# MATHEMATICAL JUSTIFICATION OF THE MACH PRINCIPLE.

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***Abstract.*** *As a mathematical justification of Mach's principle, we present a system of cosmological equations that combines the parameters of the Universe and fundamental physical constants. The equations included in the system of cosmological equations were derived independently and at different times by famous scientists: Schrödinger, Blackley, Milne, Hoyle and existed in a scattered form. The system of cosmological equations, formed on the basis of Mach's principle, allows us to derive mathematically the parameters of the observable Universe with an accuracy close to the accuracy of the gravitational constant G. This translates Mach's philosophical principle about the connection between the Universe and the microworld onto a rigorous mathematical basis. The system of cosmological equations gives the following roots (parameters of the observable Universe): mass:* ***Mu = 1.15348… •10^53 kg****; radius:* ***Ru = 0.856594…•10^26 m****; time:* ***Tu = 2.85729… •10^17 s****; acceleration:* ***Ao = 10.4922•10^-10 m/s^2****, Hubble parameter* ***$H_0$ = 1.74991x 10^-18 s-1 (53.9967 km/s/Mpc)****. The derived values of the parameters of the observable Universe confirm with high accuracy the connection between the Universe and the microworld formulated by Mach. The mathematical substantiation of Mach's principle showed that the parameters of the Universe are directly related to the parameters of the fundamental object of the microworld - the ELECTRON. Mach's principle turned out to be much deeper: not only are the inertial properties of any body determined by its interaction with the entire Universe, but the parameters of the Universe themselves are directly related to the fundamental constants of the microworld.*

***Keywords:*** *cosmological equations, system of cosmological equations, Mach's principle, Hubble parameter, parameters of the observable Universe, fine-tuning of the Universe.*

## 1. Introduction

The inspiration for this article came from the insightful comment of the renowned German scientist, Hans Hermann Otto (Professor Emeritus, Clausthal University of Technology), on one of my previous papers: "...I'm missing Mach's principle in this article..." Indeed, Mach's principle is missing from many articles, and even from the entire field of astrophysics and cosmology. The reason is that this philosophical principle lacks a mathematical basis. Mach's principle was formulated in 1883 by the Austrian physicist and philosopher Ernst Mach [1]. The term "Mach's principle" was later coined by Albert Einstein in 1918.

The essence of the principle is that the inertial properties of any body are determined by its gravitational interaction with all other masses in the universe. Mach's principle became the impetus for Einstein to create the General Theory of Relativity (GTR). Albert Einstein openly acknowledged that it was Mach's principle that inspired him to search for a new theory of gravity. Mach forced physicists to view local phenomena through the prism of the entire universe. This led to the understanding that the local properties of the microworld are not isolated intrinsic properties, but are determined by the structure and properties of the entire universe. However, this fundamental principle lacks a mathematical basis. This hinders the development of astrophysics and cosmology.

## 2. The first attempts at a mathematical justification of Mach's principle.

Mach's principle is not represented by a quantitative measure. Mach expounded it as a philosophical principle. In 1925, Erwin Schrödinger attempted to provide a mathematical justification for Mach's principle. He set out to explain Mach's philosophical principle in mathematical terms and prove that the properties of the microworld are determined by the influence of the entire Universe.

To confirm that the local properties of the microworld are not isolated intrinsic properties but are determined by the structure and properties of the entire Universe, Schrödinger proposed an original approach. To explain the connection between the Universe and the microworld, he based his explanation on the relationship between the parameters of the Universe and the fundamental physical constants. He derived the equation $\mathbf{GM_U = c^2R_U}$. In this equation, Schrödinger combined the parameters of the Universe (mass and radius) and the fundamental physical constants (**c** and **G**) [2, 3]. The emergence of the Schrödinger equation was a revolutionary step, launching the process of translating Mach's philosophical principle into mathematical language. This was the first attempt to apply a quantitative estimate to Mach's philosophical principle.

The same equation $\mathbf{GM_U = c^2R_U}$ was independently derived by Bleksley in 1931 [4]. Schrödinger's approach was further developed. Milne proposed the equation $\mathbf{GM_U = c^3T_U}$ in 1936 [5]. Hoyle derived the equation $\mathbf{M_U = c^3/2H_0G}$ [6]. Cosmological equations linking other parameters of the Universe and the fundamental physical constants were proposed by Dirac, Rice, Weinberg, Teller, and Carvalho [7-12].

All these equations, in the spirit of Mach's principle, reflect the connection between the Universe and the microworld and are based on the unification of the parameters of the Universe and high-precision fundamental physical constants.

Disparate equations made it impossible to confirm the relationship between the parameters of the Universe and the FUNDAMENTAL PHYSICAL CONSTANTS in a strictly mathematical manner. Equations containing two or more parameters of the Universe do not provide an unambiguous solution. As a result, the task of translating Mach's philosophical principle into the language of mathematics, which was begun by Schrödinger, remained unfinished.

## 3. Transition to a System of Cosmological Equations

Since the equations derived by Schrödinger, Blakeslee, Milne, and Hoyle pertain to a single physical object—the observable Universe—we grouped and combined them into a system of algebraic equations. We added the previously derived equation $\mathbf{m_eR_U^2/M_Ur_e^2 = \alpha}$ to this system, making it final. As a result, equations satisfying Mach's principle led to a system of equations based on Mach's principle. This eliminated the need for manually substituting inaccurate experimental values, as is typically done when using the Friedmann equations. Instead of substituting inaccurate experimental values, we seek a mathematical solution in which the unknown parameters satisfy the cosmological equations. The mathematical criterion becomes a selection factor for the unknown parameters of the Universe.

Transition to a deterministic system of algebraic equations for the observable Universe, where all parameters are calculated analytically, not only confirms Mach's principle but also creates a unique opportunity for a highly accurate derivation of the Universe's parameters. For the first time, the parameters of the Universe can be obtained as the roots of a system of algebraic equations. The high

accuracy of the system's roots allows us to confirm the connection between the Universe and the microworld using the relationship between the parameters of the Universe and fundamental physical constants as an example.

## 4. A system of cosmological equations to substantiate Mach's principle.

We included the following individual equations in the system of cosmological equations:

$$GM_U = c^2R_U \quad (1)$$

$$GM_U = c^3T_U \quad (2)$$

$$GM_UA_0 = c^4 \quad (3)$$

$$M_U = c^3/2H_0G \quad (4)$$

$$R_U^2m_e/M_Ur_e^2 = \alpha \quad (5)$$

Where: $M_U$ is the mass of the Universe, G is the gravitational constant, c is the speed of light, Ru is the radius of the Universe, $m_e$ is the mass of the electron, re is the electron radius, α is the fine-structure constant; Ao is the constant of the Universe's acceleration, $T_U$ is the time of the Universe, and $H_0$ is the Hubble parameter.

The result was a system of five algebraic equations containing the necessary parameters of the Universe. Since the equations include fundamental physical constants, without introducing any adjusting coefficients, the system of equations yields values of the Universe's parameters with an accuracy close to that of the gravitational constant. The parameters of the observable Universe are the roots of the system of algebraic equations.

The system of cosmological equations contains "a priori" parameters represented by the gravitational constant G and the fundamental physical constants. This leads to an unambiguous result for the entire set of parameters of the Universe.

> **"The mathematical basis for Mach's principle is a system of cosmological equations in which the equations contain the parameters of the Universe and the fundamental physical constants, and in which the number of unknown parameters coincides with the number of equations."**

## 5. Parameters of the Observable Universe as Roots of a System of Cosmological Equations.

The parameters of the observable Universe are solutions to a system of algebraic equations:

$$M_U = 1.15348\ldots \cdot 10^{53}\ \text{kg}, \quad (6)$$

$$R_U = 0.856594\ldots \cdot 10^{26}\ \text{m}, \quad (7)$$

$$T_U = 2.85729\ldots \cdot 10^{17}\ \text{s}, \quad (8)$$

$$H_0 = 1.74991\ldots \cdot 10^{-18}\ \text{s-1 (53.9967 km/s/Mpc)} \quad (9)$$

$$A_0 = 10.4922\ldots \cdot 10^{-10}\ \text{m/s2)} \quad (10)$$

All calculated values of the parameters of the Universe have an accuracy close to that of the gravitational constant. This is an unprecedented level of accuracy for the parameters of the observable

Universe. All parameters of the observable Universe are derived with an accuracy of 5-6 decimal places. The system of algebraic equations of the Universe made it possible to completely eliminate the use of imprecise experimental values of the Universe's parameters, eliminating the need to adjust the parameters of the Universe to measured values and eliminating any numerology. As a result, the roots of the system of algebraic equations are not random physical quantities, but fundamental mathematical constants and codes of the Universe.

**"The parameters of the observable Universe are the roots of a system of algebraic equations that combine the fundamental physical constants of the microworld and the parameters of the Universe."**

## 6. Mach's Principle as a Fundamental Law of Nature.

In summary, the system of equations (11) below demonstrates that Mach's Principle is not simply a philosophical principle, but a fundamental law of nature, represented mathematically. In this system of equations, every parameter of the Universe is mathematically related to the constants of the microworld.

$$\begin{cases} M_U = m_e \alpha D_0^2 \\ R_U = r_e \alpha D_0 \\ T_U = t_e \alpha D_0 \\ A_0 = r_e / t_e^2 \alpha D_0 \\ \Lambda = 1 / r_e^2 \alpha^2 D_0^2 \\ H_0 = 1 / 2 t_e \alpha D_0 \\ G = r_e^3 / t_e^2 m_e D_0 \end{cases} \quad (11)$$

Where: $M_U$ is the mass of the Universe, G is the gravitational constant, c is the speed of light, Ru is the radius of the Universe, re is the electron radius, Ao is the constant of the Universe's acceleration, $T_U$ is the time of the Universe, and $H_0$ is the Hubble parameter, $m_e$ is the mass of the electron; $r_e$ is the classical radius of the electron; α is the fine-structure constant; $t_e$ is the characteristic time of the electron; $D_0$ is the large Weyl number ($D_0 = \mathbf{4}.1\mathbf{656}1 \cdot \mathbf{10}^{\mathbf{4}2}$).

The system of cosmological equations (11) provides a mathematical justification for Mach's principle. Equations (11) reveal the profound meaning of Mach's principle, namely: every parameter of the universe is directly related to the fundamental constants of the microworld. It is noteworthy that the parameters of the universe are directly related to the parameters of the fundamental object of the microworld – the electron! This means that the connection between the universe and the microworld, formulated by Mach, is more profound: the universe is linked with mathematical precision to local objects of the microworld.

Equations (11) complete the translation of Mach's philosophical principle into mathematical language, initiated by Schrödinger. The development of Schrödinger's approach convincingly

demonstrates that the essence of Mach's principle is much deeper. Equations (11) demonstrate that not only are the inertial properties of any body determined by its interaction with the entire universe, but the parameters of the universe themselves are directly related to the fundamental constants of the microworld.

In mathematical terms, Mach's principle can be formulated as follows:

> **"The fundamental interconnection and interdependence of the Universe and the microworld is determined by the fact that the parameters of the Universe are directly related to the fundamental constants of the microworld and represent scaled parameters of a fundamental object of the microworld—the electron."**

The scaling of the parameters of the Universe and the microworld is based on a similar mathematical transformation of the electron's parameters using two dimensionless constants of electron: the fine structure constant "alpha" and the large Weyl number ($D_0 = 4.16561 \cdot 10^{42}$).

The mass of the observable universe is the scaled mass of the electron: $\mathbf{M_U = m_e \alpha D_0^2}$

The radius of the observable universe is the scaled classical radius of the electron: $\mathbf{R_U = r_e \alpha D_0}$

The characteristic time of the observable universe is the scaled characteristic time of the electron: $\mathbf{T_U = t_e \alpha D_0}$

The cosmological acceleration is the scaled acceleration constant: $\mathbf{A_0 = r_e / t_e^2 \alpha D_0}$

The cosmological constant is the reciprocal of the square of the radius of the observable universe: $\mathbf{\Lambda = 1 / r_e^2 \alpha^2 D_0^2}$

The Hubble parameter is the scaled characteristic frequency of the electron: $\mathbf{H_0 = 1/2 t_e \alpha D_0}$

The main discovery is that each parameter of the universe is mathematically represented by a combination of fundamental physical constants of the microworld. This confirms Schrödinger's conjecture that the mathematical justification for Mach's principle can be found in the relationship between the parameters of the universe and the constants of the microworld. Substitution of the derived parameters of the universe, represented by the constants of the microworld, into the Schrödinger equation confirms the validity of his equation with high accuracy.

## 7. The Hubble parameter as a root of the system of cosmological equations.

One of the roots of the system of algebraic equations of the Universe is the Hubble parameter. Its value is $\mathbf{H_0 = 1.74991\ldots \times 10^{-18}\ s^{-1}}$ **(53.9967 km/s/Mpc)**. The derived value of $H_0$ turned out to be much smaller than in the ΛCDM model. The Hoyle and Friedmann equations imply the presence of a coefficient of "2" in front of the Hubble parameter. As a result, the system of cosmological equations yields an unexpectedly low value for the Hubble parameter: $\mathbf{H_0 = 1.74991\ldots \times 10^{-18}\ s^{-1}}$ **(53.9967 km/s/Mpc)**.This indicates that the Universe is much older than is commonly believed in the ΛCDM model. From the Hubble parameter: $\mathbf{H_0 = 1.74991\ldots \times 10^{-18}\ s^{-1}}$ **(53.9967 km/s/Mpc)**, its real age follows: 18.1 billion years. The value $\mathbf{H_0 = 1.74991\ldots \times 10^{-18}\ s^{-1}}$ **(53.9967 km/s/Mpc)** is surprisingly very close to Allan Sandage's historical estimates (**50-55 km/s/Mpc**), which for a long time competed with Gérard de Vaucouleurs's high values (**100 km/s/Mpc**) at the end of the 20th century.

This raises the question of how to interpret the $T_U$ time. This is the time it takes light to travel a distance equal to the radius of the observable Universe. This is not an age "from birth," but a characteristic time scale that is rigidly tied to its geometric size.

The Hubble parameter, derived from the system of cosmological equations, is related to other parameters of the Universe as follows:

$$H_0 = \frac{GM_U}{2cR_U^2} = \frac{c\sqrt{\Lambda}}{2} = \frac{c^3}{2GM_U} = \frac{1}{2T_U} = \frac{G\hbar}{2c^2r^3} = ... = \frac{Gm_e}{2c\alpha\, r_e^2} = \frac{c}{2r_e\,\alpha D_0} = 1.74991...\bullet 10^{-18} s^{-1} (53.9967...km/s/MPs) \quad (12)$$

Where: G is the gravitational constant; Mu is the mass of the Universe; Ru is the radius of the Universe; Tu is time; c is the speed of ligh; $H_0$ is the Hubble parameter; $m_e$ is the mass of the electron; $r_e$ is the classical radius of the electron; α is the fine-structure constant; $D_0$ is the large Weyl number.

Verification of the derived parameters of the Universe (**$M_U$ = 1.15348… •$10^{53}$ kg, $R_U$ = 0.856594…•$10^{26}$ m, $T_U$ = 2.85729… •$10^{17}$ s, H0 = 1.74991…x $10^{-18}$ $s^{-1}$ (53.9967 km/s/Mpc), A0 = 10.4922…•10-10 m/$s^2$**) using the well-known constants of QED, GTR, and classical physics showed their impeccability. They have no internal contradictions and are completely consistent with fundamental physics.

## 8. Conclusion.

1. In 1925, Schrödinger first proposed an equation for the mathematical justification of Mach's principle, in which the parameters of the universe are combined with the fundamental physical constants of the microworld. Other equations, in which the parameters of the universe are combined with the fundamental physical constants of the microworld, were proposed by Dirac, Blakesley, Milne, and Hoyle.

2. Disparate equations did not allow for a mathematical justification of Mach's principle.

3. Developing Schrödinger's approach to the mathematical justification of Mach's principle, we combined the disparate equations into a system of cosmological equations.

4. The proposed system of algebraic equations of the Universe allows us to derive the values of the parameters of the observable Universe with unprecedented accuracy, close to the accuracy of the gravitational constant: Mass **$M_U$ = 1.15348… •$10^{53}$ kg,** radius **$R_U$ = 0.856594…•$10^{26}$ m,** time **$T_U$ = 2.85729… •$10^{17}$ s,** Hubble parameter **H0 = 1.74991…x $10^{-18}$ $s^{-1}$ (53.9967 km/s/Mpc),** acceleration **A0 = 10.4922…•10-10 m/$s^2$**.

5. The value of the Hubble parameter is derived theoretically for the first time. The derived value of the Hubble parameter shows that the Universe is much older than is commonly believed in the ΛCDM model. From the Hubble parameter: $H_0$ = 1.74991… x $10^{-18}$ $s^{-1}$ follows that its real age is **18.1** billion. years.

6. The values of the parameters of the observable Universe, derived from the system of cosmological equations, confirm with high accuracy the connection between the Universe and the microworld formulated by Mach.

7. The mathematical justification of Mach's principle showed that the parameters of the Universe are directly related to the parameters of the fundamental object of the microworld – the ELECTRON.

8. Mach's principle turned out to be much deeper: not only are the inertial properties of any body determined by its interaction with the entire Universe, but the parameters of the Universe themselves are directly related to the fundamental constants of the microworld.

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