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# First-principles derivation of quark masses, mixing angles, and CP-violating phase

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## Abstract

We present a first-principles derivation of quark masses, mixing angles, and the CP-violating phase, requiring no experimental input. Our model treats each quark as a point particle with respect to its charges and wavefunction (or field), aligning with the Standard Model and quantum field theory. However, we also incorporate a radius, determined by geometric constraints, which allows us to determine the quark masses without relying on experimental data. Notably, the radii of quarks are the same as those of charged leptons and W bosons. For example, the down quark's radius is equal to that of the electron, while the bottom quark's radius is equivalent to that of the tau lepton. The large variation in the quark masses is primarily attributed to the large difference in their radii. A key prediction of our theory is the absence of quarks belonging to the fourth or higher generations. Furthermore, we derive the quark mixing angles and CP-violating phase within the Cabibbo-Kobayashi-Maskawa (CKM) matrix from these same fundamental principles. The theoretical predictions for quark masses, mixing angles, and the CP-violating phase derived from first principles show excellent agreement with experimental values. The Standard Model of particle physics relies on a substantial number of experimentally determined input parameters. Our theory enhances the Standard Model by deriving these crucial parameters directly from fundamental principles without experimental input.

Keywords: fundamental particle masses, self-energy, quark masses, quark mixing angles, cp-violating phase.

## 1. Introduction

Predicting the fundamental particle masses remains a significant challenge, largely due to our incomplete understanding of their origins. While the Higgs mechanism successfully explains the relationship between fundamental particle masses and their interactions with the Higgs field, it does not provide specific predictions for those mass values [1]. Although numerous alternative models have been proposed to predict or correlate fundamental particle masses [2–12], the ultimate origin of these masses remains a subject of ongoing debate.

Experimental evidence consistently shows that electrons are point-like, lacking internal substructure [13–17]. Similarly, numerous particle interaction experiments indicate that leptons and quarks also behave as point particles without substructure [18–24]. The observed masses of fundamental particles include self-energy contributions. However, classical electrodynamics demonstrates that the self-energy of a point charge diverges. Likewise, point particle descriptions in quantum field theory lead to divergent self-energy calculations. Instead of elucidating the underlying physics at very short distances responsible for these divergences, quantum field theory employs regularization [25–28] and renormalization [29,30] techniques to manage these divergent results. Consequently, current theoretical frameworks cannot independently predict particle masses without experimental input. The prevalence of these divergences suggests potential incompleteness in our understanding of fundamental particles and highlights the need for more comprehensive theories capable of theoretically determining their masses.

Three generations of quarks are currently known, with their masses typically treated as input parameters in the Standard Model of particle physics. Lattice Quantum Chromodynamics (QCD) determines these mass values by fitting input parameters to experimental data from mesons and baryons [31–33]. Deriving these masses solely from theoretical principles, independent of experimental data, is a crucial objective for enhancing the completeness of the Standard Model. Success in this endeavor would provide profound insights into the nature of the universe and fundamental physics, potentially addressing unresolved questions such as the nature of dark matter [34–37] and the hierarchy problem [38].

In this paper, we present a derivation of quark masses, as well as the mixing angles and CP-violating phase within the Cabibbo-Kobayashi-Maskawa (CKM) matrix, based solely on theoretical principles without reliance on experimental input. We provide analytical formulas for calculating each parameter. The paper is structured as follows: Section 2 presents our approach to determining fundamental particle masses from first principles. Section 3 derives the masses of all quarks. Section 4 presents the derivation of quark mixing angles and the CP-violating phase within the CKM matrix. Finally, Section 5 offers a discussion of these findings, followed by conclusions in Section 6.

## 2. The spheres of fundamental particles

In the Standard Model of particle physics, fundamental particles possess at least one of three fundamental charges: electric, weak, and color. These charges are considered to be point charges located at the particle's position. Consistent with the Standard Model and quantum field theory, our particle theory treats fundamental particles as point particles regarding both their charges and wavefunctions. However, each fundamental particle is associated with a sphere whose radius is determined by geometric constraints. The rest energy of the particle corresponds to the energy stored within the fields beyond this sphere. For instance, an electron, being a point charge, is surrounded by an electric field, and its mass is equivalent to the energy contained within that field beyond its sphere. Generally, the mass of a charged lepton or a W boson is expressed as [39,40]:

$$m_p c^2 = \frac{g_p^2 \hbar c}{8\pi r_p}, \tag{1}$$

where $g_p$ is the coupling constant of the charged lepton or W boson, $r_p$ is its radius, $\hbar$ is the reduced Planck constant, and c is the speed of light. Eq. (1) demonstrates that the mass of a fundamental particle is determined by its radius and the coupling constant of the particle interacting with other particles. The coupling constant of the electron, $g_e$, is proportional to the square root of the fine structure constant of the electromagnetic interaction, specifically $g_e = (4\pi\alpha)^{1/2}$. Similarly, the coupling constant of the W bosons is equal to the weak coupling constant $g_w$. Notably, the coupling constant $g_\mu$ of the muon is equal to that of the electron ($g_e$). The square of the coupling constant of the tau lepton is given by the following equation:

$$g_\tau^2 = \frac{g_w^2}{2} + \frac{g_w^2}{2} + g_z^2\left(\frac{1}{4} + s_w^4\right) = g_w^2 + g_z^2\left(\frac{1}{4} + sin^4\theta_w\right), \tag{2}$$

where $g_w$ and $g_z$ represents the weak coupling constants of W and $Z^0$ bosons, respectively, and $\theta_w$ is the Weinberg angle. Based on our fundamental particle spin theory [43], the energy of a fundamental particle always propagates at the speed of light. In the rest frame of a particle, precisely half of its energy moves forward and the other half moves backward at the speed of light, creating an apparent zero velocity. The radius of the tau lepton is the radius at which the circular motion of the W boson mass, traveling at the speed of light, generates a spin angular momentum of $\hbar$. Consequently, the radius of the tau lepton is determined by the W boson mass as follows:

$$r_\tau m_w c = \hbar\,, \tag{3}$$

$$r_\tau = \frac{\hbar}{m_w c} = \frac{\hbar c}{m_w c^2}\,. \tag{4}$$

The radius the muon is related to that of the tau lepton as follows:

$$r_\mu = (1 + ln\,(\frac{g_w^2 + g_z^2(1/4 + s_w^4)}{g_e^2}))r_\tau\,. \tag{5}$$

The radii of the electron and W boson derived from first principles are $1.409\times10^{-15}$ meters and $3.982\times10^{-20}$ meters [39], respectively. Consequently, Eq. (1) yields an electron mass of 0.511 MeV/$c^2$ and a W boson mass of 79.9 GeV/$c^2$. This W boson mass corresponds to a radius of $2.469\times10^{-18}$ meters for the tau lepton and weak interaction. The mass formulas for the tau lepton and muon can be generated by substituting Eqs. (4) and (5) into Eq. (1) as follows:

$$m_\tau c^2 = \frac{g_\tau^2\hbar c}{8\pi r_\tau} = \frac{g_\tau^2 m_w c^2}{8\pi} = \frac{g_w^2 + g_z^2(1/4 + s_w^4)}{8\pi} m_w c^2\,, \tag{6}$$

$$m_\mu c^2 = \frac{g_\mu^2\hbar c}{8\pi r_\mu} = \frac{g_e^2 m_w c^2}{8\pi\left(1 + ln(\frac{g_\tau^2}{g_e^2})\right)}\,. \tag{7}$$

Similarly, the neutrino masses can be derived from first principles without experimental input. They were presented in [40], along with the formulas for the neutrino mixing angles and CP-violating phase.

Our theory generates a total of four fundamental radii for fundamental particles and their interactions. These are the radii of the W boson, tau lepton, muon, and electron. The radii of the up and down quarks, as well as the electromagnetic interaction and strong interaction, are equal to that of the electron. Throughout this paper, we will use the electron radius to represent the radii of the electron, up quark, down quark, electromagnetic interaction, and strong interaction. Similarly, the radius of the strange quark is equal to that of the muon, and we will therefore use the muon radius to denote it. These radii derived from first principles will be used to determine the masses (Section 3) and mixing angles (Section 4) of quarks.

In quantum theory, fundamental particles are typically treated as point particles, described by wavefunctions that determine the probability amplitude of finding them at a given position. In our fundamental particle theory, while maintaining the point-like nature of particles regarding their charges and wavefunctions—consistent with Standard Model and quantum field theory calculations—we model fundamental particles as spheres to determine their masses, as described in Eqs. (4) through (7). In this representation, a particle is characterized by the same wavefunction but with a spherical center defining its probability distribution. The radius of the sphere then becomes a fundamental property, alongside other known properties such as charge and spin. Because each fundamental particle possesses spherical symmetry, interactions between two particles depend solely on the distance between their centers. This explains the apparent point-like behavior observed in scattering experiments. Furthermore, as detailed in a separate manuscript [41], this spherical representation resolves the problem of divergent loop integrals that have challenged Quantum Field Theory for nearly a century.

## 3. The masses of quarks

In deriving the quark masses (Section 3), the mixing angles, and CP-violating phase (Section 4), we utilize the following coupling constants and the Weinberg angle derived from first principles without experimental input [42,43]:

$$g_w^2 = (\frac{2}{\pi})^2 , \tag{8}$$

$$g_z^2 = (\frac{2(\pi-2)}{\pi})^2 , \tag{9}$$

$$s_w^2 = sin^2\theta_w = \frac{(\pi-2)^2-1}{(\pi-2)^2} . \tag{10}$$

$$g_\tau^2 = g_w^2 + g_z^2(I_{3L}^2 + s_w^4) = \frac{4}{\pi^2}(\frac{\pi^2-4\pi+8}{4} + (\frac{\pi^2-4\pi+3}{(\pi-2)})^2) , \tag{11}$$

$$\alpha(m_\tau c^2) = \frac{g_e^2}{4\pi} = \frac{(\pi-2)^2-1}{\pi^3(\pi-2)^2} . \tag{12}$$

The theoretical value of $g_e^2$ from Eq. (12) at the energy scale of $m_\tau c^2$ is 0.0943, which closely matches the value of 0.0942 ($\alpha(m_\tau)$=1/133.45) reported in [44,45]. It is important to note that the square of the coupling constants (and therefore the fine-structure constant) is energy-dependent. The value changes to 0.0917 ($\alpha(m_e)$ = 1/137.04) at the energy scale of $m_e c^2$. While $g_e^2$ has a small effect on the cp-violating phase and the masses of up and down quarks, it has almost no effect on the masses of other quarks. To be consistent with the energy level of the up and down quarks, we will use 0.0917 at energy of $m_e c^2$. The derivation of the electroweak coupling constants is beyond the scope of this paper and is presented in a separate manuscript [43] which details the derivation of the Higgs boson mass, vacuum expectation value (VEV), electroweak coupling constants, and strong coupling constant from first principles. Here, we utilize these coupling constants derived from first principles to determine quark masses, mixing angles, and the CP-violating phase.

Similar to the masses of the W boson and charged leptons, the masses of quarks are determined by their coupling constants and radii, as shown in Eq. (1). The radii of quarks are the same as those of charged leptons and the W bosons. The coupling constants can be derived from the following Lagrangian of electroweak interaction [46]:

$$-\mathcal{L}_{electroweak} = \frac{g_w}{\sqrt{2}}\overline{\psi}_L \gamma^\mu (W_\mu^+ \tau_+ + W_\mu^- \tau_-)\psi_L$$

$$+ g_z \overline{\psi}\gamma^\mu (I_{3L} - Q s_w^2)\psi Z_\mu + g_e \overline{\psi}\gamma^\mu Q \psi A_\mu \,. \quad (13)$$

### 3.1. The masses of light quarks

#### 3.1.1. The mass of the strange quark

Based on the electroweak interaction Lagrangian in Eq. (13), at the radius of the muon, the strange quark is surrounded by the $W^+$ field, $W^-$ field, and $Z^0$ field. The electromagnetic field (the photon field), which is in the region beyond the radius of the electron, does not reach the radius of the muon. The total energy of the surrounding fields of the strange quark is the sum of the energies of these fields. Consequently, the square of the coupling constant of the strange quark for mass determination is the sum of the contributions from its weak couplings to $W^+$ field, $W^-$ field, and $Z^0$ field:

$$g_s^2 = \frac{g_w^2}{2} + \frac{g_w^2}{2} + (g_z I_{3L})^2 + (g_z Q s_w^2)^2$$

$$= \left(\frac{2}{\pi}\right)^2 \left(\frac{\pi^2 - 4\pi + 8}{4} + \frac{1}{9}\left(\frac{\pi^2 - 4\pi + 3}{\pi - 2}\right)^2\right) . \quad (14)$$

On the right-hand side of Eq. (14), the first two terms are from the $W^+$ and $W^-$ fields, whereas the last two terms, $(g_z I_{3L})^2$ and $(g_z Q s_w^2)^2$, are from the $Z^0$ field.

As described in Section 2, the radius of the electromagnetic interaction corresponds to that of the electron and down quark. The radius of the bottom quark corresponds to that of the tau lepton, while the radius of the strange quark corresponds to that of the muon. The field energy associated

with the coupling term $g_w^2$ and the coupling term $(g_z I_{3L})^2$ are considered to be pure weak field energy. In contrast, the field energy associated with the coupling term $(g_z Q s_w^2)^2$ is considered to be both the weak field energy and electromagnetic field energy. We will describe how the quark mixing angles and CP-violating phase within the CKM matrix are related to the $(g_z s_w^2)^2$ term in Sections 4. The square of the coupling constant for strange and bottom quarks is reduced by a term of $(g_z s_w^2)^2$. Therefore, the square of the actual coupling constant for the strange quark is

$$g_s^2 = (\frac{2}{\pi})^2(\frac{\pi^2 - 4\pi + 8}{4} - \frac{8}{9}(\frac{\pi^2 - 4\pi + 3}{\pi - 2})^2) . \tag{15}$$

The square of the coupling constant of the muon is $g_e^2$, which is the square of the coupling constant for the tau lepton multiplied by $g_e^2/g_\tau^2$. This drop of the coupling constant from $g_\tau^2$ to $g_e^2$ is caused by the increase in the radial distance from the radius of the tau lepton to the radius of the muon. Similarly, the square of the coupling constant of the strange quark is reduced by the same factor $g_e^2/g_\tau^2$. Moreover, the $g_e Q$ term in the Lagrangian is not in Eq. (15) because the electromagnetic field does not reach the radius of the muon. The rest energy of the strange quark is

$$\begin{aligned} m_s c^2 &= \frac{g_e^2}{g_\tau^2}\frac{g_s^2 \hbar c}{8\pi r_\mu} = \frac{g_s^2}{g_\tau^2}\frac{g_\mu^2 \hbar c}{8\pi r_\mu} = \frac{g_s^2}{g_\tau^2} m_\mu c^2 \\ &= \frac{g_s^2}{g_\tau^2}\frac{g_e^2 m_w c^2}{8\pi\left(1 + ln(\frac{g_\tau^2}{g_e^2})\right)} = \frac{g_s^2}{g_\tau^2}\frac{4\pi\alpha m_w c^2}{8\pi\left(1 + ln(\frac{g_\tau^2}{4\pi\alpha})\right)} = 93.5 \text{ MeV} . \end{aligned} \tag{16}$$

Using the W boson mass of 79.9 GeV/c$^2$ derived from first principles [39] and the fine structure constant of 1/137.04, Eq. (16) yields a theoretical strange quark mass of 93.5 MeV/c$^2$. This theoretical value closely matches the experimental value of 93.5 MeV/c$^2$ reported by the particle data group [45]. However, using the experimental muon mass of 105.7 MeV/c$^2$ in Eq. (16), the calculated theoretical strange quark mass is 95.6 MeV/c$^2$.

### 3.1.2. The mass of the down quark

For the down quark, the square of the coupling constant is:

$$\begin{aligned} g_d^2 &= g_w^2 + (g_z I_{3L})^2 + (g_z Q s_w^2)^2 + (g_e Q)^2 \\ &= \frac{1}{\pi^2}\left(\pi^2 - 4\pi + 8 + \frac{4}{9}(\frac{\pi^2 - 4\pi + 3}{\pi - 2})^2\right) + \frac{4\pi\alpha}{9} \\ &= 0.5507 . \end{aligned} \tag{17}$$

The radius of the down quark is the same as that of the electron. Since this radius is equal to the radius of electromagnetic interaction, the square of the coupling constant of the down quark in Eq. (17) includes the contribution $(g_e Q)^2$ from electromagnetic field.

The field energy associated with the coupling term $(g_z Q s_w^2)^2$ is considered to be both the weak field energy and electromagnetic field energy. The quark mixing angles and CP-violating phase within the CKM matrix are related to the $(g_z Q s_w^2)^2$ term. The radius for the $3(g_z s_w^2/3)^2 = (g_z s_w^2)^2/3$ portion of the square of the down quark's coupling constant is equal to that of the muon (the radius of the strange quark). Therefore, the rest energy of the down quark is:

$$m_d c^2 = \frac{\left(g_d^2 - \frac{(g_z s_w^2)^2}{3}\right)\hbar c}{8\pi r_e} + \frac{(\frac{g_z s_w^2)^2}{3}}{g_\tau^2}\frac{g_e^2 \hbar c}{8\pi r_\mu}$$

$$= \frac{(g_d^2 - \frac{(g_z s_w^2)^2}{3})\hbar c}{8\pi r_e} + \frac{(g_z s_w^2)^2/3}{g_\tau^2}\frac{4\pi\alpha m_w c^2}{8\pi\left(1 + ln(\frac{g_\tau^2}{4\pi\alpha})\right)} = 4.76 \text{ MeV}. \quad (18)$$

Using the W boson mass of 79.9 GeV/c$^2$ derived from first principles [39] and the fine structure constant of 1/137.04, Eq. (18) yields a theoretical down quark mass of 4.76 MeV/c$^2$. This theoretical value closely matches the observed average mass of 4.70 MeV/c$^2$ reported by the Particle Data Group [45]. However, when using the experimental muon mass of 105.7 MeV/c$^2$ in Eq. (18), the calculated theoretical down quark mass is 4.79 MeV/c$^2$.

### 3.1.3. The mass of the up quark

The W boson fields surrounding the down quark include the $W^+$ field and the $W^-$ field, with each contributing $g_w^2/2$ to the square of the coupling constant of the down quark. A down quark absorbs a $W^+$ boson to transform into an up quark. After absorbing the $W^+$, the energy stored in the $W^-$ field is converted, resulting in the up quark being surrounded by the $W^+$ field, not the $W^-$ field. Therefore, the square of the coupling constant associated with the up quark is:

$$g_u^2 = g_w^2/2 + (g_z I_{3L})^2 + (g_z Q s_w^2)^2 + (g_e Q)^2$$

$$= \frac{1}{\pi^2}\left(\pi^2 - 4\pi + 6 + \frac{16}{9}(\frac{\pi^2 - 4\pi + 3}{\pi - 2})^2\right) + \frac{16\pi\alpha}{9}. \quad (19)$$

In Eq. (19), $g_w^2/2$ is from the $W^+$ field, $(g_z I_{3L})^2$ and $(g_z Q s_w^2)^2$ are from the $Z^0$ field, and $(g_e Q)^2$ is from electromagnetic field (photon field). The mass of the up quark is therefore:

$$m_u c^2 = \frac{g_u^2 \hbar c}{8\pi r_e} = 2.16 \text{ MeV}. \quad (20)$$

This theoretical mass of the up quark is remarkably close to the average mass of 2.16 MeV/c$^2$ derived from experiments [45].

## 3.2. The masses of heavy quarks

### 3.2.1. The mass of the bottom quark

The radius of weak interaction is equivalent to that of the tau lepton. Beyond this radius, the tau lepton is surrounded by $W^+$, $W^-$ and $Z^0$ boson fields, which mediate the weak interaction. Similarly, the radius of strong interaction is equal to that of the electron. Quarks are confined within this sphere, effectively coinciding with the electron's sphere. Each quark exhibits three color states (red, green, and blue), differing only in their color charges but sharing identical properties. As fundamental particles, quarks exhibit wave-particle duality. Based on our fundamental particle spin theory [43], without interaction, the strong field energy of a quark is 220 MeV. This energy corresponds to a wavelength which is equal to the diameter of the electron's sphere. For a quark with rest energy exceeding 660 MeV, its mass encompasses the field energy associated with all three color states. Consequently, because the bottom quark's mass significantly exceeds 660 MeV/$c^2$, all of its color states contribute to its overall mass through their weak fields. Each color state within the bottom quark is associated with a specific weak field. The radius of the bottom quark is same as that of weak interaction. Like that for the strange quark, the square of the coupling constant for the bottom quark is reduced by a term of $(g_z s_w^2)^2$. Consequently, the square of the coupling constant for one color state of the bottom quark is equal to that for the strange quark. The square of the coupling constant for other two color states is equal to that of the W boson. Therefore, the square of the coupling constant for the bottom quark is

$$g_b^2 = g_s^2 + 2g_w^2 = \frac{1}{\pi^2}(\pi^2 - 4\pi + 16 - \frac{32}{9}(\frac{\pi^2 - 4\pi + 3}{\pi - 2})^2) . \quad (21)$$

Since the radius of the bottom quark is equivalent to the radius of the tau lepton, its mass is calculated as follows:

$$m_b c^2 = \frac{g_b^2 \hbar c}{8\pi r_\tau} = \frac{g_b^2}{g_\tau^2}\frac{g_\tau^2 \hbar c}{8\pi r_\tau} = \frac{g_b^2}{g_\tau^2} m_\tau c^2 = 4.204 \text{ GeV} . \quad (22)$$

With a theoretical mass of 1.799 GeV/$c^2$ derived from first principles for the tau lepton [40], the theoretical mass of the bottom quark is 4.204 GeV/$c^2$, which closely matches the experimental value of 4.183 GeV/$c^2$ [45].

### 3.2.2. The mass of the top quark

The top quark is a heavy quark, and its mass arises from the contributions of three color states. However, only one of these states participates in weak interactions with other particles. The radius associated with this color state is equal to that of weak interaction (the radius of the tau lepton). A bottom quark can transform into a top quark by absorbing a $W^+$ boson. The weak field surrounding this color state of the top quark is similar to that of an up quark. Consequently, the square of the coupling constant for this specific color state of the top quark is:

$$g_{t,1}^2 = g_w^2/2 + (g_z I_{3L})^2 + (g_z Q s_w^2)^2$$

$$= (\frac{2}{\pi})^2(\frac{\pi^2-4\pi+6}{4}+\frac{4}{9}(\frac{\pi^2-4\pi+3}{(\pi-2)})^2)\,. \quad (23)$$

The other two color states do not participate in weak interactions, implying their radii should differ from that of the tau lepton. The radii of these two color states are equal to that of the W bosons. Since only W and $Z^0$ bosons have this specific radius, these two remaining color states are surrounded by corresponding $W^-$ and $Z^0$ fields. Consequently, the top quark's mass is the sum of the contributions from all three color states:

$$m_t c^2 = \frac{g_{t,1}^2 \hbar c}{8\pi r_\tau} + \frac{g_w^2 \hbar c}{8\pi r_w} + \frac{g_z^2 \hbar c}{8\pi r_w} = \frac{g_{t,1}^2}{g_\tau^2} m_\tau c^2 + m_w c^2 + m_z c^2 = 172.2 \text{ GeV}\,. \quad (24)$$

Using the first-principles theoretical mass values of 1.799 GeV/$c^2$ for the tau lepton [40], 79.9 GeV/$c^2$ for the W boson [39], and 91.2 GeV/$c^2$ for the $Z^0$ boson [43], the calculated theoretical mass of the top quark is 172.2 GeV/$c^2$. This theoretical top quark mass closely matches the average mass of 172.6 GeV/$c^2$ derived from experimental observations [45], with an error of only 0.23%.

### 3.2.3. The mass of the charm quark

The charm quark is a heavy quark. A strange quark can transform into a charm quark by absorbing a $W^+$ boson. The weak fields surrounding all three color states of the charm quark contribute to its overall mass. However, only one of these three color states participates in weak interactions and its radius is equal to that of weak interaction (the radius of the tau lepton). The weak field surrounding this particular color state is analogous to those surrounding the up quark and the top quark. Therefore, the square of the coupling constant for this specific color state of the charm quark is:

$$g_{c,1}^2 = g_w^2/2 + (g_z I_{3L})^2 + (g_z Q s_w^2)^2$$

$$= (\frac{2}{\pi})^2(\frac{\pi^2-4\pi+6}{4}+\frac{4}{9}(\frac{\pi^2-4\pi+3}{\pi-2})^2)\,. \quad (25)$$

The other two color states do not participate in weak interactions, so their radii must differ from that of weak interaction (the radius of the tau lepton). These two color states possess radii equal to that of the muon (the radius of the strange quark). Since one color state of the charm quark is surrounded by a $W^+$ field (as indicated by the coupling constant in Eq. (25)), the remaining two color states are surrounded by a $W^-$ field and a $Z^0$ field, respectively. This implies that the coupling constants for these two color states are $g_w$ and $g_z$, respectively. Therefore, the mass of the charm quark is calculated as follows:

$$m_c c^2 = \frac{g_{c,1}^2 \hbar c}{8\pi r_\tau} + \frac{g_e^2}{g_\tau^2} \frac{(g_w^2+g_z^2)\hbar c}{8\pi r_\mu}$$

$$= \frac{g_{c,1}^2 m_w c^2}{8\pi} + \frac{g_w^2+g_z^2}{g_\tau^2} \frac{4\pi\alpha m_w c^2}{8\pi\left(1 + ln(\frac{g_\tau^2}{4\pi\alpha})\right)} = 1275 \text{ MeV}. \quad (26)$$

Using the W boson mass of 79.9 GeV/$c^2$ derived from first principles [39] and the fine structure constant of 1/137.04, Eq. (26) yields a theoretical charm quark mass of 1275 MeV/$c^2$. This theoretical value closely matches the observed average mass of 1273 MeV/$c^2$ reported by the Particle Data Group [45]. However, when using the experimental muon mass of 105.7 MeV/$c^2$ in Eq. (26), the calculated theoretical charm quark mass is 1279 MeV/$c^2$.

The six quark masses calculated from their electroweak coupling constants and radii are presented in Table 1. The experimental values are sourced from the particle data group data [45]. These calculations represent tree-level results, and future work will focus on incorporating higher-order corrections.

Table 1. Theoretical quark masses in MeV vs their experimental values. The theoretical values are calculated with the electroweak coupling constants and the experimental values are taken from [45].

| Quark | Theoretical Mass | Experimental Value |
|---|---|---|
| u | 2.16 | 2.16 |
| d | 4.76 | 4.70 |
| s | 93.5 | 93.5 |
| c | 1275 | 1273 |
| b | 4204 | 4183 |
| t | $172.2\times10^3$ | $172.6\times10^3$ |

In summary, quark masses are determined by their radii and electroweak coupling constants. While each quark possesses three color states, only one color state actively participates in the electroweak interaction at any given time. The square of the coupling constant associated with this interacting color state is the sum of all terms in the Lagrangian for the electroweak interaction. For the other two color states that do not participate in the electroweak interaction, the square of the coupling constant is $g_w^2+g_z^2$ for up-type quarks and $2g_w^2$ for down-type quarks. The radii of quarks are equal to those of charged leptons and the W bosons. The masses of quarks vary enormously across the three generations—the mass of the top quark is nearly five orders of magnitude larger than that of the up quark. This striking variation in the masses of quarks is caused by the large difference in their radii. The radius of the up quark is that of the electron, whereas the radii of two colors of the top quark are that of the W bosons. As the radius of the W bosons is more than four orders of magnitude smaller than that of the electron, the mass of the top quark is more than 4 orders of magnitude larger than the masses of the up and down quarks.

Quarks are categorized as either heavy or light based on their masses. The mass of a heavy quark arises from the contributions of electroweak fields across all three color states. Conversely, the mass of a light quark is determined by the electroweak fields of a single color state. Quarks are confined within the sphere of the strong interaction. When the wavelength equals the diameter of this sphere, the corresponding energy is 220 MeV. Since the mass of a light quark is less than this

energy, only one color state's electroweak fields contribute to its mass. In contrast, a heavy quark has a mass exceeding 660 MeV—more than three times this energy. Consequently, the electroweak fields of all three color states contribute to its mass.

The radii of heavy quarks are smaller than those of light quarks. At least one color of a heavy quark has the radius equivalent to that of the tau lepton or W boson. In contrast, the radii of light quarks are the same as those of the electron and muon. Because the radii of quarks are the same as those of the charged leptons, the masses of quarks are related to the masses of charged leptons. For example:

$$m_u c^2 = \frac{g_w^2/2 + (g_z/2)^2 + (2g_z s_w^2/3)^2 + (2g_e/3)^2}{g_e^2} m_e c^2 \,, \tag{27}$$

$$m_s c^2 = \frac{g_w^2 + (g_z/2)^2 - 8(g_z s_w^2)^2/9}{g_w^2 + (g_z/2)^2 + (g_z s_w^2)^2} m_\mu c^2 \,, \tag{28}$$

$$m_b c^2 = \frac{3g_w^2 + (g_z/2)^2 - 8(g_z s_w^2)^2/9}{g_w^2 + (g_z/2)^2 + (g_z s_w^2)^2} m_\tau c^2 \,. \tag{29}$$

The masses of fundamental particles are not arbitrary input parameters; they are determined by the spheres and the electroweak coupling constants. The radius of the tau lepton is the radius for weak interaction. Since the bottom quark's radius is equal to that of the tau lepton, a fourth-generation quark would require smaller radius for the leptons, which is not physically possible. Therefore, fourth and higher generation quarks do not exist.

## 4. The quark mixing angles and cp-violating phase

### 4.1. The quark mixing angles

There are three generations of quarks, corresponding to the three generations of charged leptons. Each generation consists of two quarks: one up-type and one down-type. The CKM quark-mixing matrix [47,48] describes how up-type quarks transition into down-type quarks (and vice versa) via weak interaction. It is parameterized by three mixing angles, which determine the probability of favor-changing transition, and one complex phase, which causes the CP violation [45]. The transition between two different generations of quarks is determined by the ratios of various terms in the Lagrangian of electroweak interaction.

As described in Section 2, the mass of the electron originates from the electromagnetic field beyond its radius (the radius of electromagnetic interaction), while the mass of the tau lepton arises from the weak fields beyond its radius. The tau lepton's radius is the radius of weak interaction for leptons and quarks. Although the muon's coupling constant is equivalent to that of the electromagnetic interaction, it is attenuated due to the muon's larger radius compared to that of the tau lepton [40]. Consequently, the field energy associated with the muon is primarily weak field energy. Because of the W boson mass, the strength of weak interaction diminishes exponentially beyond the tau lepton's radius. When the radial distance reaches the radius of the electron, the strength of the weak interaction is multiplied by approximately $\exp(-r_e/r_\tau) \approx 10^{-248}$, effectively eliminating weak interaction for leptons at the radius of the electron. Therefore, the weak field

only reaches the radius of the muon where its effective strength equals that of the electromagnetic interaction. On the other hand, since the electromagnetic interaction radius corresponds to the electron's radius, the electromagnetic field, which is beyond the radius of the electron, does not reach the radius of the muon. Therefore, the field energy in the tau lepton or muon is primarily derived from weak fields, while the field energy in the electron is predominantly electromagnetic. Similarly, the field energy within the bottom or strange quark is primarily weak field energy.

The radius of strong interaction is equivalent to that of the electron and electromagnetic interaction. Quarks are confined within spheres of this radius. For mesons and baryons, quarks carry W and $Z^0$ bosons into regions accessible within the strong interaction radius. Consequently, for quarks, the weak field can reach the radius of electromagnetic interaction (the radius of the electron). However, as demonstrated in the derivation of the down quark's mass in Section 3, the weak field energy at the electron's radius is equal to that at the radius of the tau lepton multiplied by $r_\tau/r_e$.

Because the field energy of the muon and strange quark in the second generation is weak field energy and the field energy in both electron and down quark in the first generation is electromagnetic field energy, transitions between the first- and second-generation quarks necessitate a conversion between these two energy types. According to the electroweak interaction Lagrangian in Eq. (9), the energy stored within the $Z^0$ field comprises two terms: $(g_z I_{3L})^2$ and $(g_z Q s_w^2)^2$. The $Z^0$ field energy associated with $(g_z Q s_w^2)^2$ term is of the same type as electromagnetic field energy. It represents the overlap between weak and electromagnetic energy types, meaning that the energy from this term corresponds to both weak and electromagnetic field energies. The coupling constant of the strange quark is reduced by a term of $(g_z s_w^2)^2$. In contrast, within the square of the coupling constant for the down quark, the radius associated with a portion of that square is equal to that of the muon. The probability of transition between first- and second-generation quarks is equal to the fraction of this energy within the weak field energy, effectively quantifying the conversion between weak and electromagnetic field energies. Therefore, $\sin\theta_{12}$ is calculated as follows:

$$sin^2\theta_{12} = \frac{(g_z s_w^2)^2}{g_w^2 + (g_z I_{3L})^2 + (g_z s_w^2)^2} = 0.05053\,, \tag{30}$$

$$sin\theta_{12} = \sqrt{0.05053} = 0.225\,. \tag{31}$$

This value is remarkably close to 0.225 from experiments [45].

As described in Section 3, for both strange and bottom quarks, the square of the coupling constant is reduced by a term of $(g_z s_w^2)^2$. The field energy of bottom quark (and the tau lepton) in the third generation and the strange quark (and muon) in the second generation are weak field energy, the transition between the second- and third-generation quarks does not require energy conversion. The transition probability is determined by their respective radii. Unlike charge leptons, whose weak field energy only reaches the radius of the muon, the weak field energy of quarks reaches the radius of the electron (the radius of the strong interaction) due to the strong interaction. For quarks, the ratio of the weak field energy at the electron's radius to that at the tau lepton's radius is equal to $r_\tau/r_e$. This ratio directly determines the probability of transition between the second- and third-generation quarks. Therefore, $\sin\theta_{23}$, quantifying this transition, is calculated as follows:

$$sin^2\theta_{23} = \frac{r_\tau}{r_e} = 0.001745\,, \tag{32}$$

$$sin\theta_{23} = \sqrt{0.001745} = 0.0418\ . \tag{33}$$

This value is remarkably close to 0.0418 from experiments [45].

The energy stored in the electromagnetic field is directly proportional to the square of the electromagnetic coupling constant. In contrast, the energy stored in the weak fields is directly proportional to the square of the tau lepton's coupling constant, representing the combined contributions from the $W^+$, $W^-$, and $Z^0$ bosons. The radius of the bottom (third-generation) quark corresponds to the radius of weak interaction, while the radius of the down (first-generation) quark corresponds to the radius of electromagnetic interaction. Therefore, transitions between first-generation and third-generation quarks require changes in both energy type and the radius of the interaction. The probability of this transition—representing the shift from weak field energy at the weak interaction radius to electromagnetic field energy at the electromagnetic interaction radius—is calculated as follows:

$$sin^2\theta_{13} = \frac{(g_z s_w^2)^2}{g_\tau^2}\frac{r_\tau}{r_e}\frac{g_e^2}{g_\tau^2} = 1.428\times10^{-5}\,, \tag{34}$$

$$sin\theta_{13} = 0.00378\ . \tag{35}$$

The first and second terms in Eq. (29) correspond to $\sin^2\theta_{12}$ and $\sin^2\theta_{23}$, respectively. The third term is the ratio of square of the electromagnetic coupling constant to that of weak coupling. This is the ratio of the electromagnetic field energy over the weak field energy after removing the effect of the interaction radius. The theoretical value of 0.00378 is remarkably close to 0.00373 from experiments [45].

In summary, transitions between quark generations involve both the conversion between weak and electromagnetic field energies and alterations in their respective interaction radii. The transition between the first and second generations quantifies the probability of the conversion between weak field energy and electromagnetic field energy. Conversely, the transition between the second and the third generations measures the probability of the change in the radius of interaction (from that of weak interaction to that of electromagnetic and strong interaction), while maintaining the field energy type (the weak field energy). In contrast, the transition between the first and third generations measures changes in both field energy type and the radius of interaction.

The three quark mixing angles from our first-principles calculations, based on the theoretical coupling constants and the radii of fundamental particles, are presented in Table 2. For comparison, the experimental values reported in the particle data group [45] are also included in the table. The theory accurately predicts the quark mixing angles in the Standard Model. Because the theory contains no adjustable parameters, it offers insight into the underlying physics governing the transitions between different quark generations.

Table 2. Theoretical quark mixing angles and CP-violating phase within the CKM matrix vs their experimental values. The theoretical values are calculated from first principles and the experimental values are taken from the data of the particle data group [45].

| | Theoretical Value | Experimental Value |
|---|---|---|
| $\sin\theta_{12}$ | 0.225 | 0.225 |
| $\sin\theta_{23}$ | 0.0418 | 0.0418 |
| $\sin\theta_{13}$ | 0.00378 | 0.00373 |
| $\delta$ | 1.155 | 1.147 |

### 4.2. The cp-violating phase

The CP-violating phase is from the energy conversion between the energy stored in the electromagnetic field and the energy stored in the weak field. The energy stored in the electromagnetic fields include the pure electromagnetic field energy from the square of the coupling constant $g_e^2 - (g_z Q s_w^2)^2$ which is different from the weak field energy, and the overlapping part, $(g_z Q s_w^2)^2$, which is the same type as the weak field energy. The cp-violating phase $\delta$ in the CKM matrix is directly proportional to the pure electromagnetic field energy:

$$\delta = k g^2_{purely\ electromagnetic}\,, \tag{36}$$

$$g^2_{purely\ electromagnetic} = 3[(g_e Q)^2 - (g_z Q s_w^2)^2]\,, \tag{37}$$

$$k = \frac{\pi}{2(g_z s_w^2)^2}\,, \tag{38}$$

$$\delta = \frac{\pi}{2(g_z s_w^2)^2}\frac{3}{9}(g_e^2 - (g_z s_w^2)^2)\,. \tag{39}$$

The 3/9 in Eq. (39) is from $Q^2$=1/9 multiplied by 3, the number of color states for each quark. As described in Section 3 that derives the quark masses, this factor appears in the down quark's coupling constant. While the radius of the down quark is equivalent to that of the electron, the radius for the $(g_z s_w^2)^2/3$ portion of the square of the down quark's coupling constant is equal to that of the muon. This portion of electromagnetic field energy in the down quark (whose radius is that of the electron) appears to be weak field energy at radius of muon. For $g_e^2$=0.0917 ($\alpha$=1/137.04), the cp-violating phase $\delta$ from Eq. (39) is 1.155 ($66.19^0$). The average experimental value reported by the particle data group is 1.147 [45]. The theory accurately predicts the cp-violating phase in the CKM matrix. Because the theory contains no adjustable parameters, it offers insight into the underlying physics governing the cp-violating phase.

There are three known generations of leptons, corresponding to the three generations of quarks. The neutrino mixing angles and CP-violating phase are described by in the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix. In addition to producing the CP-violating phase in the quark mixing described in this section, our theory also accurately predicts the CP-violating phase in the

neutrino mixing, as detailed in a separate manuscript [40] that presents the derivation of the masses of leptons, as well as the mixing angles and CP-violating phase of neutrinos, from first principles

## 5. Discussion

In this paper, we derive quark masses, mixing angles, and the CP-violating phase from theory alone, without relying on experimental input. We utilize a spherical representation of fundamental particles, which defines four fundamental radii for these particles and their associated interactions: the electron, muon, tau lepton, and W boson. The radii of quarks are equivalent to those of the W bosons and leptons. As demonstrated in Section 3, the QCD quark masses are the energies stored in the electromagnetic and weak fields beyond their radii. Specifically, the energy stored in each field is proportional to the square of the coupling constant that describes the quark's coupling with that field, as defined in the Lagrangian of electroweak interactions.

QCD describes the strong interaction between quarks. However, the quark masses used as input parameters in QCD calculations originate from the electroweak field energy, not from the strong field energy. The masses of mesons and baryons incorporate the electroweak field energy from their quark masses, along with the strong field energies of these quarks. The constituent quark models [49,50] often employ quark masses that differ from the QCD quark masses. The quark masses in the constituent quark models include the electroweak field energy (in the QCD quark masses) and strong field energy of the quarks. Because QCD quark masses stem from the electroweak fields, and QCD is the theory that models strong fields, QCD alone cannot determine quark masses. Lattice QCD determines quark masses by fitting input mass parameters to experimental data of mesons and baryons [31–33]. This approach is analogous to determining neutrino masses by fitting input parameters to match experimental results from neutrino oscillation experiments [51–53].

In a separate manuscript, we derive the masses of $\pi^{\pm}$, $\rho^{\pm}$, $K^{\pm}$, $K^{*\pm}$, $D^{\pm}$, $D^{*\pm}$, $B^{\pm}$, and $B^{*\pm}$ mesons without relying on experimental input. A meson's mass is determined by the sum of the rest energies of its constituent quarks and the interaction between them. These rest energies are the field energies associated with these quarks. In addition to electromagnetic and weak field energy, a quark also possesses strong field energy. Because electromagnetic field energy is proportional to the square of the charge, a quark with a charge of -e/3 requires three identical copies—its three color states—to achieve 1/3 the electromagnetic energy of a charge of -e. While each color state possesses the same electromagnetic field energy, only one interacts electromagnetically with other particles. In the Standard Model of particle physics, electromagnetic and weak fields are unified; therefore, each color state carries both electromagnetic and weak field energy. Consequently, a quark's total rest energy is the sum of the field energies associated with its three color states.

Although all three color states within a quark possess electroweak field energy, only one color state participates in electroweak interactions with other particles. In heavy quarks, the color state involved in these interactions has a radius equivalent to that of the tau lepton and a weak field energy much higher than 220 MeV. This results in a 220 MeV of strong field energy associated with the color state, which combines the weak field energies from all three color states to form the QCD quark mass. In contrast, light quarks have a color state participating in electroweak interactions with a radius equivalent to that of the electron or muon. The electroweak field energy of this color state is much lower than 220 MeV. Consequently, this color state does not possess the strong field energy necessary to combine the electroweak field energies from all three color

states. Therefore, the QCD quark mass of a light quark includes the electroweak field energy associated with the single color state involved in the electroweak interactions.

Our derivation of QCD quark masses in Section 3 is based on the Lagrangian of electroweak interactions. The electroweak coupling constants are derived from first principles, as detailed in [42,43], and do not require experimental input. This approach significantly enhances the Standard Model of particle physics.

In our particle theory, fundamental particles are described as point particles for their charges and wavefunctions, consistent with the Standard Model and quantum field theory calculations. At a specific location in three-dimensional space, in addition to the wavefunction $\psi(\mathbf{r},t)$ that provides the quantum mechanical description of the particle at that position, the particle located at the position possesses classical properties and associated fields throughout space that are described by the classical theory. For example, an electron exhibits wave-particle duality, and its spatial distribution is characterized by the wavefunction $\psi(\mathbf{r},t)$, derived from the Dirac or Schrödinger equation. Assuming the electron is at rest, the wavefunction $\psi(\mathbf{r},t)$ describes its state at a given location ($\mathbf{r}$) in space and simultaneously, at this location ($\mathbf{r}$), this electron also acts as a classical charge, generating an electric field with associated energy density throughout three-dimensional space, as described by classical electrodynamics.

Classical electrodynamics predicts infinite total electric field energy for a point charge. In our particle theory, the electron has a radius $r_e$, and its mass represents the energy of the electric field beyond this radius. Similar to the electron, each fundamental particle or color state also possesses a radius. This radius is a fundamental property alongside others such as spin and charge, defined by geometric constraints within our theoretical framework. The radius defines a sphere around the particle, and the field energy beyond this sphere represents the particle's rest energy (mass). This approach resolves the ultraviolet divergence problem in quantum field theory and allows us to determine the masses of all fundamental particles, along with electromagnetic, weak, and strong coupling constants, mixing angles, and CP-violating phases, from first principles without experimental input [39,40,42,43,54].

The radius of a fundamental particle is an absolute distance. As detailed in a separate manuscript that derive the energy-frequency relation of quantum theory and the Schwarzschild metric of general relativity from fundamental particle spin model [55], this absolute distance is the same in all inertial frames and locations in the universe. It defines the radius and sphere of a fundamental particle, a property unaffected by reference frames or gravitational fields.

## 6. Conclusions

We derive the quark masses, as well as quark mixing angles and the CP-violating phase in the CKM matrix, from theoretical principles without relying on experimental data. Quark masses are determined by their radii and electroweak coupling constants. The radii of quarks are equivalent to those of charged leptons and the W bosons. The substantial variation in quark masses is primarily attributed to differences in their radii. For heavy quarks, the electroweak fields of all three color states contribute to their masses. In contrast, for light quarks, only one color state's electroweak field contributes. The resulting theoretical predictions for masses, mixing angles, and CP-violating phase exhibit excellent agreement with experimental data. Our theory predicts that higher generations (fourth or greater) of quarks are absent. The transition probability between two

different quark generations is determined by the ratios of various terms in the Lagrangian governing electroweak interactions:

- The probability of transition between the first and second generations is equal to the ratio of the field energy from $(g_z Q s_w^2)^2$ term over the total weak field energy at the same radial distance.
- The probability of transition between the second and third generations is equal to the ratio of the weak field energy at the radius of electromagnetic interaction (the radius of the electron) over that at the radius of weak interaction (the radius of the tau lepton).
- The probability of transition between the first and third generations represents the change from the weak field energy at the radius of weak interaction to the electromagnetic field energy at the radius of electromagnetic interaction.

The CP-violating phase is directly proportional to the pure electromagnetic field energy.

In this and other manuscripts [39,40,42,43,54], we derive the parameters of the Standard Model of particle physics—including the masses of all fundamental particles, coupling constants for the electromagnetic, weak, and strong interactions, neutrino mixing angles and CP-violating phase in the PMNS matrix, and quark mixing angles and CP-violating phase in the CKM matrix—entirely from first principles, without reliance on experimental data. The Standard Model of particle physics has been considered incomplete because it relies on a large number of input parameters whose values must be determined experimentally. Our theory significantly enhances the Standard Model as it determines these input parameters from first principles.

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## Data availability statement

The data that support the findings of this article are openly available [45].

## References

[1] Bednyakov VA, Giokaris ND, Bednyakov AV. On the Higgs mass generation mechanism in the Standard Model. Phys Part Nucl 2008;39:13–36. https://doi.org/10.1134/S1063779608010024.

[2] Tang J, Tang Q. A Precise Unified Mass Scaling Law for Leptons and Quarks: Implications for Their Internal Geometric Structures governed by Hypercomplex Algebra. Int J Quantum Technol 2025;01:01–7. https://doi.org/10.66311/3068-9090.01.02.17.

[3] Xing Z, Zhang H. On the Koide-like relations for the running masses of charged leptons, neutrinos and quarks. Phys Lett B 2006;635:107–11. https://doi.org/10.1016/j.physletb.2006.02.051.

[4] Huang Y-C, Iqbal ST, Lei Z, et al. Some new symmetric relations and prediction of left- and right-handed neutrino masses using Koide's relation. Chin Phys C 2017;41:043104. https://doi.org/10.1088/1674-1137/41/4/043104.
[5] Cao F-G. Neutrino masses from lepton and quark mass relations and neutrino oscillations. Phys Rev D 2012;85:113003. https://doi.org/10.1103/PhysRevD.85.113003.
[6] Koide Y. Fermion-boson two-body model of quarks and leptons and cabibbo mixing. Lett Al Nuovo Cimento 1982;34:201–5. https://doi.org/10.1007/BF02817096.
[7] Koide Y. New view of quark and lepton mass hierarchy. Phys Rev D 1983;28:252–4. https://doi.org/10.1103/PhysRevD.28.252.
[8] Washburn J, Allahyarov E. Charged Lepton Masses from the Recognition Composition Law: A Derivation with Zero Continuously Adjustable Dimensionless Parameters. Symmetry 2026;18:962. https://doi.org/10.3390/sym18060962.
[9] Washburn J, Allahyarov E. A Complete Derivation of the Fermion Spectrum from the Recognition Composition Law 2026. https://doi.org/10.48550/arXiv.2506.12859.
[10] Mikhail I Aksman. First-Principles Derivation of the Electron Rest Mass, Spin-1/2, and Lepton-Hadron Mass Hierarchy from Discrete Heegner Vorton Dynamics 2026. https://doi.org/10.13140/RG.2.2.19764.00649.
[11] Krasnoholovets V. Direct Derivation of the Neutrino Mass. J High Energy Phys Gravit Cosmol 2024;10:621–46. https://doi.org/10.4236/jhepgc.2024.102039.
[12] Buravov LI. Simple way to calculate neutrino masses. Ann Math Phys 2022;5:135–6. https://doi.org/10.17352/amp.000053.
[13] ACME Collaboration. Improved limit on the electric dipole moment of the electron. Nature 2018;562:355–60. https://doi.org/10.1038/s41586-018-0599-8.
[14] Brodsky SJ, Drell SD. Anomalous magnetic moment and limits on fermion substructure. Phys Rev D 1980;22:2236–43. https://doi.org/10.1103/PhysRevD.22.2236.
[15] Dehmelt H. A Single Atomic Particle Forever Floating at Rest in Free Space: New Value for Electron Radius. Phys Scr 1988;T22:102–10. https://doi.org/10.1088/0031-8949/1988/T22/016.
[16] Fan X, Myers TG, Sukra BAD, et al. Measurement of the Electron Magnetic Moment. Phys Rev Lett 2023;130:071801. https://doi.org/10.1103/PhysRevLett.130.071801.
[17] Roussy TS, Caldwell L, Wright T, et al. An improved bound on the electron's electric dipole moment. Science 2023;381:46–50. https://doi.org/10.1126/science.adg4084.
[18] Aaron FD, Alexa C, Andreev V, et al. Search for contact interactions in e ± p collisions at HERA. Phys Lett B 2011;705:52–8. https://doi.org/10.1016/j.physletb.2011.09.109.
[19] Abbott B, Abolins M, Abramov V, et al. Measurement of the High-Mass Drell-Yan Cross Section and Limits on Quark-Electron Compositeness Scales. Phys Rev Lett 1999;82:4769–74. https://doi.org/10.1103/PhysRevLett.82.4769.
[20] The ALEPH Collaboration, Schael S, Barate R, et al. Fermion pair production in e+e- collisions at 189–209 GeV and constraints on physics beyond the standard model. Eur Phys J C 2007;49:411–37. https://doi.org/10.1140/epjc/s10052-006-0156-8.
[21] The ATLAS collaboration, Aad G, Abbott B, et al. Search for new non-resonant phenomena in high-mass dilepton final states with the ATLAS detector. J High Energy Phys 2020;2020:5. https://doi.org/10.1007/JHEP11(2020)005.
[22] The ATLAS collaboration, Aaboud M, Aad G, et al. Search for new high-mass phenomena in the dilepton final state using 36 fb−1 of proton-proton collision data at s = 13 $$

\sqrt{s}=13 $$ TeV with the ATLAS detector. J High Energy Phys 2017;2017:182. https://doi.org/10.1007/JHEP10(2017)182.
[23] The CMS collaboration, Sirunyan AM, Tumasyan A, et al. Search for resonant and nonresonant new phenomena in high-mass dilepton final states at $$ \sqrt{s} $$ = 13 TeV. J High Energy Phys 2021;2021:208. https://doi.org/10.1007/JHEP07(2021)208.
[24] The CMS collaboration, Sirunyan AM, Tumasyan A, et al. Search for contact interactions and large extra dimensions in the dilepton mass spectra from proton-proton collisions at $$ \sqrt{s}=13 $$ TeV. J High Energy Phys 2019;2019:114. https://doi.org/10.1007/JHEP04(2019)114.
[25] Bollini CG, Giambiagi JJ. Dimensional renorinalization : The number of dimensions as a regularizing parameter. Il Nuovo Cimento B 1972;12:20–6. https://doi.org/10.1007/BF02895558.
[26] Gnendiger C, Signer A, Stöckinger D, et al. To d , or not to d: recent developments and comparisons of regularization schemes. Eur Phys J C 2017;77:471. https://doi.org/10.1140/epjc/s10052-017-5023-2.
[27] Pauli W, Villars F. On the Invariant Regularization in Relativistic Quantum Theory. Rev Mod Phys 1949;21:434–44. https://doi.org/10.1103/RevModPhys.21.434.
[28] 'T Hooft G, Veltman M. Regularization and renormalization of gauge fields. Nucl Phys B 1972;44:189–213. https://doi.org/10.1016/0550-3213(72)90279-9.
[29] Mukhi S. Renormalisation in Quantum Field Theory 2019. https://doi.org/10.48550/ARXIV.1908.04075.
[30] Ivanov AV, Kharuk NV. Renormalization aspects of the Yang-Mills theory with a cutoff 2026. https://doi.org/10.48550/ARXIV.2606.03494.
[31] Heitger J, Joswig F, Kuberski S. Determination of the charm quark mass in lattice QCD with 2 + 1 flavours on fine lattices. J High Energy Phys 2021;2021:288. https://doi.org/10.1007/JHEP05(2021)288.
[32] Lytle AT, Davies CTH, Hatton D, et al. Determination of quark masses from n f = 4 lattice QCD and the RI-SMOM intermediate scheme. Phys Rev D 2018;98:014513. https://doi.org/10.1103/PhysRevD.98.014513.
[33] Mason Q, Trottier HD, Horgan R, et al. High-precision determination of the light-quark masses from realistic lattice QCD. Phys Rev D 2006;73:114501. https://doi.org/10.1103/PhysRevD.73.114501.
[34] Chadha-Day F, Ellis J, Marsh DJE. Axion dark matter: What is it and why now? Sci Adv 2022;8:eabj3618. https://doi.org/10.1126/sciadv.abj3618.
[35] Dasgupta B, Kopp J. Sterile neutrinos. Phys Rep 2021;928:1–63. https://doi.org/10.1016/j.physrep.2021.06.002.
[36] Freese K. Status of dark matter in the universe. Int J Mod Phys D 2017;26:1730012. https://doi.org/10.1142/S0218271817300129.
[37] Heinemeyer S, Muñoz C. Dark Matter in Supersymmetry. Universe 2022;8:427. https://doi.org/10.3390/universe8080427.
[38] Peskin ME. What is the Hierarchy Problem? Nucl Phys B 2025;1018:116971. https://doi.org/10.1016/j.nuclphysb.2025.116971.
[39] Luo X. First-principles derivation of electron and W boson masses and implications for gravity 2026. https://doi.org/10.5281/ZENODO.20181437.
[40] Luo X. Deriving lepton masses, neutrino mixing angles, and CP-violating phase from first principles 2026. https://doi.org/10.5281/ZENODO.21494729.

[41] Luo X. A novel particle interaction mechanism for resolving ultraviolet divergences in quantum field theory 2026. https://doi.org/10.5281/ZENODO.19988368.
[42] Luo X. First-principles derivation of electroweak coupling constants and Higgs boson mass 2026. https://doi.org/10.5281/ZENODO.19917946.
[43] Luo X. A First-Principles Derivation of Higgs Boson Mass, Electroweak Coupling Constants, Strong Coupling Constant, and Newtonian Gravitational Constant 2026. https://doi.org/10.5281/ZENODO.23898588.
[44] Erler J. Calculation of the QED coupling α ^ ( M Z ) in the modified minimal-subtraction scheme. Phys Rev D 1999;59:054008. https://doi.org/10.1103/PhysRevD.59.054008.
[45] Navas S, Amsler C, Gutsche T, et al. Review of Particle Physics. Phys Rev D 2024;110:030001. https://doi.org/10.1103/PhysRevD.110.030001.
[46] Nagashima Y. Elementary particle physics. Weinheim: Wiley-VCH; 2013.
[47] Cabibbo N. Unitary Symmetry and Leptonic Decays. Phys Rev Lett 1963;10:531–3. https://doi.org/10.1103/PhysRevLett.10.531.
[48] Kobayashi M, Maskawa T. *C P* -Violation in the Renormalizable Theory of Weak Interaction. Prog Theor Phys 1973;49:652–7. https://doi.org/10.1143/PTP.49.652.
[49] Amsler C. The Quark Structure of Hadrons: An Introduction to the Phenomenology and Spectroscopy. Cham: Springer International Publishing AG; 2018.
[50] Entem DR, Fernández F, Ortega PG, et al. The Constituent Quark Model 2025. https://doi.org/10.48550/arXiv.2504.07897.
[51] Capozzi F, Di Valentino E, Lisi E, et al. Unfinished fabric of the three neutrino paradigm. Phys Rev D 2021;104:083031. https://doi.org/10.1103/PhysRevD.104.083031.
[52] De Salas PF, Forero DV, Gariazzo S, et al. 2020 global reassessment of the neutrino oscillation picture. J High Energy Phys 2021;2021:71. https://doi.org/10.1007/JHEP02(2021)071.
[53] Esteban I, Gonzalez-Garcia MC, Maltoni M, et al. NuFit-6.0: updated global analysis of three-flavor neutrino oscillations. J High Energy Phys 2024;2024:216. https://doi.org/10.1007/JHEP12(2024)216.
[54] Luo X. First-principles determination of the strong coupling constant 2026. https://doi.org/10.5281/ZENODO.19885004.
[55] Luo X. Fundamental Particle Spin: Derivation of the Energy-Frequency Relation, Schwarzschild Metric, and Fine Structure Constant 2026. https://doi.org/10.5281/ZENODO.23089701.