbers ($F_n\sqrt{5}$ / 10), it is established that enforcing the even-gap rule between active bits forces the sum of Fibonacci indices to vanish identically.

2. The invariance of dual canonical forms (PFWC and PFCC) has been confirmed. It is shown that local bit folding and unfolding via the rule $X_n \leftrightarrow X_{n-1} + X_{n-2}$ introduces zero residual truncation error (ε_trans = 0), enabling dual coding to be used for hardware data self-checking without loss of precision.

3. A standardized reference weight basis has been constructed over the range $n \in [-16, 14]$. Uniform weight computation via the formula $\tilde{X}_n = L_n/10 + F_n \times 0.22360679775$ restores mirror symmetry of conjugate pairs relative to the central scale element $X_0 = 0.2$ and guarantees exact integer arithmetic in finite hardware registers without arbitrary-precision computation.

References

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APPENDICES

APPENDIX A

Table A1 – SBNS (Scaled Bergman Number System) codes for numbers 1–8, where:
GAR – Greedy Algorithm Representation, PFWC – Parity-Filter Working Code, PFCC – Parity-Filter Check Code

Number	SBNS code representation constraints	SBNS weights																
		8	7	6	5	4	3	2	1	0	–1	–2	–3	–4	–5	–6	–7	–8
		9,39574275275	5.80688837075	3.588854382	2.21803398875	1.37082039325	0.8472135955	0.52360679775	0.32360679775	0.2	0.12360679775	0.07639320225	0.0472135955	0.02917960675	0.01803398875	0.011145618	0.00688837075	0.00425724725
1	GAR						1				1			1				
	PFWC						1					1	1	1				
	PFCC							1	1			1	1	1				
2	GAR					1		1				1		1				
	PFWC					1			1	1			1	1	1	1		
	PFCC						1	1	1	1			1	1	1	1		
3	GAR				1			1		1			1			1		
	PFWC				1			1			1	1	1			1		
	PFCC					1	1	1			1	1	1			1		
4	GAR			1					1			1				1		
	PFWC			1					1			1					1	1
	PFCC				1	1			1			1					1	1
5	GAR			1		1								1		1		
	PFWC			1			1	1							1	1	1	1
	PFCC				1	1	1	1							1	1	1	1
6	GAR		1								1		1		1			1
	PFWC		1									1	1	1	1	1	1	1
	PFCC			1	1							1	1	1	1	1	1	1
7	GAR		1				1		1						1			1
	PFWC		1					1	1	1	1					1	1	1
	PFCC			1	1			1	1	1	1					1	1	1
8	GAR		1			1		1		1		1			1			1
	PFWC		1			1			1	1	1	1	1	1	1			1
	PFCC			1	1	1			1	1	1	1	1	1	1			1

Table A2 – SBNS (Scaled Bergman Number System) codes for numbers 9–15, where:
GAR – Greedy Algorithm Representation, PFWC – Parity-Filter Working Code, PFCC – Parity-Filter Check Code

Number	SBNS code representation constraints	SBNS weights																		
		8	7	6	5	4	3	2	1	0	–1	–2	–3	–4	–5	–6	–7	–8	–9	–10
		9.39574275275	5.80688837075	3.588854382	2.21803398875	1.37082039325	0.8472135955	0.52360679775	0.32360679775	0.2	0.12360679775	0.07639320225	0.0472135955	0.02917960675	0.01803398875	0.011145618	0.00688837075	0.00425724725	0.0026311235	0.00162612375
9	GAR		1		1		1				1							1		
	PFWC		1			1	1	1	1			1	1					1		
	PFCC			1	1	1	1	1	1			1	1					1		
10	GAR	1						1				1						1		
	PFWC	1							1	1			1	1					1	1
	PFCC		1	1					1	1			1	1					1	1
11	GAR	1				1				1				1				1		
	PFWC	1					1	1			1	1			1	1			1	1
	PFCC		1	1			1	1			1	1			1	1			1	1
12	GAR	1			1				1				1			1		1		
	PFWC	1			1					1	1			1	1	1			1	1
	PFCC		1	1		1	1			1	1			1	1	1			1	1
13	GAR	1		1												1		1		
	PFWC	1			1	1											1	1	1	1
	PFCC		1	1	1	1											1	1	1	1
14	GAR	1		1			1				1			1		1		1		
	PFWC	1			1	1	1					1	1	1			1	1	1	1
	PFCC		1	1	1	1	1					1	1	1			1	1	1	1
15	GAR	1		1		1		1				1		1		1		1		
	PFWC	1			1	1	1	1	1	1			1	1	1	1	1	1	1	1
	PFCC		1	1	1	1	1	1	1	1			1	1	1	1	1	1	1	1

Table A3 – SBNS (Scaled Bergman Number System) codes for numbers 16–22, where:
GAR – Greedy Algorithm Representation, PFWC – Parity-Filter Working Code, PFCC – Parity-Filter Check Code

Number	SBNS code representation constraints	SBNS weights																			
		10	9	8	7	6	5	4	3	2	1	0	–1	–2	–3	–4	–5	–6	–7	–8	–9	–10
		24.59837387625	15.2026311235	9.39574275275	5.80688837075	3.588854382	2.21803398875	1.37082039325	0.8472135955	0.52360679775	0.32360679775	0.2	0.12360679775	0.07639320225	0.0472135955	0.02917960675	0.01803398875	0.011145618	0.00688837075	0.00425724725	0.0026311235	0.00162612375
16	GAR		1							1		1			1		1		1			1
	PFWC		1							1			1	1	1			1	1	1	1	1
	PFCC			1	1					1			1	1	1			1	1	1	1	1
17	GAR		1					1			1			1			1		1			1
	PFWC		1					1			1			1			1			1	1	1
	PFCC			1	1			1			1			1			1			1	1	1
18	GAR		1				1			1					1				1			1
	PFWC		1					1	1	1					1					1	1	1
	PFCC			1	1			1	1	1					1					1	1	1
19	GAR		1			1						1							1			1
	PFWC		1			1							1	1					1			1
	PFCC			1	1	1							1	1					1			1
20	GAR		1		1				1		1					1			1			1
	PFWC		1			1	1			1	1	1	1			1			1			1
	PFCC			1	1	1	1			1	1	1	1			1			1			1
21	GAR		1			1		1		1		1		1		1			1			1
	PFWC		1			1			1	1	1	1	1	1	1	1	1	1	1			1
	PFCC			1	1	1			1	1	1	1	1	1	1	1	1	1	1			1
22	GAR		1		1				1				1				1					1
	PFWC		1			1	1			1	1			1	1			1	1			1
	PFCC			1	1	1	1			1	1			1	1			1	1			1

Table A4 – SBNS (Scaled Bergman Number System) codes for numbers 23–30, where:
GAR – Greedy Algorithm Representation, PFWC – Parity-Filter Working Code, PFCC – Parity-Filter Check Code

Number	SBNS code representation constraints	SBNS weights																					
		10	9	8	7	6	5	4	3	2	1	0	–1	–2	–3	–4	–5	–6	–7	–8	–9	–10	–11	–12
		24.59837387625	15.2026311235	9.39574275275	5.80688837075	3.588854382	2.21803398875	1.37082039325	0.8472135955	0.52360679775	0.32360679775	0.2	0.12360679775	0.07639320225	0.0472135955	0.02917960675	0.01803398875	0.011145618	0.00688837075	0.00425724725	0.0026311235	0.00162612375	0.00100499975	0.000621124
23	GAR		1		1			1		1				1			1					1		
	PFWC		1			1	1	1			1	1			1	1	1					1		
	PFCC			1	1	1	1	1			1	1			1	1	1					1		
24	GAR		1		1		1			1		1			1							1		
	PFWC		1			1	1	1	1	1			1	1	1							1		
	PFCC			1	1	1	1	1	1	1			1	1	1							1		
25	GAR	1									1			1								1		
	PFWC	1									1			1									1	1
	PFCC		1	1							1			1									1	1
26	GAR	1						1								1						1		
	PFWC	1							1	1							1	1					1	1
	PFCC		1	1					1	1							1	1					1	1
27	GAR	1					1						1		1			1				1		
	PFWC	1					1							1	1	1	1	1					1	1
	PFCC		1	1			1							1	1	1	1	1					1	1
28	GAR	1					1		1		1							1				1		
	PFWC	1					1			1	1	1	1						1	1			1	1
	PFCC		1	1			1			1	1	1	1						1	1			1	1
29	GAR	1				1				1		1		1				1				1		
	PFWC	1					1	1			1	1	1	1	1	1			1	1			1	1
	PFCC		1	1			1	1			1	1	1	1	1	1			1	1			1	1
30	GAR	1				1		1			1			1		1		1				1		
	PFWC	1					1	1	1	1	1			1			1	1	1	1			1	1
	PFCC		1	1			1	1	1	1	1			1			1	1	1	1			1	1

Table A5 – SBNS (Scaled Bergman Number System) codes for numbers 31–38, where:
GAR – Greedy Algorithm Representation, PFWC – Parity-Filter Working Code, PFCC – Parity-Filter Check Code

Number	SBNS code representation constraints	SBNS weights																					
		10	9	8	7	6	5	4	3	2	1	0	–1	–2	–3	–4	–5	–6	–7	–8	–9	–10	–11	–12
		24.59837387625	15.2026311235	9.39574275275	5.80688837075	3.588854382	2.21803398875	1.37082039325	0.8472135955	0.52360679775	0.32360679775	0.2	0.12360679775	0.07639320225	0.0472135955	0.02917960675	0.01803398875	0.011145618	0.00688837075	0.00425724725	0.0026311235	0.00162612375	0.00100499975	0.000621124
31	GAR	1			1					1					1		1			1		1		
	PFWC	1			1					1					1			1	1	1			1	1
	PFCC		1	1	1					1					1			1	1	1			1	1
32	GAR	1			1			1				1					1			1		1		
	PFWC	1			1			1					1	1				1	1	1			1	1
	PFCC		1	1	1			1					1	1				1	1	1			1	1
33	GAR	1			1		1				1				1					1		1		
	PFWC	1			1			1	1			1	1			1	1			1			1	1
	PFCC		1	1	1			1	1			1	1			1	1			1			1	1
34	GAR	1		1																1		1		
	PFWC	1			1	1															1	1	1	1
	PFCC		1	1	1	1															1	1	1	1
35	GAR	1		1					1				1			1				1		1		
	PFWC	1			1	1			1					1	1	1					1	1	1	1
	PFCC		1	1	1	1			1					1	1	1					1	1	1	1
36	GAR	1		1				1		1				1		1				1		1		
	PFWC	1			1	1			1	1	1	1			1	1	1	1			1	1	1	1
	PFCC		1	1	1	1			1	1	1	1			1	1	1	1			1	1	1	1
37	GAR	1		1			1			1		1			1			1		1		1		
	PFWC	1			1	1	1			1			1	1	1			1			1	1	1	1
	PFCC		1	1	1	1	1			1			1	1	1			1			1	1	1	1
38	GAR	1		1		1					1			1				1		1		1		
	PFWC	1			1	1	1	1			1			1					1	1	1	1	1	1
	PFCC		1	1	1	1	1	1			1			1					1	1	1	1	1	1

Table A6 – SBNS (Scaled Bergman Number System) codes for numbers 39–46, where:
GAR – Greedy Algorithm Representation, PFWC – Parity-Filter Working Code, PFCC – Parity-Filter Check Code

Number	SBNS code representation constraints	SBNS weights																							
		11	10	9	8	7	6	5	4	3	2	1	0	–1	–2	–3	–4	–5	–6	–7	–8	–9	–10	–11	–12
		39.80100499975	24.59837387625	15.2026311235	9.39574275275	5.80688837075	3.588854382	2.21803398875	1.37082039325	0.8472135955	0.52360679775	0.32360679775	0.2	0.12360679775	0.07639320225	0.0472135955	0.02917960675	0.01803398875	0.011145618	0.00688837075	0.00425724725	0.0026311235	0.00162612375	0.00100499975	0.000621124
39	GAR		1		1		1		1								1		1		1		1		
	PFWC		1			1	1	1	1	1	1							1	1	1	1	1	1	1	1
	PFCC			1	1	1	1	1	1	1	1							1	1	1	1	1	1	1	1
40	GAR	1												1		1		1		1		1			1
	PFWC	1													1	1	1	1	1	1	1	1	1	1	1
	PFCC		1	1											1	1	1	1	1	1	1	1	1	1	1
41	GAR	1								1		1						1		1		1			1
	PFWC	1									1	1	1	1					1	1	1	1	1	1	1
	PFCC		1	1							1	1	1	1					1	1	1	1	1	1	1
42	GAR	1							1		1		1		1			1		1		1			1
	PFWC	1							1			1	1	1	1	1	1	1			1	1	1	1	1
	PFCC		1	1					1			1	1	1	1	1	1	1			1	1	1	1	1
43	GAR	1						1		1				1						1		1			1
	PFWC	1							1	1	1	1			1	1					1	1	1	1	1
	PFCC		1	1					1	1	1	1			1	1					1	1	1	1	1
44	GAR	1					1				1				1					1		1			1
	PFWC	1					1					1	1			1	1				1	1	1	1	1
	PFCC		1	1			1					1	1			1	1				1	1	1	1	1
45	GAR	1					1		1				1				1			1		1			1
	PFWC	1					1			1	1			1	1			1	1	1			1	1	1
	PFCC		1	1			1			1	1			1	1			1	1	1			1	1	1
46	GAR	1				1						1				1		1				1			1
	PFWC	1					1	1					1	1			1	1	1	1			1	1	1
	PFCC		1	1			1	1					1	1			1	1	1	1			1	1	1

Table A7 – SBNS (Scaled Bergman Number System) codes for numbers 47–54, where:
GAR – Greedy Algorithm Representation, PFWC – Parity-Filter Working Code, PFCC – Parity-Filter Check Code

Number	SBNS code representation constraints	SBNS weights																						
		11	10	9	8	7	6	5	4	3	2	1	0	–1	–2	–3	–4	–5	–6	–7	–8	–9	–10	–11	–12
		39.80100499975	24.59837387625	15.2026311235	9.39574275275	5.80688837075	3.588854382	2.21803398875	1.37082039325	0.8472135955	0.52360679775	0.32360679775	0.2	0.12360679775	0.07639320225	0.0472135955	0.02917960675	0.01803398875	0.011145618	0.00688837075	0.00425724725	0.0026311235	0.00162612375	0.00100499975	0.000621124
47	GAR	1				1			1									1				1			1
	PFWC	1					1	1	1									1					1	1	1
	PFCC		1	1			1	1	1									1					1	1	1
48	GAR	1				1		1						1		1						1			1
	PFWC	1					1	1	1	1					1	1	1	1					1	1	1
	PFCC		1	1			1	1	1	1					1	1	1	1					1	1	1
49	GAR	1				1		1		1		1										1			1
	PFWC	1					1	1	1	1	1	1	1	1									1	1	1
	PFCC		1	1			1	1	1	1	1	1	1	1									1	1	1
50	GAR	1			1						1		1		1							1			1
	PFWC	1			1							1	1	1	1	1	1					1			1
	PFCC		1	1	1							1	1	1	1	1	1					1			1
51	GAR	1			1				1			1			1		1					1			1
	PFWC	1			1					1	1	1			1			1	1			1			1
	PFCC		1	1	1					1	1	1			1			1	1			1			1
52	GAR	1			1			1			1					1			1			1			1
	PFWC	1			1			1			1					1			1			1			1
	PFCC		1	1	1			1			1					1			1			1			1
53	GAR	1			1		1						1						1			1			1
	PFWC	1			1			1	1					1	1					1	1	1			1
	PFCC		1	1	1			1	1					1	1					1	1	1			1
54	GAR	1			1		1			1		1					1		1			1			1
	PFWC	1			1			1	1	1			1	1			1			1	1	1			1
	PFCC		1	1	1			1	1	1			1	1			1			1	1	1			1

Table B1 – SBNS bit weights for a 64-bit register ($n \in [-31, 32]$)

Index (n)	Lucas Number (L_n)	Fibonacci Number (F_n)	Standardized Weight $\tilde{X}_n$	Scaled Integer for ROM ($\tilde{X}_n \times 10^{11}$)
-31	-3010349	1346269	0.00000009475	9475
-30	1860498	-832040	0.00000009	9000
-29	-1149851	514229	0.00000018475	18475
-28	710647	-317811	0.00000027475	27475
-27	-439204	196418	0.0000004595	45950
-26	271443	-121393	0.00000073425	73425
-25	-167761	75025	0.00000119375	119375
-24	103682	-46368	0.000001928	192800
-23	-64079	28657	0.00000312175	312175
-22	39603	-17711	0.00000504975	504975
-21	-24476	10946	0.0000081715	817150
-20	15127	-6765	0.00001322125	1322125
-19	-9349	4181	0.00002139275	2139275
-18	5778	-2584	0.000034614	3461400
-17	-3571	1597	0.00005600675	5600675
-16	2207	-987	0.00009062075	9062075
-15	-1364	610	0.0001466275	14662750
-14	843	-377	0.00023724825	23724825
-13	-521	233	0.00038387575	38387575
-12	322	-144	0.000621124	62112400
-11	-199	89	0.00100499975	100499975
-10	123	-55	0.00162612375	162612375
-9	-76	34	0.0026311235	263112350
-8	47	-21	0.00425724725	425724725
-7	-29	13	0.00688837075	688837075
-6	18	-8	0.011145618	1114561800
-5	-11	5	0.01803398875	1803398875
-4	7	-3	0.02917960675	2917960675
-3	-4	2	0.0472135955	4721359550
-2	3	-1	0.07639320225	7639320225
-1	-1	1	0.12360679775	12360679775
0	2	0	0.2	20000000000

Index (n)	Lucas Number (L_n)	Fibonacci Number (F_n)	Standardized Weight $\tilde{X}_n$	Scaled Integer for ROM ($\tilde{X}_n \times 10^{11}$)
1	1	1	0.32360679775	32360679775
2	3	1	0.52360679775	52360679775
3	4	2	0.8472135955	84721359550
4	7	3	1.37082039325	137082039325
5	11	5	2.21803398875	221803398875
6	18	8	3.588854382	358885438200
7	29	13	5.80688837075	580688837075
8	47	21	9.39574275275	939574275275
9	76	34	15.2026311235	1520263112350
10	123	55	24.59837387625	2459837387625
11	199	89	39.80100499975	3980100499975
12	322	144	64.399378876	6439937887600
13	521	233	104.20038387575	10420038387575
14	843	377	168.59976275175	16859976275175
15	1364	610	272.8001466275	27280014662750
16	2207	987	441.39990937925	44139990937925
17	3571	1597	714.20005600675	71420005600675
18	5778	2584	1155.599965386	115559996538600
19	9349	4181	1869.80002139275	186980002139275
20	15127	6765	3025.39998677875	302539998677875
21	24476	10946	4895.2000081715	489520000817150
22	39603	17711	7920.59999495025	792059999495025
23	64079	28657	12815.80000312175	1281580000312175
24	103682	46368	20736.399998072	2073639999807200
25	167761	75025	33552.20000119375	3355220000119375
26	271443	121393	54288.59999926575	5428859999926575
27	439204	196418	87840.8000004595	8784080000045950
28	710647	317811	142129.39999972525	14212939999972525
29	1149851	514229	229970.20000018475	22997020000018475
30	1860498	832040	372099.59999991	37209959999991000
31	3010349	1346269	602069.80000009475	60206980000009475
32	4870847	2178309	974169.40000000475	97416940000000475