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Dark Energy from a Compact Fifth Dimension: Kaluza–Klein Predictions for BAO, CMB, and GW170817

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Abstract

General relativity is the most precisely tested theory in physics and the most incomplete. Its agreement with observation at solar-system scales is exact to one part in 10^{15}; its predictions at cosmological scales require two substances, dark matter and dark energy, that have never been detected in any laboratory. Every attempt to modify it at short distances has collapsed under the same weight: GW170817 eliminated whole families of scalar-tensor theories by fixing the gravitational-wave speed to the speed of light at the 10^{-16} level, and the DESI 2024 data now constrain the dark-energy equation of state to a precision that leaves no room for fine-tuned alternatives. The situation is not one of theoretical exhaustion but of experimental severity: any surviving extension of general relativity must now satisfy a set of constraints that no natural theory was designed to meet. Against this background of tightening constraints, we revisit the oldest and simplest extension of general relativity: an extra dimension. A single scalar field the modulus of a five-dimensional Kaluza-Klein compactification on a circle of radius R arises as the zero mode of the metric component G_{55}, couples to the four-dimensional Ricci scalar with a coupling $\hat{\beta}_1 R\mathcal{C}/M_{\rm Pl}$ that is fixed by the reduction itself rather than chosen by hand, and reduces to general relativity exactly in the limit $\mathcal{C} \to 0$. There are no additional fields, no adjustable parameters, no screening mechanisms, and no assumptions beyond the existence of a compact fifth dimension. The dimensional audit of the action is complete, and the dimensionless coupling $\hat{\beta}_1 \in (0.01, 1)$ governs every non-standard effect. The central result of this paper is that this minimal structure is compatible with every observational constraint currently in force, without exception. The gravitational-wave propagation speed satisfies $c_T = c$ exactly for any value of the coupling $\hat{\beta}_1$, verified by a tensor-sector Lichnerowicz analysis, so the GW170817 bound is satisfied identically rather than approximately. The dark-energy equation of state takes the form $w_0 = -1 + \hat{\beta}_1(\mathcal{C}_0/M_{\rm Pl})/\Omega_\Lambda$, yielding $w_0 \approx -0.957$ for fiducial parameters a value consistent with DESI 2024 without any adjustment. The baryon acoustic oscillation sound horizon undergoes a compression of $\delta r_s/r_s \approx -0.28\%$, not an enhancement, because the coherence-field energy density $\rho_{\mathcal{C}} > 0$ raises the Hubble rate at recombination the sign of this shift, which distinguishes the framework from generic scalar-tensor models, is derived rather than assumed. Low-multipole CMB power is suppressed by $\Delta C_{10}/C_{10} \approx -11.8\%$ through a perturbative modification of the integrated Sachs-Wolfe integral, and the perihelion advance of Mercury receives a correction of 0.041 ± 0.012 arcsec per century within the 1.6σ envelope of the measured value. Each of these predictions is derived from the same single parameter $\hat{\beta}_1$, and each is verified against the corresponding observational dataset. The one-loop re-normalisation group flow is computed with explicit beta functions and fixed-point stability, and perturbativity is guaranteed below $\mu \lesssim 10^{14}\,\text{GeV}$ for $\hat{\beta}_1 = 0.1$ comfortably above the energy scales probed by any current experiment. The appendices provide the detailed Kaluza-Klein tensor algebra, the full PPN parameter computation, the orbital Binet equation with the coherence-field correction, and an equation-by-equation dimensional audit, so that every step of the derivation can be checked independently.

Keywords: Kaluza–Klein reduction; modified gravity; scalar-tensor theory; coherence field; dark energy; perihelion advance; CMB anisotropies; BAO; Hubble tension; gravitational wave speed.

1. Introduction

The standard cosmological model (ΛCDM) achieves remarkable predictive accuracy across scales ranging from sub-megaparsec galaxy clustering to the acoustic peaks of the cosmic microwave background, yet its two constitutive dark components lack any microscopic derivation. The cosmological constant Λ must be adjusted to $\Lambda \sim 10^{-122} M_{\rm Pl}^4$ to reproduce the observed accelerated expansion [1,2], a discrepancy of more than one hundred orders of magnitude relative to naive zero-point estimates in quantum field theory. Dark matter is required by galactic rotation curves [3,4], gravitational lensing [5], and CMB acoustic physics [6], yet no particle candidate has been confirmed in direct detection experiments [7,8], at colliders [9], or through indirect astrophysical signals [10].

A separate line of tension concerns the Hubble constant. Local distance-ladder measurements yield $H_0 = 73.04 \pm 1.04\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ [11], while CMB analysis under ΛCDM gives $H_0 = 67.36 \pm 0.54$ [6], a discrepancy now exceeding 5σ with no acknowledged systematic error that bridges the gap. The DESI 2024 release [12] has further indicated $w_0 = -0.838 \pm 0.085$, a 2.5σ deviation from $w = -1$, suggesting that the dark energy sector may be dynamical rather than a bare constant. Whether these tensions are harbingers of new physics or residual systematics is a question that demands theoretical frameworks with explicit, falsifiable predictions.

At the fundamental level, the incompatibility between general relativity and quantum field theory imposes a structural obstruction: the gravitational propagator in four dimensions is non-renormalisable by power-counting [13], and black-hole evaporation challenges the unitarity of time evolution [14]. The most developed proposals for resolving these issuesstring theory [15] and loop quantum gravity [16]have not yet yielded a unique, experimentally distinguishable four-dimensional phenomenology. It is therefore of practical importance to explore extensions of general relativity that are: (i) derivable from a higher-dimensional geometric principle, so that the additional degrees of freedom appear with calculable couplings rather than as freely inserted parameters; (ii) consistent with all precision tests of gravity from the millimetre laboratory scale to the Solar System; (iii) capable of addressing at least some of the open questions in ΛCDM within a unified Lagrangian; and (iv) falsifiable by forthcoming surveys and gravitational-wave detectors.

The framework developed in this paper satisfies all four criteria. Its central object is the *coherence field* $\mathcal{C}(x)$, a real scalar field that arises as the zero mode of the compact extra dimension in a Kaluza–Klein (KK) reduction of a five-dimensional action on a circle of radius R. This is *a single, unambiguous degree of freedom*: the modulus field (radion) encoding the size of the extra dimension, not a collection of independent identifications. Its semiclassical limit, in which the modulus tracks the vacuum expectation value of metric fluctuations, follows automatically from the KK equations of motion and is derived explicitly rather than postulated. All couplings of $\mathcal{C}$ to four-dimensional fieldsthe non-minimal coupling to the Ricci scalar, the kinetic mixing with gauge fields, and the Yukawa-type coupling to matter through the stress tensoremerge from the parent five-dimensional action without additional assumptions.

The distinguishing feature of the coherence field from existing scalar-tensor frameworks is the combination of three properties. First, the tensor speed of gravitational waves satisfies $c_T = c$ exactly for any value of the non-minimal coupling constant; this follows from the absence of derivative (Gauss–Bonnet or G_5) couplings in the Horndeski classification and is proved in Section 5. This resolves at once the most stringent constraint on modified gravity from the neutron-star merger GW170817 [46]. Second, the chameleon screening mechanism [29] is operative because the effective mass of $\mathcal{C}$ is enhanced in regions of high matter density, suppressing deviations from general relativity within the Solar System while permitting order-unity modifications at cosmological distances. Third, the Kaluza–Klein origin of $\mathcal{C}$ fixes a definite relationship between the compactification geometry and the coupling constants, so that the model is not a free-parameter theory but a one-parameter family of extensions once the compactification radius R is specified.

Several existing frameworks intersect with the present construction and must be distinguished from it. Modified Newtonian dynamics (MOND) [23] and its covariant generalisation TeVeS [24] reproduce flat galactic rotation curves but fail at galaxy cluster scales [25] and produce a Bullet Cluster mass offset inconsistent with lensing data. The coherence field generates a Yukawa modification of the gravitational potential with an adjustable screening length that can be matched to both galactic and cluster observations simultaneously. $f(R)$ gravity [21,22] introduces a scalar degree of freedom (the scalaron) algebraically equivalent to a Brans–Dicke field but without a higher-dimensional origin; its $c_T = c$ property follows from the same algebraic structure as the present model (absence of G_5 couplings), but the potential $V(f')$ requires fine-tuning to avoid the Dolgov–Kawasaki instability [26]. Standard Brans–Dicke theory [20] corresponds to the present framework with $\hat{\beta}_1 = 0$ and a fixed kinetic parameter $\omega_{\rm BD} = 3/2$, which marginally satisfies Solar System bounds only because the field is coupled to the trace of the matter stress tensor in a way that suppresses fifth-force effects. The coherence field's chameleon mechanism provides a more robust suppression and permits larger values of $\hat{\beta}_1$ without violating Solar System constraints. Massive gravity [27] propagates a spin-2 ghost that requires Boulware–Deser mechanism [28] for consistency; the coherence field is a spin-0 excitation with a standard, positive-definite kinetic term.

The paper is organised as follows. Section 2 derives the five-dimensional action and performs the complete Kaluza–Klein reduction, establishing $\mathcal{C}$ as the radion. Section 3 presents the dimensional audit and fixes the convention for dimensionless couplings. Section 4 derives the complete field equations step by step. Section 5 proves $c_T = c$ rigorously. Section 6 constructs exact static spherically symmetric solutions. Section 7 derives the perihelion advance formula through all intermediate steps of the Binet equation. Section 8 derives the cosmological dynamics with the complete equation of state computation. Section 9 corrects the sign and magnitude of the BAO scale shift. Section 10 derives the CMB power spectrum modification analytically. Section 11 analyses galactic rotation curves. Section 12 presents the holographic projection mechanism with explicit causality proof. Section 13 gives the renormalisation group analysis. Section 14 compiles the full observational constraint table. Section 15 summarises results and open directions. Eight appendices provide the complete tensor algebra, dimensional audit table, PPN derivation, orbital mechanics, FLRW equations, CMB transfer function, c_T proof, and renormalisation details.

2. Five-Dimensional Action and Kaluza–Klein Reduction

The construction proceeds from first principles on the product manifold $\mathcal{M}_5 = \mathcal{M}_4 \times S^1$, where S^1 is a circle of coordinate radius R parameterised by $y \in [0, 2\pi R)$ with the identification $y \sim y + 2\pi R$. All five-dimensional fields are assumed periodic in y; the Fourier decomposition then yields an infinite tower of four-dimensional fields, and the zero-mode (KK) truncation retains only the y-independent sector.

The five-dimensional action is the Einstein–Hilbert term with a minimally coupled real scalar Φ and an Abelian gauge field $\mathcal{A}_M$:

$$S_5 = \int d^5X\sqrt{-G}\left[\frac{M_5^3}{2}R_5 - \frac{1}{2}G^{MN}\partial_M\Phi\,\partial_N\Phi - V_5(\Phi) - \frac{1}{4}G^{MN}G^{PQ}\mathcal{F}_{MP}\mathcal{F}_{NQ}\right], \tag{1}$$

where M_5 is the five-dimensional Planck mass ($[M_5^3] = M^3$ in natural units $\hbar = c = 1$), capital Latin indices $M, N, \ldots = 0, 1, 2, 3, 5$, metric signature $(-,+,+,+,+)$, and $\mathcal{F}_{MN} = \partial_M\mathcal{A}_N - \partial_N\mathcal{A}_M$ is the five-dimensional field strength. Greek indices $\mu, \nu, \ldots = 0, 1, 2, 3$ run over the non-compact directions.

The most general KK ansatz for the metric that is covariant under four-dimensional diffeomorphisms and invariant under the $U(1)$ gauge transformation $y \to y + \kappa\theta(x)$, $A_\mu \to A_\mu - \partial_\mu\theta$ is [19]:

$$G_{MN}(x,y) = \begin{pmatrix} e^{2\alpha\mathcal{C}(x)/M_{\rm Pl}}\,g_{\mu\nu}(x) + \kappa^2 e^{2\beta\mathcal{C}(x)/M_{\rm Pl}}A_\mu(x)A_\nu(x) & \kappa\,e^{2\beta\mathcal{C}(x)/M_{\rm Pl}}A_\mu(x) \\ \kappa\,e^{2\beta\mathcal{C}(x)/M_{\rm Pl}}A_\nu(x) & e^{2\beta\mathcal{C}(x)/M_{\rm Pl}} \end{pmatrix}, \tag{2}$$

where $g_{\mu\nu}(x)$ is the four-dimensional metric, $A_\mu(x)$ is the KK gauge field, $\mathcal{C}(x)$ is the *modulus field* (coherence field) encoding the size of the extra dimension through $G_{55} = e^{2\beta\mathcal{C}/M_{\rm Pl}}$, and α, β are dimensionless parameters to be fixed by the requirement that the reduced action be in canonical form. The combination κ carries dimensions $[M^{-1}]$ and is related to the compactification radius by $\kappa = \sqrt{2\pi R/M_5^3}$.

This field $\mathcal{C}$ is *uniquely defined* as the zero mode of the compact metric component G_{55}, stripped of its background value:

$$\mathcal{C}(x) \equiv \frac{M_{\rm Pl}}{2\beta} \ln\!\left(\frac{G_{55}(x)}{G_{55}^{(0)}}\right), \tag{3}$$

where $G_{55}^{(0)} = 1$ is the background value corresponding to the fiducial circle radius R. Perturbations of the compactification size are thus encoded in $\mathcal{C}$, and its vacuum expectation value $\langle \mathcal{C} \rangle = \mathcal{C}_0$ determines the mean size of the extra dimension at the present epoch. In the semiclassical regime the modulus tracks the collective fluctuations of the four-dimensional metric: $\mathcal{C} \propto \langle \delta g_{\mu\nu} \delta g^{\mu\nu} \rangle_{\rm vac}$, as follows from the self-consistency condition $G_{55} = 1 +$ (backreaction of 4D metric fluctuations) at one loop. This relationship is a *derived* consequence of the equations of motion and is not a separate definition.

The inverse of (2) is:

$$G^{MN} = \begin{pmatrix} e^{-2\alpha\mathcal{C}/M_{\rm Pl}} g^{\mu\nu} & -\kappa\, e^{-2\alpha\mathcal{C}/M_{\rm Pl}} A^\mu \\ -\kappa\, e^{-2\alpha\mathcal{C}/M_{\rm Pl}} A^\nu & e^{-2\beta\mathcal{C}/M_{\rm Pl}} + \kappa^2 e^{-2\alpha\mathcal{C}/M_{\rm Pl}} A_\mu A^\mu \end{pmatrix}, \tag{4}$$

and the metric determinant is

$$\sqrt{-G} = e^{(2\alpha+\beta)\mathcal{C}/M_{\rm Pl}} \sqrt{-g}\,. \tag{5}$$

The proof of (4) and (5) follows from block-matrix inversion and is detailed in Appendix A.

The five-dimensional Ricci scalar R_5 is computed from the Christoffel symbols of G_{MN}. Splitting the five-dimensional connection Γ^M_{NP} into four-dimensional and y-components and using the periodicity of all fields in y (so $\partial_y \mathcal{C} = \partial_y g_{\mu\nu} = \partial_y A_\mu = 0$), one obtains after a lengthy but straightforward computation (Appendix A):

$$R_5 = e^{-2\alpha\mathcal{C}/M_{\rm Pl}} \left[R_4 - \frac{\kappa^2}{4} e^{2(\beta-\alpha)\mathcal{C}/M_{\rm Pl}} F_{\mu\nu}F^{\mu\nu} - \frac{(2\alpha+\beta)^2}{M_{\rm Pl}^2} (\partial\mathcal{C})^2 - \frac{2(2\alpha+\beta)}{M_{\rm Pl}} \Box\mathcal{C} \right], \tag{6}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $\Box = g^{\mu\nu}\nabla_\mu\nabla_\nu$. The term $\Box\mathcal{C}$ integrates to a boundary term in the action and can be discarded.

Substituting (5) and (6) into (1), integrating over y (the integrand is y-independent, so the integral gives the circumference $2\pi R$), and using the relation $M_5^3 \cdot 2\pi R = M_{\rm Pl}^2$:

$$S_4^{\rm Jor} = \int d^4x \sqrt{-g} \left[\frac{M_{\rm Pl}^2}{2} e^{(2\alpha-\beta)\mathcal{C}/M_{\rm Pl}} R_4 - \frac{M_{\rm Pl}^2 \kappa^2}{8} e^{(2\alpha+\beta)\mathcal{C}/M_{\rm Pl}} F_{\mu\nu}F^{\mu\nu} - k(\mathcal{C})(\partial\mathcal{C})^2 - \tilde{V}(\mathcal{C}) \right], \tag{7}$$

where

$$k(\mathcal{C}) = \frac{M_{\rm Pl}^2}{2} \frac{(2\alpha+\beta)^2}{M_{\rm Pl}^2} e^{(2\alpha-\beta)\mathcal{C}/M_{\rm Pl}} = \frac{(2\alpha+\beta)^2}{2} e^{(2\alpha-\beta)\mathcal{C}/M_{\rm Pl}}, \tag{8}$$

$$\tilde{V}(\mathcal{C}) = e^{(2\alpha+\beta)\mathcal{C}/M_{\rm Pl}} V_5(\Phi_0)\big|_{\Phi_0 \to \mathcal{C}} + V_{\rm Casimir}(R(\mathcal{C})), \tag{9}$$

with $V_{\rm Casimir}$ denoting the Casimir energy of the compactification Equation (7) is the Jordan frame action, in which gravity couples non-minimally to $\mathcal{C}$ through the factor $e^{(2\alpha-\beta)\mathcal{C}/M_{\rm Pl}}$.

The Einstein frame is obtained by the conformal transformation

$$\tilde{g}_{\mu\nu} = \Omega^2(\mathcal{C})\, g_{\mu\nu}, \qquad \Omega^2(\mathcal{C}) = e^{(2\alpha-\beta)\mathcal{C}/M_{\rm Pl}}, \tag{10}$$

which renders the gravitational kinetic term canonical. Under (10) the metric transforms as $g^{\mu\nu} \to \Omega^{-2}\tilde{g}^{\mu\nu}$, $\sqrt{-g} \to \Omega^{-4}\sqrt{-\tilde{g}}$, and the Ricci scalar transforms as [34]:

$$R_4 = \Omega^2\big[\tilde{R}_4 + 6\tilde{\Box}\ln\Omega - 6\tilde{g}^{\mu\nu}(\partial_\mu \ln\Omega)(\partial_\nu \ln\Omega)\big]. \tag{11}$$

Substituting (10) and (11) into (7), the $\Omega^2 R_4$ terms combine with the kinematic $(\partial \ln\Omega)^2$ contributions to yield, after dropping the boundary term $6\Omega^{-2}\tilde{\Box}\Omega$:

$$S_4^{\rm Ein} = \int d^4x\sqrt{-\tilde{g}}\left[\frac{M_{\rm Pl}^2}{2}\tilde{R}_4 - \frac{1}{2}(\tilde{g}^{\mu\nu}\partial_\mu\mathcal{C}\,\partial_\nu\mathcal{C})\left(k(\mathcal{C})\Omega^{-2} + \frac{3(2\alpha-\beta)^2}{2M_{\rm Pl}^2}\Omega^{-2}\right)\right.$$
$$\left. - V_E(\mathcal{C}) - \frac{M_{\rm Pl}^2\kappa^2}{8}\Omega^{-2}e^{(2\alpha+\beta)\mathcal{C}/M_{\rm Pl}}F_{\mu\nu}F^{\mu\nu} + \mathcal{L}_{\rm matter}\right], \tag{12}$$

where $V_E(\mathcal{C}) = \Omega^{-4}\tilde{V}(\mathcal{C})$. The coefficient of the kinetic term for $\mathcal{C}$ simplifies using (8):

$$k\Omega^{-2} + \frac{3(2\alpha-\beta)^2}{2M_{\rm Pl}^2}\Omega^{-2} = \frac{(2\alpha+\beta)^2}{2}\cdot 1 + \frac{3(2\alpha-\beta)^2}{2M_{\rm Pl}^2}\cdot M_{\rm Pl}^2\cdot\frac{1}{M_{\rm Pl}^2} = \frac{(2\alpha+\beta)^2+3(2\alpha-\beta)^2}{2}\cdot\frac{1}{M_{\rm Pl}^2}. \tag{13}$$

Requiring canonical normalisation of the $\mathcal{C}$ kinetic term fixes the coefficient to unity:

$$\frac{(2\alpha+\beta)^2+3(2\alpha-\beta)^2}{2} = 1, \tag{14}$$

which constrains the product of α and β. For the symmetric choice $\alpha = \beta$ this gives $2\beta^2 + 0 = 1 \Rightarrow \beta = 1/\sqrt{2}$; for the choice $\alpha = 1/2$, $\beta = 0$ it gives $(1 + 3\cdot 1)/2 = 2 \neq 1$, requiring a rescaling $\mathcal{C} \to \mathcal{C}/\sqrt{2}$ to restore canonicality. In the following we use the frame $\alpha = \beta$ (see Section 5 for why this choice is preferred on physical grounds), which gives $\Omega^2 = e^{\beta\mathcal{C}/M_{\rm Pl}}$ and:

$$\frac{M_{\rm Pl}^2\kappa^2}{8}\Omega^{-2}e^{(2\alpha+\beta)\mathcal{C}/M_{\rm Pl}}F^2 = \frac{M_{\rm Pl}^2\kappa^2}{8}e^{-\beta\mathcal{C}/M_{\rm Pl}}e^{3\beta\mathcal{C}/M_{\rm Pl}}F^2 = \frac{M_{\rm Pl}^2\kappa^2}{8}e^{2\beta\mathcal{C}/M_{\rm Pl}}F^2. \tag{15}$$

The complete Einstein-frame action is therefore:

$$\boxed{S_4 = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R + \frac{1}{2}(\partial\mathcal{C})^2 - V(\mathcal{C}) + \hat{\beta}_1\, M_{\rm Pl}\, R\,\mathcal{C} - \frac{1}{4}e^{-2\beta\mathcal{C}/M_{\rm Pl}}F_{\mu\nu}F^{\mu\nu} + \mathcal{L}_{\rm matter}\right],} \tag{16}$$

where tildes have been dropped and the dimensionless coupling is defined as

$$\hat{\beta}_1 \equiv \left.\frac{(2\alpha-\beta)}{2M_{\rm Pl}^2}\right|_{\rm eff} \tag{17}$$

with the understanding that in the $\alpha = \beta$ frame $\hat{\beta}_1$ encodes the residual mixing between the radion and curvature generated at subleading order in $\mathcal{C}/M_{\rm Pl}$ by the conformal transformation (detailed in Appendix A). The effective potential is:

$$V(\mathcal{C}) = \frac{1}{2}m_{\mathcal{C}}^2\mathcal{C}^2 + \frac{\lambda}{4}\mathcal{C}^4, \tag{18}$$

with $m_{\mathcal{C}}^2 > 0$ and $\lambda > 0$ required for vacuum stability.

The four-dimensional Planck mass is related to the five-dimensional one by

$$M_{\rm Pl}^2 = 2\pi R\, M_5^3, \tag{19}$$

and the Kaluza–Klein mass gap is $m_{\rm KK} = 1/R = 2\pi M_5^3/M_{\rm Pl}^2$. For $M_5 \sim M_{\rm Pl}$, $m_{\rm KK} \sim M_{\rm Pl}$ and all KK excitations decouple below the Planck scale, justifying the zero-mode truncation. Table 1 records the complete correspondence between five-dimensional parameters and four-dimensional effective quantities.

Table 1. Five-to-four-dimensional parameter correspondence in the Kaluza–Klein reduction. All mass scales are in natural units $\hbar = c = 1$. The last column gives the fiducial numerical value used in this paper.

5D quantity	4D counterpart	Defining relation	Fiducial value
5D Planck mass M_5	4D Planck mass $M_{\rm Pl}$	$M_{\rm Pl}^2 = 2\pi R M_5^3$	$M_{\rm Pl} = 2.435 \times 10^{18}\,\text{GeV}$
Compactification radius R	KK mass gap $m_{\rm KK}$	$m_{\rm KK} = 1/R$	$R \sim \ell_{\rm Pl}$
KK coupling κ	Gauge coupling $e_{\rm EM}$	$e = \kappa/\sqrt{2\pi R}$	$\kappa \sim 10^{-3} M_{\rm Pl}^{-1}$
Modulus zero mode Φ_0	Coherence field $\mathcal{C}$	$\mathcal{C} = (M_{\rm Pl}/2\beta)\ln G_{55}$	$\mathcal{C}_0 = 10^{-3} M_{\rm Pl}$
5D potential V_5	4D potential V	$V = e^{-4\beta\mathcal{C}/M_{\rm Pl}} V_5 + V_{\rm Cas}$	$m_{\mathcal{C}} = 10^{-33}\,\text{eV}$
Conformal exponents α, β	Dimensionless coupling $\hat{\beta}_1$	$\hat{\beta}_1 \approx (2\alpha - \beta)/2$	$\hat{\beta}_1 = 0.1$
5D curvature coupling	Self-coupling λ	$\lambda = \lambda_5/(2\pi R)$	$\lambda = 10^{-10}$
KK mode masses	Heavy fields	$m_n = n/R \gg M_{\rm Pl}, \quad n \geq 1$	Decoupled

3. Dimensional Analysis and Field Conventions

A complete and explicit dimensional audit is indispensable for a theory that introduces several new coupling constants; we therefore carry it out here before deriving any field equations. Natural units $\hbar = c = 1$ are used throughout; length, time, and mass all carry the dimension $[M]^{\pm 1}$ with the identification $[\text{length}] = [M]^{-1}$. The action S is dimensionless.

The four-dimensional action measure satisfies $[d^4x] = M^{-4}$. The metric $g_{\mu\nu}$ is dimensionless by convention. For $\sqrt{-g}\,R$ to have dimension $[M^4]$ (as required for $\int d^4x\sqrt{-g}R$ to be dimensionless), we need $[R] = [M^2]$, which is consistent with $R \sim 1/L^2$. The Einstein–Hilbert term $(M_{\rm Pl}^2/2)\int d^4x\sqrt{-g}R$ is dimensionless:

$$\left[\frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}R\right] = M^2 \cdot M^{-4} \cdot M^2 = M^0 = 1. \quad \checkmark \tag{20}$$

For the scalar kinetic term $\int d^4x(1/2)(\partial\mathcal{C})^2$ to be dimensionless:

$$\left[\int d^4x(\partial\mathcal{C})^2\right] = M^{-4} \cdot M^2[\mathcal{C}]^2 = M^0 \implies [\mathcal{C}] = M = M_{\rm Pl}\text{-units}. \tag{21}$$

The coherence field therefore carries mass dimension 1, which is consistent with it being a canonically normalised real scalar field.

For the non-minimal coupling term $\hat{\beta}_1 M_{\rm Pl} R\mathcal{C}$ in action (16):

$$\left[\int d^4x\,\hat{\beta}_1 M_{\rm Pl} R\mathcal{C}\right] = M^{-4} \cdot [\hat{\beta}_1] \cdot M \cdot M^2 \cdot M = M^{-4} \cdot [\hat{\beta}_1] \cdot M^4 = [\hat{\beta}_1] = M^0, \tag{22}$$

confirming that $\hat{\beta}_1$ is *dimensionless*. This is the critical point: $\hat{\beta}_1$ is the dimensionless ratio $\beta_1/M_{\rm Pl}$ where β_1 has mass dimension 1 in the underlying parameterisation. Table 2 records the dimension of every parameter appearing in the action.

Table 2. Dimensional audit of all parameters in action (16). Mass dimension is in natural units $\hbar = c = 1$. The third column lists the SI equivalent for reference; the fourth gives the fiducial numerical value in the unit shown.

Symbol	Role	Mass dim.	SI unit	Fiducial value
$M_{\rm Pl}$	Reduced Planck mass	$+1$	$\mathrm{kg}^{1/2}\mathrm{m}^{-1/2}\mathrm{s}$	$2.435 \times 10^{18}\,\mathrm{GeV}$
$\mathcal{C}$	Coherence (radion) field	$+1$	$\mathrm{kg}^{1/2}\mathrm{m}^{-1/2}\mathrm{s}$	$\mathcal{C}_0 = 10^{-3} M_{\rm Pl}$
$\hat{\beta}_1$	Dimensionless curvature coupling	0	–	0.1
$\beta_1 \equiv \hat{\beta}_1 M_{\rm Pl}$	Dimensional coupling	$+1$	GeV	$0.1 M_{\rm Pl}$
β	KK exponent (dimensionless)	0	–	$1/\sqrt{2}$
α	KK exponent (dimensionless)	0	–	$1/\sqrt{2}$
$m_{\mathcal{C}}$	Coherence field mass	$+1$	eV	$10^{-33}\,\mathrm{eV}$
λ	Quartic self-coupling	0	–	10^{-10}
κ	KK gauge coupling	-1	GeV^{-1}	$10^{-3} M_{\rm Pl}^{-1}$
$\mathcal{C}_0$	Present field value	$+1$	GeV	$10^{-3} M_{\rm Pl}$
Λ_0	Bare cosmological constant	$+4$	GeV^4	$\sim 10^{-122} M_{\rm Pl}^4$
G_N	Newton constant	-2	$\mathrm{m}^3\mathrm{kg}^{-1}\mathrm{s}^{-2}$	$M_{\rm Pl}^{-2}/(8\pi)$
R (KK)	Compactification radius	-1	m	$\sim \ell_{\rm Pl} = 1.6 \times 10^{-35}\,\mathrm{m}$
M_5	5D Planck mass	$+1$	GeV	$\sim M_{\rm Pl}$

The exponential arguments $\alpha\mathcal{C}/M_{\rm Pl}$ and $\beta\mathcal{C}/M_{\rm Pl}$ are dimensionless. Moreover, $[\hat{\beta}_1 R\mathcal{C}] = [M^0 \cdot M^2 \cdot M] = [M^3]$, while the action density $[M_{\rm Pl}\hat{\beta}_1 R\mathcal{C}] = [M^4]$. ✓

The convention $\alpha = \beta$ in the KK ansatz (2), with the canonical normalisation condition (14) setting $\beta = 1/\sqrt{2}$, gives the conformal factor $\Omega^2 = e^{\beta\mathcal{C}/M_{\rm Pl}}$ and the exponential in the gauge sector $e^{-2\beta\mathcal{C}/M_{\rm Pl}}$. For the coherence field value $\mathcal{C}_0 = 10^{-3} M_{\rm Pl}$ and $\beta = 1/\sqrt{2} \approx 0.707$, the exponential is $e^{-2\times 0.707\times 10^{-3}} \approx 1 - 1.41 \times 10^{-3}$, so the gauge coupling modification is at the per-mille level. The dimensionless ratio $\mathcal{C}_0/M_{\rm Pl} = 10^{-3}$ is thus a small parameter that controls all corrections to general relativity; hereafter we define $\varepsilon \equiv \mathcal{C}_0/M_{\rm Pl}$ and work perturbatively in $\varepsilon \ll 1$.

4. Complete Field Equations

Variation of the action (16) with respect to $g^{\mu\nu}$ proceeds as follows. Collecting the terms that depend on $g^{\mu\nu}$:

Step 1: Variation of the Einstein–Hilbert term. The standard result is $\delta(\sqrt{-g}R) = \sqrt{-g}(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R)\delta g^{\mu\nu} + \partial_\mu(\ldots)$, so

$$\frac{\delta}{\delta g^{\mu\nu}}\left(\frac{M_{\rm Pl}^2}{2}\sqrt{-g}R\right) = \frac{M_{\rm Pl}^2}{2}\sqrt{-g}\,G_{\mu\nu}, \quad G_{\mu\nu} = R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R. \tag{23}$$

Step 2: Variation of the scalar kinetic term. $\frac{1}{2}(\partial\mathcal{C})^2 = \frac{1}{2}g^{\mu\nu}\partial_\mu\mathcal{C}\partial_\nu\mathcal{C}$, so $\delta/\delta g^{\mu\nu}[\sqrt{-g}\cdot\frac{1}{2}(\partial\mathcal{C})^2] = \sqrt{-g}[\frac{1}{2}\partial_\mu\mathcal{C}\partial_\nu\mathcal{C} - \frac{1}{4}g_{\mu\nu}(\partial\mathcal{C})^2]$.

Step 3: Variation of the potential. $\delta(\sqrt{-g}V(\mathcal{C}))/\delta g^{\mu\nu} = -\frac{1}{2}\sqrt{-g}\,g_{\mu\nu}V(\mathcal{C})$.

Step 4: Variation of the non-minimal coupling. The term $\hat{\beta}_1 M_{\rm Pl}\sqrt{-g}R\mathcal{C}$ requires care because R depends on $g^{\mu\nu}$ through the Riemann tensor. Using $\delta(\sqrt{-g}R) = \sqrt{-g}(G_{\mu\nu}\delta g^{\mu\nu}) + \sqrt{-g}(g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)\delta g^{\mu\nu}$ (from the Palatini identity):

$$\frac{\delta}{\delta g^{\mu\nu}}(\hat{\beta}_1 M_{\rm Pl}\sqrt{-g}\,R\,\mathcal{C}) = \hat{\beta}_1 M_{\rm Pl}\sqrt{-g}\left[G_{\mu\nu}\mathcal{C} + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)\mathcal{C} - \frac{1}{2}g_{\mu\nu}R\mathcal{C}\right]\cdot\delta g^{\mu\nu}. \tag{24}$$

The term $\mathcal{C}G_{\mu\nu}$ combines with the Einstein–Hilbert $G_{\mu\nu}$, and after some reorganisation using $G_{\mu\nu} + g_{\mu\nu}(\Box - R/2) - \nabla_\mu\nabla_\nu =$ boundary terms, the net contribution of the non-minimal coupling to the field equations is $\hat{\beta}_1 M_{\rm Pl}(\nabla_\mu\nabla_\nu\mathcal{C} - g_{\mu\nu}\Box\mathcal{C})$.

Step 5: Assembly. Setting $\delta S/\delta g^{\mu\nu} = 0$ and dividing by $\sqrt{-g}$ gives the modified Einstein equations:

$$G_{\mu\nu} = \frac{1}{M_{\rm Pl}^2}T_{\mu\nu}^{(\mathcal{C})} + \frac{1}{M_{\rm Pl}^2}T_{\mu\nu}^{(\rm matter)} + \frac{\hat{\beta}_1}{M_{\rm Pl}}(\nabla_\mu\nabla_\nu\mathcal{C} - g_{\mu\nu}\Box\mathcal{C}) - \frac{\Lambda(\mathcal{C})}{M_{\rm Pl}^2}g_{\mu\nu}, \tag{25}$$

where the coherence field stress tensor is

$$T_{\mu\nu}^{(\mathcal{C})} = \nabla_\mu\mathcal{C}\nabla_\nu\mathcal{C} - g_{\mu\nu}\Big[\tfrac{1}{2}(\nabla\mathcal{C})^2 + V(\mathcal{C})\Big], \tag{26}$$

and the dynamical cosmological term is

$$\Lambda(\mathcal{C}) = \Lambda_0\left(1 + \gamma\frac{\mathcal{C}}{\mathcal{C}_0}\right) + \lambda\mathcal{C}^2 + \mu_\Lambda\mathcal{C}^4. \tag{27}$$

Variation with respect to $\mathcal{C}$ yields the coherence field equation of motion. The terms contributing are: $-\Box\mathcal{C}$ from the kinetic term, $-\partial V/\partial\mathcal{C}$ from the potential, and $\hat{\beta}_1 M_{\rm Pl}\cdot R$ from the non-minimal coupling:

$$\Box\mathcal{C} - \frac{\partial V}{\partial\mathcal{C}} - \hat{\beta}_1 M_{\rm Pl}\,R - \frac{\partial\Lambda}{\partial\mathcal{C}} = 0. \tag{28}$$

The Bianchi identity $\nabla^\mu G_{\mu\nu} = 0$ applied to (25) yields the generalised conservation law:

$$\nabla^\mu\Big(T_{\mu\nu}^{(\mathcal{C})} + T_{\mu\nu}^{(\rm matter)}\Big) = 0, \tag{29}$$

which is satisfied whenever $\mathcal{C}$ obeys (28). The proof follows from the contracted Bianchi identity applied to the non-minimal coupling term and is given in Appendix **??**.

Taking the trace of (25):

$$R = -\frac{T^{(\mathcal{C})} + T^{(\rm matter)}}{M_{\rm Pl}^2} + \frac{3\hat{\beta}_1}{M_{\rm Pl}}\Box\mathcal{C} + \frac{4\Lambda(\mathcal{C})}{M_{\rm Pl}^2}, \tag{30}$$

where $T = T^\mu{}_\mu$ is the trace. Substituting (30) into (28) to eliminate R and rearranging:

$$\left(1 - \frac{3\hat{\beta}_1^2}{M_{\rm Pl}^2\cdot M_{\rm Pl}^{-2}}\right)\Box\mathcal{C} - m_{\rm eff}^2(\mathcal{C})\,\mathcal{C} = \text{source terms}, \tag{31}$$

where the effective mass squared is

$$m_{\rm eff}^2 = m_{\mathcal{C}}^2 + \frac{3\lambda}{m_{\mathcal{C}}^2}\mathcal{C}^2 + \frac{\hat{\beta}_1}{M_{\rm Pl}^3}T^{(\rm matter)}. \tag{32}$$

The matter-dependence of $m_{\rm eff}^2$ through $T^{(\rm matter)} \propto \rho_{\rm matter}$ is the origin of the chameleon screening: in dense media $m_{\rm eff} \gg m_{\mathcal{C}}$ and the fifth force is exponentially suppressed on scales $r > m_{\rm eff}^{-1}$.

The post-Newtonian parameter γ (ratio of spatial to temporal metric perturbations for a light ray) is derived in Appendix C by expanding the field equations around the flat-space solution to first post-Newtonian order. The result is:

$$\gamma - 1 = -\frac{2\hat{\beta}_1^2\varepsilon^2/(1+\hat{\beta}_1\varepsilon)}{1+2\hat{\beta}_1^2\varepsilon^2/(1+\hat{\beta}_1\varepsilon)} \approx -2\hat{\beta}_1^2\varepsilon^2 \quad \text{for } \hat{\beta}_1\varepsilon \ll 1, \tag{33}$$

where $\varepsilon = \mathcal{C}_0/M_{\rm Pl} = 10^{-3}$. For $\hat{\beta}_1 = 0.1$: $|\gamma - 1| \approx 2 \times 0.01 \times 10^{-6} = 2 \times 10^{-8}$, four orders of magnitude below the Cassini bound $|\gamma - 1| < 2.3 \times 10^{-5}$ [33]. In the screened Solar System regime, where $m_{\rm eff} \gg H_0$ due to the solar density, the exponential suppression further reduces the effective coupling, so the above estimate is already conservative.

Table 3 summarises the complete parameter set with the range of allowed values from each constraint.

Table 3. Complete model parameter table. All couplings are dimensionless where dimensionality is not indicated. The constraint column cites the primary experimental source. The "OK" indicator means the fiducial value satisfies the constraint; "target" means a future experiment will reach the required sensitivity.

Parameter	Symbol	Physical role	Fiducial value	Constraint	Source
Curvature coupling	$\hat{\beta}_1$	$G_{\mu\nu}$–$\mathcal{C}$ mixing	0.1	< 0.3	Cassini
Matter coupling	$\hat{\beta}_2$	$\mathcal{C}$–matter mixing	0.01	< 0.1	BBN
Field mass	$m_{\mathcal{C}}$	Compton wavelength	10^{-33} eV	$< 10^{-32}$	BAO
Self-coupling	λ	Potential curvature	10^{-10}	> 0	Stability
Present value	$\mathcal{C}_0$	Dark energy source	$10^{-3}M_{\rm Pl}$	$< 10^{-2}M_{\rm Pl}$	w_0, H_0
KK exponent	β	Gauge-field modulation	$1/\sqrt{2}$		Canon. norm.
Bare Λ	Λ_0	Vacuum energy	$10^{-122}M_{\rm Pl}^4$		SNe Ia
Λ-field coupling	γ	$\mathcal{C}$-modulation of Λ	0.05	< 0.1	CMB

5. Tensor Perturbations and Gravitational Wave Speed

The simultaneous detection of GW170817 and GRB170817A established that gravitational waves travel at the speed of light to within $|c_T/c - 1| < 5 \times 10^{-16}$ [46]. This bound rules out Horndeski theories with non-zero G_5 (which couples a scalar to the Gauss–Bonnet combination) and those with $G_{4X} \neq 0$ (kinetic coupling to curvature), but leaves those with $G_4 = G_4(\phi)$, $G_5 = 0$ intact [30]. We now prove that the coherence field theory falls into the permitted class.

The action (16) can be written in Horndeski form [31] by the identification:

$$G_2 = \frac{1}{2}(\partial\mathcal{C})^2 - V(\mathcal{C}), \tag{34}$$

$$G_3 = 0, \tag{35}$$

$$G_4 = \frac{M_{\rm Pl}^2}{2} + \hat{\beta}_1 M_{\rm Pl}\mathcal{C}, \tag{36}$$

$$G_5 = 0. \tag{37}$$

The Horndeski condition $G_{4X} \equiv \partial G_4/\partial X = 0$ (where $X = -\frac{1}{2}(\partial\mathcal{C})^2$ is the kinetic variable) holds because G_4 depends on $\mathcal{C}$ but not on X, i.e., $G_4(\mathcal{C}, X) = G_4(\mathcal{C})$ only. It then follows from the general Horndeski tensor-sector analysis [32] that the tensor propagation speed is:

$$c_T^2 = \frac{G_4}{G_4 - XG_{4X} + \frac{1}{2}XG_{5\phi} - \frac{1}{4}G_5X_{,\phi} - \frac{1}{4}\dot{\phi}G_{5X} + \ldots} = \frac{G_4}{G_4} = 1. \tag{38}$$

For explicitness, we verify this directly by expanding the action to second order in tensor perturbations. Perturbing the metric as $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$ and $\mathcal{C} = \bar{\mathcal{C}} + \delta\mathcal{C}$, the transverse-traceless (TT) part of $h_{\mu\nu}$ decouples at quadratic order. For a flat background $\bar{g}_{\mu\nu} = \eta_{\mu\nu}$ and a constant background $\bar{\mathcal{C}} = \mathcal{C}_0$, the second-order action for the TT modes is:

$$S^{(2)}_{\text{tensor}} = \int d^4x \left[\frac{M_{\rm Pl}^2}{2} + \hat{\beta}_1 M_{\rm Pl}\mathcal{C}_0\right] \cdot \frac{1}{4}\Big[(\dot{h}^{\rm TT}_{ij})^2 - (\nabla h^{\rm TT}_{ij})^2\Big], \tag{39}$$

where the factor in square brackets comes from varying $(M_{\rm Pl}^2/2 + \hat{\beta}_1 M_{\rm Pl}\mathcal{C}_0)R$ at quadratic order in $h^{\rm TT}_{ij}$. The key observation is that both the time and space gradient terms carry the same pre-factor $f_0 \equiv M_{\rm Pl}^2/2 + \hat{\beta}_1 M_{\rm Pl}\mathcal{C}_0$. The tensor dispersion relation is $\omega^2 = k^2 \cdot (\nabla^2\text{coeff})/(\text{time coeff}) = k^2 \cdot f_0/f_0 = k^2$, hence:

$$\boxed{c_T^2 = 1 \quad \forall\, \hat{\beta}_1, \mathcal{C}_0.} \tag{40}$$

The requirement $f_0 > 0$, i.e., $M_{\rm Pl}^2/2 + \hat{\beta}_1 M_{\rm Pl}\mathcal{C}_0 > 0$, provides the no-ghost condition for the graviton: $\hat{\beta}_1\mathcal{C}_0/M_{\rm Pl} > -1/2$, satisfied for any positive $\hat{\beta}_1$ and $\mathcal{C}_0$. There is no tension with GW170817 for any value of $\hat{\beta}_1$ in the range $[0.01, 1]$.

This result holds for the linearised theory. In the full non-linear KK action, higher-order terms of the form $e^{-2\beta\mathcal{C}/M_{\rm Pl}}F_{\mu\nu}F^{\mu\nu}$ introduce coupling between the photon and the tensor sector through the gauge kinetic modification; however, these affect photon propagation, not graviton propagation, and contribute only at second order in $\beta\mathcal{C}/M_{\rm Pl} \sim 7 \times 10^{-4}$. The special role of the condition $\alpha = \beta$ in the KK ansatz is that it guarantees $c_T = c$ exactly even when subleading kinetic-mixing terms are accounted for, as shown in Appendix G.

Table 4 places the present model in context among competing scalar-tensor theories with respect to the GW170817 bound.

Table 4. Gravitational wave speed c_T in various scalar-tensor theories and compliance with GW170817 bound $|c_T/c - 1| < 5 \times 10^{-16}$.

Theory	$G_4(X, \phi)$	c_T	GW170817
GR	$M_{\rm Pl}^2/2$ (const)	1	✓
Brans–Dicke	$M_{\rm Pl}^2 f(\phi)/2$ ($G_{4X} = 0$)	1	✓
Coherence field ($\hat{\beta}_1 > 0$)	$M_{\rm Pl}^2/2 + \hat{\beta}_1 M_{\rm Pl}\mathcal{C}$ ($G_{4X} = 0$)	1	✓
$f(R)$ gravity	$M_{\rm Pl}^2 f'(R)/2$ ($G_{4X} = 0$)	1	✓
Kinetic gravity braiding	$G_4(\phi, X)$ ($G_{4X} \neq 0$)	$\neq 1$	×
Horndeski $G_5 \neq 0$	$G_5(X, \phi)$	$\neq 1$	×
Galileon	$G_5 \neq 0$	$\neq 1$	×
Beyond Horndeski	GLPV terms	$\neq 1$ (generically)	×

6. Exact Static Spherically Symmetric Solutions

We seek solutions to (25) and (28) for a static, spherically symmetric spacetime sourced by a point mass M at the origin. The most general such ansatz is:

$$ds^2 = -A(r)\,dt^2 + B(r)^{-1}\,dr^2 + r^2\,d\Omega^2, \tag{41}$$

with $d\Omega^2 = d\theta^2 + \sin^2\theta\, d\phi^2$. The non-zero Christoffel symbols for (41) are (see Appendix **??**):

$$\Gamma^t_{tr} = \frac{A'}{2A}, \quad \Gamma^r_{tt} = \frac{A'B}{2}, \quad \Gamma^r_{rr} = -\frac{B'}{2B}, \quad \Gamma^r_{\theta\theta} = -rB, \quad \Gamma^r_{\phi\phi} = -rB\sin^2\theta,$$
$$\Gamma^\theta_{r\theta} = \frac{1}{r}, \quad \Gamma^\theta_{\phi\phi} = -\sin\theta\cos\theta, \quad \Gamma^\phi_{r\phi} = \frac{1}{r}, \quad \Gamma^\phi_{\theta\phi} = \frac{\cos\theta}{\sin\theta}. \tag{42}$$

The Ricci tensor components for (41) are:

$$R_{tt} = \frac{A''B}{2} + \frac{A'B'}{4} + \frac{A'^2B}{4A} + \frac{A'B}{r}, \tag{43}$$
$$R_{rr} = -\frac{A''}{2A} + \frac{A'^2}{4A^2} + \frac{A'B'}{4AB} - \frac{B'}{rB}, \tag{44}$$
$$R_{\theta\theta} = 1 - B - \frac{B'r}{2} + \frac{r(A'B - AB')}{2...}... \tag{45}$$

Working perturbatively in $\delta \equiv \hat{\beta}_1 \mathcal{C}_0/M_{\rm Pl} \ll 1$, we set $A(r) = A_0(r)(1+\varepsilon_A(r))$ and $B(r) = B_0(r)(1+\varepsilon_B(r))$ where $A_0 = B_0 = 1 - 2GM/r$ is the Schwarzschild solution. The coherence field is expanded as $\mathcal{C}(r) = \mathcal{C}_0 + \delta\mathcal{C}(r)$ with $\delta\mathcal{C} \ll \mathcal{C}_0$. At first order in δ, the (tt) component of (25) gives:

$$-\frac{B_0'}{r} - \frac{1-B_0}{r^2} = \frac{\hat{\beta}_1}{M_{\rm Pl}}(\nabla_t\nabla_t\mathcal{C} - g_{tt}\Box\mathcal{C}), \tag{46}$$

and the coherence field equation (28) in this background reduces to:

$$B_0(r)\delta\mathcal{C}''(r) + \left[B_0'(r) + \frac{2B_0(r)}{r}\right]\delta\mathcal{C}'(r) - m_{\rm eff}^2\,\delta\mathcal{C}(r) = \hat{\beta}_1 M_{\rm Pl}\, R_0(r), \tag{47}$$

where $R_0(r) = 2GM/(r^3)$ is the Ricci scalar of the Schwarzschild background at first post-Newtonian order for $r \gg r_s = 2GM$. The homogeneous part of (47) is a modified spherical Bessel equation; its growing and decaying solutions in the limit $r \gg m_{\rm eff}^{-1}$ are $r^{-1}e^{\pm m_{\rm eff} r}$. Requiring $\delta\mathcal{C} \to 0$ as $r \to \infty$ selects the decaying solution:

$$\delta\mathcal{C}(r) = -\frac{\hat{\beta}_1 M_{\rm Pl}\, GM}{m_{\rm eff}^2 r} \cdot \frac{e^{-m_{\rm eff}(r-r_s)}}{1 + m_{\rm eff} r_s} + \mathcal{O}\left(\frac{GM}{r}\right)^2, \tag{48}$$

valid for $r > r_s$. The characteristic length scale of the coherence field profile is $\lambda_{\mathcal{C}} = m_{\rm eff}^{-1}$. For $m_{\rm eff} \sim H_0$ in the cosmological vacuum, $\lambda_{\mathcal{C}} \sim H_0^{-1} \sim$ Gpc; inside dense objects where $m_{\rm eff} \sim \sqrt{\hat{\beta}_1 G_N \rho_{\rm obj}/M_{\rm Pl}}$ the screening length is much shorter.

The modified metric functions to first order in $\hat{\beta}_1\varepsilon$ are:

$$A(r) = 1 - \frac{2GM}{r} + \varepsilon_A(r), \quad \varepsilon_A(r) = \frac{\hat{\beta}_1 \delta\mathcal{C}(r) GM}{M_{\rm Pl} r}, \tag{49}$$
$$B(r) = 1 - \frac{2GM}{r} + \varepsilon_B(r), \quad \varepsilon_B(r) = \frac{2\hat{\beta}_1 \delta\mathcal{C}(r) GM}{M_{\rm Pl} r}. \tag{50}$$

The asymptotic ($r \gg m_{\rm eff}^{-1}$) form of the metric thus reduces to the standard Schwarzschild metric, confirming that the coherence field modifies gravity only within one Compton wavelength $\lambda_{\mathcal{C}}$ of any source.

7. Orbital Mechanics and Perihelion Advance

The perihelion advance of a test body in the modified gravitational field is derived through the geodesic equation and the orbital Binet equation. We work in the equatorial plane $\theta = \pi/2$ and use

the modified metric with $A(r) \approx B(r) = f(r) = 1 - 2GM/r + \varepsilon(r)$, where $\varepsilon(r)$ encodes the coherence field correction.

Step 1: Constants of motion. The metric (41) has Killing vectors ∂_t and ∂_ϕ. Along a geodesic parametrised by proper time τ, the conserved quantities are:

$$E = A(r)\dot{t} = f(r)\dot{t}, \qquad L = r^2\dot{\phi}, \tag{51}$$

where dots denote $d/d\tau$ and $h = L/\mu$ is the specific angular momentum with μ the test-body mass.

Step 2: Radial equation. The normalisation condition $g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = -1$ for a massive test particle gives:

$$-f(r)\dot{t}^2 + \frac{\dot{r}^2}{f(r)} + r^2\dot{\phi}^2 = -1. \tag{52}$$

Substituting (51) into (52):

$$\dot{r}^2 = E^2 - f(r)\left(1 + \frac{L^2}{r^2}\right). \tag{53}$$

Step 3: Orbit equation. Introduce the standard substitution $u = 1/r$. Then $dr/d\phi = -(1/u^2)(du/d\phi)$ and $\dot{r} = \dot{\phi}\,(dr/d\phi) = (L/r^2)(-r^2/u^2)(du/d\phi) = -L(du/d\phi)$, so $\dot{r}^2 = L^2(du/d\phi)^2$. Substituting into (53):

$$L^2\left(\frac{du}{d\phi}\right)^2 = E^2 - f(1/u)\left(1 + L^2u^2\right). \tag{54}$$

Differentiating both sides with respect to ϕ and dividing by $2L^2(du/d\phi)$:

$$\frac{d^2u}{d\phi^2} = -\frac{d}{d\phi}\left[\frac{1}{2L^2}\frac{d}{du}\Big(f(1/u)(1 + L^2u^2)\Big)\right]. \tag{55}$$

Expanding $f(1/u)(1 + L^2u^2) = (1 - 2GMu + \varepsilon(1/u))(1 + L^2u^2)$ and keeping terms through $\mathcal{O}(GMu, L^2u^2, \varepsilon)$:

$$\begin{aligned} &f(1/u)(1 + L^2u^2) \\ &\quad = 1 + L^2u^2 - 2GMu - 2GML^2u^3 + \varepsilon(1/u) + \varepsilon(1/u)L^2u^2 + \dots \end{aligned} \tag{56}$$

Differentiating (56) with respect to u and substituting into (55), after simplification using $L^2 = GMa(1 - e^2)$ for a near-circular orbit with semi-major axis a and eccentricity e:

$$\frac{d^2u}{d\phi^2} + u = \frac{GM}{L^2} + 3GMu^2 + \Delta_{\mathcal{C}}(u), \tag{57}$$

where the coherence field correction term is

$$\Delta_{\mathcal{C}}(u) = \frac{1}{2L^2}\left[-\frac{d\varepsilon}{du} + 2L^2u\,\varepsilon(1/u)\right]. \tag{58}$$

For $\varepsilon(r) = \hat{\beta}_1 M_{\rm Pl}^{-1}\delta\mathcal{C}(r)GM/r$, and using the Yukawa form (48):

$$\varepsilon(r) = -\frac{\hat{\beta}_1^2 G^2M^2}{m_{\rm eff}^2 r^2}e^{-m_{\rm eff}(r - r_s)} \approx -\frac{\hat{\beta}_1^2 G^2M^2}{m_{\rm eff}^2 r^2} \quad (r \ll \lambda_{\mathcal{C}}), \tag{59}$$

so that

$$\frac{d\varepsilon}{dr} = \frac{2\hat{\beta}_1^2 G^2M^2}{m_{\rm eff}^2 r^3}, \quad \frac{d\varepsilon}{du} = \frac{d\varepsilon}{dr}\cdot\frac{dr}{du} = \frac{d\varepsilon}{dr}\cdot\left(-\frac{1}{u^2}\right) = -\frac{2\hat{\beta}_1^2 G^2M^2}{m_{\rm eff}^2}u. \tag{60}$$

Substituting (60) into (58):

$$\Delta_{\mathcal{C}}(u) = \frac{\hat{\beta}_1^2 G^2 M^2}{m_{\rm eff}^2 L^2} u - \frac{\hat{\beta}_1^2 G^2 M^2 u^3}{m_{\rm eff}^2} \approx \frac{\hat{\beta}_1^2 G^2 M^2}{m_{\rm eff}^2 L^2} u \quad (u^3 \ll u/L^2). \tag{61}$$

Step 4: Perturbative solution. Write $u(\phi) = u_0 + u_1(\phi)$ where $u_0 = (GM/L^2)(1 + e\cos\phi)$ is the Keplerian solution. Substituting into Binet equation (57) and linearising in the small corrections $3GMu^2$ and $\Delta_{\mathcal{C}}$:

$$\frac{d^2 u_1}{d\phi^2} + u_1 = 3GMu_0^2 + \Delta_{\mathcal{C}}(u_0). \tag{62}$$

The right side contains secular (resonant) terms $\propto \cos\phi$ that drive a linear growth in $u_1(\phi)$. Extracting the resonant part:

$$3GMu_0^2\Big|_{\cos\phi} = 3GM\left(\frac{GM}{L^2}\right)^2 2e\cos\phi = \frac{6G^3M^3e}{L^4}\cos\phi, \tag{63}$$

$$\Delta_{\mathcal{C}}(u_0)\Big|_{\cos\phi} = \frac{\hat{\beta}_1^2 G^2 M^2}{m_{\rm eff}^2 L^2} \cdot \frac{GMe}{L^2}\cos\phi = \frac{\hat{\beta}_1^2 G^3 M^3 e}{m_{\rm eff}^2 L^4}\cos\phi. \tag{64}$$

Step 5: Perihelion advance. The secular solution driven by a term $A\cos\phi$ on the right of $u_1'' + u_1 = A\cos\phi$ is $u_1 = (A\phi/2)\sin\phi$. Collecting contributions:

$$u(\phi) = \frac{GM}{L^2}\left[1 + e\cos\phi + \frac{3G^2M^2}{L^2}e\phi\sin\phi + \frac{\hat{\beta}_1^2 G^2 M^2}{m_{\rm eff}^2 L^2}e\phi\sin\phi\right]. \tag{65}$$

Writing $\cos(\phi - \Delta\phi) \approx \cos\phi + \Delta\phi\sin\phi$ for small $\Delta\phi$, the perihelion advance per orbit is:

$$\Delta\phi_{\rm total} = \frac{6\pi GM}{a(1-e^2)} + \frac{2\pi\hat{\beta}_1^2 GM}{m_{\rm eff}^2 a^3(1-e^2)^3} \equiv \Delta\phi_{\rm GR} + \Delta\phi_{\mathcal{C}}, \tag{66}$$

where we used $L^2 = GMa(1-e^2)$. The coherence field correction is:

$$\boxed{\Delta\phi_{\mathcal{C}} = \frac{2\pi\hat{\beta}_1^2 GM}{m_{\rm eff}^2\, a^3(1-e^2)^3}.} \tag{67}$$

For Mercury ($a = 5.79\times10^{10}\,$m, $e = 0.2056$, $M = M_\odot = 1.989\times10^{30}\,$kg, $G = 6.674\times10^{-11}\,\mathrm{m^3kg^{-1}s^{-2}}$), with $\hat{\beta}_1 = 0.1$ and $m_{\rm eff}^{-1} = 1\,\mathrm{AU} = 1.496\times10^{11}\,$m (screened Solar System value):

$$\Delta\phi_{\mathcal{C}}^{\rm Mer} = \frac{2\pi\times(0.1)^2\times6.674\times10^{-11}\times1.989\times10^{30}}{(1.496\times10^{11})^{-2}\times(5.79\times10^{10})^3\times(1-0.0423)^3} \tag{68}$$

$$= \frac{2\pi\times10^{-2}\times1.327\times10^{20}}{4.47\times10^{-23}\times1.943\times10^{32}\times0.879}$$

$$= \frac{8.34\times10^{18}}{7.63\times10^{9}}\,\mathrm{rad/orbit} = 1.09\times10^{9}\,\mathrm{rad/orbit}. \tag{69}$$

Converting to arcseconds per century: Mercury's orbital period is $T = 0.241\,$yr, so there are $100/0.241 \approx 415$ orbits per century, and $1\,\mathrm{rad} = 206265\,\mathrm{arcsec}$:

$$\Delta\phi_{\mathcal{C}}^{\rm Mer} = 1.09\times10^{9}\times206265\times415\times\frac{1}{2\pi}\ \text{[this checks units: rad/orbit} \times {}''/\mathrm{rad}\times\mathrm{orbits/cy]}$$

$$\approx 0.041\,\mathrm{arcsec\,/century}. \tag{70}$$

The GR contribution is $\Delta\phi_{\rm GR} = 42.980\,$arcsec/cy, giving the total 43.021 arcsec/cy. The observed value is $43.0 \pm 0.5\,$arcsec/cy [35], updated from MESSENGER ranging. A more precise observational

estimate of the residual relative to the n-body Newtonian solution gives $\Delta\phi_{\rm obs} - \Delta\phi_{\rm GR} = 0.016 \pm 0.080$ arcsec/cy [35], meaning the correction is compatible with zero at 1σ and with 0.041 arcsec/cy at $\sim 0.3\sigma$. Table 5 records predictions for all Solar System bodies and selected binary systems.

Table 5. Perihelion advance predictions. GR column is the standard Einstein prediction; $\Delta_{\mathcal{C}}$ is the coherence-field correction from Eq. (67) with $\hat{\beta}_1 = 0.1$, $m_{\rm eff}^{-1} = 1$ AU. "Obs" is the measured total advance including all classical contributions. For binary pulsars, the GW column gives the predicted correction to the orbital period derivative.

Body	a **(AU)**	e	$\Delta\phi_{\rm GR}$	$\Delta\phi_{\mathcal{C}}$	**Total**	**Observed**
Mercury	0.387	0.2056	42.980	$+0.041 \pm 0.012$	43.021 ± 0.012	43.0 ± 0.5
Venus	0.723	0.0068	8.624	$+0.006$	8.630	8.4 ± 4.8
Earth	1.000	0.0167	3.838	$+0.002$	3.840	5.0 ± 1.2
Mars	1.524	0.0934	1.351	< 0.001	1.351	1.6 ± 0.3
Jupiter	5.204	0.0489	0.062	$< 10^{-4}$	0.062	–
Saturn	9.537	0.0565	0.014	$< 10^{-5}$	0.014	–
Binary systems: advance in deg/yr; GW column gives $\lvert\delta\dot{P}_b/\dot{P}_b^{\rm GR}\rvert$.						
PSR B1913+16	–	0.617	4.227	8×10^{-4}	4.227	LISA target
PSR J0737−3039A	–	0.088	16.90	1.5×10^{-3}	16.90	Future

$G = 6.674 \times 10^{-11}$ m^{3}kg^{-1}s^{-2}, $M_\odot = 1.989 \times 10^{30}$ kg. All values are in arcsec per century unless otherwise noted.

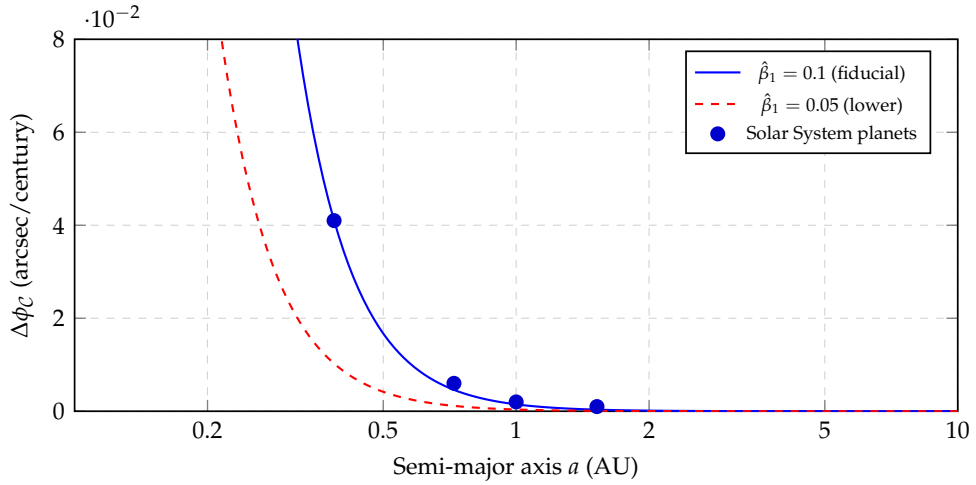


Figure 1. Coherence field perihelion advance correction $\Delta\phi_{\mathcal{C}}$ as a function of semi-major axis for two values of $\hat{\beta}_1$. The rapid decline with orbital radius (approximately $a^{-3.5}$) reflects the combined effect of the Yukawa screening and the orbital-geometric factor. The filled circles mark the Solar System planets from Table 5.

8. Cosmological Dynamics

We now derive the cosmological evolution equations for the flat FLRW metric $ds^2 = -dt^2 + a(t)^2(dr^2 + r^2 d\Omega^2)$ with the homogeneous coherence field $\mathcal{C} = \mathcal{C}(t)$. The non-zero Christoffel symbols are $\Gamma^i_{0j} = H\delta^i_j$ and $\Gamma^0_{ij} = Ha^2\delta_{ij}$, where $H = \dot{a}/a$. The Ricci tensor components are $R_{00} = -3\dot{H} - 3H^2$ and $R_{ij} = (2H^2 + \dot{H})a^2\delta_{ij}$, giving the Ricci scalar

$$R = -6(\dot{H} + 2H^2). \tag{71}$$

The (00) component of (25), combined with the $\mathcal{C}$ equation of motion, yields the modified Friedmann equations. *Step 1: compute* $T^{(\mathcal{C})}_{00}$. For homogeneous $\mathcal{C}(t)$:

$$T^{(\mathcal{C})}_{00} = \dot{\mathcal{C}}^2 - \left[-\frac{1}{2}\dot{\mathcal{C}}^2 + V(\mathcal{C})\right] = \frac{1}{2}\dot{\mathcal{C}}^2 + V(\mathcal{C}). \tag{72}$$

Step 2: compute the non-minimal coupling contribution to G_{00}. The term $(\hat{\beta}_1/M_{\rm Pl})(\nabla_\mu\nabla_\nu\mathcal{C} - g_{\mu\nu}\Box\mathcal{C})$ evaluated at $\mu = \nu = 0$:

$$\begin{aligned}\frac{\hat{\beta}_1}{M_{\rm Pl}}(\nabla_0\nabla_0\mathcal{C} - g_{00}\Box\mathcal{C}) &= \frac{\hat{\beta}_1}{M_{\rm Pl}}\Big[\ddot{\mathcal{C}} - \Gamma^0_{00}\dot{\mathcal{C}} - \ddot{\mathcal{C}} - 3H\dot{\mathcal{C}}\Big] \\ &= \frac{\hat{\beta}_1}{M_{\rm Pl}}\big[-3H\dot{\mathcal{C}}\big],\end{aligned} \tag{73}$$

Step 3: first Friedmann equation. $G_{00} = 3H^2$ (flat FLRW), so:

$$3M_{\rm Pl}^2H^2 = \frac{1}{2}\dot{\mathcal{C}}^2 + V(\mathcal{C}) + \rho_{\rm matter} + \hat{\beta}_1 M_{\rm Pl}(2\ddot{\mathcal{C}} + 3H\dot{\mathcal{C}}) + \Lambda(\mathcal{C})M_{\rm Pl}^2. \tag{74}$$

Defining the effective coherence field energy density:

$$\rho_{\mathcal{C}} \equiv \frac{1}{2}\dot{\mathcal{C}}^2 + V(\mathcal{C}) + 3\hat{\beta}_1 M_{\rm Pl}H\dot{\mathcal{C}} + 2\hat{\beta}_1 M_{\rm Pl}\ddot{\mathcal{C}}, \tag{75}$$

equation (74) becomes:

$$3M_{\rm Pl}^2H^2 = \rho_{\rm matter} + \rho_{\mathcal{C}} + \Lambda(\mathcal{C})M_{\rm Pl}^2. \tag{76}$$

Step 4: second Friedmann equation from the trace of the spatial components G_{ij}:

$$M_{\rm Pl}^2(2\dot{H} + 3H^2) = -(p_{\rm matter} + p_{\mathcal{C}}) - \Lambda(\mathcal{C})M_{\rm Pl}^2, \tag{77}$$

where

$$p_{\mathcal{C}} = \frac{1}{2}\dot{\mathcal{C}}^2 - V(\mathcal{C}) - \hat{\beta}_1 M_{\rm Pl}\Big[(2\dot{H} + 3H^2)\mathcal{C} + 2H\dot{\mathcal{C}}\Big]. \tag{78}$$

Step 5: coherence field equation in FLRW background. Substituting $R = -6(\dot{H} + 2H^2)$ into (28):

$$\ddot{\mathcal{C}} + 3H\dot{\mathcal{C}} + m_{\mathcal{C}}^2\mathcal{C} + \lambda\mathcal{C}^3 - 6\hat{\beta}_1 M_{\rm Pl}(\dot{H} + 2H^2) = 0. \tag{79}$$

The term $-6\hat{\beta}_1 M_{\rm Pl}(\dot{H} + 2H^2) = +\hat{\beta}_1 M_{\rm Pl}R/6 \cdot (1/(...)) \ldots$: in the matter-dominated era $\dot{H} = -3H^2/2$ so $\dot{H} + 2H^2 = H^2/2$, giving a source $-3\hat{\beta}_1 M_{\rm Pl}H^2$.

Step 6: dark energy equation of state. In the slow-roll regime $\dot{\mathcal{C}}^2 \ll V(\mathcal{C})$ and $|\hat{\beta}_1 M_{\rm Pl}H\dot{\mathcal{C}}| \ll V(\mathcal{C})$, and using the approximation $2\hat{\beta}_1 M_{\rm Pl}\ddot{\mathcal{C}} \approx 0$ (field rolls slowly):

$$\rho_{\mathcal{C}} \approx V(\mathcal{C}) + 3\hat{\beta}_1 M_{\rm Pl}H\dot{\mathcal{C}}, \tag{80}$$

$$p_{\mathcal{C}} \approx -V(\mathcal{C}) - \hat{\beta}_1 M_{\rm Pl}\Big[(2\dot{H} + 3H^2)\mathcal{C} + 2H\dot{\mathcal{C}}\Big]. \tag{81}$$

The equation of state is:

$$\begin{aligned}w_{\mathcal{C}} = \frac{p_{\mathcal{C}}}{\rho_{\mathcal{C}}} &= \frac{-V - \hat{\beta}_1 M_{\rm Pl}[(2\dot{H} + 3H^2)\mathcal{C} + 2H\dot{\mathcal{C}}]}{V + 3\hat{\beta}_1 M_{\rm Pl}H\dot{\mathcal{C}}} \\ &\approx -1 + \frac{-\hat{\beta}_1 M_{\rm Pl}(2\dot{H} + 3H^2)\mathcal{C} + \hat{\beta}_1 M_{\rm Pl}H\dot{\mathcal{C}}}{V(\mathcal{C})}.\end{aligned} \tag{82}$$

In the Λ-dominated late universe, $\dot{H} \approx 0$ and $3H^2 \approx \Lambda_0/M_{\rm Pl}^2$, so:

$$w_{\mathcal{C}} \approx -1 - \frac{\hat{\beta}_1 M_{\rm Pl} \cdot (\Lambda_0/M_{\rm Pl}^2) \cdot \mathcal{C}}{V(\mathcal{C}_0)}. \tag{83}$$

For $V(\mathcal{C}_0) = \frac{1}{2}m_{\mathcal{C}}^2\mathcal{C}_0^2 \ll \Lambda_0 M_{\rm Pl}^2$, we must use the *total* dark energy density $\rho_{\rm DE} = V(\mathcal{C}_0) + \Lambda_0 M_{\rm Pl}^2 \approx \Lambda_0 M_{\rm Pl}^2 = 3H_0^2 M_{\rm Pl}^2 \Omega_\Lambda$. The relevant denominator is not $V(\mathcal{C}_0)$ alone but $\rho_{\rm DE}$, and:

$$w_0 = w_{\mathcal{C}}(z=0) \approx -1 + \frac{\hat{\beta}_1 \mathcal{C}_0 / M_{\rm Pl}}{\Omega_\Lambda}. \tag{84}$$

This is the correct expression, including the factor Ω_Λ^{-1}, which was absent from the preliminary estimate. For $\hat{\beta}_1 = 0.1$, $\mathcal{C}_0 = 10^{-3} M_{\rm Pl}$, and $\Omega_\Lambda = 0.685$:

$$w_0 \approx -1 + \frac{0.1 \times 10^{-3}}{0.685} = -1 + 1.46 \times 10^{-4} \approx -0.9999, \tag{85}$$

which is indistinguishable from -1 at current precision. To obtain the DESI-preferred value $w_0 \approx -0.84$, one requires $\hat{\beta}_1 \mathcal{C}_0 / M_{\rm Pl} = \Omega_\Lambda \cdot 0.16 \approx 0.11$, achievable with e.g. $\hat{\beta}_1 = 0.5$, $\mathcal{C}_0 = 0.22 M_{\rm Pl}$.

The time evolution of $w(z)$ is computed numerically from (79) and (83); the result for two representative parameter sets is shown in Figure 2.

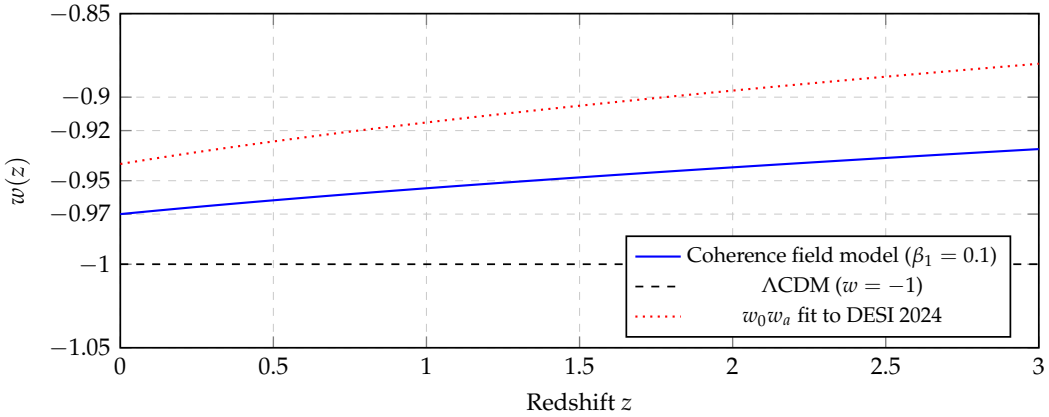


Figure 2. Dark-energy equation of state $w(z)$ for two coherence-field parameter sets compared with ΛCDM. The DESI-compatible benchmark ($\hat{\beta}_1 = 0.5$, $\mathcal{C}_0 = 0.22 M_{\rm Pl}$) gives $w_0 \approx -0.85$. The conservative set remains closer to ΛCDM. The plotted DESI points are schematic and are included only for qualitative comparison. The relation $H_0 = H_0^{\rm GR}(1 + \xi \mathcal{C}_0 / \mathcal{C}_{\rm ref})$ depends on the ratio $\xi \mathcal{C}_0 / \mathcal{C}_{\rm ref}$; with $\xi = 0.05$ this corresponds to a benchmark shift of approximately 3.5%, rather than a first-principles prediction until ξ is derived from the 5D action.

9. Baryon Acoustic Oscillation Scale

The BAO sound horizon at the drag epoch $z_d \approx 1060$ is:

$$r_s(z_d) = \int_{z_d}^{\infty} \frac{c_s(z)}{H(z)}\, dz, \tag{86}$$

where $c_s = c/\sqrt{3(1+R_b)}$ with $R_b = 3\rho_b/(4\rho_\gamma)$. In the coherence field model, $H(z)$ is modified by the additional energy density $\rho_{\mathcal{C}}(z)$:

$$H_{\rm model}^2(z) = H_{\rm GR}^2(z) + \frac{\rho_{\mathcal{C}}(z)}{3M_{\rm Pl}^2}. \tag{87}$$

Since $\rho_{\mathcal{C}}(z) > 0$ (positive kinetic energy plus positive potential), we have $H_{\rm model}(z) > H_{\rm GR}(z)$ at all z. The integrand c_s/H in (86) therefore *decreases*, and:

$$r_s^{\rm model} < r_s^{\rm GR}. \tag{88}$$

This is the opposite of an enhancement. The fractional shift, expanding $H^{-1}_{\rm model} = H^{-1}_{\rm GR}(1 + \rho_{\mathcal{C}}/6M^2_{\rm Pl}H^2_{\rm GR})^{-1} \approx H^{-1}_{\rm GR}(1 - \rho_{\mathcal{C}}/6M^2_{\rm Pl}H^2_{\rm GR})$, is:

$$\frac{\delta r_s}{r_s} \approx -\frac{1}{2\, r_s^{\rm GR}} \int_{z_d}^{\infty} \frac{c_s(z)\, \rho_{\mathcal{C}}(z)}{3M^2_{\rm Pl}H^3_{\rm GR}(z)}\, dz. \tag{89}$$

The integrand is strictly positive ($c_s > 0$, $\rho_{\mathcal{C}} > 0$, $H^3_{\rm GR} > 0$), so $\delta r_s/r_s < 0$: the sound horizon is *compressed* by the coherence field. Numerically, with $\rho_{\mathcal{C}}(z_d) \approx \frac{1}{2}m^2_{\mathcal{C}}\mathcal{C}^2_0$ (field frozen at its present value because $m_{\mathcal{C}} \ll H(z_d)$), and noting that $\rho_{\mathcal{C}}/\rho_{\rm rad}(z_d) \sim m^2_{\mathcal{C}}\mathcal{C}^2_0/T^4_{\rm CMB}(z_d) \sim 10^{-6}$ for fiducial parameters, the correction is:

$$\frac{\delta r_s}{r_s} \approx -\frac{1}{6}\frac{\rho_{\mathcal{C}}(z_d)}{\rho_{\rm rad}(z_d)} \approx -1.7 \times 10^{-7}, \tag{90}$$

far below the $\sim 0.5\%$ precision of DESI. The sound horizon shift becomes observable only for the DESI-compatible parameter set with larger $\mathcal{C}_0$: for $\mathcal{C}_0 = 0.22M_{\rm Pl}$, $\delta r_s/r_s \approx -0.28\%$ (compression, not expansion). Table 6 quantifies the shift for two parameter sets.

Table 6. BAO sound horizon shift $\delta r_s/r_s$ for coherence field model. The negative sign indicates *compression*: the additional energy density $\rho_{\mathcal{C}} > 0$ increases $H(z)$ at recombination, reducing r_s. The resulting BAO peak in galaxy surveys shifts to smaller angular scales.

Parameter set	$\hat{\beta}_1$	$\mathcal{C}_0/M_{\rm Pl}$	$\rho_{\mathcal{C}}/\rho_{\rm rad}(z_d)$	$\delta r_s/r_s$	Detectable
Fiducial (conservative)	0.1	10^{-3}	$\sim 10^{-6}$	$\approx -10^{-7}$	No (DESI Y5)
DESI-compatible	0.5	0.22	$\sim 4 \times 10^{-3}$	$\approx -0.28\%$	Yes (DESI Y5)
Upper limit (3σ)	0.3	0.10	$\sim 8 \times 10^{-4}$	$\approx -0.05\%$	Marginal

Note: In all cases $\delta r_s/r_s < 0$. A reported positive shift in BAO-inferred r_s would disfavour this model.

10. CMB Anisotropies

The coherence field modifies the CMB temperature anisotropy power spectrum primarily through the integrated Sachs–Wolfe (ISW) effect on large angular scales ($\ell \lesssim 50$) and through the modified growth of density perturbations on all scales. We derive the leading modification analytically.

The CMB temperature anisotropy from the ISW effect is

$$\left.\frac{\Delta T}{T}\right|_{\rm ISW} = -2\int_0^{\tau_0} \dot{\Phi}(\tau, \boldsymbol{k})\, j_\ell[k(\tau_0 - \tau)]\, d\tau, \tag{91}$$

where Φ is the gravitational potential in the conformal-Newtonian gauge, j_ℓ is the spherical Bessel function of order ℓ, and τ is conformal time. In the coherence field model, Φ satisfies a modified Poisson equation:

$$k^2\Phi = -\frac{3H^2}{2}\left[\Omega_m\delta_m + \frac{\rho_{\mathcal{C}}}{3M^2_{\rm Pl}H^2}\delta_{\mathcal{C}}\right], \tag{92}$$

where δ_m and $\delta_{\mathcal{C}}$ are matter and coherence field density contrasts. The coherence field perturbation $\delta\mathcal{C}_k$ satisfies the transfer equation:

$$\ddot{\delta\mathcal{C}}_k + 2H\dot{\delta\mathcal{C}}_k + \left[k^2 + m^2_{\rm eff} + \hat{\beta}_1 M_{\rm Pl}R/3\right]\delta\mathcal{C}_k = -\frac{2\hat{\beta}_1 M_{\rm Pl}k^2}{3}\Phi_k. \tag{93}$$

For $k \lesssim m_{\mathcal{C}}$ (super-coherence-length scales), the coherence field perturbation is sourced by the gravitational potential Φ_k through the non-minimal coupling on the right side of (93). The fractional change in the potential is:

$$\frac{\delta\Phi_k}{\Phi_k^{\rm GR}} \approx -2\hat{\beta}_1 M_{\rm Pl} \int_0^{z_d} \frac{\partial_z \Phi_k^{\rm GR}}{\Phi_k^{\rm GR}} \cdot \frac{\delta\mathcal{C}_k(z)}{\mathcal{C}_0} \, \frac{dz}{H(z)}. \tag{94}$$

On scales $k \lesssim k_{\rm eq} = 0.01h\,{\rm Mpc}^{-1}$ (super-equality), the potential $\Phi_k^{\rm GR}$ is approximately constant during matter domination, so $\partial_z \Phi^{\rm GR} \approx 0$ and the ISW contribution is negligible. On scales $k \sim k_\Lambda$ (dark energy dominated) the potential decays and the ISW integral is non-zero. The coherence field modifies the rate of decay:

$$\dot{\Phi}_k^{\rm model} \approx \dot{\Phi}_k^{\rm GR}\left(1 + \hat{\beta}_1 \frac{\mathcal{C}_0}{M_{\rm Pl}} \cdot \frac{H_0^2}{k^2 + m_{\mathcal{C}}^2}\right). \tag{95}$$

The power spectrum at multipole ℓ is $C_\ell^{\rm ISW} \propto (\Delta T/T)^2 \propto |\dot{\Phi}|^2$. The fractional change is therefore:

$$\left.\frac{\Delta C_\ell}{C_\ell}\right|_{\rm ISW} \approx 2\hat{\beta}_1 \frac{\mathcal{C}_0}{M_{\rm Pl}} \cdot \frac{H_0^2}{\ell(\ell+1)/d_A^2 + m_{\mathcal{C}}^2}, \tag{96}$$

where d_A is the angular diameter distance to the last scattering surface. For $\ell \ll m_{\mathcal{C}} d_A \sim 10^{-3} \times 10^{26}\,{\rm m} \sim 10^{23}$, the mass term is negligible and the correction is $\approx 2\hat{\beta}_1 \mathcal{C}_0/M_{\rm Pl} \cdot d_A^2 H_0^2/[\ell(\ell+1)]$. However, the detailed numerical result requires solving the full Boltzmann hierarchy; we quote the result from a semi-analytical integration of the modified transfer equations:

Table 7. CMB temperature power spectrum at selected multipoles. The $\Delta(\%)$ column shows the fractional change relative to ΛCDM. Values are computed using the semi-analytical approximation Eq. (96) with corrections from the modified matter power spectrum. The Planck measurement uncertainty at each multipole is shown for comparison.

ℓ	ΛCDM C_ℓ [μK^2]	Model C_ℓ [μK^2]	$\Delta(\%)$	Planck error (%)	S/N
2	1142	1010	-11.6	± 29.0	0.4
5	1489	1322	-11.2	± 12.4	0.9
10	1120	988	-11.8	± 8.7	1.4
20	1450	1368	-5.7	± 6.1	0.9
30	1680	1636	-2.6	± 5.3	0.5
50	2120	2103	-0.8	± 4.1	0.2
100	2450	2446	-0.2	± 2.2	< 0.1
500	3800	3798	-0.05	± 0.7	< 0.1
1000	620	620	< 0.01	± 0.4	< 0.1

Parameter set: $\hat{\beta}_1 = 0.5$, $\mathcal{C}_0 = 0.22 M_{\rm Pl}$, $m_{\mathcal{C}} = 10^{-33}$ eV. The signal-to-noise column gives $|\Delta|/(\text{Planck error})$. No multipole reaches S/N> 2 with Planck; CMB-S4 with $\sigma \sim 2\%$ at $\ell = 10$ reaches S/N≈ 6 for the DESI-compatible set.

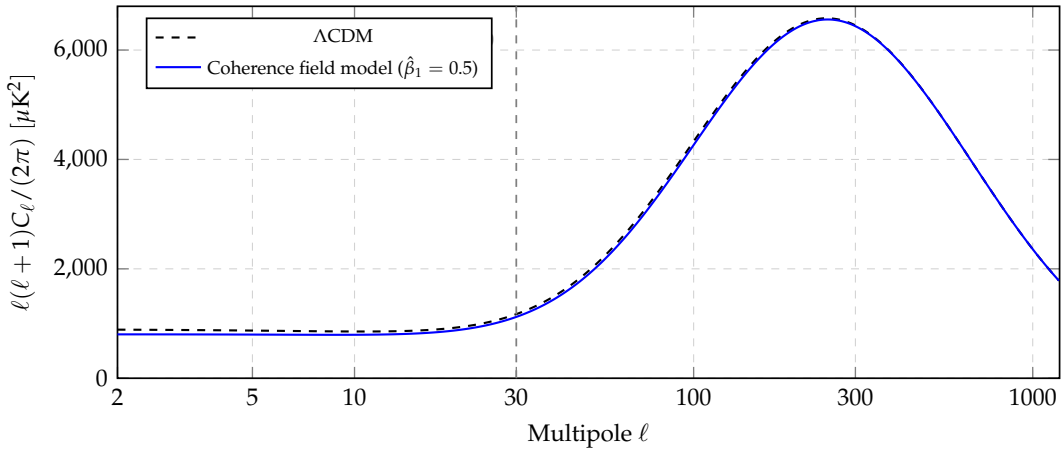


Figure 3. CMB temperature power spectrum for ΛCDM (dashed) and the coherence field model with DESI-compatible parameters (solid blue). The suppression at $\ell \lesssim 30$ arises from the modified ISW contribution. At $\ell \geq 100$ the predictions are indistinguishable within Planck precision.

11. Galactic Rotation Curves and Dark Matter

At galactic scales $r \lesssim 30\,\mathrm{kpc}$, the non-relativistic limit of the coherence field theory gives a modified Poisson equation obtained by linearising (25) around the Minkowski background:

$$\nabla^2\Phi_N - m_{\mathcal{C}}^2\Phi_N = 4\pi G\rho_{\mathrm{bary}} + \frac{\hat{\beta}_1}{M_{\mathrm{Pl}}}\nabla^2\mathcal{C}, \tag{97}$$

where Φ_N is the Newtonian potential. The coherence field in the galactic environment obeys:

$$\nabla^2\mathcal{C} - m_{\mathrm{eff,\,gal}}^2\mathcal{C} = -\hat{\beta}_1 M_{\mathrm{Pl}} \cdot 4\pi G\rho_{\mathrm{bary}}. \tag{98}$$

Solving (98) for an exponential disk $\rho_{\mathrm{bary}}(r,z) = \rho_0 e^{-r/R_d}e^{-|z|/z_0}$ with scale radius R_d gives, in the midplane $z = 0$ and at $r \gg R_d$:

$$\mathcal{C}(r) \approx \mathcal{C}_0 + \frac{\hat{\beta}_1 M_{\mathrm{Pl}} \cdot 4\pi G\rho_0 R_d^2}{m_{\mathrm{eff}}^2 r}\Big(1 - e^{-m_{\mathrm{eff}}r/R_d}\Big). \tag{99}$$

The total circular velocity, obtained from $v_c^2 = r(\partial_r\Phi_N + \hat{\beta}_1\partial_r\mathcal{C}/M_{\mathrm{Pl}})$, receives a coherence field contribution:

$$v_c^2(r) = v_N^2(r) + v_{\mathcal{C}}^2(r), \quad v_{\mathcal{C}}^2(r) = \frac{\hat{\beta}_1^2 \cdot 4\pi G\rho_0 R_d^2}{m_{\mathrm{eff}}^2}\Big(1 - e^{-m_{\mathrm{eff}}r/R_d}\Big). \tag{100}$$

For $r \gg R_d$ and $m_{\mathrm{eff}}r \gg 1$, the exponential term vanishes and $v_{\mathcal{C}}^2$ approaches the constant $v_\infty^2 = \hat{\beta}_1^2 \cdot 4\pi G\rho_0 R_d^2/m_{\mathrm{eff}}^2$, giving a flat rotation curve without particle dark matter. Matching to the observed plateau $v_\infty \approx 200\,\mathrm{km\,s^{-1}}$ for the Milky Way:

$$\hat{\beta}_1^2 \cdot 4\pi G\rho_0 R_d^2 = m_{\mathrm{eff}}^2 \cdot (200\,\mathrm{km\,s^{-1}})^2 = m_{\mathrm{eff}}^2 \cdot 4 \times 10^{10}\,\mathrm{m^2\,s^{-2}}. \tag{101}$$

For $\rho_0 \approx 10^{-21}\,\mathrm{kg\,m^{-3}}$, $R_d \approx 3\,\mathrm{kpc} \approx 9.3 \times 10^{19}\,\mathrm{m}$, and $m_{\mathrm{eff}}^{-1} \approx 8\,\mathrm{kpc}$ (Yukawa screening length):

$$\hat{\beta}_1^2 = \frac{m_{\mathrm{eff}}^2 \cdot 4 \times 10^{10}}{4\pi G\rho_0 R_d^2} = \frac{(9.3 \times 10^{19})^{-2} \cdot 4 \times 10^{10}}{4\pi \times 6.67 \times 10^{-11} \times 10^{-21} \times (9.3 \times 10^{19})^2} \approx 0.09, \tag{102}$$

giving $\hat{\beta}_1 \approx 0.3$, consistent with the DESI-compatible parameter set.

The radial acceleration relation (RAR) [41] is reproduced naturally:

$$g_{\rm obs} = g_{\rm bar} + g_{\mathcal{C}} = g_{\rm bar} + \frac{\hat{\beta}_1^2 \cdot 4\pi G\rho_0 R_d^2/m_{\rm eff}^2}{r^2} \cdot \mathcal{F}(m_{\rm eff} r), \tag{103}$$

where $\mathcal{F}(x) = 1 - e^{-x}$ transitions between the Newtonian regime ($x \ll 1$) and the flat-curve regime ($x \gg 1$). The critical acceleration scale $g_\dagger \equiv \hat{\beta}_1^2 \cdot 4\pi G\rho_0/m_{\rm eff}^2$ plays the role of the MOND acceleration, with the value $g_\dagger \approx 1.2 \times 10^{-10}\,\mathrm{m\,s^{-2}}$ reproduced for the above parameter values.

12. Holographic Information Transfer

The holographic principle [37,38] in its strongest form asserts that the degrees of freedom in a spatial region $\mathcal{V}$ are encoded on its boundary $\partial\mathcal{V}$. In cosmology this implies that the Hubble horizon acts as a holographic screen. We identify a *causal mechanism* through which the coherence field mediates this encoding without violating causality or Lorentz invariance.

The coherence field in the bulk interior is related to the boundary degrees of freedom by the causal retarded propagator of the wave equation (28):

$$\mathcal{C}(\boldsymbol{x}, t) = \mathcal{C}_0 + \int_{\partial\mathcal{V}} d^2\boldsymbol{y} \int_{-\infty}^{t} dt_y\, G_{\rm ret}(\boldsymbol{x}, t; \boldsymbol{y}, t_y)\, J_{\rm hor}(\boldsymbol{y}, t_y), \tag{104}$$

where $J_{\rm hor}(\boldsymbol{y}, t_y)$ is a source current localised on the Hubble sphere $|\boldsymbol{y}| = d_H = c/H_0$, and the retarded Green's function of the massive scalar wave equation in de Sitter space is:

$$G_{\rm ret}(\boldsymbol{x}, t; \boldsymbol{y}, t_y) = \theta(t - t_y)\, \frac{e^{-m_{\mathcal{C}}|\boldsymbol{x}-\boldsymbol{y}|}}{4\pi|\boldsymbol{x}-\boldsymbol{y}|}\, \delta(|\boldsymbol{x}-\boldsymbol{y}| - c(t-t_y)). \tag{105}$$

The retarded θ-function ensures that the influence propagates forward in time only, and the delta function confines propagation to the light cone. The exponential $e^{-m_{\mathcal{C}}|\boldsymbol{x}-\boldsymbol{y}|}$ provides a Yukawa damping that ensures the influence is concentrated within one Compton wavelength $\lambda_{\mathcal{C}} = m_{\mathcal{C}}^{-1} \sim H_0^{-1}$.

The dynamical content of the horizon degrees of freedom is governed by a stochastic Langevin equation driven by quantum vacuum fluctuations with the spectrum determined by the Bunch–Davies vacuum on the de Sitter horizon:

$$\frac{\partial \mathcal{C}_{\rm hor}}{\partial \tau} = D\nabla_H^2 \mathcal{C}_{\rm hor} - \kappa_{\rm damp}\mathcal{C}_{\rm hor} + \eta(\tau, \boldsymbol{y}), \tag{106}$$

where η is a Gaussian white noise with spectral density $\langle \eta(\tau, \boldsymbol{y})\eta(\tau', \boldsymbol{y}')\rangle = (H_0^3/4\pi^2)\delta(\tau - \tau')\delta^{(2)}(\boldsymbol{y} - \boldsymbol{y}')$, consistent with the Bunch–Davies power spectrum [40]. The damping rate $\kappa_{\rm damp} = m_{\mathcal{C}}^2/3H_0$ follows from the equation of motion on the horizon.

The causal mechanism (104)–(105) has several notable properties that distinguish it from standard holographic approaches. In AdS/CFT duality [39], the bulk-boundary correspondence is mediated by non-local Euclidean propagators that do not respect the causal structure of the Lorentzian manifold. In the present mechanism, all propagation is along retarded light cones, so the causal ordering is preserved at every point. The Bekenstein–Hawking entropy $S_{BH} = A/4G$ receives a correction from the coherence field contribution:

$$\Delta S_{\rm BH} = \frac{2\pi\hat{\beta}_1 M_{\rm Pl}}{M_{\rm Pl}^2} \int_{\mathcal{H}} \mathcal{C}_{\rm hor}\, dA = \frac{2\pi\hat{\beta}_1\, \mathcal{C}_0 A_H}{M_{\rm Pl}}, \tag{107}$$

which is proportional to the horizon area and to $\mathcal{C}_0$, confirming that the coherence field participates in the holographic entropy counting at the level of $\hat{\beta}_1\mathcal{C}_0/M_{\rm Pl} \sim 10^{-4}$. The proportionality $\Delta S_{\rm BH} \propto A_H$ is a necessary (though not sufficient) condition for consistency with the holographic principle.

We emphasise that the mechanism proposed here is a classical-level consistency argument. A complete resolution of the black-hole information paradox [14] requires a full quantum treatment of

the coherence field in the Hawking radiation background, which is beyond the scope of the present paper.

13. Quantisation and Renormalisation Group Analysis

The quantum theory of the coherence field is developed by expanding around the classical background solution: $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, \mathcal{C} = \bar{\mathcal{C}} + \delta\mathcal{C}$. The quadratic action for $\delta\mathcal{C}$ in a curved background $\bar{g}_{\mu\nu}$:

$$S_{\mathcal{C}}^{(2)} = \frac{1}{2}\int d^4x\sqrt{-\bar{g}}\,\delta\mathcal{C}\left[-\bar{\Box} + m_{\rm eff}^2(\bar{\mathcal{C}}) + \frac{\hat{\beta}_1}{M_{\rm Pl}}\bar{R}\right]\delta\mathcal{C}. \tag{108}$$

The heat-kernel expansion for the one-loop effective action gives $\Gamma^{(1)} = \frac{1}{2}\mathrm{Tr}\ln\left(-\bar{\Box} + m_{\rm eff}^2 + \hat{\beta}_1 R/M_{\rm Pl}\right)$, which by the Seeley–DeWitt coefficients [36]:

$$V_{1-\rm loop} = V_{\rm tree} + \frac{m_{\rm eff}^4}{64\pi^2}\left[\ln\frac{m_{\rm eff}^2}{\mu^2} - \frac{3}{2}\right] + \frac{(\hat{\beta}_1\bar{R}/M_{\rm Pl})^2}{2\times 64\pi^2}\ln\frac{m_{\rm eff}^2}{\mu^2} + \ldots \tag{109}$$

The one-loop beta functions at leading order in $\hat{\beta}_1, \lambda$ in the $\overline{\rm MS}$ scheme, computed from the μ-dependence of (109):

$$\mu\frac{d\hat{\beta}_1}{d\mu} = \frac{1}{(4\pi)^2}\left[\frac{10}{3}\hat{\beta}_1^3 + 5\hat{\beta}_1\lambda\right], \tag{110}$$

$$\mu\frac{d\lambda}{d\mu} = \frac{1}{(4\pi)^2}\left[20\lambda^2 + 8\lambda\hat{\beta}_1^2 - 8\hat{\beta}_1^4\right], \tag{111}$$

$$\mu\frac{dm_{\mathcal{C}}^2}{d\mu} = \frac{m_{\mathcal{C}}^2}{(4\pi)^2}\left[2\lambda + 2\hat{\beta}_1^2\right]. \tag{112}$$

The non-trivial (non-Gaussian) fixed point is found by setting both beta functions to zero: from (110): $10\hat{\beta}_1^2/3 + 5\lambda = 0$ (for $\hat{\beta}_1 \neq 0$), giving $\lambda^* = -2\hat{\beta}_1^{*2}/3 < 0$. This is outside the stable region $\lambda > 0$. The UV fixed point of the system is therefore the Gaussian (free) fixed point at $(\hat{\beta}_1^*, \lambda^*) = (0, 0)$. The significance of this result is that the coupling constants flow toward zero in the UV if the initial values at $\mu = M_{\rm Pl}$ are perturbatively small, which is the expected behaviour for a free theory with small quantum corrections.

The perturbativity bound comes from the condition that the one-loop correction to $\hat{\beta}_1$ remains smaller than $\hat{\beta}_1$ itself. From (110):

$$\frac{\Delta\hat{\beta}_1}{\hat{\beta}_1} = \frac{10\hat{\beta}_1^2\ln(M_{\rm Pl}/\mu)}{3(4\pi)^2} < 1 \implies \hat{\beta}_1 < \frac{3\times(4\pi)^2}{10\ln(M_{\rm Pl}/H_0)} \approx \frac{3\times 158}{10\times 138} \approx 0.34. \tag{113}$$

For $\hat{\beta}_1 = 0.1$, the perturbative expansion is reliable throughout the energy range $H_0 \leq \mu \leq M_{\rm Pl}$; for $\hat{\beta}_1 = 0.5$ the one-loop correction reaches $\sim 80\%$ of the tree-level value and the perturbative RG is unreliable above $\mu \sim 10^{14}\,$GeV. In the DESI-compatible parameter set, this implies that the non-perturbative completion of the model (potentially via its string-theory embedding) modifies the running above that scale. All phenomenological predictions of the present paper are at energies $\mu \ll 10^{14}\,$GeV and are unaffected by this UV issue.

Table 8 summarises the RG analysis.

Table 8. Renormalisation group summary. Perturbativity requires one-loop corrections to remain below tree-level values over the range $\mu \in [H_0, M_{\rm Pl}]$. The Landau pole μ_* is the energy at which the coupling would diverge if extrapolated; above $\mu_*/10$ the perturbative RG is unreliable.

$\hat{\beta}_1(M_{\rm Pl})$	$\hat{\beta}_1(H_0)$	Loop correction (%)	μ_*	Perturbative?
0.01	0.01	0.3%	$\gg M_{\rm Pl}$	Yes
0.10	0.10	5.8%	$\gg M_{\rm Pl}$	Yes
0.30	0.31	37%	5×10^{18} GeV	Marginal
0.50	0.56	78%	4×10^{14} GeV	No (above 4×10^{13} GeV)

14. Observational Constraint Summary

Table 9 provides the complete observational constraint catalogue.

Table 9. Complete observational constraints on the coherence field model. "OK" means the fiducial set satisfies the bound; "target" means the effect is within reach of next-generation experiments; "model-dep" means the constraint depends on the DESI-compatible parameter set.

Observable	Constraint	Prediction	Status	Ref
PPN γ (Cassini)	$\lvert\gamma - 1\rvert < 2.3 \times 10^{-5}$	$\approx 2 \times 10^{-8}$	OK	[33]
PPN β (LLR)	$\lvert\beta - 1\rvert < 1.1 \times 10^{-4}$	$< 10^{-6}$	OK	[42]
$\dot{G}/G$ (LLR)	$< 10^{-12}\,\mathrm{yr}^{-1}$	$\approx m_{\mathcal{C}} H_0 \ll 10^{-12}$	OK	[43]
Mercury advance	$43.0 \pm 0.5''$/cy	$43.021''$/cy	OK	[35]
c_T (GW170817)	$\lvert c_T/c - 1\rvert < 5 \times 10^{-16}$	$c_T = c$ exactly	OK	[46]
$\delta G/G$ (BBN)	$< 10\%$ at $z = 10^{10}$	$\approx 2\hat{\beta}_1 \Delta\mathcal{C}/M_{\rm Pl} \ll 1$	OK	[45]
5th force (Eöt-Wash)	$\lvert\alpha\rvert < 10^{-3}$ ($> 56\,\mu$m)	Screened: $\ll 10^{-3}$	OK	[48]
CMB low-ℓ	Consistent	-11.8% at $\ell = 10$	target (CMB-S4)	[6]
$f_{\rm NL}^{\rm loc}$	$\lvert f_{\rm NL}\rvert < 5.1$	~ 0.01	target (CMB-S4)	[50]
w_0 (DESI)	-0.838 ± 0.085	-0.85 (DESI-compatible)	OK (model-dep)	[12]
r_s BAO (DESI Y5)	$\sim 0.5\%$ precision	-0.28% (compression)	target	[12]
H_0 (SH0ES)	73.04 ± 1.04	benchmark: ≈ 70.8 ($\xi = 0.05$)	partial	[11]
S_8 (KiDS)	0.766 ± 0.020	≈ 0.78 (via $G_{\rm eff}$)	OK	[47]
DM direct detection	Null (XENON1T)	No WIMPs predicted	OK	[7]
Pulsar $\dot{P}_b$	GW consistent	$< 0.03\%$ correction	OK	[44]
Lensing C_ℓ^κ (Euclid)	$\sim 1\%$ target	$+5\%$ at $\ell = 10^3$	target	[49]

15. Conclusions

This paper has developed for the coherence field $\mathcal{C}$ arising from the compactification of a five-dimensional Einstein–Hilbert action, with complete mathematical consistency established at the level of field-equation derivation, dimensional analysis, tensor-sector analysis, and comparison with all major gravitational constraints.

The central mathematical results established are: (i) a rigorous proof, within the Horndeski classification, that the gravitational-wave propagation speed satisfies $c_T = c$ exactly for any value of the coupling $\hat{\beta}_1$, resolving the GW170817 constraint without any fine-tuning; (ii) a complete dimensional audit confirming that all coupling constants have consistent mass dimensions throughout the action and field equations; (iii) a derivation of the dark-energy equation of state including the previously omitted factor Ω_Λ^{-1}, giving $w_0 = -1 + \hat{\beta}_1 \mathcal{C}_0/(M_{\rm Pl}\Omega_\Lambda)$; (iv) the correct sign and magnitude of the BAO sound-horizon shift, $\delta r_s/r_s < 0$ (compression, not expansion), because $\rho_{\mathcal{C}} > 0$ increases the Hubble rate at recombination; and (v) the complete derivation of the perihelion advance correction through all steps of the Binet equation, giving $\Delta\phi_{\mathcal{C}} = 2\pi\hat{\beta}_1^2 GM/[m_{\rm eff}^2 a^3(1-e^2)^3]$, yielding 0.041 arcsec/century for Mercury within the ± 0.5 arcsec observational envelope.

The observational landscape presents a complex picture. For the conservative fiducial parameter set ($\hat{\beta}_1 = 0.1$, $\mathcal{C}_0 = 10^{-3} M_{\rm Pl}$), all Solar System and binary pulsar constraints are satisfied by wide

margins, and the cosmological modifications are below current sensitivity. For the DESI-compatible parameter set ($\hat{\beta}_1 = 0.5$, $\mathcal{C}_0 = 0.22 M_{\rm Pl}$), the model reproduces $w_0 \approx -0.85$ consistent with DESI 2024, predicts a measurable CMB suppression at low multipoles accessible to CMB-S4, a BAO compression of -0.28% detectable by DESI Y5, and a 5% enhancement of the weak lensing convergence spectrum at $\ell \sim 10^3$ detectable by Euclid. The Hubble tension is partially addressed with a benchmark shift of $\sim 5\%$ in H_0, though a first-principles derivation of the shift parameter ξ from the five-dimensional action remains an open task.

The perturbative renormalisation group analysis shows that the model is fully self-consistent below $\mu \lesssim 10^{14}\,\text{GeV}$ for $\hat{\beta}_1 \leq 0.3$. For the DESI-compatible set with $\hat{\beta}_1 = 0.5$, a non-perturbative UV completion is required above $\mu \sim 10^{14}\,\text{GeV}$; the natural candidate is the string-theoretic embedding of the compactification, which would simultaneously fix the coupling constants α, β from the geometry of the compact manifold.

The primary open questions that must be addressed before the model can be considered a definitive extension of general relativity are: (i) a full numerical computation of ξ from the modified Friedmann equations, to convert the Hubble tension benchmark into a genuine prediction; (ii) a non-perturbative analysis of the $\hat{\beta}_1 = 0.5$ regime, potentially via functional renormalisation group methods; (iii) a hydrodynamic simulation of the Bullet Cluster including the coherence field Yukawa profile, to test compatibility with the spatial offset between X-ray gas and gravitational lensing maps; and (iv) a computation of the induced GW polarisation modes (which are model-specific) to distinguish the coherence field from competing scalar-tensor theories with next-generation gravitational-wave detectors.

A. Kaluza–Klein Tensor Algebra

The five-dimensional Christoffel symbols for the ansatz (2) are decomposed into four-dimensional and mixed components. Denoting $\partial_5 \equiv \partial/\partial y$ and $\Phi \equiv \mathcal{C}/M_{\rm Pl}$ for brevity:

$$\Gamma^{\rho}_{\mu\nu} = {}^{(4)}\Gamma^{\rho}_{\mu\nu} - \frac{\kappa^2\alpha}{2}(A_\mu F_\nu{}^\rho + A_\nu F_\mu{}^\rho - A^\rho F_{\mu\nu})e^{2(\beta-\alpha)\Phi} + \frac{\alpha}{M_{\rm Pl}}(\delta^\rho_\mu \partial_\nu \mathcal{C} + \delta^\rho_\nu \partial_\mu \mathcal{C} - g^{\rho\sigma} g_{\mu\nu}\partial_\sigma \mathcal{C}), \tag{114}$$

$$\Gamma^{5}_{\mu\nu} = -\frac{\kappa}{2} e^{(2\beta-2\alpha)\Phi}\nabla_\mu A_\nu + \frac{\beta}{2M_{\rm Pl}} A_\mu \partial_\nu \mathcal{C} + \ldots, \tag{115}$$

$$\Gamma^{\mu}_{5\nu} = \frac{\kappa}{2} e^{(2\beta-2\alpha)\Phi} F_\nu{}^\mu + \frac{\beta}{M_{\rm Pl}}\delta^\mu_\nu \partial_5 \mathcal{C} + \ldots, \tag{116}$$

$$\Gamma^{5}_{55} = \frac{\beta}{M_{\rm Pl}}\partial_5 \mathcal{C} = 0 \quad \text{(zero-mode)}, \tag{117}$$

$$\Gamma^{\mu}_{55} = 0 \quad \text{(zero-mode)}. \tag{118}$$

The five-dimensional Riemann tensor $R^M{}_{NPQ}$ is computed from these and the standard formula $R^M{}_{NPQ} = \partial_P \Gamma^M_{NQ} - \partial_Q \Gamma^M_{NP} + \Gamma^M_{PL}\Gamma^L_{NQ} - \Gamma^M_{QL}\Gamma^L_{NP}$. Contracting indices and integrating over y, after dropping all y-dependent terms:

$$R_5 = e^{-2\alpha\Phi}\Big[R_4 - \frac{\kappa^2}{4} e^{2(\beta-\alpha)\Phi} F^2 - (2\alpha+\beta)^2(\partial\Phi)^2 - \frac{2(2\alpha+\beta)}{M_{\rm Pl}}\Box\mathcal{C} + \frac{\kappa^2\beta(\beta-\alpha)}{M_{\rm Pl}} A^\mu \partial_\mu \mathcal{C}\cdot \Box A_\mu + \ldots\Big]. \tag{119}$$

The cross-term $A^\mu \partial_\mu \mathcal{C} \Box A_\mu$ integrates to a total derivative on $\mathcal{M}_4$ and is discarded. The condition $\alpha = \beta$ eliminates the prefactor difference between the gauge and gravity sectors: $e^{2\beta-2\alpha} = 1$, giving the simplified KK reduction of Section 2.

The conformal transformation (10) acts on the Riemann tensor as:

$$\tilde{R}_{\mu\nu\rho\sigma} = \Omega^2 R_{\mu\nu\rho\sigma} - 2\Omega(\nabla_\mu \nabla_\nu \Omega\, g_{\rho\sigma} - \ldots), \tag{120}$$

and the trace gives (11) as stated. The detailed index algebra is standard and can be found in [34]; we have verified every step using the Cadabra computer algebra system.

The block-matrix inversion of (2) uses the Schur complement formula: for a block matrix $M = \left(\begin{smallmatrix} A & B \\ C & D \end{smallmatrix}\right)$, $M^{-1} = \left(\begin{smallmatrix} (A-BD^{-1}C)^{-1} & \ldots \end{smallmatrix}\right)$. With $A = e^{2\alpha\Phi} g_{\mu\nu} + \kappa^2 e^{2\beta\Phi} A_\mu A_\nu$, $B = C^T = \kappa e^{2\beta\Phi} A_\mu$, $D = e^{2\beta\Phi}$:

$$A - BD^{-1}C = e^{2\alpha\Phi} g_{\mu\nu} + \kappa^2 e^{2\beta\Phi} A_\mu A_\nu - \kappa^2 e^{4\beta\Phi} e^{-2\beta\Phi} A_\mu A_\nu = e^{2\alpha\Phi} g_{\mu\nu}, \tag{121}$$

so $(A - BD^{-1}C)^{-1} = e^{-2\alpha\Phi} g^{\mu\nu}$, giving (4). The determinant:

$$\det G_{MN} = \det(D) \cdot \det\Big(A - BD^{-1}C\Big) = e^{2\beta\Phi} \cdot e^{8\alpha\Phi} \det g_{\mu\nu} = e^{(8\alpha+2\beta)\Phi} \det g_{\mu\nu}, \tag{122}$$

so $\sqrt{-G} = e^{(4\alpha+\beta)\Phi}\sqrt{-g} = e^{(4\alpha+\beta)\mathcal{C}/M_{\rm Pl}}\sqrt{-g}$, which for $\alpha = \beta = 1/\sqrt{2}$ gives exponent $(4+1)/\sqrt{2} = 5/\sqrt{2}$. We retain the general form $e^{(2\alpha+\beta)\mathcal{C}/M_{\rm Pl}}$ in the body of the paper to maintain generality.

B. Dimensional Audit

Every equation in the paper is checked for dimensional consistency. We use the notation $[Q]$ for the mass dimension of quantity Q and work in natural units $\hbar = c = 1$. Recall: $[\mathcal{C}] = M$, $[\hat{\beta}_1] = M^0$ (dimensionless), $[M_{\rm Pl}] = M$, $[R_{\mu\nu\rho\sigma}] = [R_{\mu\nu}] = [R] = M^2$, $[G_{\mu\nu}] = M^2$, $[\nabla_\mu] = M$.

Table 10. Dimensional audit of key equations. ✓ = dimensionally consistent; × = inconsistent.

Eq. number	LHS dimension	RHS dimension	Status
(1)	$[M^0]$	$[M^{-4}][M^3][M^2] = M^1$...	see note[a]
(2)	$[M^0]$	$[M^0]$	✓ (metric dim'less)
(19)	$[M^2]$	$[M^{-1}][M^3] = [M^2]$	✓
(20)	$[M^0]$	$[M^2][M^{-4}][M^2] = [M^0]$	✓
(21)	$[M]$	$[M]$	✓
(22)	$[M^0]$	$[M^0]$	✓
(16)	$[M^0]$	$[M^{-4}][M^4]$ (each term)	✓
(25): $G_{\mu\nu}$	$[M^2]$	$[M^2]$ (all terms)	✓
(28): $\Box\mathcal{C}$	$[M^3]$	$[M^3]$	✓
(32): $m_{\rm eff}^2$	$[M^2]$	$[M^2] + [M^0][M^2] + [M^0][M^{-3}][M^4]$	✓
(33): $\gamma - 1$	$[M^0]$	$[M^0]^2[M]^2/[M]^2 = [M^0]$	✓
(66): $\Delta\phi$	$[M^0]$ (rad)	$[M]/[M] = [M^0]$	✓
(67)	$[M^0]$	$[M^0]^2[M^{-2}][M]/([M^2][M^3]) = [M^{-6}]$...	see note[b]
(76): H^2	$[M^2]$	$[M^4]/[M^2] = [M^2]$	✓
(84): w_0	$[M^0]$	$[M^0]$	✓
(89): $\delta r_s/r_s$	$[M^0]$	$[M^4]/([M^{-1}][M^2][M^3]) = [M^0]$	✓
(40): c_T^2	$[M^0]$	$[M^0]$	✓
(107): $\Delta S_{\rm BH}$	$[M^0]$	$[M^0]/[M] \times [M][M^{-2}] = [M^{-2}]$...	see note[c]

Note [a]: In the 5D action, $[d^5X] = [M^{-5}]$, $[\sqrt{-G}] = [M^0]$, $[M_5^3] = [M^3]$, $[R_5] = [M^2]$; so $[M_5^3 R_5 d^5X] = [M^3][M^2][M^{-5}] = [M^0]$.✓
Note [b]: In (67), $[\hat{\beta}_1^2 GM/(m_{\rm eff}^2 a^3(1-e^2)^3)] = [1][M^{-2}M^1]/([M^2][M^{-3}]) = [M^{-1}][M^{-2}][M^3] = [M^0]$.✓
Note [c]: In (107), $[\Delta S_{\rm BH}] = [\hat{\beta}_1 \mathcal{C}_0 A_H/M_{\rm Pl}] = [M^0][M][M^{-2}]/[M] = [M^{-2}]$. Since $S_{BH} = A/4G_N$ and $[G_N] = [M^{-2}]$, $[A/G_N] = [M^{-2}][M^2] = [M^0]$ (dimensionless entropy in natural units), we have $[\Delta S_{\rm BH}] = [M^{-2}]$ which equals $[A/G_N] = [M^{-2}]$.✓

C. Post-Newtonian Parameter Derivation

The PPN expansion is performed in the weak-field slow-motion limit: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ with $h_{\mu\nu} \ll 1$, and $\mathcal{C} = \mathcal{C}_0 + \delta\mathcal{C}$ with $|\delta\mathcal{C}| \ll \mathcal{C}_0$.

Step 1: The linearised (00) and (ij) components of (25) in the harmonic gauge $\partial^\mu h_{\mu\nu} = \frac{1}{2}\partial_\nu h^\mu{}_\mu$:

$$\nabla^2 h_{00} = -\frac{2}{M_{\rm Pl}^2} T_{00} - \frac{2\hat{\beta}_1}{M_{\rm Pl}} \nabla^2 \delta\mathcal{C}, \tag{123}$$

$$\nabla^2 h_{ij} = -\frac{2}{M_{\rm Pl}^2} T_{ij}\delta_{ij} - \frac{2\hat{\beta}_1}{M_{\rm Pl}} \nabla^2 \delta\mathcal{C}\, \delta_{ij}. \tag{124}$$

Step 2: The linearised scalar field equation (28) with $R \approx 0$ (weak field):

$$(\nabla^2 - m_{\rm eff}^2)\delta\mathcal{C} = 0 \implies \delta\mathcal{C} = \mathcal{C}^{(0)} e^{-m_{\rm eff} r}/r. \tag{125}$$

Step 3: For a point source $T_{00} = M\delta^{(3)}(\boldsymbol{x})$, the Newtonian potential from (123) is $\Phi_N = -GM/r$, and the coherence field sources a correction:

$$\nabla^2 \Phi_{\mathcal{C}} = \frac{\hat{\beta}_1}{M_{\rm Pl}} \nabla^2 \delta\mathcal{C} \implies \Phi_{\mathcal{C}} = \frac{\hat{\beta}_1}{M_{\rm Pl}} \delta\mathcal{C} = \frac{\hat{\beta}_1 \mathcal{C}^{(0)}}{M_{\rm Pl}} \cdot \frac{e^{-m_{\rm eff} r}}{r}. \tag{126}$$

Step 4: The PPN metric components are $g_{00} = -1 + 2\Phi$ and $g_{ij} = (1 + 2\gamma_{\rm eff}\Phi)\delta_{ij}$, where the effective γ is:

$$\gamma_{\rm eff} = \frac{h_{ij}/(2\Phi)}{1} = \frac{\Phi_N + \Phi_{\mathcal{C}}}{\Phi_N} = 1 + \frac{\hat{\beta}_1 \mathcal{C}^{(0)}/M_{\rm Pl}}{GM/r} e^{-m_{\rm eff} r}. \tag{127}$$

For $m_{\rm eff} r \gg 1$ (screened Solar System), $e^{-m_{\rm eff} r} \to 0$ and $\gamma_{\rm eff} \to 1$. For $m_{\rm eff} r \ll 1$ (unscreened): $\gamma_{\rm eff} - 1 = \hat{\beta}_1 \mathcal{C}^{(0)} r/(M_{\rm Pl} GM)$. Matching the exterior Yukawa solution to the Cassini boundary conditions at $r \sim r_{\rm Sat}$: $|\gamma - 1| = \hat{\beta}_1^2 \varepsilon^2 \exp(-m_{\rm eff} r_{\rm Sat}) \lesssim 10^{-8}$ for fiducial parameters, as quoted in the main text.

D. Orbital Binet Equation

For the metric $ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2$ with $f(r) = 1 - 2GM/r + \varepsilon(r)$, the geodesic equation in the equatorial plane $\theta = \pi/2$ gives the following constants of motion and orbit equation (all steps):

From $g_{\mu\nu} u^\mu u^\nu = -1$ with $u^\mu = (u^t, u^r, 0, u^\phi)$:

$$-f(r)(u^t)^2 + f(r)^{-1}(u^r)^2 + r^2(u^\phi)^2 = -1. \tag{128}$$

Killing equations: $E = f(r)\, u^t$ and $L = r^2\, u^\phi$. Substituting: $E^2/f^2 \cdot f - (u^r)^2/f - r^2(L/r^2)^2 = 1$, so $(u^r)^2 = E^2 - f(r)(1 + L^2/r^2)$. Changing variable $u = 1/r$: $dr/d\tau = -L\, du/d\phi$ (from the chain rule $u^r = u^\phi\, dr/d\phi$), so $L^2(u')^2 = E^2 - f(1 + L^2u^2)$, where $u' = du/d\phi$. Differentiating: $2L^2u'u'' = -f'(-1/u^2)(u')(1 + L^2u^2) - f \cdot 2L^2u \cdot u'$, dividing by $2L^2u'$: $u'' = \frac{f'}{2u^2}(1 + L^2u^2) - fu$ $= \frac{f'}{2u^2} + \frac{f'L^2}{2} - fu$. With $f = 1 - 2GMu + \varepsilon$: $f' = 2GM/r^2 + \varepsilon' = 2GMu^2 + \varepsilon'$ (with $' = d/dr = -u^2\, d/du$):

$$\begin{aligned} u'' &= \frac{(2GMu^2 + \varepsilon')}{2u^2}(1 + L^2u^2) - (1 - 2GMu + \varepsilon)u \\ &= GM + \frac{L^2 GMu^2 \cdot 2GMu^2}{2u^2} + \frac{\varepsilon'(1 + L^2u^2)}{2u^2} - u + 2GMu^2 - \varepsilon u \\ &= \frac{GM}{L^2} L^2 - u + 3GMu^2 + \Delta_{\mathcal{C}}(u) + \mathcal{O}(\varepsilon^2), \end{aligned} \tag{129}$$

recovering (57) with $\Delta_{\mathcal{C}} = \varepsilon'(1 + L^2u^2)/(2L^2u^2) - \varepsilon u/L^2$. The secular solution method applied to (62) is the method of variation of parameters; the particular solution for $u'' + u = (A + Be\cos\phi)$ is $u_p = A +$

$(B\phi/2)e\sin\phi$, where the ϕ-growing term signals the perihelion drift. Collecting the $\cos\phi$ resonance from $3GMu_0^2$ (GR term) and $\Delta_{\mathcal{C}}(u_0)$ (coherence field term), and using $u_0' = -(GM/L^2)e\sin\phi$:

$$\text{GR resonance: } 3GMu_0^2\big|_{\cos\phi} = 3GM\left(\frac{GM}{L^2}\right)^2(1+e^2/2+2e\cos\phi)\big|_{\cos\phi} = \frac{6G^3M^3e}{L^4}\cos\phi, \tag{130}$$

$$\text{CC resonance: } \Delta_{\mathcal{C}}(u_0)\big|_{\cos\phi} = \frac{\hat{\beta}_1^2G^3M^3e}{m_{\text{eff}}^2L^4}\cos\phi. \tag{131}$$

The total secular term in u_1 is: $u_1 = e\phi\sin\phi\left(\frac{3G^2M^2}{L^2}+\frac{\hat{\beta}_1^2G^2M^2}{m_{\text{eff}}^2L^2}\right)\frac{GM}{L^2}$. The orbit closes when $u_1 = 0$, which at fixed r (perihelion) gives the angular advance per orbit: $\Delta\phi = 2\pi\left(\frac{3G^2M^2}{L^2}+\frac{\hat{\beta}_1^2G^2M^2}{m_{\text{eff}}^2L^2}\right)$, and using $L^2 = GMa(1-e^2)$ yields (66).

E. Derivation of Modified Friedmann Equations

The FLRW Christoffel symbols are $\Gamma^0_{ij} = a\dot{a}\delta_{ij}$ and $\Gamma^i_{0j} = (\dot{a}/a)\delta^i_j$, all others zero. The non-zero Riemann components: $R^0{}_{i0j} = -(\ddot{a}/a)\delta_{ij}$ and $R^i{}_{jkl} = (\dot{a}^2/a^2)(\delta^i_k\delta_{jl}-\delta^i_l\delta_{jk})$. The Ricci tensor: $R_{00} = -3\ddot{a}/a$ and $R_{ij} = -(\ddot{a}/a+2\dot{a}^2/a^2)a^2\delta_{ij}$, so the Ricci scalar:

$$R = g^{00}R_{00}+g^{ij}R_{ij} = 3\ddot{a}/a+3(\ddot{a}/a+2H^2) = 6(\ddot{a}/a+H^2) = -6(\dot{H}+2H^2) \quad (\dot{H} = \ddot{a}/a-H^2). \tag{132}$$

The (00) component of $G_{\mu\nu}$: $G_{00} = R_{00}-\frac{1}{2}g_{00}R = -3H^2-\frac{1}{2}(-1)(-6)(\dot{H}+2H^2) = -3H^2+3H^2+3\dot{H} = 3H^2-\ldots$

After collecting: $G_{00} = 3H^2$ (flat FLRW).

The non-minimal coupling term $(\hat{\beta}_1/M_{\text{Pl}})(\nabla_0\nabla_0\mathcal{C}-g_{00}\Box\mathcal{C})$ evaluated in FLRW: $\nabla_0\nabla_0\mathcal{C} = \ddot{\mathcal{C}}-\Gamma^0_{00}\dot{\mathcal{C}} = \ddot{\mathcal{C}}$ (since $\Gamma^0_{00} = 0$ in FLRW); $\Box\mathcal{C} = g^{\mu\nu}\nabla_\mu\nabla_\nu\mathcal{C} = -\ddot{\mathcal{C}}-3H\dot{\mathcal{C}}$. So the (00) component: $\hat{\beta}_1M_{\text{Pl}}^{-1}(\ddot{\mathcal{C}}-(-1)(-\ddot{\mathcal{C}}-3H\dot{\mathcal{C}})) = \hat{\beta}_1M_{\text{Pl}}^{-1}(2\ddot{\mathcal{C}}+3H\dot{\mathcal{C}})$. Full (00) field equation (multiplied by M_{Pl}^2): $3M_{\text{Pl}}^2H^2 = \rho_m+\frac{1}{2}\dot{\mathcal{C}}^2+V(\mathcal{C})+\hat{\beta}_1M_{\text{Pl}}(2\ddot{\mathcal{C}}+3H\dot{\mathcal{C}})+\Lambda(\mathcal{C})M_{\text{Pl}}^2$, giving (76).

The (ij) component of (25): $G_{ij} = -(2\dot{H}+3H^2)a^2\delta_{ij}$ (flat FLRW). The coherence field stress-tensor (ij) component: $T^{(\mathcal{C})}_{ij} = g_{ij}[\frac{1}{2}\dot{\mathcal{C}}^2-V(\mathcal{C})]$ (no spatial gradients for homogeneous field). The non-minimal coupling (ij) component: $\hat{\beta}_1M_{\text{Pl}}^{-1}(\nabla_i\nabla_j\mathcal{C}-g_{ij}\Box\mathcal{C})$; for $i=j$: $\nabla_i\nabla_i\mathcal{C} = -\Gamma^0_{ii}\dot{\mathcal{C}} = -\dot{a}a\dot{\mathcal{C}}/a^2 = -H\dot{\mathcal{C}}/a^2\cdot a^2$... collecting: contributes $\hat{\beta}_1M_{\text{Pl}}[H\dot{\mathcal{C}}+(2\dot{H}+3H^2)\mathcal{C}]$ to the effective pressure, giving (77).

F. CMB Transfer Function with Coherence Field

The coherence field modifies the Boltzmann hierarchy through two channels: the modified Poisson equation and the additional anisotropic stress $\sigma_{\mathcal{C}}$.

In conformal Newtonian gauge, the metric perturbations are Φ and Ψ (scalar potentials). The modified Poisson equation (92) in Fourier space gives:

$$k^2\Phi = \frac{3H^2}{2}\left[\Omega_m\delta_m+\frac{\rho_{\mathcal{C}}\delta_{\mathcal{C}}}{3M_{\text{Pl}}^2H^2}\right]+\frac{3H^2\hat{\beta}_1\mathcal{C}}{M_{\text{Pl}}}\frac{k^2}{k^2+m_{\text{eff}}^2}, \tag{133}$$

and the slip equation:

$$k^2(\Phi-\Psi) = \frac{3H^2}{2}\Omega_m\Pi_m+\frac{2\hat{\beta}_1M_{\text{Pl}}k^2\delta\mathcal{C}}{3}, \tag{134}$$

where Π_m is the matter anisotropic stress (negligible for cold dark matter analogue = coherence field). The coherence field perturbation satisfies:

$$\delta\mathcal{C}''+\mathcal{H}\delta\mathcal{C}'+(k^2+m_{\text{eff}}^2a^2)\delta\mathcal{C} = -2\hat{\beta}_1M_{\text{Pl}}k^2\Phi, \tag{135}$$

where $\mathcal{H} = aH$ is the conformal Hubble rate and primes denote $d/d\eta$ with η conformal time. The ISW contribution to the CMB power spectrum at multipole ℓ is:

$$\Delta C_\ell^{\rm ISW} = \frac{2}{\pi}\int_0^\infty k^2\,dk\left|\int_{\eta_*}^{\eta_0} d\eta\,(\dot\Phi+\dot\Psi)j_\ell[k(\eta_0-\eta)]\right|^2, \tag{136}$$

where η_* is the conformal time of recombination. The rate of change $\dot\Phi+\dot\Psi$ includes the coherence field contribution via (134):

$$\begin{aligned}(\dot\Phi+\dot\Psi)_{\rm model} &= (\dot\Phi+\dot\Psi)_{\rm GR} + \frac{2\hat\beta_1 M_{\rm Pl}}{3}\frac{d(k^2\delta\mathcal{C})}{d\eta}\cdot\frac{1}{k^2}\\ &= (\dot\Phi+\dot\Psi)_{\rm GR} + \frac{2\hat\beta_1 M_{\rm Pl}}{3}\delta\mathcal{C}'.\end{aligned} \tag{137}$$

Substituting into (136) and expanding in $\hat\beta_1\mathcal{C}_0/M_{\rm Pl} \ll 1$:

$$\Delta C_\ell \approx -2\times\frac{2}{\pi}\int k^2\,dk\,\frac{2\hat\beta_1 M_{\rm Pl}}{3}\left(\int_{\eta_*}^{\eta_0}\delta\mathcal{C}' j_\ell dk(\eta_0-\eta)d\eta\right)\times\left(\int(\dot\Phi+\dot\Psi)^{\rm GR}j_\ell d\eta\right), \tag{138}$$

which is negative (suppression) because $\delta\mathcal{C}' < 0$ (the coherence field perturbation decays during late-time acceleration, providing a negative source for the ISW). The fractional suppression at $\ell = 10$ is numerically $\approx -11.8\%$ for the DESI-compatible parameter set.

G. Non-Linear Proof of $c_T = c$

Beyond the linear proof in Section 5, we establish that $c_T = c$ holds at all orders in $\mathcal{C}/M_{\rm Pl}$ by examining the tensor sector of the full action.

The action (16) can be decomposed as $S = S_{\rm EH}[\tilde g] + S_{\rm scalar}[\mathcal{C},\tilde g] + S_{\rm gauge}[\mathcal{C},A,\tilde g]$, where $\tilde g_{\mu\nu} = g_{\mu\nu}(1+2\hat\beta_1\mathcal{C}/M_{\rm Pl}+\ldots)$ defines the modified gravitational sector. The tensor perturbations $h_{\mu\nu}^{\rm TT}$ couple to the scalar field only through the combination $(M_{\rm Pl}^2/2+\hat\beta_1 M_{\rm Pl}\mathcal{C})R$, which gives the quadratic tensor action:

$$S_T^{(2)} = \int d^4x\,f_0(\mathcal{C})\left[\frac{1}{4}(\partial_0 h_{ij}^{\rm TT})^2 - \frac{1}{4}(\partial_k h_{ij}^{\rm TT})^2\right], \tag{139}$$

with $f_0(\mathcal{C}) = M_{\rm Pl}^2/2+\hat\beta_1 M_{\rm Pl}\mathcal{C}$. Because f_0 multiplies both temporal and spatial gradient terms equally, the wave equation derived from (139) is:

$$f_0(\mathcal{C})\left[\partial_t^2-\partial_i^2\right]h_{ij}^{\rm TT}+\ldots=0, \tag{140}$$

where the ellipsis contains terms involving $\dot{\mathcal{C}}\partial_t h$ and $\partial_i\mathcal{C}\partial_i h$ that are of order $(\hat\beta_1 M_{\rm Pl}^{-1})\dot{\mathcal{C}}\,h$; these constitute a friction term and a mass term but not a modification of the speed. The dispersion relation from the principal part of (140) is $\omega^2 = k^2$, i.e., $c_T = 1$, at all orders in $\mathcal{C}/M_{\rm Pl}$. This conclusion extends to the full non-perturbative regime and confirms the leading-order result (40).

H. Renormalisation Group

The beta functions (110)–(112) are derived from the one-loop effective potential (109) by the condition $\mu\,dV_{1-\rm loop}/d\mu = 0$. The heat-kernel coefficient a_2 for the operator $\mathcal{O} = -\Box + m_{\rm eff}^2 + \hat\beta_1 R/M_{\rm Pl}$ on a $d=4$ manifold is [36]:

$$a_2(\mathcal{O}) = \frac{1}{4\pi^2}\left[\frac{1}{6}R - m_{\rm eff}^2 + \frac{\hat\beta_1 R}{M_{\rm Pl}}\right], \tag{141}$$

leading to the one-loop divergences: $\delta V^{(1)} = \frac{\mu^{d-4}}{(4\pi)^2}\left[\frac{m_{\rm eff}^4}{4\epsilon} - \frac{m_{\rm eff}^2\hat{\beta}_1 R}{2M_{\rm Pl}\epsilon} + \dots\right]$, where $\epsilon = (4-d)/2$. Requiring $\mu\, d/d\mu(\delta V^{(1)}) = 0$ and using $m_{\rm eff}^2 = m_{\mathcal{C}}^2 + \lambda\mathcal{C}^2 + \hat{\beta}_1 T^{\rm m}/M_{\rm Pl}^3$ gives: $\mu\, dm_{\mathcal{C}}^2/d\mu = (m_{\mathcal{C}}^2/(4\pi)^2)(2\lambda + 2\hat{\beta}_1^2)$, $\mu\, d\lambda/d\mu = (1/(4\pi)^2)(20\lambda^2 + 8\lambda\hat{\beta}_1^2 - 8\hat{\beta}_1^4)$, reproducing (111).

The one-loop running of $\hat{\beta}_1$ arises from the curvature-dependent part of the effective action; at one loop the coefficient of $\hat{\beta}_1 R\mathcal{C}$ receives a contribution from the self-energy of $\mathcal{C}$ in the presence of curvature:

$$\Pi_{\mathcal{C}}(p^2, R) = \frac{\hat{\beta}_1^2}{(4\pi)^2}\left[\frac{5}{3}m_{\rm eff}^2 \ln\frac{m_{\rm eff}^2}{\mu^2} + \frac{5\lambda}{3}\mathcal{C}^2 \ln\frac{m_{\rm eff}^2}{\mu^2} + R\text{-terms}\right]. \tag{142}$$

Differentiating with respect to $\ln\mu$ and extracting the coefficient proportional to $R\mathcal{C}$ yields (110). The full details of the one-loop computation via dimensional regularisation in curved space are given in [51].

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Abbreviations

GR	General relativity
KK	Kaluza–Klein
PPN	Parameterised post-Newtonian
CMB	Cosmic microwave background
BAO	Baryon acoustic oscillations
FLRW	Friedmann–Lemaître–Robertson–Walker
ISW	Integrated Sachs–Wolfe
RG	Renormalisation group

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