The Proposed Conservation Law of Combined Energy-Momentum in Gravitational Field: A Response to an Objection

Mueiz Gafer KamalEldeen, Nov. 2026

Abstract

A proposal has been put forward to define energy and momentum in such a way that their total is conserved in a gravitational field. This conservation can be verified directly in Schwarzschild spacetime. In Kerr spacetime, however, the proposed conservation law appears to fail. Yet, once the transfer of energy–momentum from the source of the gravitational field to the test body is taken into account, it becomes clear that the proposed conservation law remains valid in this case as well

Introduction

Among the most fascinating and stimulating cases when proposing solutions to problems in theoretical physics whether those proposals are correct or incorrect are those in which an objection that initially appears to be evidence of a proposal's failure instead becomes a support for it, contributing to its development and deepening our understanding of it.

This paper represents one such case. The proposal concerns the law of conservation of energy and momentum in a gravitational field, presented by the author elsewhere[1]. Its mathematical and physical validity was demonstrated clearly and simply in Schwarzschild spacetime[2], as summarized below.

However, this proposal appears to face a challenge when applied to the Kerr metric[3], as the author was informed by some critics of the proposal. Nevertheless, a simple mathematical analysis—addressing an aspect overlooked by the objector—shows that the law remains applicable in Kerr spacetime as well. This result provides further support for the proposal and offers deeper insight into its concepts, as well as into the concepts of Kerr spacetime itself.

Summary of the Proposed Law

Proposed Definition of Energy and Momentum:

$$\mathbf{E}_g = m^2\gamma^2c^2\left(g_{00}v^0v^0 - g_{00}U^0U^0\right)$$

$$\mathbf{P}_g = \sum_{i,j=1,2,3} m^2\gamma^2c^2\left(g_{ij}v^iv^j - g_{ij}U^iU^j\right)$$

Where:

$\boldsymbol{v}^P$ is the velocity of the body as measured by local observer stationary relative to the gravitational source .which also used to calculate the Lorentz factor $\boldsymbol{\gamma}$,

$\boldsymbol{v}^P = d\boldsymbol{x}^P / d\boldsymbol{x}^0$

$\boldsymbol{dx}^P$ is the change in distance as measured by local observer..

$d\boldsymbol{x}^0$ change in time as mearured by local observer, we have $\boldsymbol{v}^0 = 1$

$\boldsymbol{U}^p$ is a purely radial vector quantity whose magnitude is given by:

$$\boldsymbol{U} = \sqrt{\frac{2GM}{r}}$$

To understand the meaning of the components of this quantity at a particular point in a gravitational field, let us consider a body initially at rest and located far away from the gravitational source. This means that all the components of its velocity are zero, except for the temporal component, which is equal to one.

As the body falls and reaches the specified point, it acquires a new velocity. The radial component becomes equal to the velocity described above, while the temporal component remains equal to one. In other words, the body does not acquire any additional velocity in the time direction as a result of the gravitational field.

Therefore, the value of the component in the temporal direction is zero. Thus, these components represent the velocities acquired by the body due to the gravitational field as it falls from outside the field

$$\boldsymbol{U^t = 0} \ , \ \boldsymbol{U^r} = \frac{\boldsymbol{U}}{\sqrt{\mathrm{g_{rr}}}} \ , \ \boldsymbol{U^\theta = 0} \ , \ \boldsymbol{U^\emptyset = 0}$$

The proposed law states that the total energy and momentum of a test body, as defined in the proposal, remain conserved: this con be proved easly in In Schwarzschild spacetime

We have:

$$\boldsymbol{U^2} = \sum_{i,j=1,2,3} \mathrm{g_{ij}} \boldsymbol{U^i U^j}$$

$$\boldsymbol{V^2} = \sum_{i,j=1,2,3} \mathrm{g_{ij}} \boldsymbol{V^i V^j}$$

$$\mathrm{g_{tt}} = 1 - \boldsymbol{U}^2/\mathrm{c}^2$$

This leads to:

$$\mathbf{E}_g + \mathbf{P}_g = \boldsymbol{m^2 \gamma^2 c^2 \left(c^2 - U^2\right) + m^2 \gamma^2 c^2 (\, U^2 - V^2) = m^2 c^4}$$

- **E_g and P_g are conserved for a body moving in geodesy in static gravitational field.**
- **$E_g + P_g$ is conserved for a body in all cases in gravitational field.**

In Schwarzschild spacetime, the gravitational field causes an exchange of energy and momentum among the internal components of the test body, but it does not cause a net exchangeof energy or momentum between the test body and the source of gravity. Therefore, the conservation law can be applied to the test body alone.

The Objection

It is clear that the Kerr metric presents the proposal with two major problems.First, there is a mathematical problem: the sum of energy and momentum, as defined in the proposal, may not lead to the claimed conservation law .Second, there is a conceptual problem: the Kerr metric contains non-zero mixed spacetime components, g_{03} . g_{30} Using the same method used to define energy and momentum, one could construct quantities analogous to momentum and energy from these mixed components, and these quantities would not necessarily vanish.Ignoring these quantities and restricting the proposed conservation law to the purely temporal components and the purely spatial components may appear to violate the equivalence of the four dimensions.

This conceptual problem also exists in Schwarzschild spacetime, but it has no practical effect there because all mixed components vanish. Consequently, whether one includes or excludes the mixed components makes no practical difference in the Schwarzschild case.

Response to the Objection

The error in this objection lies in its neglect of a very important distinction between Schwarzschild spacetime and Kerr spacetime.

In Schwarzschild spacetime, the test body exchanges energy and momentum among its own internal components, but there is no exchange of energy or momentum between the test body and the source of gravity. For this reason, the conservation law can be applied to the test body alone.

In Kerr spacetime, however, the test body acquires part of the angular momentum of the gravitational source. This transferred quantity appears in the mixed components, as well as in the purely temporal and purely spatial components. Therefore, the conservation law cannot be applied to the test body alone; it must be applied to the test body and the gravitational source together.

Since the gravitational source exchanges a certain amount of energy-momentum with the test body, its components deviate from the values they would have in the Schwarzschild case. The quantities transferred from the gravitational source to the test body disappear from the conservation equation applied to both bodies together, because they have not been created or destroyed; they have merely been transferred from one body to the other. Thus, if these transferred quantities are removed from the test

body’s energy and momentum, what remains is the energy and momentum as defined in Schwarzschild spacetime, whose total is known to be conserved for the test particle.

This can be summarized as follows: we separate the effect of the rest mass of the gravitational source from the effect of its rotation. The effect of the rest mass leads to an exchange of energy and momentum among the components of the test body itself, without violating their conservation. While the effect of the source’s rotation leads to a transfer of energy and momentum from the gravitational source to the test body.

This leaves us with two equivalent options: The first is to apply the conservation law to the test body and the gravitational source together. The second is to remove, from every component of the test body’s energy-momentum, the portion transferred from the gravitational source, and then apply the conservation law to the remaining quantities.

Physical Implications

This does not merely represent a way of escaping the objection. It also sheds light on the meaning of the mixed components in the matrix representing energy and momentum. It explains why neglecting these mixed components does not affect the definition of energy and momentum or the conservation law: those components represent quantities transferred between the rotating source and the test body, rather than an independent failure of conservation.

This analysis also adds a new and complementary understanding of the effect of rotating gravitational sources on surrounding objects. That effect can be divided into two processes The first is an exchange of quantities among the internal components of the test body.The second is an exchange of quantities between the gravitational source and the test body.

Conclusion

- If a body exists in a region with no gravitational field, every component of its energy and momentum retains its own value.
- If the body exists in a static or time-dependent gravitational field, the field causes an exchange among the body’s internal components of energy and momentum.
- If the source of gravity rotates about its own axis, it transfers a certain amount of energy-momentum from its components to the test body.
- In all cases, the total energy and momentum as defined by the proposal , across all components remain conserved, provided that the conservation law is applied either to the complete system—test body plus gravitational source—or to the test body after subtracting the quantity transferred from the source.

References

[1] *The Expansion of the Universe is a Transformation of Energy into Momentum, Mueiz Gafer KamalEldeen,viXra:;251r.0077.*

[2] Schwarzschild, K. (1916). Über das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie. Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin, 189–196.

[3] Kerr, R. P. (1963). Gravitational Field of a Spinning Mass as an Example of Algebraically Special Metrics. Physical Review Letters, 11(5), 237–238.

.