A New Lorentz Transformation with Geometry Satisfying the Principle of Relativity and Evaluating the Application of Field Theory to Special Relativity

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Abstract

The classical Lorentz transformation provides the mathematical framework for Minkowski spacetime symmetries; however, its operational derivation implicitly assumes that space and time exhibit identical symmetric projections. This paper proposes a comprehensive, dual-pathway formulation to address the structural paradoxes within special relativity. First, by analyzing the impact of signal propagation latency on measurements across distinct inertial frames, time is decomposed into four separate operational parameters. Based on this temporal distinction, we reconstruct a new Lorentz transformation characterized by a spatial scaling factor of $\boldsymbol{k = 1}$, thereby fully preserving the principle of relativity and the invariant constancy of the speed of light $\boldsymbol{c}$. This formulation is geometrized through a novel, localized "time cone" architecture that moves alongside the observers, demonstrating how equiangular coordinate axis rotation structurally ensures exact reciprocity. Second, to address the principle of relativity within a broader context, this framework is extended by relaxing its absolute validity at extreme scales. This alternative pathway introduces a static four-dimensional spacetime background field to account for the physical origins of length contraction and time dilation. Within this field-theoretic approach, the traditional Lorentz transformation is recovered not as an invariant kinematic symmetry, but as a dynamical consequence of vacuum field resistance acting upon moving excitations. Ultimately, evaluating the application of field theory to explain the operational principles of special relativity requires a rigorous, case-by-case analysis to achieve an objective theoretical conclusion.

R'esum'e

La transformation de Lorentz classique fournit le cadre mathématique des symétries de l'espace-temps de Minkowski ; cependant, sa dérivation opérationnelle suppose implicitement que l'espace et le temps présentent des projections symétriques identiques. Cet article propose une formulation complète à double voie pour aborder les paradoxes structurels au sein de la relativité restreinte. Premièrement, en analysant l'impact de la latence de propagation des signaux sur les mesures à travers différents référentiels inertiels, le temps est décomposé en quatre paramètres opérationnels distincts. Sur la base de cette distinction temporelle, nous reconstruisons une nouvelle transformation de Lorentz caractérisée par un facteur d'échelle spatiale de $\boldsymbol{k = 1}$, préservant ainsi pleinement le principe de relativité et la constance invariante de la vitesse de la lumière $\boldsymbol{c}$. Cette formulation est géométrisée à travers une nouvelle architecture de cône de temps localisée qui se déplace aux côtés des observateurs, démontrant comment la rotation équiangulaire des axes de coordonnées assure structurellement une réciprocité exacte. Deuxièrement, pour aborder le principe de relativité dans un contexte plus large, ce cadre est étendu en relaxant sa validité absolue aux échelles extrêmes. Cette voie alternative introduit un champ de fond statique d'espace-temps à quatre dimensions pour rendre compte des origines physiques de la contraction des longueurs et de la dilatation du temps. Dans le cadre de cette approche de la théorie des champs, la transformation de Lorentz traditionnelle est retrouvée non pas comme une symétrie cinématique invariante, mais comme une conséquence dynamique de la résistance du champ de vide agissant sur les excitations en mouvement. En fin de compte, l'évaluation de l'application de la théorie des champs pour expliquer les principes opérationnels de la relativité restreinte exige une analyse rigoureuse, au cas par cas, afin de parvenir à une conclusion théorique objective.

Keywords: Modified Lorentz transformations, Co-moving time cone, Spacetime symmetry, Constancy of the speed of light, Principle of relativity, Field-theoretic approach

1 Introduction

The Lorentz transformation forms the mathematical foundation of special relativity, leading to profound kinematic consequences—such as time dilation and length contraction—for reference frames moving at relativistic speeds. Although these effects have been rigorously confirmed by experimental evidence, fundamental challenges arise when analyzing how localized operational measurements scale across different systems. Specifically, kinematic descriptions often implicitly rely on the choice of a conventional rest frame for a given observer; this role cannot simply be symmetrically interchanged with a second observer. This structural asymmetry between observers involving such reciprocal choices can lead to the classic paradoxes of special relativity, namely the twin paradox and the ladder paradox. While numerous explanations for

these paradoxes exist within the literature, they frequently presuppose a fixed, preferred reference framework; conversely, the core tenet of special relativity asserts that a localized observer cannot discern their own absolute velocity.

From a foundational perspective, these paradoxes necessitate a closer examination of how coordinate transformations physically manifest the principle of relativity—particularly the absolute physical equivalence of all inertial reference frames. A detailed analysis of the standard formulation of the Lorentz transformation reveals a subtle empirical distinction between the proper time of a moving reference frame and the coordinate time recorded by a distant observer. This discrepancy poses an interesting challenge for the consistent application of the relativity postulate across measurement systems. To address this conceptual conflict, this paper examines a comprehensive theoretical framework comprising two parallel approaches:

- **Preserving the Relativity Postulate through Descriptive Geometry:** Based on the assumption of the absolute validity of physical equivalence between inertial reference frames, we demonstrate that coordinate transformation equations can be reformulated to ensure strict symmetry. By decoupling spatial scale variations from signaling propagation delays using linear geometry, we construct a modified transformation with a structural factor of $k = 1$. This architecture employs a "time cone" that moves alongside localized observers, ensuring that the light pulse wavefront consistently maintains a fixed angular bisector position by design, thereby resolving classical coordinate paradoxes.
- **An Alternative Theoretical Framework Based on Field Theory:** In an alternative direction—assuming that standard relativistic relations undergo modifications at extreme scales—we investigate a broader approach that relaxes the strict principle of relativity. Within this alternative framework, field-theoretic mechanisms are introduced to provide a dynamical explanation for the traditional Lorentz transformation. By modeling material entities as excitations within a static four-dimensional spacetime background field, the properties of traditional transformations are explained as physical consequences of moving through an underlying vacuum medium possessing finite propagation limits.

2 The Traditional Lorentz Transformation Does Not Satisfy the Principle of Relativity

2.1 Proof That Two Reference Frames in Relative Motion Generate Four Distinct Operational Times

An object located at a spatial distance always manifests within the causal past of a localized observer. For instance, since the Sun is situated approximately 8 light-minutes away from the Earth, any optical observation recorded on Earth structurally perceives the Sun as it existed 8 minutes in the observer's causal past:

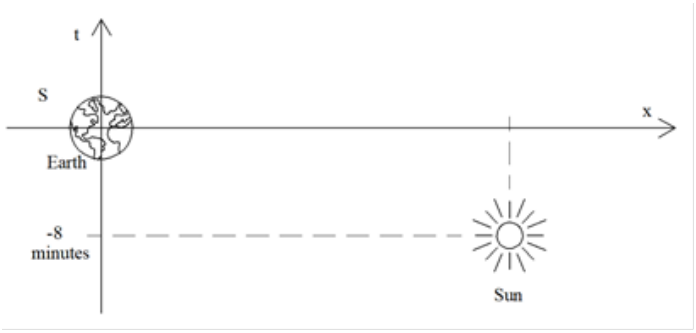


Fig. 1 Operational diagram of signaling latency between distant spacetime events [1, 2].

Consider two inertial reference frames, S (Earth) and S' (Spaceship), where S' moves at a constant uniform velocity v relative to S along their common x-axis. Within each reference frame, a master clock is positioned at the spatial origin alongside its respective localized observer. At the initial temporal intersection, the two coordinate systems are synchronized such that $t = t' = 0$. Throughout the continuous observation process, the two reference frames systematically exchange encoded temporal data via electromagnetic signaling.

Because the signal propagation velocity is finite and fundamentally bounded by the speed of light c, when the observer in S registers a received timestamp value of t', the coordinate time within the rest frame S has inevitably progressed beyond the initial emission instant t [3]. Consequently, a rigorous operational decomposition reveals the existence of four distinct, non-coincident temporal parameters rather than the traditionally assumed two:

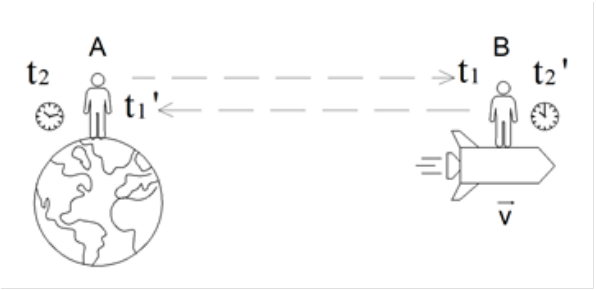


Fig. 2 Geometric configuration and structural notation of the four distinct operational time parameters between moving inertial reference frames [3].

- t_1 represents the coordinate time in reference frame S when the moving origin of S' is physically located at the spatial position $x = vt_1$, which operationally corresponds to a localized reading of t'_1 on the co-moving clock in S' [3].
- t_2 represents the coordinate time in reference frame S at the exact instant observer A physically receives the temporal data packet indicating that the clock reading on the moving spaceship was t'_1 [3].

- t'_1 represents the proper time recorded in reference frame S' corresponding to the instantaneous spatial configuration at time t_1 as defined on Earth [3].
- t'_2 represents the proper time recorded in reference frame S' at the exact instant observer B physically receives the temporal data packet indicating that the clock reading on Earth was t_1 [3].

Analytically, it is evident that the temporal variables t_1 and t'_1 correspond precisely to the conventional, non-differentiated concepts of time t and t' utilized in the traditional derivation of the Lorentz transformation [3]:

$$t \equiv t_1 \quad \text{and} \quad t' \equiv t'_1 \tag{1}$$

However, when observer A at the origin of S receives the inbound information signal from the moving spaceship, the master clock on Earth has already advanced to a retarded coordinate time governed by signaling latency [3]:

$$t_2 = t_1 + \frac{vt_1}{c} = t_1\left(1 + \frac{v}{c}\right) \tag{2}$$

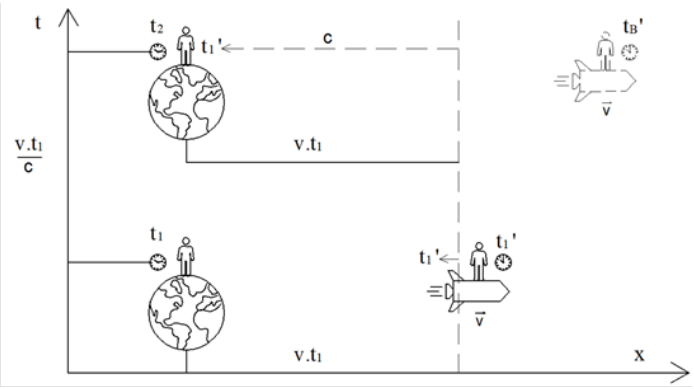


Fig. 3 The functional dependency between information transmission latency and coordinate time progression across reciprocal reference systems [3].

From this structural architecture, it is entirely evident that the temporal parameters t_1 and t_2 are physically and analytically distinct ($t_2 \neq t_1$). Consequently, when establishing the mathematical foundations of coordinate transitions, these operational values must be explicitly differentiated. Failing to separate the emission time from the reception time introduces a fundamental mathematical conflation that ultimately prevents the traditional Lorentz transformation from fully satisfying the principle of relativity across the entire spacetime manifold [3].

2.2 Step-by-Step Derivation Analysis: Tracking the Inconsistency Between the Traditional Lorentz Transformation and the Principle of Relativity

The classical Lorentz transformation is historically constructed upon two foundational postulates originally formalized by Einstein [4]:

- The principle of relativity (the absolute physical equivalence of all inertial reference frames).
- The constancy of the speed of light in all inertial reference frames.

The standard step-by-step derivation of the transformation equations between two inertial reference frames S and S' (where S' moves with a uniform velocity v relative to S along their common x-axis) is summarized below, following the pedagogical review detailed in prior literature [1, 2]:

Step 1: Postulate of Spatial Linearity. We assume that the coordinate transformation between the two systems is linear. Based on uniform relative motion, the spatial mappings are parameterized by a velocity-dependent structural coefficient k:

$$x' = k(x - vt) \tag{3}$$

$$x = k(x' + vt') \tag{4}$$

where k is an undetermined scaling factor depending solely on the relative velocity v.

Step 2: Operational Light-Signal Propagation. Using the axiom that light always propagates at a constant velocity c across all inertial reference frames, let the spatial origins of frames S and S' coincide at the initial temporal alignment $t = t' = 0$. At this exact instance, a ray of light is projected from the common origin:

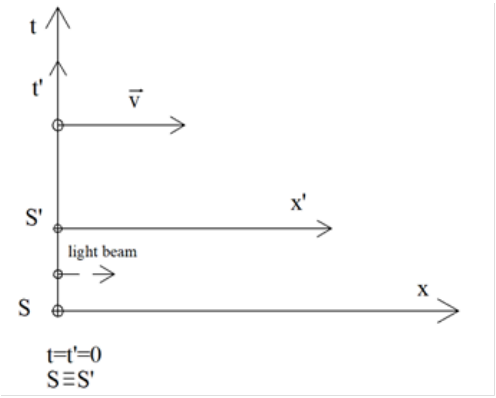


Fig. 4 Initial temporal alignment and spatial coincidence of the origins of frames S and S' at $t = t' = 0$ [1, 2].

At coordinate time t, when the light front reaches position x in the rest frame S, the invariant light-cone trajectories dictate that:

$$x' = ct' \tag{5}$$
$$x = ct \tag{6}$$

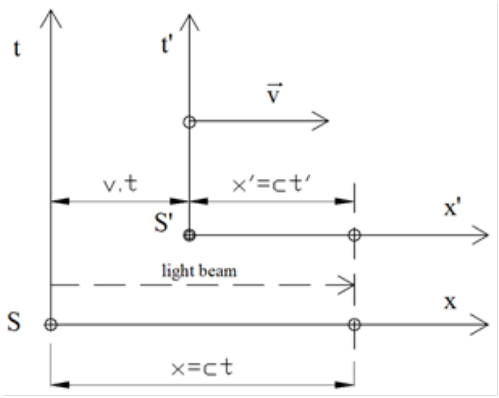


Fig. 5 Kinematic separation of the reference frames and propagation of the light signal at coordinate time $t > 0$ [1, 2].

Step 3: Algebraic Substitution. Substituting these light-cone conditions (5) and (6) into equations (3) and (4) yields:

$$ct' = k(ct - vt) = kt(c - v) \tag{7}$$
$$ct = k(ct' + vt') = kt'(c + v) \tag{8}$$

Step 4: Evaluation of the Structural Factor. Multiplying equations (7) and (8) side-by-side simultaneously eliminates the temporal coordinate variables t and t':

$$c^2 tt' = k^2 tt'(c - v)(c + v) \tag{9}$$

Assuming $t, t' \neq 0$, this directly simplifies to:

$$c^2 = k^2(c^2 - v^2) \quad \Longrightarrow \quad k^2 = \frac{c^2}{c^2 - v^2} = \frac{1}{1 - \frac{v^2}{c^2}} \tag{10}$$

Extracting the positive root, we obtain the definitive classical structural result:

$$k = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \equiv \gamma \tag{11}$$

where γ is identified as the standard Lorentz factor. By incorporating this factor back into the kinematics, the corresponding temporal coordinate transformations are structurally deduced as [4]:

$$t' = \gamma \left(t - \frac{vx}{c^2} \right) \tag{12}$$

$$t = \gamma \left(t' + \frac{vx'}{c^2} \right) \tag{13}$$

An explicit operational examination of this derivation reveals that at a given coordinate time t, there exists only a singular pair of points, denoted as x_c and x'_c, that simultaneously satisfies the light-cone conditions [1, 2]:

$$x_c = ct \quad \text{and} \quad x'_c = ct' \tag{14}$$

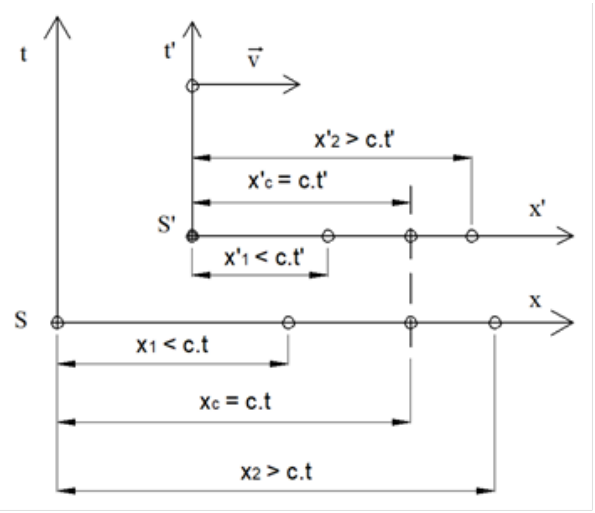


Fig. 6 At coordinate time t, the position of the light ray in the two reference frames maps to an exclusive pair of points designated as x_c and x'_c [1, 2].

This restriction implies that the structural coefficient k reduces to the standard Lorentz factor γ exclusively at these specific positions (x_c, x'_c). To analyze this geometric setting across the entire spacetime manifold, we evaluate arbitrary spatial coordinates via a scale parameter α:

$$x = \alpha x_c, \quad x' = \alpha x'_c, \quad \text{where } \alpha \in \mathbb{R} \tag{15}$$

Repeating the construction steps above under these generalized spatial intervals yields:

$$k = \alpha\gamma \tag{16}$$

Furthermore, it can be formally demonstrated that for any spatial position $x \neq ct$, executing a reciprocal coordinate mapping between the inertial reference frames will reveal a mathematical contradiction such that $x' \neq ct'$.

This discrepancy stems from a fundamental structural asymmetry: the temporal parameter t' dictated by the light-signal kinematics $x' = ct'$ does not analytically coincide with the coordinate time t' embedded within the reciprocal coordinate transformation $x = k(x' + vt')$. Consequently, this structural mismatch violates the strict operational equivalence required by the principle of relativity between two inertial reference frames [1, 2].

From a foundational perspective, coordinate transitions must preserve a strict uniqueness condition: a physical entity must possess a unique worldline, while observers in localized moving frames merely register distinct coordinate projections of its position and time. The physical validity of this uniqueness can be rigorously verified by introducing an independent third observer, for whom the observed entity must remain uniquely defined.

The conventional derivation of the Lorentz transformation, however, unveils a subtle operational ambiguity regarding this uniqueness. Under specific kinematic constraints, the transformed coordinates mathematically manifest as two distinct physical states across different reference frames—occupying identical spatial locations but exhibiting a temporal bifurcation. This desynchronization becomes explicitly observable either by adopting the perspective of a third co-moving frame of reference, or more simply, by positioning a third observer at a spatial offset from the origin of frame S.

To illustrate this foundational tension, the following thought experiment is proposed:

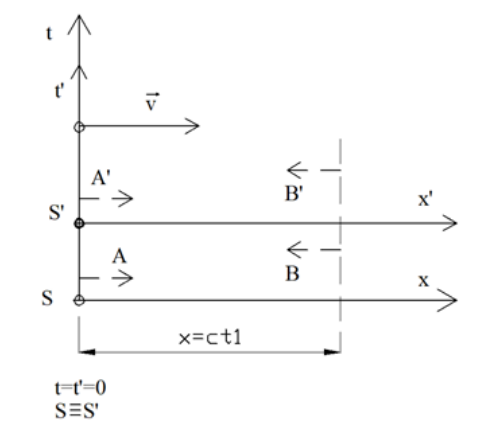


Fig. 7 Initial simultaneous projection of four independent light signals denoted as A, A', B, and B' [1, 2].

At the initial temporal alignment $t = t' = 0$ and spatial coincidence $x = x' = 0$, two forward-propagating light signals are emitted from the origins: ray A is associated with frame S, and ray A' is associated with frame S'. Simultaneously, at a distant

spatial coordinate $x_1 = ct_1$, two backward-propagating light signals are emitted: ray B is associated with frame S, and ray B' is associated with frame S' [1, 2]:

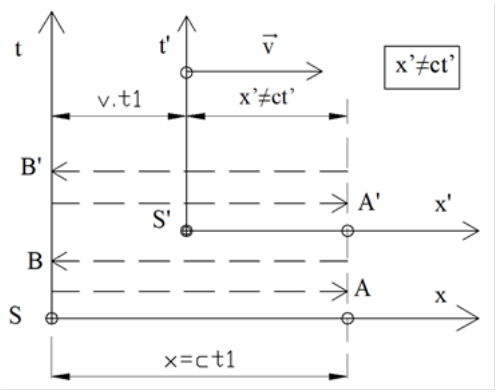


Fig. 8 Coordinate distributions of the four light signals A, A', B, and B' at coordinate time $t = t_1$ [1, 2].

Upon reaching the temporal milestone $t = t_1$, rays A and A' arrive at the designated coordinate x_1, while rays B and B' intersect the spatial origin S_0. According to the standard Lorentz formulation, after the interval $t = t_1$, the signal B' is mathematically required to reach the moving origin S_0'. In physical reality, however, it coincides with the stationary origin S_0. This physical contradiction rigorously validates the condition that $x' \neq ct'$ under general coordinate evaluations, proving an operational conflict with the principle of relativity [1, 2].

Therefore, we conclude that no physical entity characterized by real spacetime coordinates within the intersecting domains of S and S' can simultaneously satisfy the mathematically overdetermined system of equations [1, 2]:

$$x' = k(x - vt) \tag{17}$$
$$x = k(x' + vt') \tag{18}$$
$$x' = ct' \tag{19}$$
$$x = ct \tag{20}$$

Conclusion: The traditional constraint enforcing that the structural coordinate scaling factor k must identically equal the classical Lorentz factor γ (i.e., $k \equiv \gamma$) is the fundamental cause of the operational asymmetry. This constraint introduces a hidden mathematical restriction that ultimately prevents the conventional Lorentz transformation from unconditionally satisfying the principle of relativity across arbitrary positions on the spacetime manifold.

3 Construction of a New Lorentz Transformation Ensuring Spacetime Symmetry via Signaling Latency

3.1 Method 1: Analytical Derivation via Classical Kinematics

We formally demonstrate that the intrinsic propagation velocity of an information-carrying measuring signal does not introduce structural scale distortions into the physical measurement of a proper length interval. Let us consider two alternative operational frameworks for determining the proper length l_0 of a stationary rigid object A [1, 2]:

- **First Operational Framework (Optical Signaling):** Utilizing light signals propagating at a constant invariant velocity c:

$$x_1 = ct_1 \tag{21}$$
$$x_2 = ct_2 \tag{22}$$
$$l_0 = x_1 - x_2 = c\Delta t \tag{23}$$

- **Second Operational Framework (Acoustic Signaling):** Utilizing sound signals propagating at a constant medium-bound velocity v_s:

$$x_1 = v_s t_1' \tag{24}$$
$$x_2 = v_s t_2' \tag{25}$$
$$l_0 = x_1 - x_2 = v_s \Delta t' \tag{26}$$

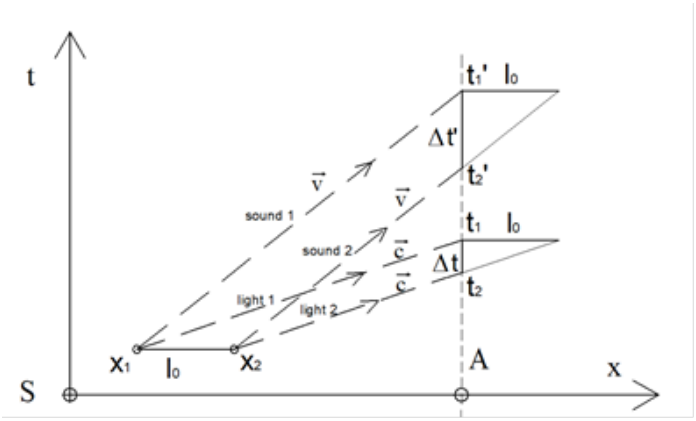


Fig. 9 Comparative length measurement methodology utilizing optical and acoustic signaling baselines [1, 2].

Analytically, within both observational baselines, the structural governing equation for the proper length l_0 manifests as a first-degree linear equation mapping directly to its respective localized temporal duration Δt (or $\Delta t'$). Consequently, the invariant physical quantity l_0 remains strictly independent of the specific propagation velocity characteristic of the information-carrying wave signal [1, 2].

This invariance implies that the structural scaling coefficient k embedded within the spatial relativity coordinate transformation must identically reduce to unity ($k \equiv 1$). Under this exact physical constraint ($k = 1$), the spatial coordinate transformations between two moving inertial reference frames correspond formally to the classical Galilean mapping structure [1, 2]:

$$x' = x - vt \tag{27}$$

$$x = x' + vt' \tag{28}$$

This derivation reveals that the constancy of the speed of light c across all inertial reference frames is structurally decoupled from the spatial scaling factor k and the spatial coordinate mapping itself; rather, it is governed exclusively by the non-trivial, retarded temporal transformation group between the frames. To satisfy the constancy of the speed of light postulate under the $k = 1$ spatial boundary condition, the corresponding temporal transformation equations are formalized as follows:

$$t' = t - \frac{vx}{c^2} \tag{29}$$

$$t = t' + \frac{vx'}{c^2} \tag{30}$$

3.2 Method 2: Structural Derivation via Linear Descriptive Geometry

The geometric representation of the classical Galilean relativity transformation is mapped on a sheared spacetime manifold:

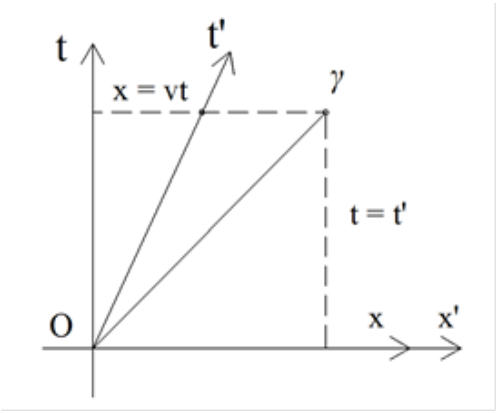


Fig. 10 Geometric coordinate grid deformation under a classical Galilean relativity transformation [3].

Within the Galilean transformation group, time is an absolute invariant parameter ($t = t'$) because the speed of information propagation is assumed to be instantaneous ($c \to \infty$). Conversely, within the conventional relativistic framework, the registered coordinate time t' experiences an operational transmission delay relative to the emission time t equal to the latency interval $\frac{vx}{c^2}$, because electromagnetic information propagates at a finite velocity c [3].

To model this transition geometrically, we replace the independent temporal axes t and t' with the normalized, dimensionally unified coordinates ct and ct', and define the non-dimensional velocity ratio parameter:

$$\beta = \frac{v}{c} \tag{31}$$

Under uniform kinematic shear, the linear algebraic projections yield:

$$ct' = ct - \beta x \tag{32}$$

$$x' = x - \beta ct \tag{33}$$

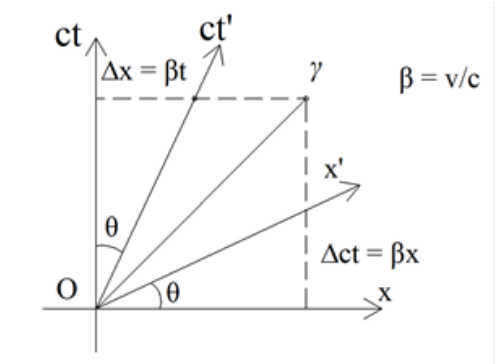


Fig. 11 Spacetime axis deformation incorporating finite signal transmission latency [3].

From the diagrammatic grid structure, it is evident that this formulation is a linear transformation that preserves the path invariance of a light pulse wavefront across the spacetime manifold:

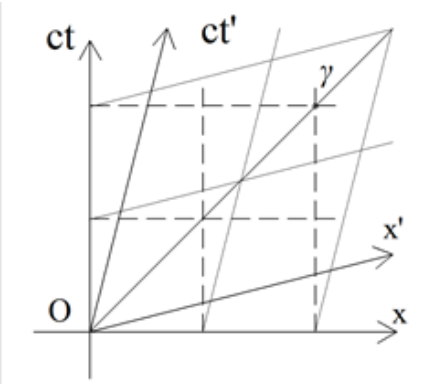


Fig. 12 The Lorentz transformation modeled as a linear geometric mapping preserving light-front invariance [3].

This structural path invariance ensures that the measured velocity of light remains an invariant constant across all reciprocal systems [3].

In the generalized case, a linear coordinate transformation comprises not only the shear deformation of space and time but also a spatial scaling factor. Consequently, in its general undetermined form, the transformation group is parameterized by a scaling coefficient k [3]:

$$ct' = k(ct - \beta x) \tag{34}$$

$$x' = k(x - \beta ct) \tag{35}$$

Applying the principle of relativity, the symmetrical inverse coordinate mapping yields:

$$ct = k(ct' + \beta x') \tag{36}$$
$$x = k(x' + \beta ct') \tag{37}$$

Here, the undetermined structural factor k characterizes the scaling parameters of the coordinate grid framework [3].

Derivation of the Structural Factor k

Multipivating equations (35) and (37) side-by-side establishes the coordinate product relation:

$$xx' = k(x' + \beta ct') \cdot k(x - \beta ct) \tag{38}$$
$$\Leftrightarrow xx' = k^2(x' + vt')(x - vt) \tag{39}$$

Let us evaluate a light ray emitted from the origin of the moving spaceship. Because the local propagation speed is bounded by c relative to the moving reference frame, its trajectory maps strictly to the sub-manifold [3]:

$$x' = ct'_1 = ct' \tag{40}$$

Upon reaching the rest observer, the light ray propagates at velocity c relative to the frame S. However, according to the four-parameter operational time framework established in Section 2.1, the coordinate time at which the signal arrives is no longer the emission instant t_1 but has progressed to the retarded reception time t_2 due to transmission latency [3]:

$$x = ct_2 = c\left(t_1 + \frac{vt_1}{c}\right) = ct_1 + vt_1 = ct + vt \tag{41}$$

Substituting these explicit dynamic time parameters into the coordinate product relation (39) yields:

$$(ct + vt)ct' = k^2(ct' + vt')(ct + vt - vt) \tag{42}$$
$$\Leftrightarrow (c + v)ctt' = k^2(c + v)ctt' \tag{43}$$
$$\Leftrightarrow k^2 = 1 \tag{44}$$

Because the structural scale coefficient must strictly preserve orientation and take positive values ($k > 0$), it follows that:

$$k \equiv 1 \tag{45}$$

By incorporating this boundary value back into the coordinate systems, we isolate the complete modified relativity transformation group [3]:

$$x' = x - vt \tag{46}$$

$$t' = t - \frac{vx}{c^2} \tag{47}$$

$$x = x' + vt' \tag{48}$$

$$t = t' + \frac{vx'}{c^2} \tag{49}$$

This dynamic transformation group satisfies both foundational postulates of special relativity. To evaluate its complete physical implementation and establish how this algebraic structure unconditionally preserves coordinate reciprocity, a compatible geometric framework must be explicitly formulated.

4 Constructing a New Geometric Description for the Modified Lorentz Transformation

4.1 Inconsistencies Between Minkowski Geometry and the Lorentz Transformation Mapping

Within standard relativistic transformations, two localized observers are conventionally defined to track physical phenomena:

- A first observer who remains strictly at rest relative to the spatial origin of reference frame S.
- A second observer who remains strictly at rest relative to the spatial origin of the moving reference frame S'.

However, when visualizing and illustrating these reference frames on a coordinated spacetime diagram featuring an explicit temporal axis, a subtle epistemological challenge arises. Constructing a single unified diagram essentially requires the conceptual introduction of a third observer situated outside of the spacetime manifold itself, who acts as an external meta-observer monitoring both coordinate grids simultaneously [3]. In classical physics or Galilean kinematics, conflating this external third observer with the localized first observer does not introduce mathematical errors because information is assumed to propagate instantaneously ($c \to \infty$):

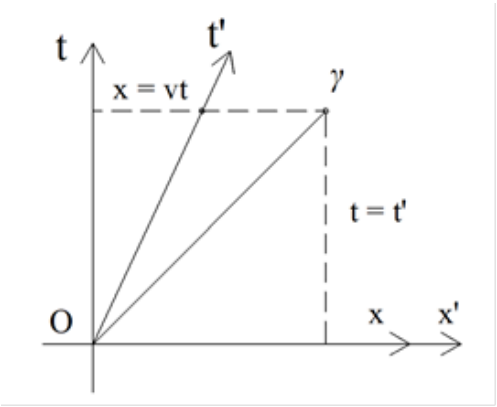


Fig. 13 Spacetime diagram illustrating the conventional coordinate projections recorded by the first observer [3].

Conversely, within conventional special relativity, the light cone is introduced to govern causality, establishing a fundamental boundary in spacetime that strictly confines all events capable of exerting a physical influence on a given localized observer to its interior. In Minkowski geometry, this causal light cone structure is mathematically formulated as a static global background boundary [3]:

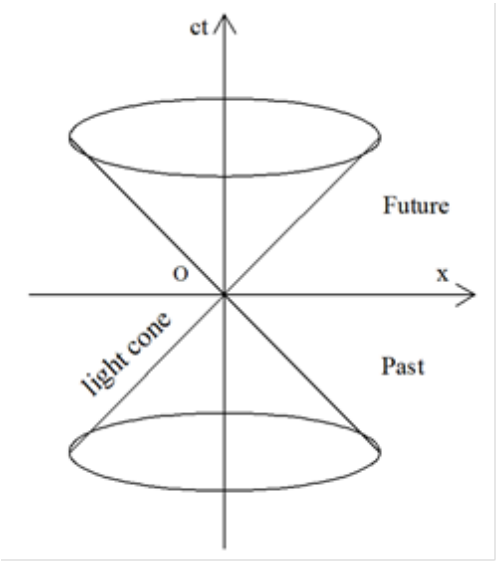


Fig. 14 The static geometric configuration of the light cone in conventional Minkowski spacetime [3].

Within the standard framework of Minkowski geometry, the first observer and moving material objects are modeled as moving along independent, localized worldlines relative to this fixed background cone [3]:

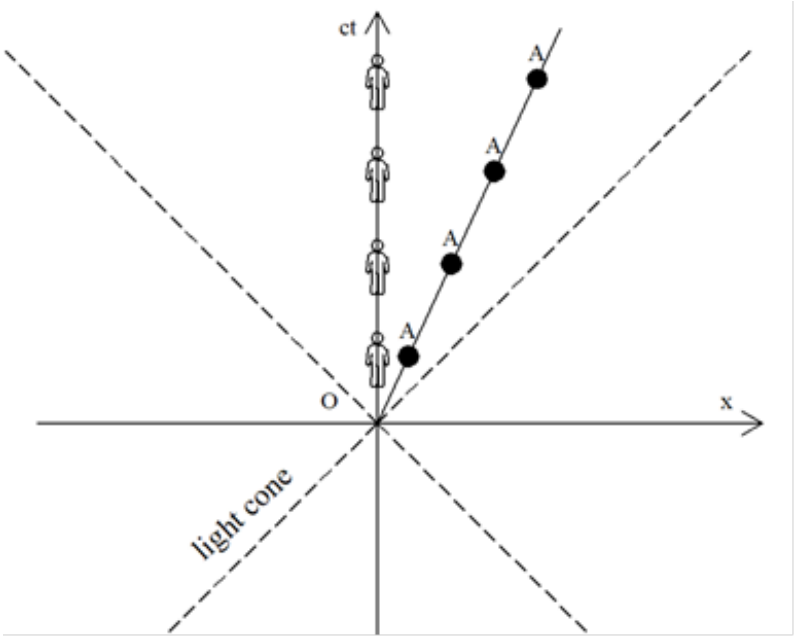


Fig. 15 Geometric configuration demonstrating the standard Minkowski light cone remaining decoupled from the kinematic worldline of the localized observer.

The logical and structural flaw in this conventional geometric representation lies in the fact that the first observer's causal light cone remains strictly locked at the initial spatial origin, while the physical observer progresses continuously forward along the temporal axis. Consequently, Minkowski geometry suffers from a profound representational error by conflating the localized, signal-dependent role of the first observer with the global, omnipresent role of the abstract third observer [3].

In order to accurately preserve the physical meaning of localized measurements and maintain consistency with the postulates of special relativity, a correct geometric representation demands that the vertex of the causal light cone must translate continuously along with the co-moving worldline of its respective localized observer [3].

4.2 Formulation of the New Geometry Describing the Modified Lorentz Transformation

4.2.1 Developing a Co-moving Time-Cone Model

We establish the following observations:

- The light cone can only be explicitly formulated for the first observer. Furthermore, this light cone continuously translates through spacetime alongside the first observer [3].
- It is well established that from the perspective of the first observer, increasing spatial distance from their instantaneous position corresponds to a regression into the past; hence, the vertex of the light cone precisely marks the current position and temporal instant of the first observer [3].

Based on these premises, the geometric representation of the light cone is formulated as follows: **Step 1:** Representing the First Observer's Perspective Using Euclidean Geometry [3] Consider an observer positioned at the spatial origin O. At the initial time $t = 0$, the observer launches an object A and simultaneously emits a light ray γ [3].

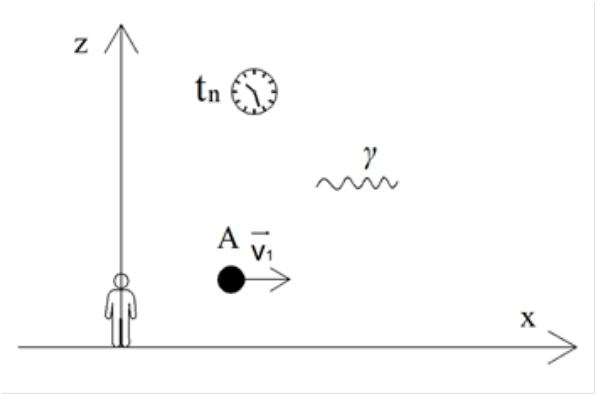


Fig. 16 Object A and the light ray are moving relative to the first observer [3].

From the perspective of this first observer, time is captured strictly as a instantaneous moment:

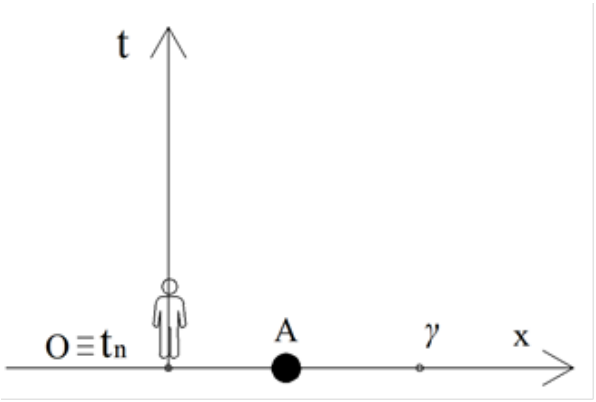


Fig. 17 In the classical Galilean transformation, the first observer with object A and the light ray have the same time t_n. [3].

Step 2: The Temporal Framework from the First Observer's Perspective Takes a Conical Form with Its Vertex at the Observer's Position [3]

We project the spatial positions of the observed objects onto this light cone [3]:

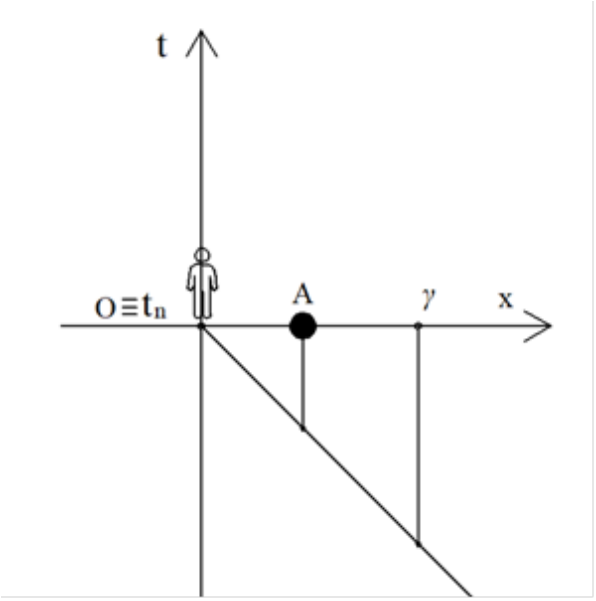


Fig. 18 Representation of the position of object A and the light ray, accounting for the measurement signal delay experienced by the first observer [3].

Consequently, we obtain the geometric representation of the light cone. All observable objects are situated strictly on the surface of this light cone, because the first observer captures physical phenomena only as instantaneous moments:

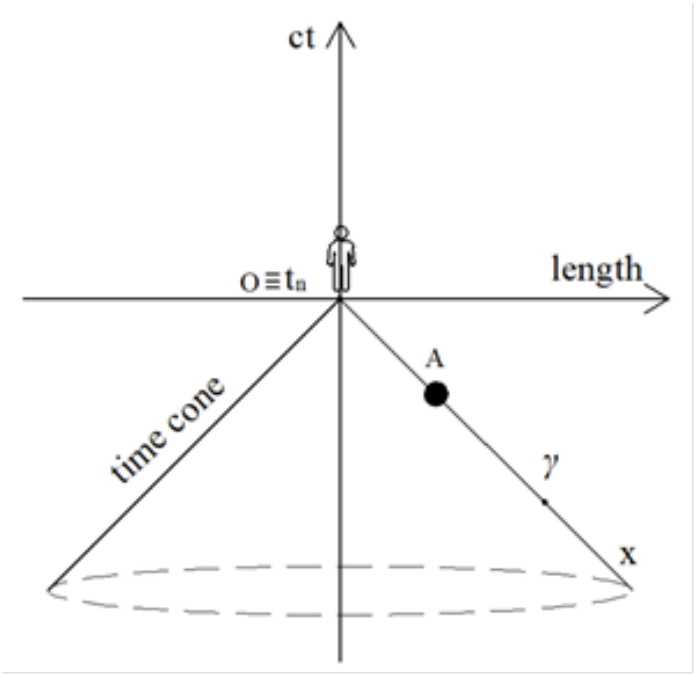


Fig. 19 Time cone.

This light cone can be formally designated as a "time cone" because as the spatial distance from the first observer increases, one progressively regresses into the past of the respective reference frame [3].

4.2.2 Geometric Representation of the Modified Lorentz Transformation via Co-moving Time Cones

The representation is formulated as follows:

- The first observer remains stationary in space and propagates along the temporal axis. This observer is intrinsically anchored to the time cone, which structurally constitutes the spatial axis x [3].
- Observed object A is associated with the second reference frame S'; its light cone is analogous to that of the observer in S. Object A moves along world line K' [3].

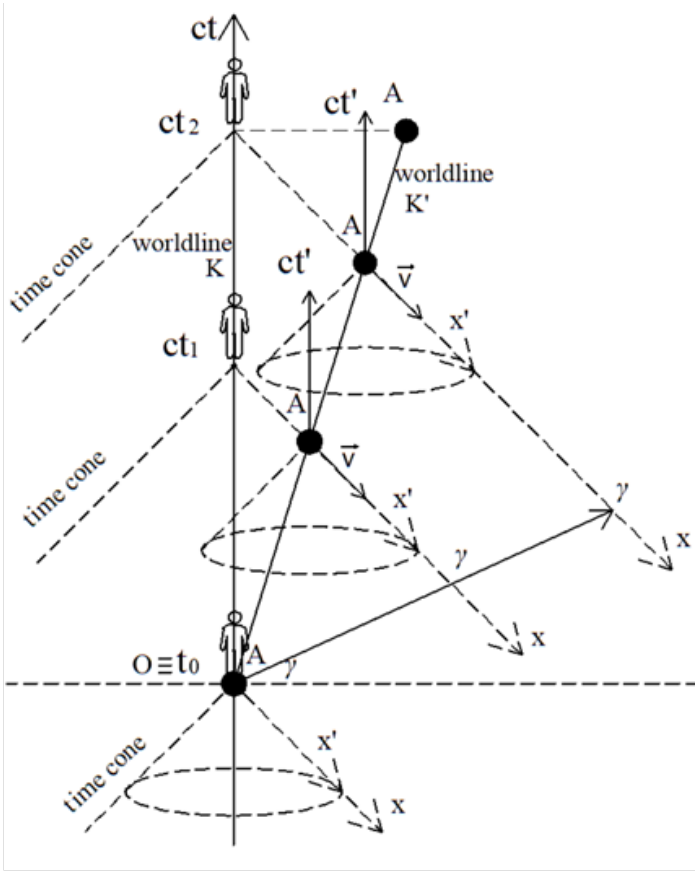


Fig. 20 Visualizing the Lorentz transformation using the time cone [3].

When switching the roles of observers 1 and 2—treating reference frame S' as stationary and reference frame S as moving—we obtain the following diagram :

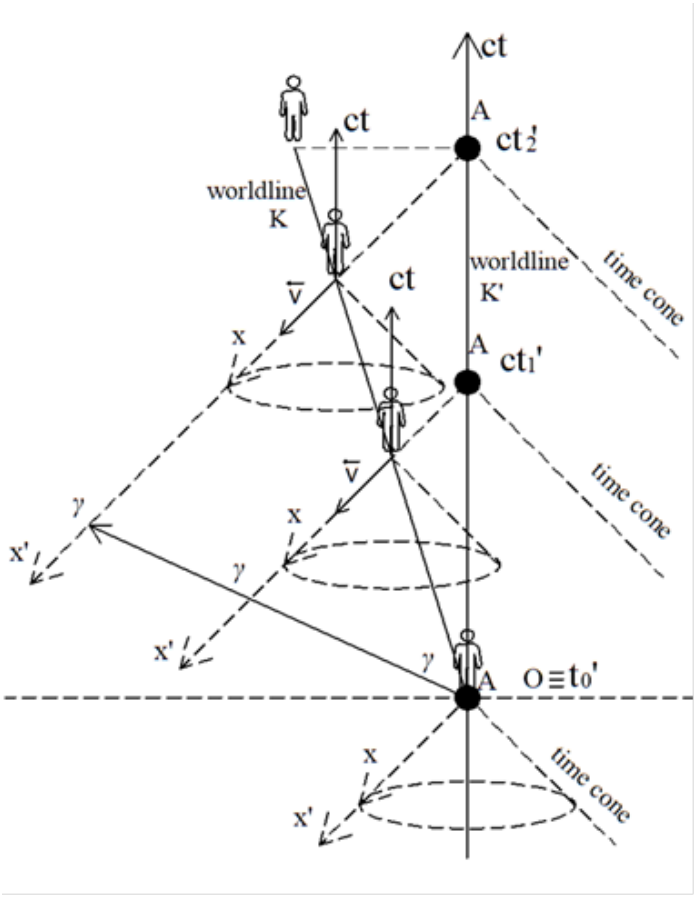


Fig. 21 Demonstrating the Lorentz transformation for the case where the second observer is stationary while the first observer is in motion [3].

We observe that when using reference frame S' (a distant reference frame), we typically examine phenomena occurring in the vicinity of its origin; therefore, we can omit the representation of S''s time cone to simplify the diagram [3]:

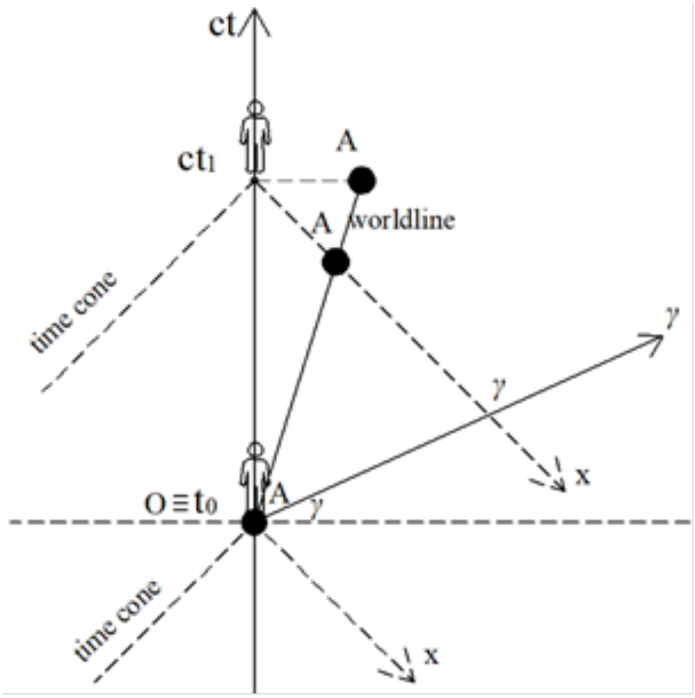


Fig. 22 Illustration of the Lorentz transformation with the second observer's time cone omitted [3].

From this diagram, it is evident that the trajectories of all light rays in both the first and second reference frames are parallel to one another. Thus, the diagram successfully illustrates that the speed of light remains constant across reference frames [3].

Based on this diagram, we can apply a coordinate axis rotation:

- Step 1: Rotate the ct' axis to align with the worldline, and rotate the x' axis by the same angle but in the opposite direction to the rotation of the ct' axis [3].
- Step 2: Rotate the x axis to a position perpendicular to the ct axis [3].

This co-axial angular alignment successfully recovers the geometric configuration illustrating the modified Lorentz transformation from the localized perspective of the first observer, structurally matching the linear algebraic system derived in Section 3.2:

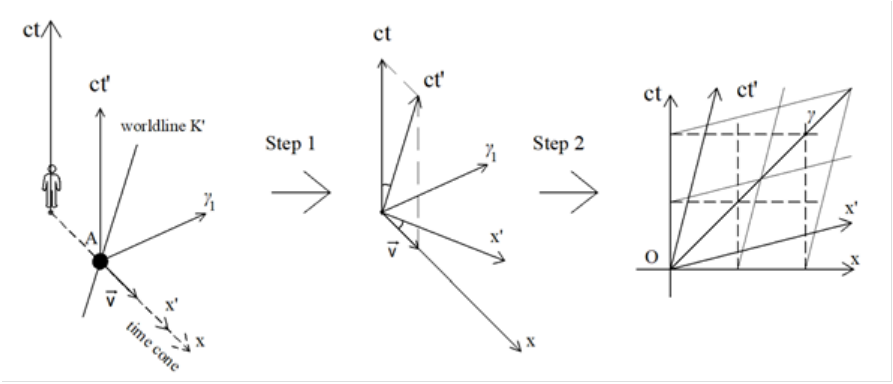


Fig. 23 Equiangular coordinate axis rotation recovering the reciprocal linear transformation group via symmetric structural tilting.

4.3 Proof of Consistency with the Principle of Relativity

To formally demonstrate that the modified relativity transformation group satisfies the principle of relativity, we analyze the structural and operational significance of the temporal coordinate equation:

$$t' = t - \frac{vx}{c^2} \tag{50}$$

Here, the kinematic product vt defines the instantaneous spatial separation distance between the spatial origin of the moving reference frame S' and the origin of the stationary rest frame S. Consequently, the structural term $\frac{vx}{c^2}$ is operationally interpreted as the signaling propagation latency or information transmission delay for data packets transmitted from an observer B positioned at the spatial origin of S' to an observer A at the origin of S. Therefore, the coordinate time parameter t' does not represent the intrinsic proper time within the moving frame S'. Rather, it functions as a retarded coordinate time registered by observer A, compiled exclusively from incoming information signals emitted by the moving system S' [3].

According to Section 4.2.1 the spacetime in the first observer's frame of reference has the structure of a light cone:

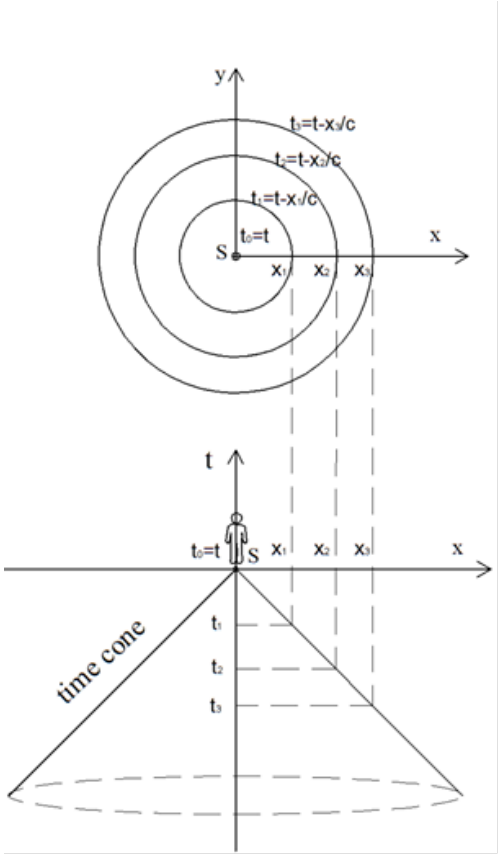


Fig. 24 Geometric representation of the temporal asymmetric time cone around an observer.

As illustrated in the diagrammatic field above, time exhibits a fundamental structural asymmetry compared to space. Consequently, a relativistic temporal coordinate transformation cannot be physically executed by merely swapping the mathematical sign operator from subtraction to addition via a naive algebraic inversion. Therefore, the conventional inverse temporal formulation,

$$t = t' + \frac{vx'}{c^2} \tag{51}$$

is physically incompatible, as it fails to represent the actual retarded coordinate time registered by a co-moving observer O' receiving incoming tracking signals from frame S. Accordingly, this asymmetric inverse formulation is omitted from the modified relativity transformation group [3].

To determine the measurements registered by a localized observer embedded within reference frame S', a coordinate transition must be executed to swap the operational roles of the systems, whereby S' is treated as the stationary rest system and S becomes the moving frame. Therefore, the modified Lorentz transformation group applies the

rigorous measurement of space and time in reference frame S' to the observer in reference frame S via the coupled system:

$$x' = x - vt \tag{52}$$

$$t' = t - \frac{vx}{c^2} \tag{53}$$

It is worth noting that this temporal coordinate formulation is initially constructed under the strict boundary condition where the two spatial origins coincide at the synchronization epoch $t = t' = 0$. For a generalized kinematic scenario featuring an initial spatial offset x_0 at the initial epoch, the temporal transformation is formalized as:

$$t' = t - \frac{vx}{c^2} - \frac{x_0}{c} \tag{54}$$

From this generalized relation, we derive the apparent temporal progression rate registered by the rest observer tracking a moving spacecraft relative to the intrinsic frame duration:

$$\Delta t' = \left(1 - \frac{v}{c}\right) \Delta t \tag{55}$$

In this operational formulation, the velocity v acts strictly as a directional vector parameter: $v > 0$ when the frame S' is receding from S, and $v < 0$ when S' is approaching S. We analytically evaluate these two distinct kinematic scenarios:

Case 1: Receding Kinematics. Consider a spacecraft A attached to frame S' moving away from the origin of frame S. An observer at the origin of S continuously receives temporal data transmitted from the spacecraft. Since the relative distance between the two reference frames increases monotonically over time, the information signals transmitted from the spacecraft require a progressively longer propagation duration to arrive; consequently, the observer registers a dilated temporal rate, perceiving time in frame S' to elapse more slowly.

Case 2: Approaching Kinematics. Conversely, as the spacecraft A reverses its trajectory and approaches the spatial origin, the propagation time required for the signals to reach the observer decreases monotonically; consequently, the observer registers an accelerated temporal rate, perceiving the clock on spacecraft A to tick faster.

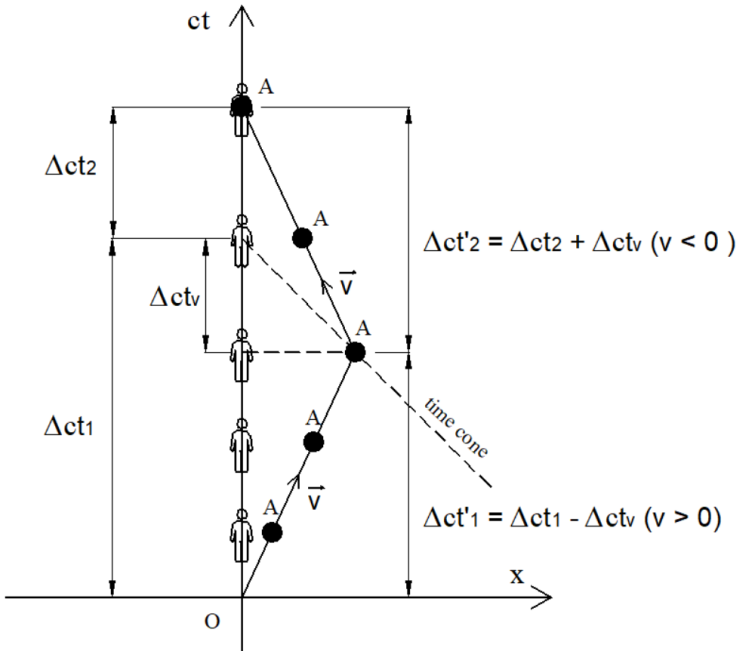


Fig. 25 Relativistic temporal configurations and proper time passage during the outbound and return trajectories of spacecraft A.

In conclusion, the operational analysis establishes that:

- When frames S and S' recede from each other, an observer in S perceives the temporal progression in S' to slow down, and due to structural symmetry, an observer in S' registers an identical dilation for clocks in S.
- When frames S and S' approach each other, an observer in S perceives the temporal progression in S' to accelerate, while an observer in S' symmetrically registers a corresponding acceleration for clocks in S.

Consequently, this reciprocal coordinate symmetry explicitly demonstrates that the newly formulated modified Lorentz transformation group fully preserves and satisfies the principle of relativity across all measurement systems [3].

5 Integrating a Four-Dimensional Spacetime Field to Explain the Traditional Lorentz Transformation

5.1 The Lorentz Transformation Under Static Background Field Constraints

5.1.1 The Speed of Light as the Upper Velocity Limit

Within the framework of quantum field theory, all material entities manifest as localized excitations of their respective underlying fields. Massless quanta propagate as

fundamental field waves and invariably travel at the speed of light, c, due to the total absence of intrinsic inertial mass. Consequently, the speed of light constitutes the absolute upper velocity limit within the spacetime manifold, as its propagation encounters zero structural resistance from the underlying background field [5].

5.1.2 Constancy of the Speed of Light Across Inertial Reference Frames

Regardless of whether a light signal is emitted from a stationary source or a moving body, its propagation velocity remains invariantly c relative to the fundamental background field. This phenomenon is macroscopically analogous to hydrodynamic waves generated by a moving vessel; irrespective of the vessel's localized velocity, the resulting wave packets propagate at a fixed, constant velocity relative to the underlying water surface itself [5]:

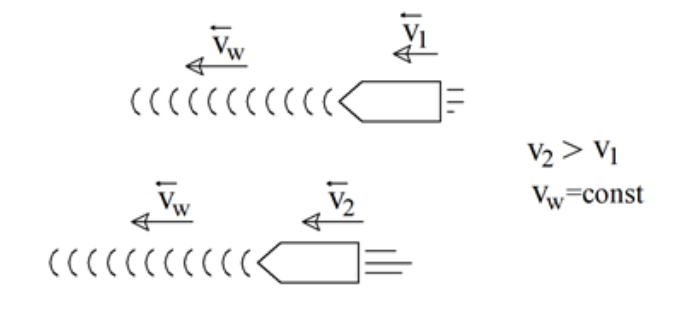


Fig. 26 Hydrodynamic waves propagating at a constant velocity relative to the background medium surface, independent of the vessel's velocity [1, 5].

Furthermore, since the signal propagation velocity is finite and bounded by c, the mechanical rate of recession from the emission source is precisely balanced by the progressive accumulation of the signaling propagation delay. Consequently, an observer within an arbitrarily moving frame of reference will invariantly measure the speed of light to be c, strictly independent of the frame's relative velocity [5].

5.1.3 Spacetime Dilation Mechanics

When a massive body moves through the background field, its physical interaction with the vacuum field intensifies asymptotically as its velocity approaches c. This field-induced resistance consequently results in physical length contraction and temporal dilation [5]. For an observer at rest relative to the background field, the phenomenon of time dilation perfectly coincides with the conventional inverse Lorentz transformation:

$$x = \gamma(x' + vt') \tag{56}$$

$$t = \gamma\left(t' + \frac{vx'}{c^2}\right) \tag{57}$$

Assuming the Earth is approximately at rest relative to the cosmic spacetime field, a spacecraft on either an outbound or a return journey will invariably experience a slower rate of elapsed proper time compared to the coordinate time recorded on Earth [5]:

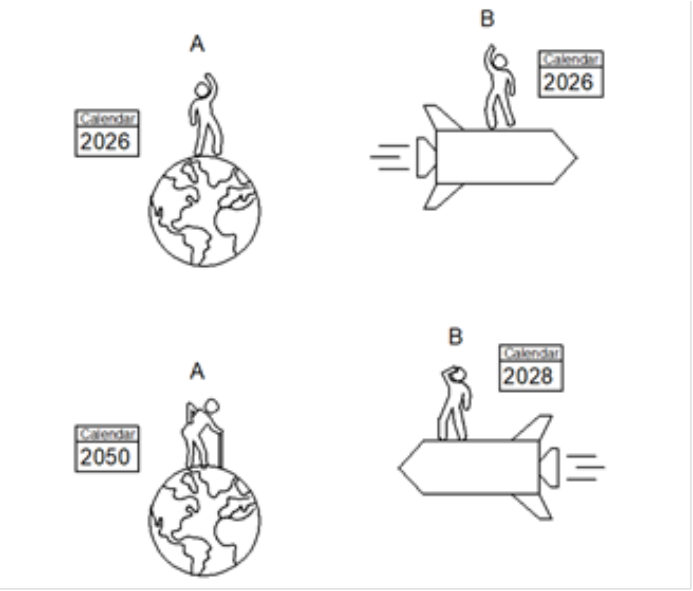


Fig. 27 Temporal progression on a relativistic spacecraft elapsing more slowly than on Earth during both the outbound and return trajectories [1, 5].

However, if the observer is also in motion relative to the background field, the aforementioned time dilation no longer conforms to the standard Lorentz formulation. For instance, consider two reference frames in relative motion to each other; if they maintain identical speeds relative to the background field, the rate of proper time passage within both frames remains strictly equal. This phenomenon is operationally analogous to two satellites orbiting a central gravitational mass in opposite directions, where the onboard atomic clocks record identical elapsed proper times [5].

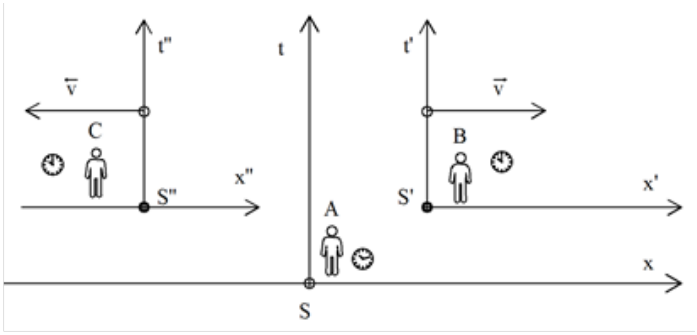


Fig. 28 Relativistic temporal symmetry between counter-orbiting satellites maintaining identical velocities relative to the background field rest frame [5].

5.2 Operational Feasibility of Determining the Absolute Velocity of a Reference Frame Relative to the Background Field

Assuming an observer is stationed within reference frame A and seeks to determine the absolute velocity of frame A relative to the background spacetime field, the following experimental methodology is proposed. Two space probes, designated as B and C, are dispatched in diametrically opposite directions such that their relative velocities with respect to frame A are strictly identical. Three synchronized atomic clocks are utilized, with one positioned at the base station A and the remaining two mounted onboard the respective space probes [5]. The operational steps are formalized as follows:

Step 1: Determination of the Orthogonal Axis. Rotate the orientation of the space probes' launch axis until the elapsed proper times recorded by the two onboard atomic clocks are observed to be strictly equal. The objective of this step is to isolate the spatial axis that is orthogonal to the absolute direction of motion of reference frame A relative to the field [5].

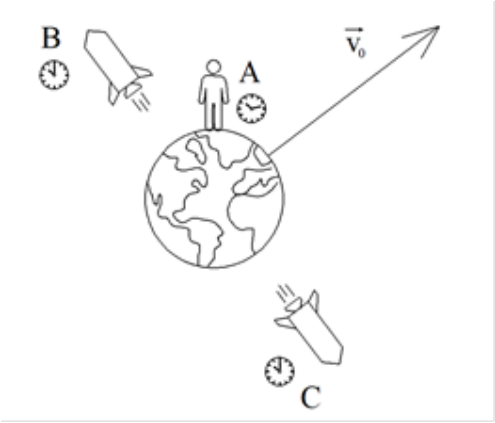


Fig. 29 Methodology for determining the spatial axis orthogonal to the absolute velocity vector of the reference frame relative to the stationary field [5].

Step 2: Alignment and Velocity Calculation. We align the axes of motion of space probes B and C so that they are perpendicular to the orthogonal axis isolated in Step 1 (thereby aligning them parallel to the directional motion of frame A). By analyzing the respective rates of the three atomic clocks, the absolute velocity of frame A relative to the background field can be calculated via two kinematic cases [5]:

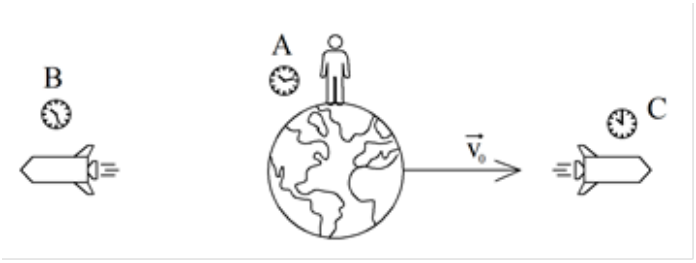


Fig. 30 Kinematic alignment of space probes B and C launched parallel to the absolute velocity vector of frame A [5].

- **Case 1: Zero Absolute Velocity.** If the two onboard clocks record identical elapsed times and run slower than the master clock at reference frame A, it can be concluded that reference frame A is strictly at rest relative to the background field [5].
- **Case 2: Non-Zero Absolute Velocity.** If the two onboard clocks display a measurable discrepancy where one runs faster than the other, let us assume space probe B carries the faster clock. By launching a subsequent sequential series of space probes in the direction of B at varying velocities, we can identify the specific probe carrying the clock with the maximum tick rate. This maximum rate indicates that the respective probe has achieved a net velocity of zero relative to the background

field. Consequently, it can be concluded that the absolute velocity of frame A relative to the background field is exactly equal in magnitude to the relative velocity of this specific space probe with respect to A [5].

6 Discussion

It is evident that applying a field-theoretic framework to coordinate transformations brings a high degree of conceptual unity to physical phenomena such as length contraction and time dilation within relativity. However, as demonstrated through the operational analysis of signaling latencies, the traditional Lorentz transformation reveals severe structural limitations and fails to maintain reciprocity under arbitrary coordinate boundary conditions. Therefore, to fundamentally resolve these underlying contradictions, it is necessary to strictly uphold the principle of relativity—the core postulate of special relativity—and formulate a modified relativistic transformation group characterized by a structural scaling factor of $k \equiv 1$.

Concurrently, the physical mechanism governing field-induced time dilation must be systematically integrated into a more comprehensive unified field theory. Such an extended formulation holds the potential to serve as a vital conceptual bridge connecting the deterministic, geometric descriptions of general relativity with the probabilistic, localized structures of quantum mechanics.

Crucially, this framework completely neutralizes standard empirical objections regarding apparent time-dilation verifications. The traditional mainstream reliance on circular storage rings or orbital satellite data (such as the GPS framework) to validate kinematic special relativity is fundamentally deconstructed in our companion work [6], which demonstrates that such temporal variances are exclusively driven by absolute angular motion (ω) and directional acceleration field-stress rather than relative speed. Similarly, straight-line high-velocity validations—including the historic Ives-Stilwell and Frisch-Smith baselines—are explicitly resolved not as geometric alterations of time, but as dynamic alterations of internal particle transition rates governed by the localized energy dispersion profile (D_{90}) and kinetic coupling with the static field substrate ($k = 1$) [7]. Consequently, the empirical body of physics is fully preserved and re-explained without requiring standard kinematic space-time scaling.

Furthermore, this newly formulated modified Lorentz transformation offers a tangible pathway toward experimental verification. By systematically comparing high-precision temporal measurements across multiple, co-moving reference frames, empirical setups can clearly distinguish between measurement-induced signaling time lag—as mapped via the localized co-moving time cone—and the actual physical slowing of proper time caused by the dynamical interaction between a moving object and the underlying spacetime background field.

7 Conclusion

In this paper, we have re-examined the foundational structure of spacetime coordinate transformations. When the operational time variables registered by distinct observers

are explicitly distinguished based on signaling latencies, the traditional Lorentz transformation is found to exhibit a conceptual asymmetry that prevents it from fully satisfying the principle of relativity across all measurement frameworks.

If the strict relativity postulate is relaxed and field theory is alternatively applied to interpret the operational mechanisms of special relativity, the conventional Lorentz transformation holds true only as a limiting case under highly specific kinematic constraints. Therefore, a more robust and unified approach is to modify the coordinate transformation group directly, ensuring unconditional alignment with the principle of relativity. The formalized $k = 1$ transformation combined with the co-moving time cone geometry offers a consistent and physically intuitive framework that preserves the core postulates of special relativity while resolving long-standing kinematic paradoxes.

Declarations

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Competing Interests

The author has no relevant financial or non-financial interests to disclose.

Author Contributions

As the sole author, Q. H. Ngo was responsible for the conceptualization, mathematical derivations, formal analysis, thought experiments, draft preparation, and final editing of this manuscript. All research components presented in this study were independently developed by the author.

Data Availability

Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study.

Prior Publication and Preprint Notice

The conceptual parameters and spatial reciprocity frameworks evaluated in Sections 2 and 3 are significantly expanded from the foundational theoretical concepts originally outlined in the author's self-published monograph titled *"THE UNIVERSE IS A UNIFIED OBJECT: Book 1: Building a New Unified Theory with eraser and ballpoint pen"* [1].

Additionally, preliminary iterations, independent structural components, and geometric formalisms of this research have been formally deposited as open-access preprints on Zenodo. Specifically, the analytical proof regarding light-speed constancy and spacetime interval invariance is detailed in [2] (https://doi.org/10.5281/zenodo.22108473); the initial formulation regarding dynamic field-theoretic mechanisms is archived in [5] (https://doi.org/10.5281/zenodo.22655441); and the co-moving

time cone linear descriptive geometry framework is thoroughly documented in [3] (https://doi.org/10.5281/zenodo.23037113). The present manuscript represents the definitive, integrated framework characterising the generalized analytical and field-theoretic structures of this research, deposited for open peer review and academic academic dissemination.

Acknowledgments

The author would like to thank the open-science community at Zenodo for providing an accessible platform to archive preliminary scientific research and foster independent foundational physics studies.

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