Quasiperiodic Enumeration from Primitively Equidistant Square Triplet

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Abstract

Any prime number congruent to 1 (mod 4) can be factored into Gaussian integers. Such Gaussian factor—when raised to even powers—could be treated as a seed for enumerating further equidistant square triplets, while simultaneously being the mean of such triplet. Such array could be reshaped into 2D array of pixels with arbitrary ratio, exhibiting quasiperiodicity as they encode the Abs–Arg relation of their Binomial Expansion from its seed rational prime raised to n-th power. Two methods of enumeration are defined based on the permutation criterion of such triplet, i.e. unordered vs ordered.

1. Primal Equidistant Square Triplet

Three square integers, say x^2, p^2, y^2 are *primal* equidistant square triplet if $y^2 – p^2 = p^2 – x^2$, where p is a rational prime [1][2] that is, a prime number congruent to 1 (mod 4). Since p is exactly the mean of x^2 and y^2, one may say that p^2 being the centre of such square triplet.

Proposition: the Pythagorean triplet from prime Gaussian factorisation a,b,p generates three equidistant squares x^2, p^2, y^2, with y = a+b and x = a–b.

Proof: let $a^2 + b^2 = p^2$ then $(a-b)^2$, p^2, $(a+b)^2$ are equidistant. This is because

$p^2 = a^2 + b^2$, thus

$(a+b)^2 – p^2 = p^2 – (a–b)^2$

$a^2 + b^2 + 2ab – a^2 – b^2 = a^2 + b^2 – a^2 – b^2 – (– 2ab)$

$2ab = 2ab$

the equation checks out, spooling back to $(a+b)^2 – p^2 = p^2 – (a–b)^2$

one observes that y = a+b and x = a–b.

1.1. Primitively Equidistant Square Triplet

The primal equidistant triplet x^2, p^2, y^2 can be generalised further into *primitively* equidistant square triplet x^2, q^2, y^2 where q is allowed to be prime powers, i.e. $\omega(q) = 1$, with $\omega(n)$ being unique factor count of the integer n. Letting $q = p^k$ one must obtain $\omega(q) = 1$ **and** $\Omega(q) = k$, with $\Omega(n)$ being the factor count of the integer n including multiplicity.

Proposition: the primitively equidistant triplet is generated via its Gaussian factorisation.

Proof: consider the rational prime with norm $(a+b\,i)(a–b\,i)$ with $i^2 = –1$ i.e. such norm is the rational prime p [2] therefore let $q = p^n$, then $x_n = |\mathrm{Re}((a+b\,i)^n) – \mathrm{Im}((a+b\,i)^n)|$, $y_n = |\mathrm{Re}((a+b\,i)^n) + \mathrm{Im}((a+b\,i)^n)|$ yields the triplet, x_n^2, p^{2n}, y_n^2, where $|z|$ denotes the absolute value (magnitude) of complex integer z.

2. Binomial Expansion

The primitively (not primal, since $p^n = q$ is prime powers, rather than prime p) equidistant square triplet; x^2, p^{2n}, y^2, with $|x| < |p^n| < |y|$ can be simply generated via rational prime factorisation and then assigning $x = |\mathrm{Re}((a+b\,i)^n) - \mathrm{Im}((a+b\,i)^n)|$, $q = p^n$, $y = |\mathrm{Re}((a+b\,i)^n) + \mathrm{Im}((a+b\,i)^n)|$. For instance p = 5 that is $p = (2+1\,i)(2-1\,i)$, the list of x,q,y would be; (1,5,7), (17,25,31), (73,125,161), (191,625,863), …, i.e. $(1,5^1,7)$, $(17,5^2,31)$, $(73,5^3,161)$, $(191,5^4,863)$,… notice the mean value q as prime power of 5. The notion of *primitively* is important, since one could simply multiply p from the former triplet, say for p = 5, since (17,25,31) is a valid equidistant square triplet, so does 5(17,25,31) = (85,125,155) however this isn't interesting since 85 and 155 are not co-prime! Thus the proper, i.e. primitive continuation of such triplet shall be (73,125,161) instead of (85,125,155) despite such being valid triplet. Of course p could be any rational prime say $(3+2\,i)(3-2\,i) = 13$, $(4+1\,i)(4-1\,i) = 17$, $(5+2\,i)(5-2\,i) = 29$, $(6+1\,i)(6-1\,i) = 37$, $(5+4\,i)(5-4\,i) = 41$, $(7+2\,i)(7-2\,i) = 53$, $(6+5\,i)(6-5\,i) = 61$, and on and on. **However** such assignment ignores the sign which is important when one studies the combinatorial properties of such triplet. One needs a finer method, which respects the sign i.e. not committing Abs(), thus preserving their Arg(), which is the prescribed recursion as follows

Proposition: each x and y are generated from the recursion $p^4\,f[n] + (\xi^2 - p^2)\,f[n+2] + f[n+4] = 0$, with f being the sought x or y (requiring 4 initial conditions) and $\xi = |\mathrm{Re}((a+b\,i)^2) - \mathrm{Im}((a+b\,i)^2)|$.

Proof: consider the differential relation $\alpha\,d\hat{}2(2)\,f + \beta\,d\hat{}2(1)\,f + \gamma\,d\hat{}2(0)\,f = 0$, i.e. an order 4 ODE from order 4 linear recursion. The terms 2,1,0 comes from the initial two-terms recursion, yet doubled as square instead of mere integer. From the proposition in section 1, it is easy to see that $\alpha = 1$, $\beta = \xi^2 - p^2$, and $\gamma = p^4$, coming from the exponentiation of the rational prime raised to the 0,1,2 with $\xi = |\mathrm{Re}((a+b\,i)^2) - \mathrm{Im}((a+b\,i)^2)|$. Thus, said recursion is simply the ODE $d^4f + (\xi^2 - p^2)\,d^2f + p^4\,f = 0$.

Comparing the sign from solving such recursion (for example, p = 5, then the needed initial conditions are x[0] = 1, x[1] = 1, x[2] = 17, x[3] = 73 and y[0] = 1, y[1] = 7, y[2] = 31, y[3] = 161), a colour could be enumerated, say when the sign agrees (+,+) it is colour_1 when one disagrees (–,+) it is colour_2 etc. etc. up to 4 (unordered) or 8 colours when the permutation (a,b) vs (b,a) matters, i.e. being ordered.

2.1. History of Pascal's Triangle

The coefficients of Binomial Expansion was already known at least from 23 to 24 centuries ago via Piṅgala's "method of pyramidal expansion" (*meru-prastāra*) or alternatively "Staircase of Mt. Meru" [3][4] Another Indian mathematician and astronomer Varāhamihira later (that is 15 centuries ago) described a recursive method for computing such binomial coefficients, although arranged in a different orientation [5]. Roughly three centuries after Varāhamihira, the Persian mathematician al–Karajī wrote a book describing explicit construction of Pascal's triangle [6][7][8][9], however such book is now lost and only came preserved thanks to the Persian polymath Omar Khayyám [10] who committed manual repeat way before the existence of XeroX. Speaking of some name starting with the letter X, such Binomial eXpansion was known to Jia **X**ian in China *circa* millenium ago (accurate up to decade) in which two centuries later, **Y**ang Hui famously defined such triangular combinatorial structure, so that the modern Chinese (**Z**H) translation for Pascal's triangle is literally Yang Hui's triangle instead [11]. Anyway, there is convenient XYZ mnemonics there.

2.2. Fresh No More (Awake)

After incorrectly referring Freshman's Dream as Sophomore's one, yet somehow the AI assistant mentioned in some unrelated allegation did not correct this (which is understandable false positive much like daily errand), in this section, no more dream is being discussed. Time to wake up and expand the coefficient already.

Start with $(a+b)^n$, set n as positive integer. Below is the expansion up to n = 21 :

$a^1 + b^1$
$a^2 + 2\,a^1b^1 + b^2$
$a^3 + 3\,a^2b^1 + 3\,a^1b^2 + b^3$
$a^4 + 4\,a^3b^1 + 6\,a^2b^2 + 4\,a^1b^3 + b^4$
$a^5 + 5\,a^4b^1 + 10\,a^3b^2 + 10\,a^2b^3 + 5\,a^1b^4 + b^5$
$a^6 + 6\,a^5b^1 + 15\,a^4b^2 + 20\,a^3b^3 + 15\,a^2b^4 + 6\,a^1b^5 + b^6$
$a^7 + 7\,a^6b^1 + 21\,a^5b^2 + 35\,a^4b^3 + 35\,a^3b^4 + 21\,a^2b^5 + 7\,a^1b^6 + b^7$
$a^8 + 8\,a^7b^1 + 28\,a^6b^2 + 56\,a^5b^3 + 70\,a^4b^4 + 56\,a^3b^5 + 28\,a^2b^6 + 8\,a^1b^7 + b^8$
$a^9 + 9\,a^8b^1 + 36\,a^7b^2 + 84\,a^6b^3 + 126\,a^5b^4 + 126\,a^4b^5 + 84\,a^3b^6 + 36\,a^2b^7 + 9\,a^1b^8 + b^9$
$a^{10} + 10\,a^9b^1 + 45\,a^8b^2 + 120\,a^7b^3 + 210\,a^6b^4 + 252\,a^5b^5 + 210\,a^4b^6 + 120\,a^3b^7 + 45\,a^2b^8 + 10\,a^1b^9 + b^{10}$

$a^{11} + 11\,a^{10}b^1 + 55\,a^9b^2 + 165\,a^8b^3 + 330\,a^7b^4 + 462\,a^6b^5 + 462\,a^5b^6 + 330\,a^4b^7 + 165\,a^3b^8 + 55\,a^2b^9 + 11\,a^1b^{10} + b^{11}$

$a^{12} + 12\,a^{11}b^1 + 66\,a^{10}b^2 + 220\,a^9b^3 + 495\,a^8b^4 + 792\,a^7b^5 + 924\,a^6b^6 + 792\,a^5b^7 + 495\,a^4b^8 + 220\,a^3b^9 + 66\,a^2b^{10} + 12\,a^1b^{11} + b^{12}$

$a^{13} + 13\,a^{12}b^1 + 78\,a^{11}b^2 + 286\,a^{10}b^3 + 715\,a^9b^4 + 1287\,a^8b^5 + 1716\,a^7b^6 + 1716\,a^6b^7 + 1287\,a^5b^8 + 715\,a^4b^9 + 286\,a^3b^{10} + 78\,a^2b^{11} + 13\,a^1b^{12} + b^{13}$

$a^{14} + 14\,a^{13}b^1 + 91\,a^{12}b^2 + 364\,a^{11}b^3 + 1001\,a^{10}b^4 + 2002\,a^9b^5 + 3003\,a^8b^6 + 3432\,a^7b^7 + 3003\,a^6b^8 + 2002\,a^5b^9 + 1001\,a^4b^{10} + 364\,a^3b^{11} + 91\,a^2b^{12} + 14\,a^1b^{13} + b^{14}$

$a^{15} + 15\,a^{14}b^1 + 105\,a^{13}b^2 + 455\,a^{12}b^3 + 1365\,a^{11}b^4 + 3003\,a^{10}b^5 + 5005\,a^9b^6 + 6435\,a^8b^7 + 6435\,a^7b^8 + 5005\,a^6b^9 + 3003\,a^5b^{10} + 1365\,a^4b^{11} + 455\,a^3b^{12} + 105\,a^2b^{13} + 15\,a^1b^{14} + b^{15}$

$a^{16} + 16\,a^{15}b^1 + 120\,a^{14}b^2 + 560\,a^{13}b^3 + 1820\,a^{12}b^4 + 4368\,a^{11}b^5 + 8008\,a^{10}b^6 + 11440\,a^9b^7 + 12870\,a^8b^8 + 11440\,a^7b^9 + 8008\,a^6b^{10} + 4368\,a^5b^{11} + 1820\,a^4b^{12} + 560\,a^3b^{13} + 120\,a^2b^{14} + 16\,a^1b^{15} + b^{16}$

$a^{17} + 17\,a^{16}b^1 + 136\,a^{15}b^2 + 680\,a^{14}b^3 + 2380\,a^{13}b^4 + 6188\,a^{12}b^5 + 12376\,a^{11}b^6 + 19448\,a^{10}b^7 + 24310\,a^9b^8 + 24310\,a^8b^9 + 19448\,a^7b^{10} + 12376\,a^6b^{11} + 6188\,a^5b^{12} + 2380\,a^4b^{13} + 680\,a^3b^{14} + 136\,a^2b^{15} + 17\,a^1b^{16} + b^{17}$

$a^{18} + 18\,a^{17}b^1 + 153\,a^{16}b^2 + 816\,a^{15}b^3 + 3060\,a^{14}b^4 + 8568\,a^{13}b^5 + 18564\,a^{12}b^6 + 31824\,a^{11}b^7 + 43758\,a^{10}b^8 + 48620\,a^9b^9 + 43758\,a^8b^{10} + 31824\,a^7b^{11} + 18564\,a^6b^{12} + 8568\,a^5b^{13} + 3060\,a^4b^{14} + 816\,a^3b^{15} + 153\,a^2b^{16} + 18\,a^1b^{17} + b^{18}$

$a^{19} + 19\,a^{18}b^1 + 171\,a^{17}b^2 + 969\,a^{16}b^3 + 3876\,a^{15}b^4 + 11628\,a^{14}b^5 + 27132\,a^{13}b^6 + 50388\,a^{12}b^7 + 75582\,a^{11}b^8 + 92378\,a^{10}b^9 + 92378\,a^9b^{10} + 75582\,a^8b^{11} + 50388\,a^7b^{12} + 27132\,a^6b^{13} + 11628\,a^5b^{14} + 3876\,a^4b^{15} + 969\,a^3b^{16} + 171\,a^2b^{17} + 19\,a^1b^{18} + b^{19}$

$a^{20} + 20\ a^{19}b^{1} + 190\ a^{18}b^{2} + 1140\ a^{17}b^{3} + 4845\ a^{16}b^{4} + 15504\ a^{15}b^{5} + 38760\ a^{14}b^{6} + 77520\ a^{13}b^{7} + 125970\ a^{12}b^{8} + 167960\ a^{11}b^{9} + 184756\ a^{10}b^{10} + 167960\ a^{9}b^{11} + 125970\ a^{8}b^{12} + 77520\ a^{7}b^{13} + 38760\ a^{6}b^{14} + 15504\ a^{5}b^{15} + 4845\ a^{4}b^{16} + 1140\ a^{3}b^{17} + 190\ a^{2}b^{18} + 20\ a^{1}b^{19} + b^{20}$

$a^{21} + 21\ a^{20}b^{1} + 210\ a^{19}b^{2} + 1330\ a^{18}b^{3} + 5985\ a^{17}b^{4} + 20349\ a^{16}b^{5} + 54264\ a^{15}b^{6} + 116280\ a^{14}b^{7} + 203490\ a^{13}b^{8} + 293930\ a^{12}b^{9} + 352716\ a^{11}b^{10} + 352716\ a^{10}b^{11} + 293930\ a^{9}b^{12} + 203490\ a^{8}b^{13} + 116280\ a^{7}b^{14} + 54264\ a^{6}b^{15} + 20349\ a^{5}b^{16} + 5985\ a^{4}b^{17} + 1330\ a^{3}b^{18} + 210\ a^{2}b^{19} + 21\ a^{1}b^{20} + b^{21}$

And so on for n>22, the terms resulted from the Binomial Expansion of $(a+b)^n$ gets more and more verbose. Notice that after one collects each term from the original binomial expansion based on the powers of b, i.e. b^n with n being congruent to: 0 (mod 4) belongs to Re part, 1 (mod 4) belongs to Im part, 2 (mod 4) belongs to negated Re part, 3 (mod 4) belongs to negated Im part. Therefore the Binomial Expansion for Re part looks like the following :

a^{1}

$a^{2} - b^{2}$

$a^{3} - 3\ a^{1}b^{2}$

$a^{4} - 6\ a^{2}\ b^{2} + b^{4}$

$a^{5} - 10\ a^{3}\ b^{2} + 5\ a^{1}b^{4}$

$a^{6} - 15\ a^{4}\ b^{2} + 15\ a^{2}\ b^{4} - b^{6}$

$a^{7} - 21\ a^{5}\ b^{2} + 35\ a^{3}\ b^{4} - 7\ a^{1}b^{6}$

$a^{8} - 28\ a^{6}\ b^{2} + 70\ a^{4}\ b^{4} - 28\ a^{2}\ b^{6} + b^{8}$

$a^{9} - 36\ a^{7}\ b^{2} + 126\ a^{5}\ b^{4} - 84\ a^{3}\ b^{6} + 9\ a^{1}b^{8}$

$a^{10} - 45\ a^{8}\ b^{2} + 210\ a^{6}\ b^{4} - 210\ a^{4}\ b^{6} + 45\ a^{2}\ b^{8} - b^{10}$

$a^{11} - 55\ a^{9}\ b^{2} + 330\ a^{7}\ b^{4} - 462\ a^{5}\ b^{6} + 165\ a^{3}\ b^{8} - 11\ a^{1}b^{10}$

$a^{12} - 66\ a^{10}\ b^{2} + 495\ a^{8}\ b^{4} - 924\ a^{6}\ b^{6} + 495\ a^{4}\ b^{8} - 66\ a^{2}\ b^{10} + b^{12}$

$a^{13} - 78\ a^{11}\ b^{2} + 715\ a^{9}\ b^{4} - 1716\ a^{7}\ b^{6} + 1287\ a^{5}\ b^{8} - 286\ a^{3}\ b^{10} + 13\ a^{1}b^{12}$

$a^{14} - 91\ a^{12}\ b^{2} + 1001\ a^{10}\ b^{4} - 3003\ a^{8}\ b^{6} + 3003\ a^{6}\ b^{8} - 1001\ a^{4}\ b^{10} + 91\ a^{2}\ b^{12} - b^{14}$

$a^{15} - 105\ a^{13}\ b^{2} + 1365\ a^{11}\ b^{4} - 5005\ a^{9}\ b^{6} + 6435\ a^{7}\ b^{8} - 3003\ a^{5}\ b^{10} + 455\ a^{3}\ b^{12} - 15\ a^{1}b^{14}$

$a^{16} - 120\ a^{14}\ b^{2} + 1820\ a^{12}\ b^{4} - 8008\ a^{10}\ b^{6} + 12870\ a^{8}\ b^{8} - 8008\ a^{6}\ b^{10} + 1820\ a^{4}\ b^{12} - 120\ a^{2}\ b^{14} + b^{16}$

$a^{17} - 136\ a^{15}\ b^{2} + 2380\ a^{13}\ b^{4} - 12376\ a^{11}\ b^{6} + 24310\ a^{9}\ b^{8} - 19448\ a^{7}\ b^{10} + 6188\ a^{5}\ b^{12} - 680\ a^{3}\ b^{14} + 17\ a^{1}b^{16}$

$a^{18} - 153\ a^{16}\ b^{2} + 3060\ a^{14}\ b^{4} - 18564\ a^{12}\ b^{6} + 43758\ a^{10}\ b^{8} - 43758\ a^{8}\ b^{10} + 18564\ a^{6}\ b^{12} - 3060\ a^{4}\ b^{14} + 153\ a^{2}\ b^{16} - b^{18}$

$a^{19} - 171\ a^{17}\ b^{2} + 3876\ a^{15}\ b^{4} - 27132\ a^{13}\ b^{6} + 75582\ a^{11}\ b^{8} - 92378\ a^{9}\ b^{10} + 50388\ a^{7}\ b^{12} - 11628\ a^{5}\ b^{14} + 969\ a^{3}\ b^{16} - 19\ a^{1}b^{18}$

$a^{20} - 190\ a^{18}\ b^{2} + 4845\ a^{16}\ b^{4} - 38760\ a^{14}\ b^{6} + 125970\ a^{12}\ b^{8} - 184756\ a^{10}\ b^{10} + 125970\ a^{8}\ b^{12} - 38760\ a^{6}\ b^{14} + 4845\ a^{4}\ b^{16} - 190\ a^{2}\ b^{18} + b^{20}$

$a^{21} - 210\ a^{19}\ b^{2} + 5985\ a^{17}\ b^{4} - 54264\ a^{15}\ b^{6} + 203490\ a^{13}\ b^{8} - 352716\ a^{11}\ b^{10} + 293930\ a^{9}\ b^{12} - 116280\ a^{7}\ b^{14} + 20349\ a^{5}\ b^{16} - 1330\ a^{3}\ b^{18} + 21\ a^{1}b^{20}$

while the Im part, is built when n is congruent to 1 (mod 4) or negated when congruent to 3 (mod 4) :

b^{1}

$2\ a^{1}b^{1}$

$3\ a^{2}\ b^{1} - b^{3}$

$4\ a^{3}\ b^{1} - 4\ a^{1}b^{3}$

$5\ a^4\ b^1 - 10\ a^2\ b^3 + b^5$
$6\ a^5\ b^1 - 20\ a^3\ b^3 + 6\ a^1 b^5$
$7\ a^6\ b^1 - 35\ a^4\ b^3 + 21\ a^2\ b^5 - b^7$
$8\ a^7\ b^1 - 56\ a^5\ b^3 + 56\ a^3\ b^5 - 8\ a^1 b^7$
$9\ a^8\ b^1 - 84\ a^6\ b^3 + 126\ a^4\ b^5 - 36\ a^2\ b^7 + b^9$
$10\ a^9\ b^1 - 120\ a^7\ b^3 + 252\ a^5\ b^5 - 120\ a^3\ b^7 + 10\ a^1 b^9$
$11\ a^{10}\ b^1 - 165\ a^8\ b^3 + 462\ a^6\ b^5 - 330\ a^4\ b^7 + 55\ a^2\ b^9 - b^{11}$
$12\ a^{11}\ b^1 - 220\ a^9\ b^3 + 792\ a^7\ b^5 - 792\ a^5\ b^7 + 220\ a^3\ b^9 - 12\ a^1 b^{11}$
$13\ a^{12}\ b^1 - 286\ a^{10}\ b^3 + 1287\ a^8\ b^5 - 1716\ a^6\ b^7 + 715\ a^4\ b^9 - 78\ a^2\ b^{11} + b^{13}$
$14\ a^{13}\ b^1 - 364\ a^{11}\ b^3 + 2002\ a^9\ b^5 - 3432\ a^7\ b^7 + 2002\ a^5\ b^9 - 364\ a^3\ b^{11} + 14\ a^1 b^{13}$
$15\ a^{14}\ b^1 - 455\ a^{12}\ b^3 + 3003\ a^{10}\ b^5 - 6435\ a^8\ b^7 + 5005\ a^6\ b^9 - 1365\ a^4\ b^{11} + 105\ a^2\ b^{13} - b^{15}$
$16\ a^{15}\ b^1 - 560\ a^{13}\ b^3 + 4368\ a^{11}\ b^5 - 11440\ a^9\ b^7 + 11440\ a^7\ b^9 - 4368\ a^5\ b^{11} + 560\ a^3\ b^{13} - 16\ a^1 b^{15}$

$17\ a^{16}\ b^1 - 680\ a^{14}\ b^3 + 6188\ a^{12}\ b^5 - 19448\ a^{10}\ b^7 + 24310\ a^8\ b^9 - 12376\ a^6\ b^{11} + 2380\ a^4\ b^{13} - 136\ a^2\ b^{15} + b^{17}$

$18\ a^{17}\ b^1 - 816\ a^{15}\ b^3 + 8568\ a^{13}\ b^5 - 31824\ a^{11}\ b^7 + 48620\ a^9\ b^9 - 31824\ a^7\ b^{11} + 8568\ a^5\ b^{13} - 816\ a^3\ b^{15} + 18\ a^1 b^{17}$

$19\ a^{18}\ b^1 - 969\ a^{16}\ b^3 + 11628\ a^{14}\ b^5 - 50388\ a^{12}\ b^7 + 92378\ a^{10}\ b^9 - 75582\ a^8\ b^{11} + 27132\ a^6\ b^{13} - 3876\ a^4\ b^{15} + 171\ a^2\ b^{17} - b^{19}$

$20\ a^{19}\ b^1 - 1140\ a^{17}\ b^3 + 15504\ a^{15}\ b^5 - 77520\ a^{13}\ b^7 + 167960\ a^{11}\ b^9 - 167960\ a^9\ b^{11} + 77520\ a^7\ b^{13} - 15504\ a^5\ b^{15} + 1140\ a^3\ b^{17} - 20\ a^1 b^{19}$

$21\ a^{20}\ b^1 - 1330\ a^{18}\ b^3 + 20349\ a^{16}\ b^5 - 116280\ a^{14}\ b^7 + 293930\ a^{12}\ b^9 - 352716\ a^{10}\ b^{11} + 203490\ a^8\ b^{13} - 54264\ a^6\ b^{15} + 5985\ a^4\ b^{17} - 210\ a^2\ b^{19} + b^{21}$

Similar construction of Re and Im part for n > 21 is exactly done using the aforementioned $(a+b)^n$ partitioning using n (mod 4) based on the b^n term. For instance, the ratio 89 x 121 requires n = 10769.

2.3. Sample Plot

This section presents some figures generated using SageMath matrix_plot. The enumerated array captures the sign of (x,y) when generating the triplet (x,q,y) recursively. It is originally 1D but can be customised with any number of w and h of pixels to generate varieties of tiling. The quasiperiodicity arises due to aforementioned recursion produces pair of integers x and y, as in the triplet (x,q,y), such x/y or y/x did approximate an irrational number. Since no irrational number could be represented as fraction, therefore irregularity will always be present in the following tile based construction that relies on such x,y. Both Fig. 2 and Fig 3. use the same rational prime = 5, except the in Fig. 3, their order being respected, yielding 8 possibilities (coloured C,M,Y,K,R,G,B,W for 1,2,3,4,5,6,7,8). Still in Fig. 3, the ratio of 89 x 121 was used, observe the band featuring white diagonal stripes, every black diagonal stripe's left side features blue pixels over yellow pixels. Counting such pixel in each stripe one expects 3,3,3,... until suddenly it becomes 2, only to turn back 3,3,3,... thus ...3,3,3,2,3,3,3,... Merry Christmas, pattern fools ya! Notice also Fig. 6, when the rational prime p gets large, such array is deceptively regular, since its defects and imperfection getting harder to spot. The larger the p, the wider the range will be needed in order for one to encounter such impurity.

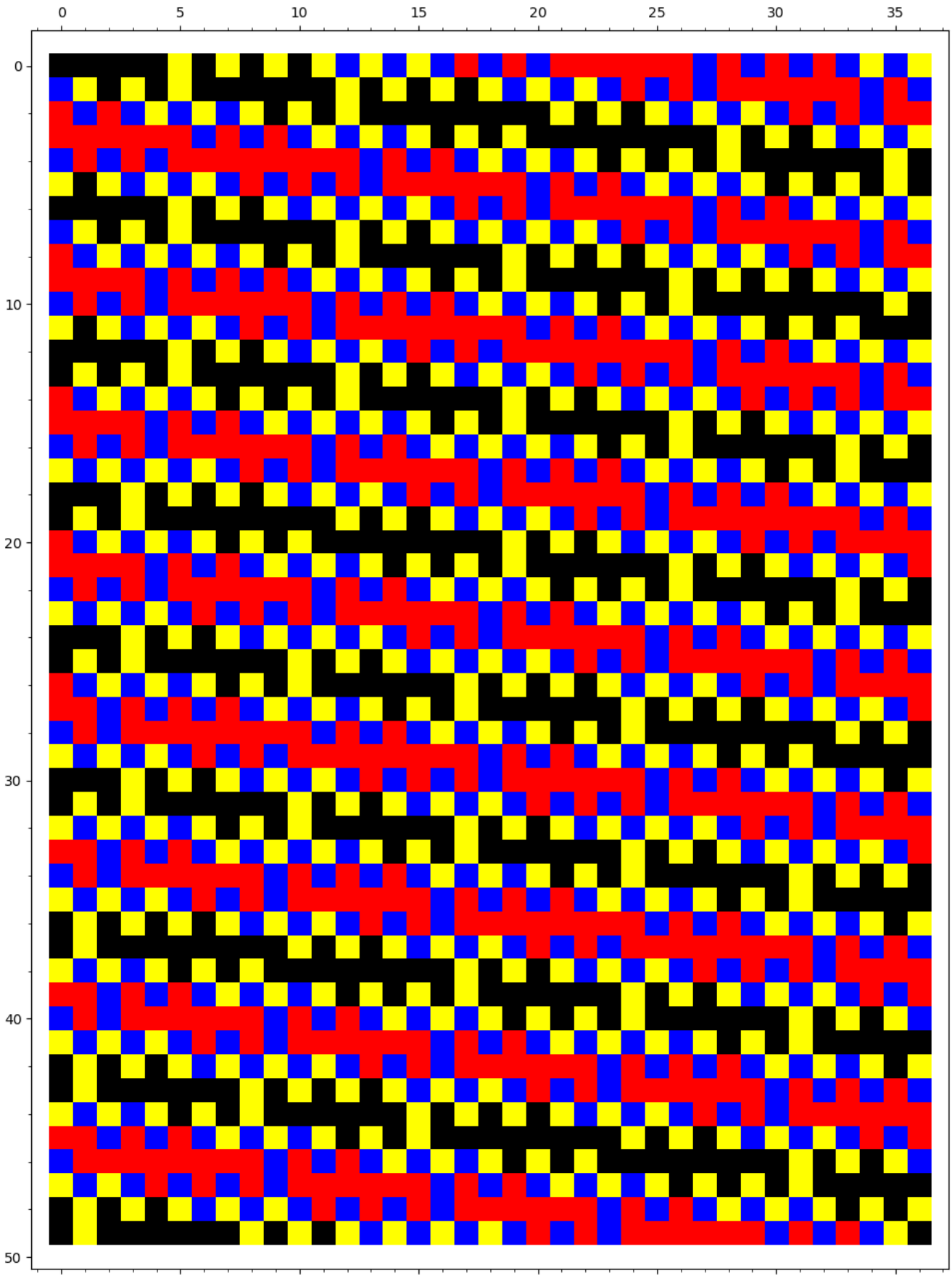

Figure 1. Width 37 px Height 50 px, p = 5, Unordered Pair

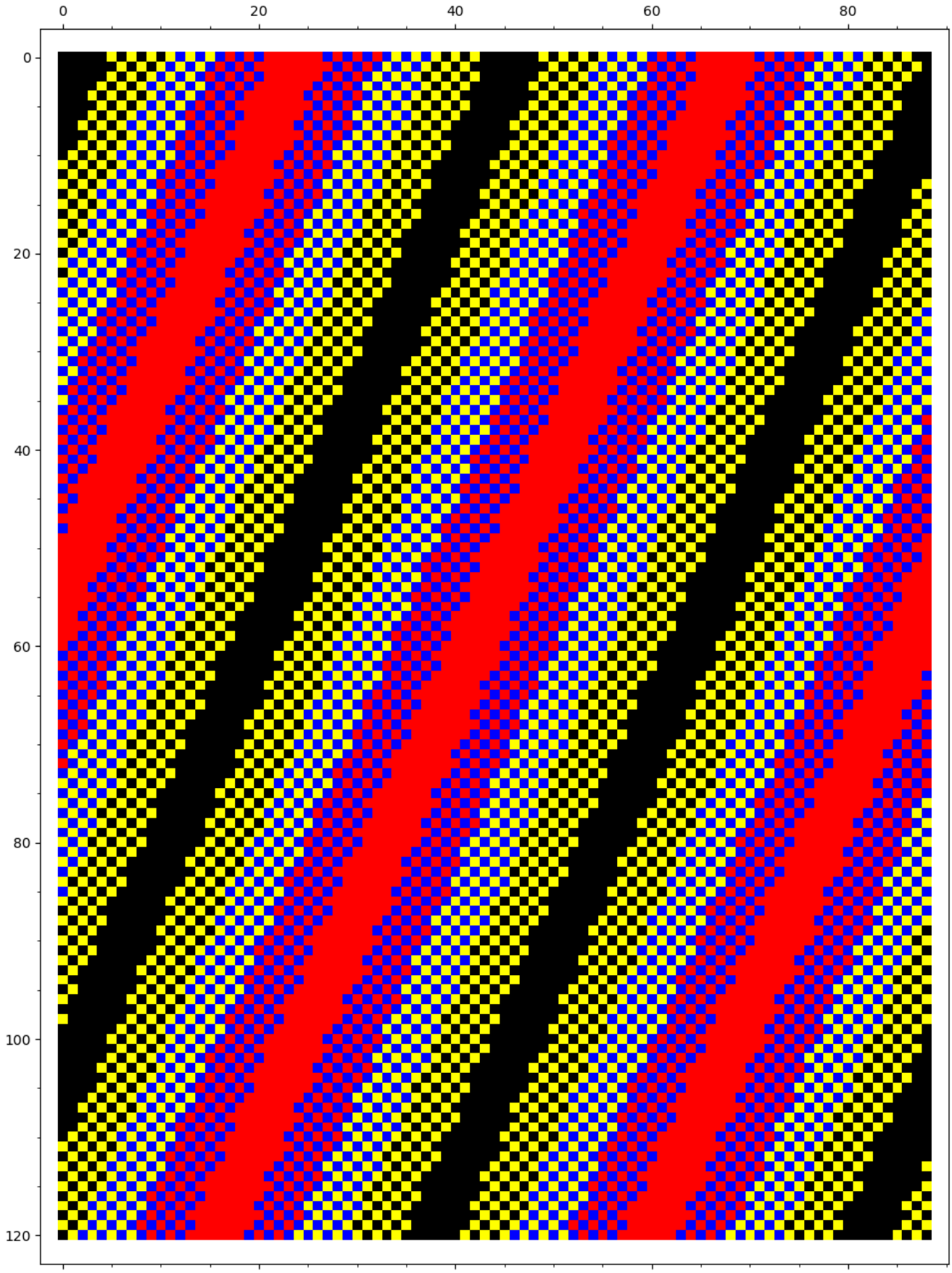

Figure 2. Width 89 px Height 121 px, p = 5, Unordered Pair

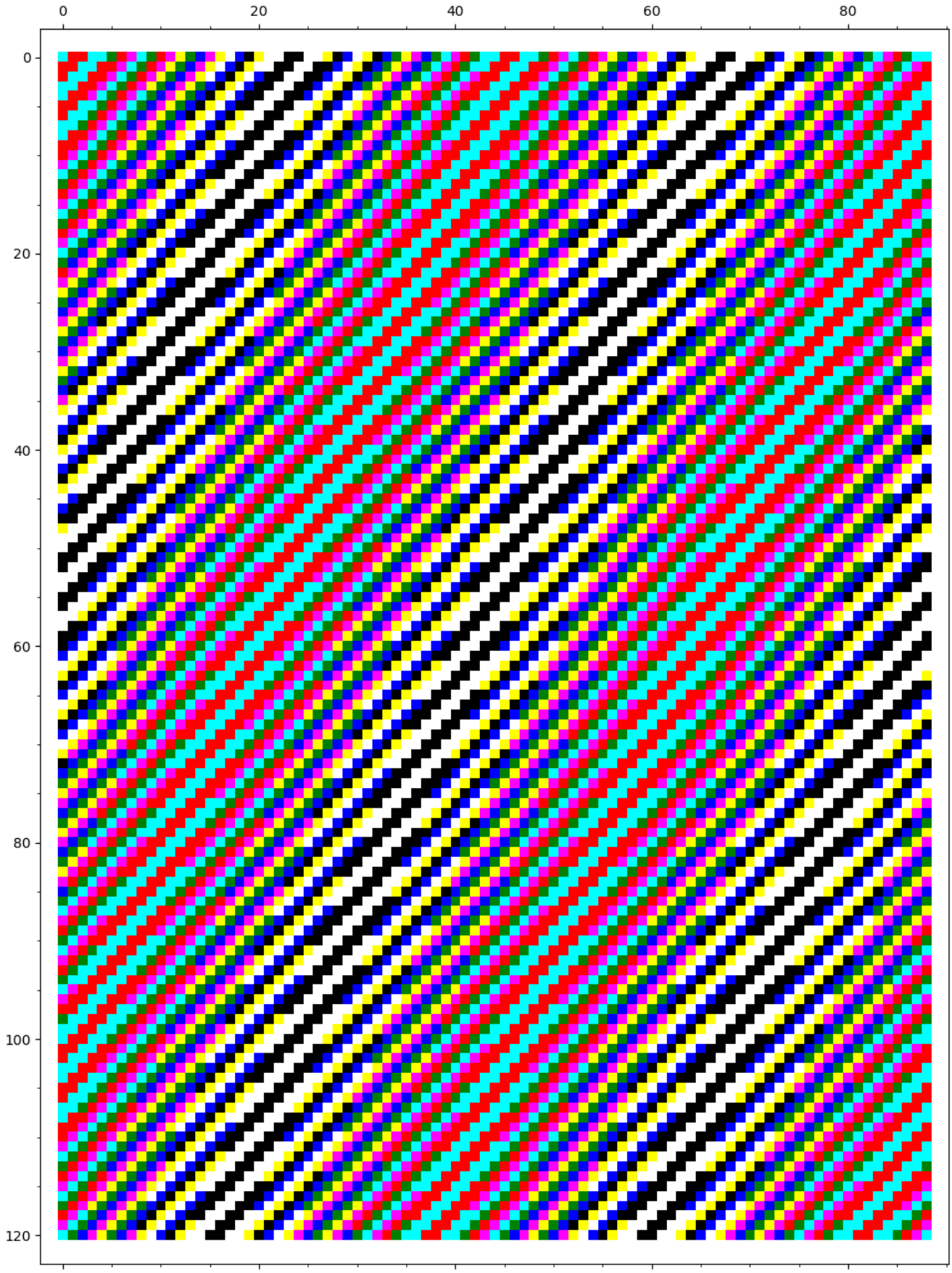

Figure 3. Width 89 px Height 121 px, p = 5, Ordered Pair

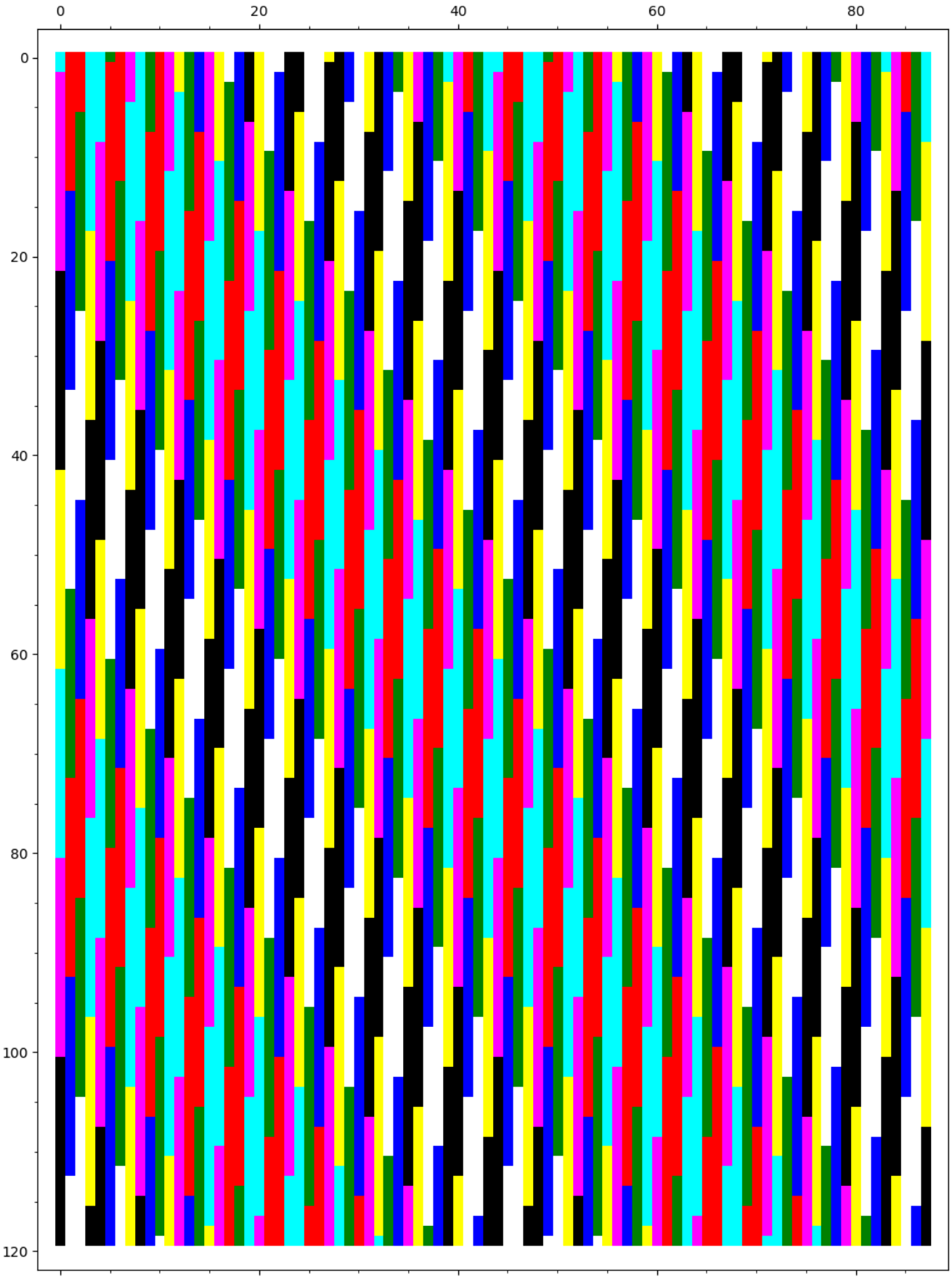

Figure 4. Width 88 px Height 120 px, p = 5, Ordered Pair

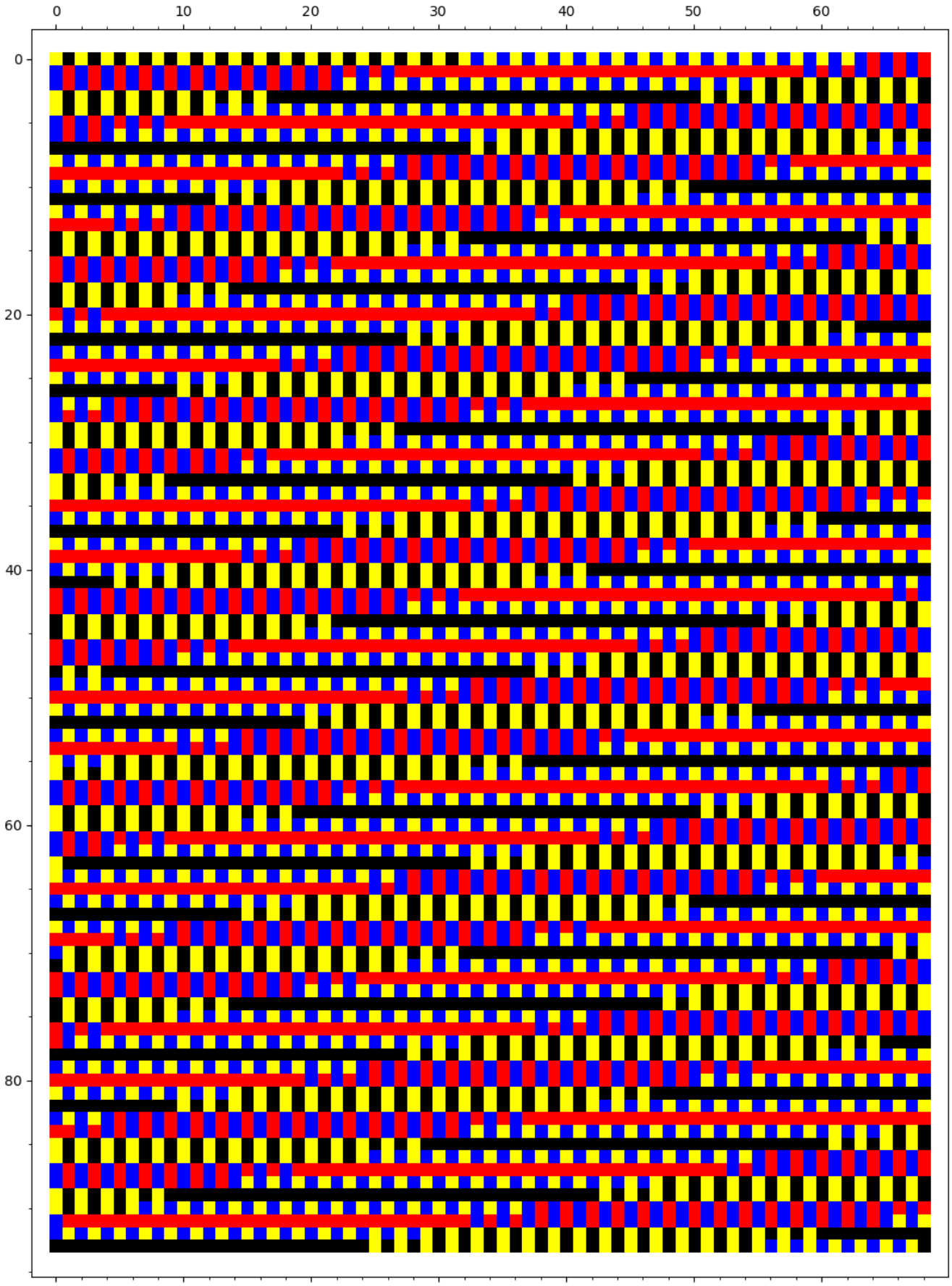

Figure 5. Width 69 px Height 94 px, p = 29, Unordered Pair

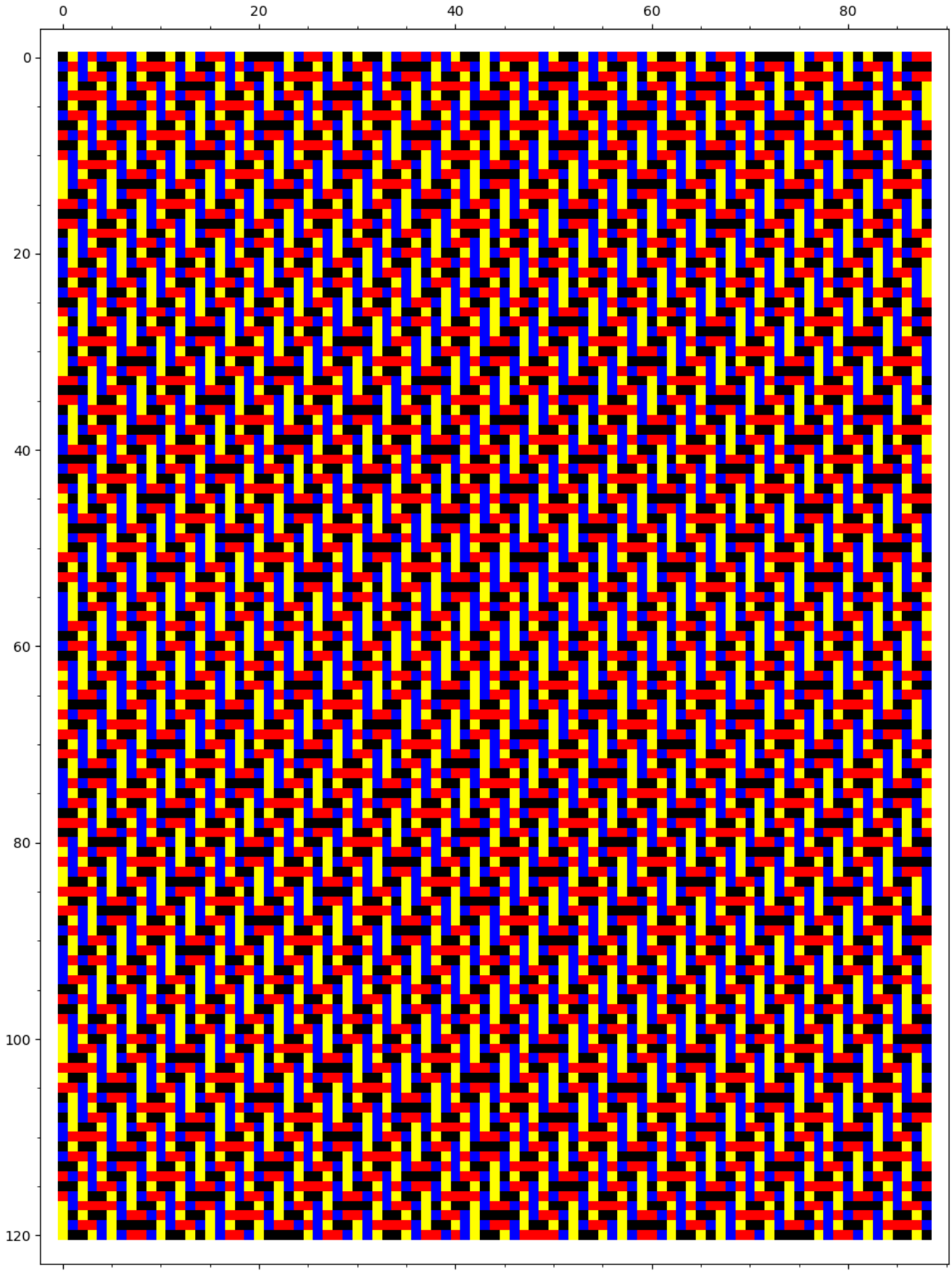

Figure 6. Width 89 px Height 121 px, p = 61, Unordered Pair

Manuscript Clarification

While manuscript originally refers to written article, these days it could also refer for a document that had been typed manually, i.e. neither generated nor formatted using AI. Minimal amount of AI assistance was used, if at all, assuming spell-checker or typo correction and typesetting counts as being an automation assistance. The Author has no access to a single agentic workflow, let alone swarm of bot agents. If the Author had, then it will be very welcome, the LEAN certificate (and possibly other theorem provers, such as Coq, Isabelle/HOL, Agda, ….) shall be produced in such case, and the AI generated work will be submitted on ai.viXra.org or the paid submission rxiVerse.org. Sadly these days purchasing computing related components (dead replacement) is already not affordable thanks to scalpers, let alone those expensive agentic plan, for context, the recent anecdotal example of 88 hours "real stab" on Buckmaster's Millenium problem.

To clarify the authenticity of the projects, manuscript, etc., the reader can visit the codeberg.org/Fian site where some of the hashed files are listed. This anticipates the case of account impersonation (as someone submitting an eprint, project, …) and mitigating for further compromise like hijacked account, or other exploits using recently unveiled security vulnerabilities.

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