

A First-Principles Derivation of the QCD Lagrangian and of Color Confinement from the Microscopic Model

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Abstract

We present a derivation of Quantum Chromodynamics (QCD) and color confinement emerging from a 7+1-dimensional microscopic theory. In this theory quarks are excitations of fundamental 7+1D fermions (the Tetrons) which are condensed into a 3+1D monolayer vacuum structure, and gluons arise as collective topological link excitations. The $SU(3)_C$ gauge symmetry is not postulated ad hoc, but emerges dynamically as an effective macroscopic description of the topological transport on a discrete 3-dimensional ‘monolayer’ (=the 3D physical universe). Furthermore, we show that color confinement arises naturally via a modified dual Meissner effect. Unlike standard phenomenological models that rely on the explicit condensation of magnetic monopoles, the confining medium in this framework is dynamically generated by the geometric frustration of the background Tetron lattice. The discrete isomagnetic nature of the vacuum forces emergent color-electric flux lines into one-dimensional tubes, yielding a rigorous microscopic origin for the linear confining potential. Also included: (i) the emergence of 3+1D Lorentz invariance from the discrete structures, (ii) a comparison between the geometric nature of gluons and of the electroweak gauge bosons and (iii) an analysis of the CP invariance of the vacuum.

1 Introduction

The formulation of Quantum Chromodynamics (QCD) as the exact gauge theory of strong interactions has achieved remarkable success in the high-energy regime due to asymptotic freedom. However, at low energies color confinement has not been proven for the continuum version of QCD, and is one of the most profound unresolved problems in theoretical physics. While the dual superconductor picture[1] proposed in the 1970s provides a compelling macroscopic analogy by attributing confinement to a dual Meissner effect via monopole condensation, the microscopic origin of this superconducting vacuum medium has historically remained elusive. Standard approaches often require the introduction of ad hoc scalar fields or topological defects without a fundamental geometric derivation[2, 3]. In this paper, we resolve this structural deficit by deriving both the $SU(3)_C$ gauge symmetry and a modified Meissner effect directly from the geometric first principles of the Tetron model[4].

The Tetron framework postulates a fundamental 7+1-dimensional spacetime governed by $SO(7, 1)$ symmetry, wherein Standard Model fermions emerge as composite states of elementary Dirac fermions called Tetrons¹. Spontaneous symmetry breaking reduces this high-dimensional space to a 3+1-dimensional macroscopic monolayer, defined by a rigid, discrete lattice of tetrahedral ‘all-out’ isospin configurations. This chiral vacuum topology has previously been shown to successfully dictate the electroweak flavor hierarchy, giving a microscopic interpretation to the Higgs mechanism and generating quark and lepton masses[4].

Here, we extend the macroscopic consequences to include the strong interaction. We identify the origin of the $SU(3)_C$ color degrees of freedom with the internal permutation symmetries of Tetrons inside the isomagnetic unit cells. At distances much larger than the lattice constant (the Planck length), the discrete phase shifts accumulated by Tetrons propagating through the tetrahedral boundary conditions manifest as a continuous $SU(3)_C$ gauge theory.

¹In addition to the 7+1D tetron model there is a 6+1D variant, the details of which were the focus of previous work[4]. Here I have chosen to take the $SO(7, 1)$ variant as starting point, mainly in order to for once give a full account of this variant, but could have followed the same lines of reasoning in the $SO(6, 1)$ version.

Crucially, the translation from the discrete bulk to the continuous effective field theory reveals that the isomagnetic tetrahedral all-out vacuum inherently acts as a topological color-superconductor. The modified dual Meissner effect derived in this work is driven not by magnetic monopoles or scalar Higgs-like condensates, but by isomagnetic couplings in the 4 extra dimensions. The bulk lattice strictly expels emergent color-electric fields, i.e. the geometric incompatibility between the lattice rigidity and the color flux forces the gauge fields into flux tubes between two Tetron triplet excitations (quarks).

By detailing the exact transition from the microscopic lattice Hamiltonian to the macroscopic QCD Lagrangian, we provide a unified geometric mechanism in which the strong force and color confinement are inevitable structural consequences of the chiral Tetron vacuum.

2 Details of the Model

Within the 7+1 dimensional spacetime there are fermions transforming under the fundamental 16D representation $8_L + 8_R$, where 8_L and 8_R are the two complex spinorial representations[5] of $SO(7,1)$, one with left handed and the other with right handed chirality. When decomposing $SO(7,1)$ into the Lorentz symmetry $SO(3,1)$ of a ‘base’ Minkowski spacetime and an ‘internal’ $SO(4)$, describing the symmetry group of the 4 extra dimensions, the representations decompose as

$$\begin{aligned}
SO(7,1) &\rightarrow SO(3,1) \times SU(2)_L \times SU(2)_R \\
8_L &\rightarrow ((1,2), (2_L,1)) + ((2,1), (1,2_R)) \\
8_R &\rightarrow ((1,2), (1,2_R)) + ((2,1), (2_L,1)) \\
8_L \oplus 8_R &\rightarrow ((1,2) + (2,1), (1,2_R) + (2_L,1))
\end{aligned} \tag{1}$$

where the covering group $SU(2)_L \times SU(2)_R$ of $SO(4)$ has been introduced. $(2,1)$ and $(1,2)$ denote left and right handed Weyl spinor representations of $SO(3,1)$ while the left and right handed $SO(4)$ spinors are denoted by $(2_L,1)$ and $(1,2_R)$. Note, the chiralities of all the doublets appearing in (1) are intertwined because they follow from the chiralities of the parent representations 8_L and 8_R .

These fermions are called Tetrons Ψ , and it is assumed that the observed quarks and leptons arise as vibrations $\delta\vec{Q}$ ('iso-magnons') of isospin vectors in the 4 extra dimensions:

$$\vec{Q}_L = \frac{1}{4}\Psi^\dagger(1 - \Gamma_5)\vec{\tau}\Psi = \frac{1}{2}\Psi_L^\dagger\vec{\tau}\Psi_L \quad \vec{Q}_R = \frac{1}{4}\Psi^\dagger(1 + \Gamma_5)\vec{\tau}\Psi = \frac{1}{2}\Psi_R^\dagger\vec{\tau}\Psi_R \quad (2)$$

In order to obtain the complete vibrational degrees of freedom, the decomposition of $8_L \oplus 8_R$ in the last line of (1) should be considered. The corresponding Tetron field represents a Dirac field $(1, 2) + (2, 1)$ in the $SO(3, 1)$ base part, with 2 $SO(4)$ doublets $\Psi_L = (U_L, D_L)$ and $\Psi_R = (U_R, D_R)$ on top.

Note that the indices L and R refer to the chiralities in the 4 extra dimensions. As well known, any rotation in $SO(4)$ can be described by a pair of 3D angular momentum vectors $\vec{Q}_L$ and $\vec{Q}_R$, corresponding to 2 simultaneous rotations of 2 planes in R^4 . (The notion of a rotation axis is not meaningful in 4D.) In the special case $\vec{Q}_L = \vec{Q}_R$ this reduces to one rotation in just one plane.

The ground state of the system has been discussed at length in previous publications[4, 6]. According to this, above Fermi scale distances our universe is a discrete structure consisting of aligned isospin tetrahedrons extending into the 4 extra dimensions. More precisely, the ground state consists of unit cells extending into the extra dimensions where the isospin system of the Tetrons forms an all-out configuration $|T_{L,out}\rangle \otimes |T_{R,out}\rangle$, with 2 aligned² tetrahedrons T_L and T_R each formed by 4 isospin vectors Q_L and Q_R , respectively, and living in the two 3D isospin spaces mentioned above. This means, at each tetrahedral corner $i = I, II, III, IV$, vectors $Q_{L,i}$ and $Q_{R,i}$ are parallel and point in the tetrahedral easy axis direction³.

Another very important point to note is that the cells are point-like in physical 3D space and only extend into the 4D = 7D - 3D internal space. Furthermore, we do

²This is the vacuum using the notation of [4], where the 'raw' definition of right-handed isospinors has been used. In Appendix A this same configuration will be called 'out-in' or $[L_{out}, R_{in}]$. The point is that depending whether one uses the 'raw' or 'tilde' definition of right-handed isospinors, an isospin vector changes its orientation. This means, in Appendix A we are using 'tilde' spinors $\tilde{\Psi} = i\tau_y \Psi_{raw}^*$ to define the right-handed isospin vectors.

³Appendix A shows that in the $SO(7, 1)$ framework the $[L_{out}, L_{in}]$ configuration is preferable over $[L_{out}, R_{in}]$. This actually makes the $SO(7, 1)$ vacuum more similar to that of the $SO(6, 1)$ version, in that there is only one relevant 3D isospin space, namely the L-space. In any case, for the derivation of QCD, as presented in the main text, these choices do not make a difference at all.

not know how the 4 tetrons are distributed spatially inside the 4D cell! We only know that the cell consists of 4 tetrons, and that these span 2 tetrahedrons of isospin vectors in the $\mathbb{R}_L^3 \times \mathbb{R}_R^3$ representation space of $SU(2)_L \times SU(2)_R$.

Due to the pseudovector property of the isospin vectors, the point group of the tetrahedral unit cell is a Shubnikov group. The tetrahedrons T_L and T_R separately are symmetric under the group

$$G_4 := A_4 + T(S_4 - A_4) \quad (3)$$

where $A_4(S_4)$ is the (full) tetrahedral symmetry group, and T the time reversal operator. This means one copy of G_4 operates on the representation space R_R^3 of $SU(2)_L$, and the other on the representation space R_R^3 of $SU(2)_R$.

Since the unit cell contains both a T_L and T_R tetrahedron, and T_L is formed by left handed 8_L and T_R by right handed fields 8_R with $8_R = 8_L^c$, the charge conjugation operator is a symmetry of the system transforming T_L and T_R into each other, thus acting as an additional symmetry operation inside the unit cell.

$$\begin{aligned} C &: L \leftrightarrow R \\ P_{4D} &: L \leftrightarrow R \quad \wedge \quad in \leftrightarrow out \end{aligned} \quad (4)$$

The behavior under C , P and T is summarized in the table.

	C	P_{4D}	T
all-in $\leftrightarrow$ all-out	$\times$	$\checkmark$	$\checkmark$
$R \leftrightarrow L$	$\checkmark$	$\checkmark$	$\times$

Table 1: Actions of C , P_{4D} and T on the internal isomagnetic structure in the $R_L^3 \times R_R^3$ representation space.

G_4 is an unbroken symmetry of the Tetron model which holds down to the lowest energies and contains a Z_3 family symmetry as a subgroup. It has only 1- and 3-dimensional representations and describes all 24 SM quarks and leptons after the SSB.

With respect to 3+1D physical space the unit cells appear pointlike, and distributed statistically/amorphously (like in a quantum liquid or an amorphous material), with

the average distance between the points being the Planck length. For the question how this can lead to an emerging continuous Lorentz invariance see the next section.

While gravity can be attributed to the elasticity of the coordinate bonds[7], the phenomena of particle physics arise from the interactions between the isospin vectors forming the tetrahedrons.

Isospins Q_L and Q_R (2) form the generators of the group $SU(2)_L \times SU(2)_R$, which acts on the 4 extra dimensions. This 4D isospin space on which the tetrahedral unit cells are hosted is a subspace of the higher-dimensional 7D manifold. Adjacent unit cells are distributed exclusively in the dimensions orthogonal to the extra $\mathbb{R}^4$. Consequently, the all-out tetrahedral array forms a topological monolayer within the 7D bulk. This monolayer constitutes our physical universe; in the continuum limit, as the spatial lattice (but not the isospin structure) becomes unresolvable, it maps onto the familiar 3+1 dimensional Minkowski spacetime, cf. the details given in the next section.

Clearly, the expansion into a relativistic spacetime continuum introduces a significant constraint: the tetrahedral isospin symmetry is strictly defined only in the rest frame of the lattice. However, the quasiparticle excitations which make up for the ordinary matter and propagate along the monolayer exhibit different symmetry properties. Because these excitations are wave-like solutions within the 3+1 dimensional effective manifold (the monolayer), they manifest as Lorentz $SO(3,1)$ covariant objects. In this interpretation, while the fundamental vacuum is non-covariant and discrete, the observable physical sector – consisting of collective excitations – obeys the laws of special relativity. These particles ‘glide’ over the monolayer substrate, treating the underlying non-invariant isospin grid as a relativistic medium.

3 The Spatial Continuum Limit and the Recovery of Lorentz Invariance

In this work we are considering particle physics effects only, which amounts to switching off gravity (and the corresponding elasticity of the spatial bonds) by considering the continuum limit with respect to physical space, while keeping the discrete tetra-

hedral isospin configuration intact. Actual calculations should be done in the rest system of the tetrahedral configurations, which on cosmic scales amounts to the so-called CMB rest system[8]. The expanding universe thus is not the expansion of an empty space, but of the increasing distance between the tetrahedral cells, corresponding to the uniform expansion of the monolayer grid.

It is well known that the fundamental spacetime constants c , $\hbar$ and G can be used to define the Planck length, time and mass L_P , T_P and M_P which describe the basic properties of space[m], time[s] and matter[kg]

$$c = \frac{L_P}{T_P} \quad \hbar = E_P T_P \quad \kappa = \frac{L_P}{E_P} \quad (5)$$

where $E_P = M_P c^2$ is the Planck energy and $\kappa = G/c^4$ the Einstein constant.

Our monolayer is a ‘Planck lattice’ – more precisely a discrete amorphous structure – in the sense that we identify the average spacing between 2 unit cells as the Planck length L_P . Since we are not interested in gravitational but only in particle physics effects, we are allowed to take the limit $G \rightarrow 0$ while keeping c and $\hbar$ fixed. This removes the spatial elasticity, at the same time allowing to consider the continuum limit $L_P \rightarrow 0$ without giving up the discrete structure in isospin space.

According to (5) this limit implies $E_P \rightarrow \infty$ and corresponds to the observed large value of the Planck energy, which is interpreted as the binding energy of the spatial ‘lattice’, while the isomagnetic couplings – to be discussed later after Eq. (6) – with magnitude $O(E_F)$ are the much smaller interaction energies among the isospins.

In other words, this limit sends G to zero while simultaneously allowing to use the continuum limit for the spatial coordinates and considering a system of discrete isospin vectors forming aligned tetrahedrons in the ground state.

As $E_P \rightarrow \infty$, the medium (which may be considered a kind of ‘Lorentz ether’) becomes infinitely stiff against gravitational deformations. What remains are the internal degrees of freedom of the symmetry group $SU(2)_L \times SU(2)_R$. If we consider E_P as the gigantic binding energy of the lattice (10^{19} GeV) and E_F as the gentle interaction energy of the isospins (10^2 GeV), we find ourselves in a regime where the 3D lattice structure is rigid but disordered (amorphous), while the isospin degrees of freedom act upon it like an ordered and relatively smooth structure. It is a sort of nematic system, where ‘molecules’ consisting of 4 isomagnets are in an

ordered/aligned phase but their positions in 3D physical space are disordered, similar to a glassy liquid[9].

We are now in the position to recover Lorentz invariance as an emergent symmetry of the low-energy sector[10]. The preferred frame of the Planck-lattice exists, but it is hidden behind the energy barrier E_P . For the matter excitations (quarks and leptons), the geometry of the monolayer effectively behaves as a $3 + 1$ dimensional Minkowski space.

More in detail, the vacuum of the present theory is a structured ‘isospin crystal’, with a lattice constant L_P being fixed up to the elasticity effects from gravitation. While this lattice constitutes a preferred frame, the observable excitations obey Lorentz invariance due to the following four principles:

- The continuum limit (non-resolvability): Since the lattice is unresolvable for wavelengths $\lambda \gg L_P$, the discrete differences between lattice sites smooth out into continuous fields. The underlying absolute grid disappears from the equations of motion, replaced by a smooth spacetime manifold where only the relative coordinates of the wave-packets matter.

- Universal limiting velocity: The ‘speed of light’ in the model is not an independent constant but an emergent property of the medium. It is determined by the ratio of the elasticity of the Tetron interactions and their density. Since all excitations propagate through the same medium, they all share the same maximum propagation speed. This universal speed limit is the foundation of the Lorentz transformation.

- Hyperbolic dispersion relations: The excitations in the monolayer are not classical particles but collective modes which obey Lorentz-invariant wave equations like the Klein-Gordon or Dirac equation. Due to the restoration force of the ground state, these modes follow a relativistic dispersion relation: $E^2 = (pc)^2 + (mc^2)^2$.

- The internal nature of the observer: Since the observers (and their measuring devices) are themselves made of these same excitations, they are locked into the same wave dynamics. An observer moving through the monolayer cannot detect the static isospin cells because their own clocks and rulers change in unison. This is the Principle of Relativity realized within a medium.

4 The Isomagnetic Interactions among Tetrons

How to obtain the above described ground state from a dynamics? It all starts with some unknown fundamental 7+1D interaction among tetrons, and then one has to assume that this induces exchange integrals which in turn lead to Heisenberg and DMI interactions among the isospin vectors, which finally are responsible for the tetrahedral order.

When trying to adapt these isospin interactions to the observed quark and lepton spectrum, a dominance of DMI over Heisenberg shows up[11, 12, 4], with the DMI Hamiltonian

$$\begin{aligned}
 H_{DMI} = & -D_{LL} \sum_{i \neq j=I}^{IV} (\vec{Q}_{Li} \times \vec{Q}_{Lj})^2 - D_{LR} \sum_{i \neq j=I}^{IV} (\vec{Q}_{Li} \times \vec{Q}_{Rj})^2 \\
 & - D_{RR} \sum_{i \neq j=I}^{IV} (\vec{Q}_{Ri} \times \vec{Q}_{Rj})^2
 \end{aligned} \tag{6}$$

Actually one should distinguish the cases for inter- and intra-cell interactions, so all of the couplings D appear in a D_{inter} and a D_{intra} version, both always being of the same order of magnitude. The result of an extensive analysis[4] shows, that the top mass is essentially determined by $D_{LL,intra}$, and the Fermi scale and the masses of the weak bosons by $D_{LL,intra}$. $D_{RR,intra}$ mainly measures the mass of the b-quark and $D_{LR,intra}$ of the τ -lepton. As explained below, there are also Heisenberg interactions in the system with much smaller couplings $\leq 1\text{GeV}$ and responsible for the masses of the second family.

Note that one has here a ‘quadratic’ type of DMI enforced by applying Moriya’s rules[13] to the tetrahedral isospin configuration, in contrast to the ‘linear DMI’ form $\vec{D}_{ij}(\vec{Q}_i \times \vec{Q}_j)$ encountered in other situations[15, 16]. While the linear DMI is a chiral interaction, the quadratic DMI effectively is $\sim (\vec{Q}_i \vec{Q}_j)^2 - (\vec{Q}_i)^2 (\vec{Q}_j)^2$ and thus non-chiral, which means there is no apriori preference of the all-out configuration (which is left-handed by definition) over the all-in configuration from the side of the Hamiltonian, but the choice of the all-out over the all-in configuration must be due to a spontaneous symmetry breaking[4, 6].

Another important point is that the sums exclude $i = j$, i.e. interactions of the Tetron wave function with itself are not allowed. This is important, because oth-

erwise the interaction $\sim (\vec{Q}_{Li} \times \vec{Q}_{Ri})^2$, which drives $\vec{Q}_{Li}$ and $\vec{Q}_{Ri}$ away from each other, would prevent the alignment of the $\vec{Q}_L$ and $\vec{Q}_R$ isospin structures.

As quantified in [4], the masses of the third family roughly correspond to the values of the DMI couplings

$$D_{LL,intra} \approx m_t \quad D_{LR,intra} \approx m_\tau \quad D_{RR,intra} \approx m_b \quad (7)$$

As mentioned above, there is also a small Heisenberg Hamiltonian active on the system

$$H_H = -J_{LL} \sum_{i \neq j=I}^{IV} \vec{Q}_{Li} \vec{Q}_{Lj} - J_{LR} \sum_{i \neq j=I}^{IV} \vec{Q}_{Li} \vec{Q}_{Rj} - J_{RR} \sum_{i \neq j=I}^{IV} \vec{Q}_{Ri} \vec{Q}_{Rj} \quad (8)$$

(again in an inter- and an intra-version). The Heisenberg couplings turn out to be responsible for the masses of the second family with

$$J_{LL,intra} \approx m_c \quad J_{LR,intra} \approx m_\mu \quad J_{RR,intra} \approx m_s \quad (9)$$

and are therefore necessarily much smaller than the DMI couplings.

One may then say that the tetrahedral ground state is a DMI-dominated isomagnet with a small admixture of Heisenberg couplings.

The full relevance of the Heisenberg couplings for quark confinement will become clear later in Sections 11 and 12. At this point, I would like to anticipate a very important fact, namely that $J_{LR,inter}$ turns out to determine both the value of the QCD string tension and of Λ_{QCD} . This surprising feature has to do with the Q_L - Q_R condensate which is essential for the tetron interpretation of QCD confinement (cf. Table 2 and Sections 11 and 12)

While a classic Heisenberg-dominated magnet favors symmetric, collinear alignment of spins, a quadratic DMI-dominated lattice penalizes collinearity and forces the vacuum into a highly frustrated, non-collinear all-out tetrahedral configuration rather than a flat, ordered state. Because the quadratic DMI $(\vec{Q}_i \times \vec{Q}_j)^2$ is 100 times stronger than the Heisenberg term, the system's ground state is deeply trapped in a steep and narrow potential well in configuration space, effectively freezing the vacuum and making any transverse fluctuations out of this tetrahedral alignment energetically unfavorable. Mathematically, this dominant quadratic DMI acts as

a symmetric operator that concentrates the massive $O(100)$ GeV energy scale essentially onto a single eigenvalue (the top mass), providing a natural origin for the heavy third-generation particles. Conversely, the lighter families remain protected and mass-free in this limit, escaping the crushing DMI scale because the quadratic interaction avoids the entangling degeneracies of standard linear DMI.

Note that each corner of the tetrahedron has 3 neighboring edges. If $(\vec{Q}_i \times \vec{Q}_j)^2$ is maximized (i.e. H_{DMI} minimized), every isospin tries to stand perpendicular to its 3 neighbors. The only configuration in the 3D spaces defined by Q_L and Q_R , that simultaneously optimizes this ‘perpendicularity’ condition, for all corners, is the all-out tetrahedral ground state. So actually, D_{LL} is the driving force of the SSB in the system and stabilizes the 2 aligned tetrahedrons T_L and T_R within the unit cell, as well as the inter-cell alignment, more than any Heisenberg interaction could ever do.

In other words, the secret of the DMI alignment is, that it does not – like the Heisenberg interaction – favor parallel or anti-parallel configurations, but maximal orthogonality in all components. Since the DMI is quadratic (proportional to the sine squared of the angle between the 2 vectors), the restoring force is even larger than for the linear DMI making the ground state isospin tetrahedrons extremely rigid structures.

5 Tetronic Origin of the Strong Interaction

A priori, to the microscopic model, QCD seems more of a foreign body. As we shall see, however, it may be derived as a compelling emergent extension, both local (at high energies) and non-local (at low energies) in nature and – at low energies – involving many unit cells, which confine any triplet(=quark) excitation. In essence this is an alternative, discrete approach to the observed strong forces, including the phenomena of asymptotic freedom and confinement, the latter having never been proven rigorously within the framework of continuous QCD. In fact, it is well known, and the proof will be done below, that a phenomenon like confinement is much easier to prove in a discrete environment like the Tetron monolayer than in the continuum.

Consider the 24 possible excitations of one unit cell. There are 6 singlet states (the

leptons) and 6 triplets (corresponding to the 6 quark flavors), all transforming under the tetrahedral Shubnikov group G_4 [4, 6]. The 3 wave functions of a triplet can be understood as arising from symmetry functions adapted to the 4 tetrahedral corners as combinations $++--$, $+-+-$ and $+-+-$, and actually can be traced back to the 3 rotations by 180° corresponding to the permutations 2143, 3412 and 4321 of the tetrahedral/permutation group A_4 ⁴, while the singlets(=leptons) in this scheme correspond to $++++$.

More in detail, the contribution to the up-type mass eigenstates from one corner I (and analogously for the corners II, III and IV) of the tetrahedron is

$$\begin{aligned}
u_I &= \frac{1}{\sqrt{3}\sqrt{1+\epsilon_1^2}} [(|Q_{LIx}\rangle + \epsilon_1 |Q_{RIx}\rangle) + (|Q_{LIy}\rangle + \epsilon_1 |Q_{RIy}\rangle) \\
&\quad + (|Q_{LIz}\rangle + \epsilon_1 |Q_{RIz}\rangle)] \\
c_I &= \frac{1}{\sqrt{3}\sqrt{1+\epsilon_2^2}} [(|Q_{LIx}\rangle + \epsilon_2 |Q_{RIx}\rangle) + \omega(|Q_{LIy}\rangle + \epsilon_2 |Q_{RIy}\rangle) \\
&\quad + \bar{\omega}(|Q_{LIz}\rangle + \epsilon_2 |Q_{RIz}\rangle)] \\
t_I &= \frac{1}{\sqrt{3}\sqrt{1+\epsilon_3^2}} [(|Q_{LIx}\rangle + \epsilon_3 |Q_{RIx}\rangle) + \bar{\omega}(|Q_{LIy}\rangle + \epsilon_3 |Q_{RIy}\rangle) \\
&\quad + \omega(|Q_{LIz}\rangle + \epsilon_3 |Q_{RIz}\rangle)]
\end{aligned} \tag{10}$$

and for the down quarks

$$\begin{aligned}
d_I &= \frac{1}{\sqrt{3}\sqrt{1+\epsilon_1^2}} [(|Q_{RIx}\rangle - \epsilon_1 |Q_{LIx}\rangle) + (|Q_{RIy}\rangle - \epsilon_1 |Q_{LIy}\rangle) \\
&\quad + (|Q_{RIz}\rangle - \epsilon_1 |Q_{LIz}\rangle)] \\
s_I &= \frac{1}{\sqrt{3}\sqrt{1+\epsilon_2^2}} [(|Q_{RIx}\rangle - \epsilon_2 |Q_{LIx}\rangle) + \omega(|Q_{RIy}\rangle - \epsilon_2 |Q_{LIy}\rangle) \\
&\quad + \bar{\omega}(|Q_{RIz}\rangle - \epsilon_2 |Q_{LIz}\rangle)] \\
b_I &= \frac{1}{\sqrt{3}\sqrt{1+\epsilon_3^2}} [(|Q_{RIx}\rangle - \epsilon_3 |Q_{LIx}\rangle) + \bar{\omega}(|Q_{RIy}\rangle - \epsilon_3 |Q_{LIy}\rangle) \\
&\quad + \omega(|Q_{RIz}\rangle - \epsilon_3 |Q_{LIz}\rangle)]
\end{aligned} \tag{11}$$

where ω and $\bar{\omega}$ are the 120° phases of the Z_3 subgroup of G_4 given by $\omega = e^{i\frac{2\pi}{3}}$ and $\bar{\omega} = \omega^2$.

Three coefficients $\epsilon_{1,2,3}$ appear in these equations, which depend on the quark and

⁴The three π rotations together with the unit element form the Klein four-group V_4 , which is a normal subgroup of the alternating group. The quotient group A_4/V_4 is isomorphic to the cyclic group Z_3 .

even on the lepton masses. They can be calculated within the model[4] to be

$$\epsilon_i = \frac{1}{6} \frac{M_{Li}}{M_{Ui} + M_{Di}} \quad (12)$$

where M_{Ui} , M_{Di} and M_{Li} denote the corresponding masses within family $i = 1, 2, 3$. These equations only reveal the weak isopin and family structure of the triplet excitation, based on the $Q_L \leftrightarrow Q_R$ and the $Z_3 \subset G_4$ symmetries of the unit cell, respectively. In order to distinguish the Shubnikov ‘color’ states one must include all 4 corners I, II, III and IV of the tetrahedron to form the symmetry adapted functions

$$\begin{aligned} |q_{++--}\rangle &= \frac{1}{\sqrt{4}}[q_I + q_{II} - q_{III} - q_{IV}] \\ |q_{+--+}\rangle &= \frac{1}{\sqrt{4}}[q_I - q_{II} - q_{III} + q_{IV}] \\ |q_{-+-+}\rangle &= \frac{1}{\sqrt{4}}[q_I - q_{II} + q_{III} - q_{IV}] \end{aligned} \quad (13)$$

for any $q = u, c, t, d, s, b$ of the expressions (10) and (11).

According to (6)ff our tetrahedral system is dominated by the quadratic DMI, and the energy of the vacuum is minimized when the all-out condition is met. If one locally excites the system by a singlet or a triplet, one creates a local deviation from this vacuum. In particular, the triplet excitations create a divergence of isomagnetic charges across the tetrahedron. By flipping two corners relative to the other two, like in the excitation $++--$, there is a conflict with the global all-out vacuum, which must be compensated for by the neighboring cells. When a triplet alters the ground state across many cells, it leaves a track in the medium. A second triplet – note this must be an antiparticle wrt the base space – can then travel along this track with less energy expenditure. This represents the classical definition of a long-range attraction at low energy. It corresponds to a strong feedback topological effect.

So when a single triplet enters, it is a deformation that cannot just terminate abruptly. It spreads out extensively. Only when 2 triplets combine to a singlet (a meson), their isomagnetic charges add up to zero, corresponding to a G_4 symmetric state. In the near field between the two triplets, the field remains strongly distorted, whereas in the far field (outside the meson), the crystal returns to a perfect, undisturbed all-out state. The triplet pair effectively encapsulates the distortion.

Note, at this point this is just the qualitative idea. The details, how the ‘color flux’ is defined within the model, and squeezed into flux tubes will be described in the section about flux tubes.

Much in contrast, the singlet modes $(++++)$ represent a uniform scaling or a global rotation of all four isospins in the tetrahedron, in the sense that every isospin vector is moved in exactly the same way relative to the all-out ‘axis’. Since the movement is symmetric across all four corners of the tetrahedron (rotational symmetry of the singlets), the total isomagnetic flux generated by this excitation sums to zero within the unit cell. As a result, a singlet does not create a source or sink of flux.

Building on the state definitions in Eq. (13), we can define the color-electric charge of a quark within the isomagnetic lattice. The color charge is not a continuous $SU(3)$ generator, but rather the topological asymmetry of the four internal vertices. While the macroscopic vacuum state as well as the lepton sector exhibit a fully symmetric, unfrustrated configuration $(++++)$, each of the three quark states represents a 180° rotation (a flip) or, equivalently, an internal sign permutation with two positive and two negative vertices. When embedded into the vacuum, the internal asymmetry of such a the state $|q\rangle$ necessarily induces a frustration at the cell boundary. It is precisely this topological boundary divergence that defines the color charge, as it forces the cell to emit a compensating, directed chain of non-trivial link operators (gluons) forming the color-electric flux tube. This will be worked out in detail in the section about flux tubes.

6 QCD Gluons Emerging from the Model

8 Shubnikov ‘gluons’ may be identified for the system. At first sight they are different than the $SU(3)$ gluons of QCD, but also non-abelian in nature, because moving e.g. from a $++--$ state to a $+-+-$ state involves a rotation that doesn’t commute with other tetrahedral rotations. In G_4 they decompose as $1 + 1 + 3 + 3$, with a vector triplet 3 carrying the ‘magnetic’ flux, and an axial triplet providing currents that wrap around the flux and, as shown in the section about flux tubes, confine it.

In this section I will explain, how QCD gluons arise as gauge fields from the discrete geometric intuition. As noted before, at this point color is just the quantum number

that distinguishes the 3 states within triplets of the G_4 Shubnikov group, and the gluons are identified as the transition operators that transform one state of a triplet into another. Mathematically, such an operator is a 3 times 3 matrix

$$\hat{G}_{\alpha\beta} = |q_\alpha\rangle\langle\bar{q}_\beta| - \frac{1}{3}\delta_{\alpha\beta} \sum_\gamma |q_\gamma\rangle\langle\bar{q}_\gamma| \quad (14)$$

acting on the 3-dimensional space spanned by the quark states (13)

$$\alpha, \beta \in \{++--,+-+,-+-,---\} \quad (15)$$

How does the bilinear form $\hat{G}_{\alpha\beta}$ look geometrically between two of the tetrahedral unit cells? A quark state is a very specific, collective isospin twist (a phase texture) of the Tetron isospins. The bilinear form, on the other hand, corresponds to a dynamic oscillation or a tunneling process in which the twist of one cell on axis β propagates to another cell while tilting to axis α .

On this level, the bilinear forms represent 8 Shubnikov dof, and as stated, the Shubnikov 8 representation is not the same as the adjoint 8 of the continuous Lie group SU(3). However, one must distinguish the symmetry G_4 of the bare lattice from the symmetry of the emergent dynamics in the continuum limit of the discrete amorphous spatial Planck structure, cf. Section 3.

To understand what I mean, consider a real cubic crystal in 3D space. It only possesses the discrete symmetry of a cubic group O_h . A cube has no continuous rotational symmetry. It cannot be rotated by arbitrary angles without altering the lattice. Yet, if we consider the sound waves in this crystal, whose wavelength is much larger than the lattice spacing, the sound wave only sees a continuous, elastic medium, and its equations of motion follow the emergent symmetry of SO(3), which arises from smoothing over many unit cells.

Similarly here: if one could microscopically resolve the unit cell, one would see that the 8 vibrational modes do not have perfect SU(3) symmetry, but split into different symmetry classes according to $8 = 1 + 1 + 3 + 3$. The fact that we measure the perfect, irreducible 8 of SU(3) in the laboratory where all 8 gluons possess exactly the same strong coupling, is an illusion of the continuum limit.

Why, in contrast to sound waves, do we arrive at SU(3) instead of SO(3)? The answer is that the discrete quark states $|q_\alpha\rangle$ span a basis of a three-dimensional

complex vector space ($\mathbb{C}^3$), because the isospins of the Tetrons are mathematically defined via complex spinors. As the lattice undergoes macroscopic, continuous deformation due to the propagation of the quarks, this effective $\mathbb{C}^3$ space of isospin deformations transforms accordingly. The mathematical group that describes unitary i.e. probability-preserving transformations in a complex $\mathbb{C}^3$ space is precisely $SU(3)$.

It must be stressed, that the deformations we are talking about do not concern the spatial 3D or 3+4D discrete structure but is purely a distortion of the internal isospin-tetrahedrons relative to one another.

7 The Gluon Field as a Gauge Field

The vacuum of the model is determined by the isospin couplings J and D . If one introduces a quark into one of the unit cells, this excitation forces a distortion of the surrounding all-out configuration, so that any arriving vibrational mode is scattered and phase-shifted. We want to relate the local gradient distortion of the crystal to a gauge potential, and its rigidity to the strong coupling constant. To that end we start with the matrix $\hat{G}$ defined in (14).

In the unperturbed vacuum, $\hat{G}$ formed by the Shubnikov quark states has a homogeneous reference value, which we define as $\hat{G}^{(0)}$. Since the modes are orthonormal (or can be orthonormalized), both $\hat{G}$ and $\hat{G}^{(0)}$ are unitary matrices (up to normalization). The local $SU(3)$ gauge element $g(\vec{r}, t)$ is then the unitary transition matrix that rotates the unperturbed reference modes of the global vacuum into the actually distorted local modes of the cell at position $\vec{r}$:

$$\hat{G}(\vec{r}, t) = g(\vec{r}, t) \cdot \hat{G}^{(0)} \iff g(\vec{r}, t) = \hat{G}(\vec{r}, t) \cdot \left(\hat{G}^{(0)}\right)^{-1} \quad (16)$$

Any thermal or quantum mechanical fluctuation that locally tilts or twists the isospin vectors $\vec{Q}_a(\vec{r}, t)$ of the cell alters the wave functions $\hat{G}(\vec{r}, t)$ and thus directly modulates the local field $g(\vec{r}, t)$.

It may be noted, that in the continuous limit $g(\vec{r}, t) \in SU(3)_C$ describes the local gauge transformation, whereas at the level of the Tetron lattice it is nothing other

than the local orientation matrix of the isospin tetrahedron relative to the global vacuum background.

One can now use the apparatus of standard gauge theory and define the gauge potential via a Maurer-Cartan form. Geometrically, in the Tetron model this means, if the orientation of the tetrahedra changes in time or spatially from one lattice cell to the next, a gradient distortion (shear/torsion) of the isospin vacuum arises. Mathematically, this is described by the derivative of g and leads to the gauge potential

$$A_\mu = -\frac{i}{g_s} g^\dagger \partial_\mu g = -\frac{i}{g_s} \hat{G}^\dagger \partial_\mu \hat{G} \quad (17)$$

If one goes back to the definition of $\hat{G}$ in (14), one sees that the gluon field can be interpreted as the generalized non-Abelian Berry connection (or geometric phase) of the Shubnikov quark states[17, 18]. When a quark state propagates through the isospin lattice, the gluon field it experiences is not an independent, fundamental force. It is the purely geometric phase resulting from the inner Shubnikov states $|q_\alpha(r, t)\rangle$ slightly rotating from one lattice site to another in spacetime. The ‘color charge’ is simply the information indicating which of these three orthogonal states the particle currently occupies.

Concerning the time component, the term $g^{-1} \partial_t g$ describes the local angular velocity (or the quantum mechanical phase velocity) with which the isospin vectors in the cell $\vec{r}$ oscillate around their equilibrium position. In classical mechanics, this corresponds to gyroscopic motion. A_0^a then is the projection of this local precession velocity onto the Gell-Mann matrices.

Spacetime covariance of A_μ is not an artificial requirement here, but results directly from the fact that the waves of isospin distortions propagate as collective modes in the monolayer with the effective ‘speed of sound’ of the amorphous lattice, which in the continuous limit takes on the role of the speed of light c .

If one writes down the equation of motion for an elastic isomagnetic wave in such an already pre-deformed system, the ordinary derivative automatically transforms into a covariant derivative

$$D_{\mu, \alpha\beta} = \partial_\mu \delta_{\alpha\beta} + i g_s A_\mu^a \lambda_{\alpha\beta}^a \quad (18)$$

The gluon fluctuations transfer momentum between the quarks because any movement of a quark compresses or stretches the vibrational modes of the surrounding Tetron crystal. The system permanently attempts to minimize the elastic energy of the Heisenberg and DMI couplings, which macroscopically manifests on the monolayer as the observed $SU(3)$ color force.

8 Flavor Independence

When describing the gluon in terms of 2-cell $q\bar{q}$ excitation states like in (14), one must actually be cautious to guarantee flavor independence. A rigorous lattice description must ensure the flavor universality of the strong interaction: the eight $SU(3)_c$ gluons must mediate identical interactions for all six quark flavors (u, d, s, c, b, t); i.e. the gluon should be understood topologically as the universal process of color transfer across any inter-cell boundary Σ , only dependent on the V_4 transition, but independent of which Z_3 state (10) or (11) one chooses.

When 2 colored fermions at cell A and cell B combine, the gluon wave function $\Psi_g^a(A, B)$ emerges as the overlap integral

$$\Psi_g^a(A, B) \approx \int_{\Sigma} d^4\xi [\bar{\chi}_A(\xi) \cdot \lambda^a \cdot \chi_B(\xi)] \quad (19)$$

Here, $\chi(\xi)$ represents the internal cellular quark wavefunction, and λ^a are the eight Gell-Mann generators of $SU(3)_c$.

The $SU(2)$ representation spaces, on which the isospin tetrahedrons live, are 3-dimensional, and therefore 2 adjacent unit cells meet at a shared 2D interface. (The boundary between two 3D volumes is mathematically a 2D surface.) Crucially, the transition dynamics across this interface are not determined by the internal flavor state of the quark (such as its generational excitation mode), but exclusively by the fundamental lattice couplings of the vacuum at the boundary – specifically the symmetric exchange couplings J_{inter} and the antisymmetric interactions D_{inter} defined after (6). The color operators λ^a strictly commute with both the Z_3 flavor rotations and the $SU(2)_{L/R}$ isospin operators[4]. Just as the boundary couplings J_{inter} and D_{inter} they depend solely on the geometry of the tetrahedral interface.

To make this more explicit one should consider Eq. (13) which shows that the property to be a Shubnikov quark triplet has nothing to do with the $Z_3 \subset G_4$ symmetry in (10) (describing the family structure) nor with pseudospin transitions $T_L \leftrightarrow T_R$ (corresponding to weak isospin $(10) \leftrightarrow (11)$) but solely refers to the three 180° -rotations of the tetrahedral Shubnikov group G_4 which connect the states (13). When a gluon is formed at the wall between 2 cells A and B (as a 2-cell excitation), only this property gets encoded into the gluon.

The critical distinction lies in how these topological properties project across the 2D inter-cell boundary. When the gauge fields are evaluated via the overlap integral of the wavefunctions at the shared interface, the boundary color operator couples exclusively to the color polarization inversions dictated by the three 180° -rotations of the tetrahedral unit cell. Because this acts as a normal subgroup – the famous Klein four-group V_4 of A_4 – its operations strictly commute with both the cyclic Z_3 generation phases $(1, \omega, \bar{\omega})$ and the $L \leftrightarrow R$ amplitudes in (10) and (11). Consequently, while the W boson actively mediates an $L \leftrightarrow R$ transition that projects across Z_3 flavor axes[4], the gluon operator acts entirely within the decoupled color subspace of G_4 . When the gluon transition amplitude is integrated across the wall, the internal Z_3 flavor phases and isospin parameters mathematically factor out, geometrically guaranteeing that the exact same fundamental mode of color flux is excited regardless of the quark's flavor.

9 Asymptotic Freedom

It is a well-known exercise to derive the standard QCD Lagrangian from an $SU(3)$ gauge structure with covariant derivative (18). Then, in ordinary continuous QCD, the β -function tracks how virtual particle loops hide the short-distance infinities of smooth space. Because the Tetron model is fundamentally discrete at the Planck scale, albeit amorphous wrt physical space, the continuous running of the coupling constant is replaced by an effective field theory description:

–At all experimentally accessible distances up to a length scale $L_E (\gg L_P)$, which lies between a PeV and the GUT scale, the random distribution of cells averages out perfectly. The vacuum looks like a smooth continuum. Here, one may use a standard

effective β -function to describe how the coupling changes because one is averaging over trillions of cells. Only very much above the weak scale, at temperatures where entire regions of isomagnetic order begin to melt or a multi-phase mixture forms, this picture loses its meaning.

–Then, in between L_E and L_P , the concept of a running coupling constant breaks down. Even more, the ordered isospin vacuum ceases to exist, together with all physical particles (its excitations). Only the bare structural couplings of the discrete Tetron units remain. Even though the cells can fluctuate and are distributed randomly due to their elastic interactions, they cannot squeeze infinitely close together, because the direct Tetron-Tetron fermionic coupling remains like a hard-core repulsive potential on the Planck scale.

–Finally, approaching the Planck energy, the monolayer evaporates into a 7D Tetron gas.

Crucially, because the physical unit cells of the 3D base space remain localized up to L_E , the quarks behave as perfectly point-like Dirac particles for all experimentally accessible energy regimes, bypassing experimental constraints on spatial quark compositeness. Of course, one may ask whether effects e.g. in jet cross sections can arise from the tendency to ‘melt’ of the local tetrahedral isospin order even below L_E , when 2 high energy particles hit at TeV-energies.

However, this concern can be countered by considering that a single high-energy particle (even at 100 TeV) cannot simply melt a local tetrahedral unit cell, as this cell is topologically and energetically anchored within the infinite collective of its amorously distributed neighboring cells.

10 The Strong CP Problem

In the Tetron model, the color flux (the gluons) at the 2D cell wall arises exclusively through the overlap of the V_4 states (13). These linear combinations are strictly real (the coefficients are merely +1 and –1).

For physical CP violation via the well-known $F\tilde{F}$ term, a complex phase in the strong sector is strictly required (often manifesting as an imaginary part in the determinant of the quark mass matrix, which couples to the gauge field via the chiral

anomaly). However, because the operator structure defining the strong interactions at the wall is generated by the purely real 180° color polarization flips (± 1), imaginary components simply do not appear in the color space. The rigidity of the real Klein four-group V_4 geometrically forces the complex-valued topological phase term θ_{QCD} to be exactly zero, and so the appearance of the $F\tilde{F}$ term is forbidden in the microscopic model.

This may be contrasted to the situation in the weak sector. Whereas the strong interaction is based on the real V_4 symmetry, the weak interaction mixes the states of the Z_3 family symmetry. As seen in (10) and (11), the Z_3 symmetry contains the complex phases ω and $\bar{\omega}$ (with $\omega = e^{2\pi i/3}$). When the W boson projects across the tilted axes, it acquires these complex phases. As shown in [4], these complex Z_3 phases are the basis for the construction of the PMNS and CKM matrices and the associated CP-violation within the present model.

In conclusion, the Tetron model separates CP conservation in the strong interaction from CP violation in the weak interaction purely through the fundamentally different algebraic geometry (real 180° flips vs. complex 120° rotations) within the 4D isospin space.

Actually, there is another necessary condition to be fulfilled, if one wants to completely exclude the appearance of an $F\tilde{F}$ term in the model, namely it must be guaranteed that CP is not dynamically broken anywhere in the theory. Before we begin to study this question, an important point to note is that when we are talking about strong CP violation, P really means P_{3D} , i.e. parity wrt the physical 3D space (not wrt the extra dimensions). This is because QCD in my model is an effective theory in the emergent continuum limit. P_{3D} just inverts the ordinary 3 spatial coordinates, effectively exchanging the positions of neighboring lattice cells, $A \leftrightarrow B$, and we are now going to confirm that this spatial invariance holds universally across all terms in the lattice Hamiltonian, namely:

–the DMI interaction is governed by the squared cross product of the isospins, $\propto (\vec{Q}_A \times \vec{Q}_B)^2$. Under P_{3D} , the cell exchange yields an odd cross product, $\vec{Q}_B \times \vec{Q}_A = -(\vec{Q}_A \times \vec{Q}_B)$. Upon squaring, the negative sign is eliminated, rendering both the LL and RR DMI terms strictly symmetric. Furthermore, the mixed LR inter-cell terms $D_{LR}(\vec{Q}_{LA} \times \vec{Q}_{RB})^2 + D_{RL}(\vec{Q}_{RA} \times \vec{Q}_{LB})^2$, are inherently symmet-

ric. Because the geometric overlap integral of an one tetrahedron in cell A with another tetrahedron in cell B is structurally identical to the reverse configuration, $D_{LR} = D_{RL}$ holds strictly. Under spatial reflection, P_{3D} merely maps $(\vec{Q}_{LA} \times \vec{Q}_{RB})^2$ to $(-\vec{Q}_{RA} \times \vec{Q}_{LB})^2 = (\vec{Q}_{RA} \times \vec{Q}_{LB})^2$, mapping the terms onto each other and leaving the Hamiltonian entirely invariant.

–The argument extends straightforwardly to the Heisenberg interactions, which are based on the scalar product $J_{LL}(\vec{Q}_{LA} \cdot \vec{Q}_{LB}) + J_{RR}(\vec{Q}_{RA} \cdot \vec{Q}_{RB})$. Since the scalar product is naturally commutative ($\vec{Q}_{LB} \cdot \vec{Q}_{LA} = \vec{Q}_{LA} \cdot \vec{Q}_{LB}$), these terms are inherently P_{3D} -even and do not require squaring to preserve spatial parity.

By contrast, the charge conjugation operator C exchanges the left- and right-handed sectors ($L \leftrightarrow R$). Applying C to the Hamiltonian transforms the coupling weights, mapping $D_{LL} \rightarrow D_{RR}$ and $J_{LL} \rightarrow J_{RR}$, and vice versa. Due to the spontaneous geometric asymmetry where $D_{LL} \neq D_{RR}$ (and $J_{LL} \neq J_{RR}$), the Hamiltonian explicitly violates C .

Consequently, under the combined macroscopic operation CP_{3D} , the system exhibits a clear symmetry violation. However, this violation is driven exclusively by the C -violation inherent in the chiral asymmetry of the couplings, effectively mirroring the parity-violating $V - A$ structure required for the electroweak sector, and actually this specific mechanism of CP_{3D} violation provides a robust structural shield against the strong CP problem. The continuum topological $F_{\mu\nu}\tilde{F}^{\mu\nu}$ term of QCD is a pseudoscalar and therefore inherently odd under P_{3D} . For such a term to exist or be radiatively generated in the effective continuum limit, the underlying lattice Hamiltonian must violate P_{3D} . Because all spatial interactions in this model – whether governed by strictly quadratic DMI cross products or commutative Heisenberg scalar products – perfectly preserve P_{3D} , the generation of a macroscopic P_{3D} -odd term is algebraically forbidden across all sectors. The theory successfully accommodates the necessary CP violation in the chiral/electroweak sector via the C -violation evident in the asymmetric effective couplings $D_{LL} \neq D_{RR}$ (which is driven by the spontaneous formation of the all-out system which prefers the LL-exchange integral over the RR exchange integral), while the strong interaction remains structurally immune to a topological θ -angle, naturally resolving the strong CP problem without requiring an axion.

11 Confinement and the Formation of Flux Tubes

As seen in the above sections, the naked gauge invariant QCD Lagrangian can be proven to arise in the continuum limit of the Tetron theory. It remains to reflect, whether the inherent discreteness of the system modifies the understanding of the low energy behavior of quarks and gluons. As discussed on the following pages, the mechanism of confinement is somewhat different than in the Mandelstam interpretation of low energy QCD. This alternative interpretation is not in contradiction to the QCD Lagrangian (which describes the local kinematics of the gauge fields and is entirely dictated by symmetry) but describes a different non-perturbative global vacuum state.

The basic idea is that the quark confinement scale is determined by the inter-cell Heisenberg couplings J_{inter} which enforce the color exchange against the rigid tetrahedral DMI lattice. As will be proven below by an explicit calculation, the much larger inter-cell DMI couplings $D_{inter} = O(100)$ GeV do not affect the string tension relevant for confinement⁵ because the DMI is blind to the formation of the tube.

When one pulls quarks apart, one is not overpowering the massive 100 GeV DMI frame; one is merely stretching the softer 1 GeV Heisenberg springs that connect the Isospins within that frame. This is why the strong interaction dynamics manifests at the 1 GeV scale, even though the structural integrity of the vacuum itself is safely anchored at 100 GeV by the DMI.

Before I go into the details of these considerations I want to compare 3 approaches to Meissner-type formation of flux tubes:

(i) Inside a standard type-II superconductor a condensate of charged particles (Cooper pairs) expels magnetic flux (Meissner effect), so that magnetic flux tubes (Abrikosov-Vortices) are formed. These flux tubes do *not* connect electric charge, but they guide and enclose a quantized magnetic field ($\Phi_0 = h/2e$), while superconducting circular currents (the Cooper pairs) enclose and circulate around the non-superconducting core of a flux tube. An interesting point is that if magnetic monopoles would exist and put in a Type-II superconductor they would be connected by Abrikosov-Vortices.

⁵Due to the phenomenon of induced ordering[19] the relevant scale is $D_{LL,inter}$ and not $D_{RR,inter}$ or $D_{LR,inter}$.

So the magnetic field lines between them do not disperse throughout space like in vacuum, but are squeezed into the extremely thin, thread-like channel of the flux tube. Because the energy of the flux tube grows linearly with its length, a constant, attractive force arises between the north and south monopole that does not decrease even at infinite distance. You can never separate the two monopoles from each other (confinement). Note, Type-II superconductivity occurs almost exclusively on a solid-state substrate with a regular lattice structure, because it is based on phonons attracting 2 electrons to form a Cooper pair.

(ii) Let me compare this to the standard picture of QCD confinement. Since the QCD Lagrangian views quarks as ‘electric’ color charges, an inverted model was proposed by Mandelstam and others[1, 20] to explain confinement: the ‘dual’ Meissner Effect. There it is assumed that the vacuum is a condensate of magnetic monopoles (instead of Cooper pairs) and that this condensate expels color-electric fields, so that color-electric flux tubes are formed, and that these connect the quark’s color-electric charges (instead of the magnetic monopoles) and lead to the linear confinement of quarks observed in the strong interactions. Note that in contrast to superconductors, the QCD vacuum may be modeled as an amorphous, fluid-like condensate of magnetic monopoles. Magnetic monopoles are assumed to condense in the vacuum just like electrons form Cooper pairs in a standard superconductor. Unlike a crystalline solid, however, this quantum vacuum is completely disordered, isotropic, and fluid-like, without any fixed lattice structure (‘amorphous’). The big drawback of the Mandelstam picture is the rather artificial assumption of the existence of a monopole vacuum.

(iii) In the present model, confinement occurs through a dual Meissner effect, just as in (ii). The ‘knack’, however, is that the tetrahedral all-out system is already a vacuum condensate of oriented Q_L - Q_R isospin pairs. These pairs do not play the same role as the Cooper pairs in a superconductor, but represent local phase locking at the 2D intercell walls, keeping the all-out isospin tetrahedrons globally aligned in parallel. The idea for confinement is then that since the vacuum disfavors any isomagnetic flux that contradicts the all-out order, it does not let the isomagnetic flux lines spread out on the monolayer, but squeezes them into a narrow flux tube. The quarks in the Tetron model are not inserted foreign particles, but isomagnetic vibrations corresponding to tiny tetrahedral misalignments, which prevent the vac-

uum from dispersing the color lines in all directions. Similar to the monopoles in the Mandelstam QCD vacuum fluid picture, the unit cells are distributed amorphously on physical space.

	Condensate/Vacuum	Confined Charges
(i) Superconductor	electric Cooper pairs embrace magnetic flux line	magnetic monopoles
(ii) Mandelstam QCD	color-magnetic monopoles form fluid-like vacuum	color-electric quarks
(iii) Tetron Model	$Q_L - Q_R$ pairs stabilize the tetrahedral vacuum	triplet G_4 excitations

Table 2: Comparison between type-II superconductivity, the Mandelstam QCD vacuum, and the Tetron model.

In the following I shall take some time to detail the approach described in (iii). Starting point of the discussion will be the underlying Tetron dynamics, with its strict energetic hierarchy between the macroscopic chiral ordering and the local inter-cell exchange interaction ($D_{inter} \gg J_{inter}$).

Because the chiral $Q_L - Q_R$ alignment is stabilized by the dominant scale D_{inter} , the tetrahedral lattice structure is so rigid that the vacuum condensate does not 'evaporate' or melt inside the flux tube. Instead, the extreme rigidity acts as the primary confinement mechanism, by forbidding the color flux from spreading isotropically into the 3D monolayer. Because deforming the D_{inter} order is energetically prohibited, the vacuum is forced to compress the color field into a one-dimensional channel.

While D_{inter} dictates the tubular geometry of the confined flux, the actual energy density (the QCD string tension of 0.3 GeV/fm) is governed by the softer Heisenberg scales J_{inter} . At the 2D boundary between adjacent cells, the quantum mechanical overlap of the highly directional isospin vectors is naturally described by the Heisenberg exchange interaction $J_{inter}(\vec{Q}_A \cdot \vec{Q}_B)$. When a quark and an anti-quark are separated, they induce a discrete 180° topological flip in the V_4 Shubnikov states (13) along the connecting path. This color-flip abruptly inverts the sign of the local geometric overlap across the cell boundary, generating a sharp, localized

energy penalty dictated entirely by J_{inter} .

Therefore, the phase inside the flux tube is a color-frustrated vacuum. The flux tube consists of a sequence of physically stable unit cells whose internal V_4 color orientation is topologically misaligned relative to the surrounding transverse vacuum. The macroscopic string tension observed in strong interactions is the accumulated geometric 'friction' – the integral of the discrete Heisenberg exchange penalties – generated across the lateral cell walls where the frustrated Shubnikov states interface with the unperturbed, rigid lattice.

A single quark (=triplet excitation) without an isospin partner would generate a shear stress in the tetrahedral order that cannot simply vanish into nothing. Instead, the elastic distortion of the field propagates far into the crystal. Since the stiffness D_{inter} is extremely high, this long-range perturbation costs a massive amount of energy, about 100 GeV. Normally, it would send its isospin flux lines out in all directions. However, since the DMI implies a massive energy penalty, the vacuum pushes back on the flux lines from every side. The flux lines find that they can minimize the damage by staying as close together as possible. Instead of spreading into all 3 base space dimensions, the flux is squeezed into a 1D line (a tube) connecting the triplet to an anti-triplet.

The quantitative proof of those statements will be given in the next section. For the moment, I will turn to the singlet modes (leptons l_{++++}), which escape the fate of confinement, because they represent a uniform scaling or a global rotation of all four isospins in the tetrahedron. Since the movement is symmetric across all 2×4 corners in the ground state of the unit cell, the total isospin flux generated by this excitation sums to zero. So there is no need for a singlet to be connected to another excitation by a flux tube. It can travel through the monolayer as a solitary wave. The energy cost is purely local. To move to the next cell, it just has to transfer its local deformation. This leads to a standard dispersion $E(k)$ rather than a linear potential $V(r) \propto r$.

12 Gluon Clouds and the String Tension

To establish a rigorous mathematical foundation for the confinement mechanism within the Tetron lattice, we must analyze the exact energetic contributions on the microscopic level, i.e. at the boundary between adjacent unit cells. A defining feature of the model is the vast hierarchy between the macroscopic vacuum rigidity scale ($D_{inter} \approx 100$ GeV) and the Heisenberg couplings ($J_{inter} = J_{LR} \approx 0.1$ GeV). Note once again that we are assuming throughout this paper that intra- and inter-cell couplings have the same order of magnitude, cf. the remark after (6).

Now we demonstrate that the quadratic DMI (6) and the Heisenberg exchange (8) couple to fundamentally different topological degrees of freedom. This asymmetry guarantees that the DMI scale acts solely as a geometric confiner of the color flux tube thickness, while the linear string tension is dictated exclusively by the Heisenberg scale.

Let us define the discrete boundary between Cell A and Cell B, coupled at their four corresponding vertices $i \in \{I, II, III, IV\}$ as in the rhs of (13). The isospin vectors at these vertices are denoted as $\vec{Q}_A^{(i)}$ and $\vec{Q}_B^{(i)}$. We analyze two transitions:

(i) the Vacuum-to-Vacuum transition. In this case Cell A and Cell B are both in the unfrustrated all-out vacuum state abbreviated as $++++$. Thus, $\vec{Q}_B^{(i)} = +\vec{Q}_A^{(i)} = \langle \vec{Q}^i \rangle$ for all $i = I, II, III, IV$, where the angle brackets denote the vacuum expectation value.

(ii) the vacuum-to-quark transition. In this case Cell A is in the vacuum state $(++++)$, while Cell B contains a frustrated Shubnikov color state (e.g., $++--$). Thus, two vertices are perfectly parallel, and two are perfectly anti-parallel: $\vec{Q}_B^{(i)} = \pm \vec{Q}_A^{(i)}$. More precisely, $\vec{Q}_A^{(i)} = \langle \vec{Q}^i \rangle$ as before, but $\vec{Q}_B^{(i)} = \delta \vec{Q}^i = q_i$ in the notation of the rhs of (13).

To identify the string tension as originating from the inter-cell Heisenberg interaction, we compare the transition energy with the DMI case. The Heisenberg Hamiltonian is given in terms of the scalar product promoting parallel alignment of the isospin vectors in cell A and cell B

$$H_{inter}^J = -J_{inter} \sum_{i=I}^{IV} \vec{Q}_A^{(i)} \cdot \vec{Q}_B^{(i)} \quad (20)$$

Evaluating this for the vacuum-to-vacuum transition yields the ground state energy

$$E_{\text{vac} \rightarrow \text{vac}}^J = -J_{\text{inter}} \sum_{i=I}^{IV} \vec{Q}_A^{(i)} \cdot (+\vec{Q}_A^{(i)}) = -4J_{\text{inter}} \quad (21)$$

In contrast, for the vacuum-to-quark transition, where two vertices flip their sign, the scalar products sum to zero

$$E_{\text{vac} \rightarrow \text{quark}}^J = -J_{\text{inter}} (1 + 1 - 1 - 1) = 0 \quad (22)$$

The transition from a trivial vacuum link to a topological color flux link thus incurs a strict energetic penalty of

$$\Delta E^J = E_{\text{vac} \rightarrow \text{quark}}^J - E_{\text{vac} \rightarrow \text{vac}}^J = +4J_{\text{inter}} \quad (23)$$

Because any macroscopic flux tube connecting a quark-antiquark pair over a distance R requires a discrete Wilson line of such 180° cell-wall flips, this fixed energy penalty per cell wall directly generates the linear confinement potential $V(R) = \sigma R$. The string tension σ is thus governed entirely by J_{inter} .

Boundary Transition	ΔE^{DMI} (Rigidity)	ΔE^J (String Tension)
Vacuum $\rightarrow$ Vacuum	0	0
Vacuum $\rightarrow$ Quark	0	$+4J_{\text{inter}}$

Table 3: Relative energetic penalties at the unit cell boundary. The unfrustrated vacuum link is defined as the zero-energy baseline. The D_{inter} term is completely blind to the discrete 180° color flip, leaving the J_{inter} term as the sole generator of the linear confinement potential.

Now for the quadratic DMI Term, which is responsible for the rigid potential walls of the isospin lattice. This term penalizes non-collinear alignments via the square of the cross product

$$H_{\text{inter}}^{\text{DMI}} = D_{\text{inter}} \sum_{i=I}^{IV} \left(\vec{Q}_A^{(i)} \times \vec{Q}_B^{(i)} \right)^2 \quad (24)$$

Evaluating this term for the pure vacuum transition (0° alignment) yields a trivial zero

$$E_{\text{vac} \rightarrow \text{vac}}^{\text{DMI}} = D_{\text{inter}} \sum_{i=I}^{IV} \left(\vec{Q}_A^{(i)} \times (+\vec{Q}_A^{(i)}) \right)^2 = 0 \quad (25)$$

Crucially, evaluating this Hamiltonian for the topological color flip (180° anti-alignment for two vertices) also yields exactly zero, as the cross product of any vector with its exact opposite vanishes:

$$E_{\text{vac} \rightarrow \text{quark}}^{\text{DMI}} = D_{\text{inter}} \sum_{i=I}^{IV} \left(\vec{Q}_A^{(i)} \times (\pm \vec{Q}_A^{(\alpha)}) \right)^2 = 0 + 0 + 0 + 0 = 0 \quad (26)$$

So we conclude $\Delta E^{\text{DMI}} = 0$ which means there is no energetic penalty from the DMI term on the formation of the color flux tube.

In conclusion, the vacuum-to-vacuum transition is energetically preferred wrt to vacuum-to-quark by $4J_{\text{inter}}$, while for the DMI there is no energetic difference between the 2 transitions. While the Heisenberg term is sensitive to discrete 180° rotations ($=V_4$ sign permutations), the quadratic DMI is blind to them. Since $D_{\text{inter}} \gg J_{\text{inter}}$ the DMI acts as an immense energetic funnel. Because the penalty scales as $D_{\text{inter}} \sin^2(\theta)$, it violently suppresses any continuous, soft spatial twisting of the isospin field ($\theta \neq 0, \pi$). It forces the lattice dynamics to remain collinear. Consequently, to avoid the large D_{inter} penalty, topological color flux cannot propagate via smooth continuous deformations. The lattice is rigidly forced to route the flux through the only loophole available: the discrete 180° flips. The fact that this happens in a 1D-channel and does not spread out radially into the monolayer, has a simple geometric reason: a 1D-chain of N frustrated unit cells necessarily contains less internal walls than a 2D- or 3D-Cluster of the same number of such cells, and thus the total energy (penalty) of the flux configuration given by $N \times 4J_{\text{inter}}$ is smaller for a 1D-chain.

Within this enforced 1D channel, the energetic cost of the flux is handed over entirely to the Heisenberg scalar product, correctly producing a string tension somewhat below the 1 GeV scale of J_{inter} . This understood, it may be asked, what the role of the DMI coupling is wrt confinement. The answer is that the large value of the DMI coupling $D_{\text{inter}} \approx 100 \times J_{\text{inter}}$ fixes the rigidity of the vacuum configuration, so that it plays the role of a ‘guardian’ of the system, because any transverse spreading of the color-electric flux would induce a quadratic penalty from the vacuum rigidity $\propto D_{\text{inter}} \sin^2(\theta)$. Consequently, the flux is forced to propagate exclusively via discrete 180° phase flips, with energy $N \times 4J_{\text{inter}}$ given above.

If we explain quark confinement this way one may ask what is the connection to

the way gluons were defined as 2-cell excitations in (14). Our system has a little resemblance with lattice gauge theory, and in the language of LGT the question can be answered by saying that quarks live on the sites (the cell volumes) because a quark is a frustrated single isospin cell (e.g., $++--$), while gluons live on the links (the 2D cell walls). As bilinears, gluons act as transition operators between cells. A gluon (14) contains the information how the color state changes as one crosses the wall from Cell A (α) to Cell B (β). The frustrated wall energy we have calculated above (vacuum to quark, or traversing the flux tube) carries a color mismatch. This mismatch is the $\bar{q}_\alpha q_\beta$ bilinear excitation.

At the end of this section I want to point to a problem which may seem to arise in the framework just presented. Namely, since an energy difference of $4J_{inter}$ appears at each cell wall (provided it contains a 180° flip), one has a huge ‘string tension’ of $4J_{inter}/L_P$ in a ‘flux tube’ consisting of only 2 cells. Now, if a 1 femtometer flux tube were constructed as a literal one-dimensional chain of bare Planck length cells, the number of 180° phase boundaries would be of the order $N \sim 10^{20}$. Given that a single microscopic boundary costs an energy $J_{inter} \sim \mathcal{O}(0.1 \text{ GeV})$, this naive geometric counting yields a bare string tension of $\sim 10^{18} \text{ GeV}^2$ – obviously an unphysically large value.

This paradox is resolved by recognizing that a single topological defect (e.g., a $++--$ quark state against the vacuum $++++$, or a $++--$ quark against a $+-+-$ quark) does not remain pointlike. Analogous to a polaron in solid-state physics – where a bare electron dynamically distorts the surrounding crystal lattice to form a macroscopic dressed quasiparticle – the bare phase defect strongly polarizes the surrounding $++++$ vacuum condensate. This interaction dresses the defect in a ‘gluon cloud’, swelling its effective physical size to the Compton wavelength dictated by the characteristic energy scale of the interaction itself, $R_{\text{cloud}} \sim \hbar c/J_{inter} \approx 1 \text{ fm}$.

Consequently, the QCD flux tube connecting two quarks is not a dense sequence of 10^{20} point-like Planck cells, all with 180° cell-wall flips. Instead, it is a chain of dressed condensate states, formed by a discrete sequence of these ‘macroscopic’ $\mathcal{O}(0.1) \text{ GeV}$ ‘polaron’-like excitations involving clouds of gluons. The fundamental degrees of freedom that constitute the string are these extended condensate clouds, each occupying a volume of $\sim (1 \text{ fm})^3$ and carrying an energy of roughly J_{inter} .

When the flux tube is modeled as a chain of these femtometer effective cells rather than bare Planck cells, the string tension $\sigma = E/L$ naturally yields the experimentally observed value

$$\sigma \approx \frac{J_{inter}}{R_{cloud}} \approx \frac{(0.1 \text{ GeV})}{1 \text{ fm}} \sim 0.2 \text{ GeV}^2 \quad (27)$$

The catastrophic UV divergence driven by L_P completely vanishes because the coherent state of the gluon cloud dynamically cuts off the localization of the topological defect at the femtometer scale.

In the tetron model the value of the Heisenberg coupling strength J_{inter} – and thereby the energy scale of the flux tube – is determined through a 7D exchange integral. As discussed in [4], due to the form of the 7D Coulomb potential $\sim r^{-5}$ the tetron wave functions inside a unit cell are strongly localized at the corners of the tetrahedrons. The exchange integral $J_{inter}(r) \equiv J_{LR}(r)$ of these localized wavefunctions is not a freely adjustable running coupling, but a physical consequence of the spatial distribution of the condensate clouds

$$J_{inter}(r) \propto \int d^7x f_1(\mathbf{x}) f_2(\mathbf{x} - \mathbf{r}) \frac{1}{r^5} f_2(\mathbf{x}) f_1(\mathbf{x} - \mathbf{r}) \quad (28)$$

where f_1 is a tetron wave function which belongs to a tetrahedron T_L in one cell and f_2 a tetron wave function which belongs to a tetrahedron T_R in a neighboring cell. Since we are including only next neighbor interactions, the correct value for J_{LR} in (8) to be chosen is $J_{LR} = J_{inter}(L_P)$, which according to (9) may be identified as the muon mass. This means, evaluating the integral yields a fixed, discrete topological energy cost of $\approx 0.1 \text{ GeV}$ for a single 180° phase flip.

The mechanism to give reasonable values for the QCD string tension relies on the internal structure of the ‘polaron’ cloud. Within such a cloud occupying a volume of $\sim (1 \text{ fm})^3$, the ‘polarization’ of the lattice is organized such that it contains essentially only one cell wall with a 180° spin flip. The rest within the cloud’s volume is perfectly aligned (with whatever state is dominant) and therefore contributes nothing to the exchange energy.

This discrete structure naturally explains the macroscopic string tension as the quarks are separated. The flux tube is not a static object but a one-dimensional chain of these polaron-like clouds. As two quarks are pulled apart, the system must

bridge the increasing geometric gap by forming additional clouds along the axis of separation. Each newly formed cloud introduces exactly one additional microscopic 180° phase flip into the lattice, costing an additional fixed energy of $J_{inter}(L_P)$. Therefore, a flux tube of length L consists of $N = L/(1 \text{ fm})$ clouds. The total energy grows linearly with distance: $E(L) = N \cdot J_{inter}(L_P)$, yielding a constant macroscopic string tension $\sigma = E/L \approx J_{inter}(L_P)/(1 \text{ fm})$.

13 Conclusions

In this paper, we have demonstrated that Quantum Chromodynamics and its phenomena can be derived as emergent macroscopic properties of a 7+1-dimensional Tetron model. By analyzing the topological and geometric properties of the discrete tetrahedral vacuum, we have bridged the gap between the discrete lattice dynamics of fundamental Dirac Tetrons and the continuous effective field theory of the strong interaction.

In this approach, the local $SU(3)_C$ color gauge symmetry is not postulated as an a priori physical input, but rather emerges naturally from the internal degrees of freedom of the composite quark states. Within the Tetron framework, quarks are formulated as excitations of $SU(2)_L \times SU(2)_R$ tetrahedrons, while gluons emerge from a topological transport as the localized phase transitions of excitations hopping between adjacent tetrahedral cells. In the continuum limit ($L_P \rightarrow 0$), the discrete phase factors accumulate during this transport as the standard gauge connections A_μ^a of a $SU(3)$ gauge theory.

The model provides a unique, UV-complete physical mechanism for both low-energy confinement and high-energy asymptotic freedom. We have shown that the Heisenberg coupling $J_{inter} \leq 1 \text{ GeV}$, which governs the scalar ‘hopping’ energy between next-neighbor cells, is physically identified with the strong interaction scale Λ_{QCD} and with the string tension of the color-electric flux tubes which have been explicitly constructed within the model.

While the standard continuous QCD Lagrangian is successfully recovered in the continuum limit due to the universality of the dynamically generated $SU(3)_C$ gauge symmetry, the Tetron model departs from the standard Mandelstam picture in its

microscopic IR mechanism. Rather than postulating a phenomenological condensate of fundamental magnetic monopoles, the dual Meissner effect in the Tetron framework makes use of the already existing isomagnetic Q_L - Q_R pairing. The further analysis has shown that it is a direct consequence of the geometric frustration and chiral boundary conditions of the all-out isospin tetrahedral structure.

So the confining medium here is the rigid, frustrated geometry of the isospin lattice itself which is formed by isomagnetic condensates. This vacuum strictly expels color-electric fields, forcing the emergent gauge flux into discrete, one-dimensional channels. In the macroscopic limit, these channels smooth out into isotropic flux tubes, yielding the characteristic linear potential $V(r) \propto r$ and a rigorous proof of color confinement.

In summary, QCD is the effective, low-energy macroscopic limit of topological Tetron dynamics. The strong force, color charge, and confinement are no longer viewed as arbitrary, postulated properties of elementary particles. Instead, they are revealed to be the inevitable consequences of a tetrahedral vacuum structure in $SO(4)$ isospin space. This framework opens up new pathways for calculating non-perturbative QCD observables directly from the discrete topological invariants of the tetrahedral lattice.

Appendix A: Analyzing CP-Invariance of Tetrahedral Unit Cells in 3+1, 6+1 and 7+1 Dimensions

As discussed in Section 5 the ‘mild’ CP violation of the Standard Model can be traced back to the form of the complex wave functions of the tetron *excitations*. Therefore, to ensure that the tetron model reflects the Standard Model symmetry structure as closely as possible, it is advisable to demand that the *vacuum* symmetry of a unit cell is CP invariant. As shown below this condition is fulfilled for the out-in state of the $SO(6, 1)$ variant of the tetron model, while it cannot be fulfilled for the $SO(7, 1)$ model. Therefore, in the $SO(7, 1)$ case one has to resort to a CP violating vacuum, the consequences of which will be discussed at the end of this appendix.

To understand how a CP-invariant vacuum cell can emerge, we must go from the

kinetic origins (1) of the tetrons to their crystallization into tetrahedral structures.

In the first part (A1) of this Appendix, we strip away all extra dimensions and internal isospin spaces and consider a unit cell formed by 2 tetrahedrons of 4 electron and 4 positron spin vectors in ordinary 3+1 dimensions, in order to show that the in-out configuration is CP invariant. Afterwards, in (A2) we consider 2 tetrahedrons of tetron and antitetron isospin vectors in 6+1 dimensions and prove CP invariance of the isospin in-out system in an analogous fashion. Finally, in (A3) and (A4) we treat the more complicated situation in 7+1D with the 2 tetrahedrons living in different isospin spaces R_L^3 and R_R^3 .

(A1) 3+1D

Working with Weyl spinors of the Minkowski base space as in (1), is not a good idea when one wants to consider a static geometric shape, such as a localized tetrahedron, because Weyl spinors lack a rest frame. In that case a transition from the chiral Weyl basis to the Dirac basis is required. In the rest frame, spatial momentum vanishes ($\vec{p} = 0$), and the Lorentz boosts freeze out, so that the Lorentz group collapses into the purely spatial rotation group $SU(2)_{spin}$. The four-component Dirac spinor Ψ splits neatly into two static, two-component Pauli spinors $\Psi_{rest} = (\phi, \chi)$.

Here, ϕ describes the localized particle (the electron, e^-) and χ describes the localized antiparticle (the positron, e^+). In this rest frame, both particles and antiparticles transform under the exact same spatial rotation group $SU(2)_{spin}$, allowing us to anchor them to physical coordinates.

With a defined rest frame, we can now populate a single unit cell of the vacuum lattice. We construct this cell using four electrons and four positrons. We arrange the four e^- into a geometric all-out tetrahedron T_{e^-} , and the four e^+ into an all-in tetrahedron T_{e^+} , sharing the same center of mass and the same tetrahedral easy axis.

To test the symmetries of this cell, we must strictly define how 3D Parity (P_{3D}) and Charge Conjugation (C) act on these static Pauli spinors.

–3D Parity (P_{3D}): Parity inverts the physical coordinates ($\vec{r} \rightarrow -\vec{r}$). However, spin is an axial vector (generated by angular momentum, $\vec{r} \times \vec{p}$); because both $\vec{r}$ and $\vec{p}$ flip signs under parity, the spin vector itself remains invariant ($\vec{S} \rightarrow \vec{S}$). Consequently,

if a fermion is originally ‘All-Out’ ($\vec{S}$ parallel to $\vec{r}$), the inversion of $\vec{r}$ means the spin now points opposite to the new position vector. Therefore, P_{3D} geometrically inverts the tetrahedra: All-Out becomes All-In, and vice versa.

–Charge Conjugation (C): This operator transforms an electron into a positron. Mathematically, it requires taking the complex conjugate of the spinor, but to preserve the rotational isotropy of the 3D space, it must be multiplied by the metric of $SU(2)$, which is $i\sigma_2$. The transformation $\psi \rightarrow i\sigma_2\psi^*$ ensures that the physical spin vector $\vec{S}$ does not flip its direction under charge conjugation. Therefore, C swaps the identity of the particles ($e^- \leftrightarrow e^+$) but strictly preserves their geometric orientation (Out remains Out, In remains In).

Let us assume the vacuum cell is formed by an anti-aligned pair: an All-Out electron tetrahedron and an All-In positron tetrahedron. We denote this state as $T_{e^-}(\text{Out}) \otimes T_{e^+}(\text{In})$. To analyze its behavior under CP we first apply C and get an intermediate state $T_{e^+}(\text{Out}) \otimes T_{e^-}(\text{In})$. Then applying P_{3D} we finally obtain $T_{e^+}(\text{In}) \otimes T_{e^-}(\text{Out})$. Since the order of the constituent tetrahedra in the tensor product of the cell is commutative, the final state $T_{e^-}(\text{Out}) \otimes T_{e^+}(\text{In})$ is perfectly identical to the starting state.

A static 3 + 1D vacuum cell constructed from Dirac fermions is thus strictly CP -invariant if and only if the particle and antiparticle tetrahedra possess opposite topological chiralities (In-Out). An Out-Out or In-In configuration would maximally violate CP symmetry under the same operational sequence.

It may further be noted that the $T_{e^+}(\text{Out}) \otimes T_{e^-}(\text{Out})$ and $T_{e^+}(\text{In}) \otimes T_{e^-}(\text{In})$ configurations violate CP.

(A2) 6+1D

Building upon the geometric logic of the physical rest frame, we can now apply the exact same mechanism to the internal symmetries of the $SO(6,1)$ model. In this framework, the fundamental building block is the 8-dimensional complex Dirac representation of $SO(6,1)$. When the original 6+1-dimensional space decomposes into a 3 + 1D physical spacetime and a 3D internal isospin space ($SO(3)_{iso}$), the relativistic branching of the fundamental spinor is given by

$$\mathbf{8}_{\mathbb{C}} \longrightarrow ((2,1) \oplus (1,2), 2_{iso}) \quad (29)$$

Here, the Lorentz group components (the Weyl spinors) are coupled to an isospin doublet. However, just as in the $3+1D$ case, Weyl spinors cannot form a static lattice. To construct a stationary unit cell, we must transition into the non-relativistic rest frame. In the Dirac basis, the Lorentz boosts freeze out, and the $\mathbf{8}_{\mathbb{C}}$ -representation neatly decomposes into a particle (Tetron) and an antiparticle (Antitetron) sector, both transforming under the spatial spin and internal isospin $SU(2)$ groups:

$$\mathbf{8}_{\mathbb{C}} \longrightarrow (2_{spin}, 2_{iso})_{tetron} \oplus (2_{spin}, 2_{iso})_{antitetron} \quad (30)$$

In this rest frame, the unit cell occupies a single, dimensionless node in the physical $3D$ space. Because the cell has no spatial extension in the physical universe, the physical spatial parity P_{3D}^{spin} (or P_{3D}^{phys}) plays absolutely no role in its internal structure.

Instead, the geometric volume of the cell unfolds entirely within the $3D$ internal isospin space. The unit cell is constructed from two interpenetrating tetrahedra: one formed by 4 Tetrons and one by 4 Antitetrons. Analogous to the electron-positron tetrahedra discussed in (A1), these isospin tetrahedra can be classified by the alignment of their isospin vectors in the isospin space: All-Out or All-In. This means, within the $6+1D$ model we have the comfortable situation of an analogy with the electron-positron configuration discussed in A1, and the same conclusion can be drawn, namely that a Tetron tetrahedron with an All-Out geometry and an Antitetron tetrahedron with an All-In geometry together give a CP-invariant unit cell.

(A3) $7+1D$

In the $SO(7, 1)$ variant of the teron model there is a $4D$ isospin space with symmetry $SO(4)_{iso} \cong SU(2)_L \times SU(2)_R$, meaning it geometrically splits into two disjoint $3D$ representation spaces: the left-handed $\mathbb{R}_L^3$ and the right-handed $\mathbb{R}_R^3$.

When forming a static vacuum cell in the physical rest frame, we must again rely on Dirac spinors that couple left- and right-chiral Lorentz representations. The structurally balanced, CP-invariant unit cell in this model turns out to consist of two tetrahedra occupying both internal spaces: one All-Out tetrahedron in the left isospin space, and one All-In tetrahedron in the right isospin space. To rigorously demonstrate the CP invariance of the vacuum state and its physical necessity

for a stationary lattice, we must dissect the transformation properties of the composite state $[(1, 2) \oplus (2, 1), L, out] \otimes [(1, 2) \oplus (2, 1), R, in]$ within the $SO(7, 1)$ framework. This state couples two distinct isospin tetrahedra, each carrying a full symmetric sum of left- and right-chiral Lorentz representations.

In order to apply the complete CP transformation, we have to consider how each operator acts on the three properties of a given state:

- The parity operator P_{3D}^{phys} in physical space mirrors the physical 3D space ($\vec{x} \rightarrow -\vec{x}$). In spinor kinematics, this spatial inversion flips spatial chirality, mapping $(1, 2) \leftrightarrow (2, 1)$. Because it only acts on the physical spacetime, the internal 4D isospin space (L/R) and the geometric orientation of the tetrahedra (in/out) remain completely unaffected.

- The charge conjugation operator C transitions particles into antiparticles by applying complex conjugation. In quantum field theory, complex conjugating a spinor mathematically forces the Lorentz chirality to flip again: $(1, 2) \leftrightarrow (2, 1)$. Crucially, C does not alter the isospin representation ($L \rightarrow L, R \rightarrow R$) because the fundamental $SU(2)$ representations are pseudo-real ($2^* \cong 2$). Furthermore, the intrinsic $i\sigma_y$ metric embedded in the definition of C prevents the pseudovector spins from flipping, preserving the exact geometry of the tetrahedra ($in \rightarrow in, out \rightarrow out$).

- the internal parity operator P_{4D}^{iso} inverts the 4D isospin space. Algebraically, this spatial inversion swaps the two disjoint representation spaces as $L \leftrightarrow R$. Geometrically, mirroring the coordinate system of the 3D internal tetrahedra within isospin space turns them inside out as $in \leftrightarrow out$, while the physical Lorentz representation remains entirely untouched.

Now, we are in the position to consider the full $CP \equiv C \cdot P_{4D}^{iso} \cdot P_{3D}^{phys}$ transformation. To simplify expressions let us denote the $SO(3, 1)$ Lorentz representation $(1, 2) \oplus (2, 1)$ as D . Our initial vacuum state is therefore written as $[D, L, out] \otimes [D, R, in]$.

- The operator P_{3D}^{phys} flips the Lorentz components within D . However, since D is the sum of both chiralities, flipping them does not modify D . Result: The state remains $[D, L, out] \otimes [D, R, in]$.

- Charge conjugation flips the Lorentz components within D , which once more maps $D \rightarrow D$. The isospin and geometry are preserved. As a result the state remains $[D, L, out] \otimes [D, R, in]$.

- The operator P_{4D}^{iso} leaves D untouched but transforms internal space. It swaps

$L \leftrightarrow R$ and $in \leftrightarrow out$ for both parts of the tensor product simultaneously. The first part $[D, L, out]$ transforms into $[D, R, in]$, the second $[D, R, in]$ into $[D, L, out]$.

In conclusion the final transformed state is $[D, R, in] \otimes [D, L, out]$. Because the order of the distinct tetrahedra within the vacuum cell tensor product is symmetric and interchangeable, this is structurally identical to the initial state. The vacuum state $\{L_{out}, R_{in}\}$ is flawlessly CP invariant. This means, similar to the (A1) and (A2) cases, the $SO(7, 1)$ model allows for a perfectly $CP \equiv C \cdot P_{4D}^{iso} \cdot P_{3D}^{phys}$ invariant ground state.

(A4) The case of $\{L_{out}, L_{in}\}$ in 7+1D

Unfortunately the unit cell presented in (A3) does not meet the phenomenological needs of the weak interactions. The weak V-A structure means that it couples both to left-handed particles and to right-handed antiparticles. If one considers the $\{L_{out}, R_{in}\}$ vacuum and the particles live in L_{out} and antiparticles in R_{in} , particles and antiparticles would feel different geometries. More precisely, antiparticles would not interact with the same chiral gauge interaction as particles, as described by the SM.

Therefore, as an alternative to (A3), one may consider the vacuum configuration $\{L_{out}, L_{in}\} \equiv [D, L, out] \otimes [D, L, in]$, in which both isospin tetrahedra live within the left-handed sector $\mathbb{R}_L^3$. This state explicitly embeds the observed left-handed chirality of the Standard Model (SM), as it dynamically populates the $SU(2)_L$ isospin space while leaving the $SU(2)_R$ sector completely uncondensed.

As seen from the discussion in (A3) the CP -transformed configuration of $\{L_{out}, L_{in}\}$ is $\{R_{out}, R_{in}\}$ and thus fully lives in the R -sector. This means, the $\{L_{out}, L_{in}\}$ ground state violates CP at the vacuum level. Formally, this is an explicit breaking but note that the formation of this state – including the CP violation – has occurred as a consequence of a SSB, namely in the condensation of the isospin vectors to form tetrahedrons in the early universe.

Note further that such a ground state also has important consequences for cosmology because the CP violation fulfills one of Sakharov's conditions and thus offers a mechanism for baryogenesis directly at the Fermi scale. Furthermore, it circumvents the need for fine-tuned high-scale scalar potentials by grounding cosmic asymmetry

in the topology of the lattice condensation. On the other hand, a potential drawback arises from the fact that low-energy SM gauge interactions preserve CP symmetry, restricting CP violation to complex phases within the CKM matrix of excitations. Reconciling the $\{L_{\text{out}}, L_{\text{in}}\}$ state with observation thus requires a protective mechanism to shield pure gauge dynamics from the background CP violation.

In summary, we know that the weak interactions couple to left-handed particles and to right-handed antiparticles. Within the $SO(7, 1)$ model this is only possible with the choice $\{L_{\text{out}}, L_{\text{in}}\}$. On the other hand, in $SO(6, 1)$ it is possible with the choice (A2), i.e. choosing the tetron tetrahedron and antitetron tetrahedron anti-aligned (because a right-handed particle in an inverted lattice behaves identically to a left-handed particle in the original lattice).

We are thus in the situation that the favorable unit cell (from the viewpoint of the V-A weak interaction) in the $SO(7, 1)$ model is CP violating, while in the $SO(6, 1)$ model the favorable unit cell is CP conserving.

Appendix B: Comparing the Nature of Gluons and Electroweak Gauge Bosons within the Tetron Model

For any impatient reader, I start this appendix with a concise summary of its content. In short, both gluons and electroweak bosons are 2-cell excitations, i.e they operate on the interface between 2 cells A and B, and they arise from interactions between a tetron in one cell and an antitetron in the other. While gluons refer to deformations of the corners I, II, III, IV of the tetrahedron and the associated V_4 symmetry functions (13), the W bosons are in-phase vibrations defined with respect to the full original continuous $SO(4)$ internal symmetry group.

As described more fully in the following subsections (B1), (B2) and (B3), W boson, Higgs particle and photon may be interpreted as 2-cell vibrations of the following kind:

- changes of direction (vibrations by angle): these cost transverse energy and lead to the massive $W^\pm$.
- changes of length (vibrations of the thickness of the inter-cell bond): these cost

longitudinal energy and lead to the massive Z and the Higgs.

–vibrations of phases (without changing the isospin vectors): these cost no energy and correspond to the massless photon.

The question of mixing of photon and Z and perhaps with the ‘X-photon’ $U(1)$ field of an underlying 7+1D QED will be discussed at the end of the Appendix. The idea is that the exchange integrals, which determine the DMI and Heisenberg couplings of the model according to (28), originally are induced by a 7+1D electrodynamics through its r^{-5} Coulomb potential. This line of reasoning will lead us directly to the famous formula[21]

$$Q = T_3^L + T_3^R + \frac{B - L}{2} \quad (31)$$

where the $B - L$ photon is nothing else than the $U(1)$ field X of the fundamental 7+1D QED.

(B1) The Higgs Doublet

The Higgs doublet emerges as a collective excitation of the isospin vectors. Unlike the Standard Model, where the Higgs is an elementary field, here it represents the radial and angular fluctuations of the tetrahedrally ordered isospin cells.

To understand the Higgs, we must consider the R^4 representation space of $SO(4)$ instead of the $R_L^3 \times R_R^3$ representation space of $SU(2)_L \times SU(2)_R$.

In the Standard Model, the Higgs field H is a complex $SU(2)_L$ doublet containing exactly four real scalar fields $(\phi_1, \phi_2, \phi_3, \phi_0)$. It is of the form

$$H(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_2(x) + i\phi_1(x) \\ \phi_0(x) - i\phi_3(x) \end{pmatrix} \quad (32)$$

The Standard Model Higgs potential

$$V(H) = \mu^2(H^\dagger H) + \lambda(H^\dagger H)^2 \quad (33)$$

depends exclusively on the squared norm

$$H^\dagger H = \frac{1}{2}(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_0^2) \quad (34)$$

which is mathematically identical to the squared length of a vector in R^4 . Because the potential only depends on this ‘radial distance’, it is completely invariant under any 4D rotation. The Higgs potential thus inherently possesses an exact, global

$SO(4)$ symmetry which is called ‘custodial’. During the electroweak SSB it gets broken to a global vectorial $SO(3)_V = SU(2)_V/Z_2$ which is called the proper custodial symmetry group.

The Higgs particle (associated with the radial component of the doublet) acts as the volume or amplitude resonance of the algebraic boundary (=‘cell wall’). It corresponds to a ‘breathing’ mode where the effective isospin vectors of the two coupled cells pulsate, modulating the macroscopic field length. Since this purely radial fluctuation does not select any preferred rotational direction within the internal $SO(4)$ representation space and preserves the rigid tetrahedral geometry, it represents the simplest and most universal coupling.

This can be made explicit by the well known polar representation of the doublet

$$H(x) = \exp\left(i\frac{\vec{\tau} \cdot \vec{\phi}(x)}{v}\right) \begin{pmatrix} 0 \\ \frac{v+h(x)}{\sqrt{2}} \end{pmatrix} \approx \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{i}{2}(\phi_1 - i\phi_2) \\ v + h - \frac{i}{2}\phi_3 \end{pmatrix} \quad (35)$$

where $h(x)$ explicitly appears as a radial contribution, but also in leading order of the field variables as a disturbance of ϕ_0 .

As a vibration of the $SO(4)$ symmetric length (34), the Higgs oscillation is absolutely consistent with this symmetry. It is ‘massive’ because the system has an energetically preferred length v , the vacuum expectation value of H . Physically, v can be traced back to the length of the isospins in their ground state $|\langle\vec{Q}_L\rangle| = |\langle\vec{Q}_R\rangle|$, and to the strength of the order inducing exchange coupling D_{LL} , so that it fulfills

$$v = D_{LL}|\langle\vec{Q}_L\rangle| \quad (36)$$

It may be noted that the other 3 excitations, i.e. the Goldstone modes ϕ_1, ϕ_2, ϕ_3 (which effectively disappear in the unitary gauge and also in the SM particle spectrum) can be interpreted as isospin excitations, too, and that this interpretation will find its way into the geometric picture of the W bosons below.

$(\phi_1, \phi_2, \phi_3, \phi_0)$ can be seen as the 4 coordinates spanning the $SO(4)$ representation space R^4 . In the ground state they look like static coordinates. However, in quantum field theory, they are fields, i.e. functions of space and time, each coordinate corresponding to a specific combination of isospin displacements at the vertices of the tetrahedron. If these displacements oscillate in time, they describe excitations

and in fact quasi particle fields in the physical base space (the monolayer), and since the isospin system as a whole is elastic with stiffness D_{LL} , these local oscillations propagate as a wave. In quantum mechanics, every such wave is a particle.

So the ground state is given by

$$\langle(\phi_1, \phi_2, \phi_3, \phi_0)\rangle = (0, 0, 0, v) \quad (37)$$

but if one, for example, changes the value of the modulus $\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_0^2$ of the vector, one creates a Higgs particle. On the other hand, if one displaces the field in the direction of $\phi_{1,2,3}$, i.e twists Q_L against Q_R , one creates angular fluctuations corresponding to a triplet particle.

Actually, one can be much more specific about the topological nature of these twists. Namely, after the formation of the 2 rigid $\vec{Q}_L$ and $\vec{Q}_R$ tetrahedrons T_L and T_R , the $SO(4) = SO(3)_L \times SO(3)_R$ transformations emerge from the independent rotatabilities T_L and T_R . Under this interpretation one can unravel the two $SO(3)$ subgroups in the following way:

- First, the ‘axial’ $SO(3)_A$ where the $\vec{Q}_{Li}$, $i = 1-4$, of the Q_L tetrahedron are rotated opposite to the $\vec{Q}_{Ri}$. This is the symmetry broken by the electroweak interaction.
- And secondly, the diagonal (the proper ‘custodial’) $SO(3)_V$ of joint identical rotations, i.e. where $\vec{Q}_{Li}$ and $\vec{Q}_{Ri}$ are rotated together by the same angle.

The former corresponds to the symmetry under rotations in the 10, 20, and 30 planes. This is broken when the 2 tetrahedrons lock in (both on the intra- and inter-cell level) and leads to the three would-be Goldstones ϕ_1 , ϕ_2 and ϕ_3 in the Higgs doublet. The latter corresponds to rotations in the 12, 13, and 23 planes and is a true symmetry of each unit cell under rotations of isospin space and even remains a symmetry when one considers the entirety of all unit cells after the SSB, i.e. after the all-out configurations of all unit cells have aligned.

The dominance of the quadratic DMI has an important consequence for the ϕ fields. It means the potential does not have a linear term but a quadratic and a quartic one - exactly what is needed for the SM Higgs potential: a potential that punishes any deviation from the all-out ground state regardless which of the 3 triplet directions $\phi_{1,2,3}$ the excitation takes.

(B2) The Massive Gauge Bosons

In this case we have to go over from the 4D representation space R^4 of $SO(4)$ to the 6D representation space $R_L^3 \times R_R^3$ of $SO(3)_L \times SO(3)_R$ ⁶.

Since the amorphous monolayer extends throughout the physical 3D base space, while each point-like cell contains an internal 3D+3D isospin structure ($R_L^3 \times R_R^3$), any coupling between cell A and cell B occurs simultaneously across all corresponding vertices. In order for an excitation (a boson) to exist stably and propagate, it must be an eigenmode of this discrete interface (the ‘cell wall’). The exchange between two neighboring cells is not a simple scalar contact, but a strict coherence condition across the internal tetrahedral boundaries, and it acts as a geometric filter: only those collective oscillation patterns that maintain constructive phase coherence across all four coupled vertices can survive and emerge as macroscopic fields.

This is related to the universality of the weak interactions. The restriction to precisely Higgs, $W^\pm$, W_z and B field arises from the discrete topology of the 4-vertex interface and its corresponding $SO(4)$ symmetry within the internal isospin space.

⁶It may be remembered that the 6D space is the space of rotation axes of the 4D space. In 4D space ($\phi_1, \phi_2, \phi_3, \phi_0$) there are 6 independent planes 01, 02, 03, 12, 13, 23, which can be rotated. If one selects one of these planes, a second, completely independent plane remains that is orthogonal to the first. If one rotates in both planes at the same speed, it is called an isoclinic rotation. Then, the 6 possible planes of rotation decompose into two independent classes of 3 dimensions each: (i) left-isoclinic rotations, rotating mathematically positive in both planes and forming the space R_L^3 , (ii) right-isoclinic rotations, rotating positively in one plane and negatively in the other and forming the space R_R^3 . Actually, this geometric discussion may be translated into group theory by recognizing that the Lie groups $SO(4)$ and $SO(3) \times SO(3)$ are locally isomorphic but globally distinct, meaning they share the same Lie algebra but have different overarching structures. Specifically, $SO(4)$ is doubly covered by $\text{Spin}(4) \cong SU(2) \times SU(2)$, which is why their fundamental representations act on different vector spaces. The fundamental representation of $SO(4)$ is defined as the standard, irreducible action of 4×4 rotation matrices on $\mathbb{R}^4$. In contrast, the standard representation of $SO(3) \times SO(3)$ acts reducibly on $\mathbb{R}^3 \oplus \mathbb{R}^3 = \mathbb{R}^6$ by rotating the first and last three coordinates independently. To bridge this dimensional gap, the 4-dimensional representation can be constructed as the complex tensor product of two 2-dimensional $SU(2)$ representations ($\mathbb{C}^2 \otimes \mathbb{C}^2$). In particle physics, the Higgs field lives in this geometric $\mathbb{R}^4$ representation space, which exhibits a global $SO(4)$ symmetry. However, physicists usually write the Higgs field as a complex doublet ($\mathbb{C}^2$) with two components containing four real degrees of freedom. This doublet terminology specifically highlights how the field transforms under the $SU(2)_L$ gauge factor of the Standard Model. Therefore, calling the Higgs (i) a doublet or (ii) a $\mathbb{R}^4$ vector describes the same mathematical object from two different group-theoretic perspectives.

Specifically, the three weak vector bosons $\vec{W}$ (similar to the components $\phi_{1,2,3}$, but this time with spin-1) represent the shear or rotational resonances of the interface. In the internal $SO(4)$ space, these correspond to the generators of $SU(2)$ rotations. The inter-cell DMI couplings dictate that the internal orientation of cell A must be strongly correlated with cell B. The W fields thus represent the three fundamental ways to tilt or rotate the isospin vector during the transition across the discrete boundary.

A priori, in $SO(4) = (SU(2)_L \times SU(2)_R)/Z_2$ there are 6 generators, 3 for Left and 3 for Right. In the Standard Model, however, we only see the 3 weak bosons ($W^\pm, Z$) and the photon. Where are the others? In the tetron model, this contradiction is resolved by the chiral symmetry breaking performed by the all-out (=left-handed) configuration as opposed to all-in (=right-handed). While all-out couples constructively to the left-handed fermion excitations, in the $SU(2)_R$ sector there is destructive interference with the all-out state, and the corresponding bosons W_R of the right sector are forbidden. Because they would have to oscillate ‘against the grain’ of the vacuum handedness, they are absent from the observable energy spectrum.

The situation becomes even clearer if one chooses the vacuum introduced in Section (A4). In that case the two tetrahedrons $T_{L,in}$ and $T_{L,out}$ inside a unit cell both exist in the R_L^3 part, thus breaking spontaneously the left-right symmetry of the dynamics, and this implies that W_R bosons do not exist at all, whereas the W_L bosons coherently fit into the structure of the cell wall and thus can propagate inside the monolayer.

The W_L bosons can be viewed as topological transition states at the inter-cell boundary, mediating the isospin flux between two fermionic excitations. Consider, for example, the weak interaction process $\bar{\nu}_{eL} + e_L \rightarrow W^-$. In the tetron model, these individual cell excitations (representing the fermions) merge at the discrete cell interface into a collective two-cell excitation. Following the derivation in [4], electron, muon and τ actually are optical mode $\chi_{\text{opt}} \propto \delta(\vec{Q}_L + \vec{Q}_R)$ within one cell A, whereas the neutrinos act as acoustic modes $\chi_{\text{ac}} \propto \delta(\vec{Q}_L - \vec{Q}_R)$ in another cell B.

At the interface between adjacent cells A and B, the wave functions of these modes overlap and fuse. Because the W boson carries weak isospin, the coupling across the

boundary must precisely capture the isospin difference of the participating fermions. Formally, rather than a continuous spatial integral over an extra-dimensional hypersurface, this transition is described topologically as a discrete overlap sum over the four coupled tetron vertices ($i = I, II, III, IV$)

$$\Psi_W^\mu(A, B) \propto \sum_{i=1}^4 \left[\bar{\chi}_{ac,A}^{(i)} \cdot \mathbb{T}^\mu \cdot \chi_{opt,B}^{(i)} \right] \quad (38)$$

where $\mathbb{T}^\mu$ is the $SU(2)_L$ isospin generator that dictates the internal rotation of the rigid all-out configuration from cell A to cell B.

This formulation provides a natural microscopic foundation for weak universality: the exact same W boson field emerges universally for all generations (e, μ, τ and the quarks) because the interface operator $\mathbb{T}^\mu$ depends exclusively on the invariant geometry of the tetrahedron and the fundamental inter-cell exchange couplings (such as D_{LL} and J_{LL}). Consequently, any fermion pair exhibiting the appropriate chiral isospin symmetry will excite the exact same fundamental mode of isospin flux when transitioning across the algebraic cell boundary.

In summary, a W boson is not a local (i.e., single-cell) excitation, but rather the collective process of isospin transfer from tetrahedron T_{LA} to tetrahedron T_{LB} . If cell A carries a specific rotational excitation of its all-out configuration, this rotational phase is seamlessly mediated to cell B via the inter-cell DMI couplings, since their internal isospin vectors are perfectly aligned in the vacuum ground state.

Any more complex excitation (e.g., a mode with higher isospin or a more complicated deformation) would interfere destructively at the 4-point interface (=wall region), as the individual coupled vertices would no longer oscillate synchronously.

In principle, the composite nature of the tetrahedral unit cell would imply the theoretical presence of higher-order internal excitations, e.g. five quadrupole (isospin-2) degrees of freedom in addition to the three isospin-1 W-bosons. However, the manifestation of any mode as observable particles in the continuum limit $L_P \rightarrow 0$ is strictly governed by the topological coherence conditions at the discrete interfaces between adjacent cells. This means, for any lattice excitation to propagate as a macroscopic field – such as the W bosons – it must maintain strict phase coherence across the 4D cell walls.

W-bosons are rigid collective rotations of the isospin vectors where the entire cell (all four tetrons) is translated or rotated as a coherent unit and all corners oscillate in phase. As such they satisfy the boundary coherence condition, allowing the uniform transmission of the wave front across the lattice. In contrast, more complex internal deformations inherently disrupt this spatial continuity. When a quadrupole mode attempts to propagate, the complex relative phase shifts of the internal vertices can no longer match synchronously across the multi-point boundary of adjacent cells. Over the macroscopic integration of these interfaces, the mismatched phases interfere destructively, completely canceling the amplitude of the transition. As a result, the cell boundaries act as a stringent topological filter: they allow only collective isospin-0 and isospin-1 modes to propagate while entirely suppressing the macroscopic continuum propagation of higher-isospin fields.

Although these quadrupole modes cannot travel as continuous waves, they could in principle exist as localized, highly massive resonances. However, this is prevented by a collective stabilization mechanism among the cells. In the densely packed amorphous 3D monolayer (with extremely small average lattice spacing L_P !) any single cell has a vast number N of stabilizing, topologically entangled neighboring cells. To induce a local geometrical deformation, one cannot merely break a single local bond; rather, one must work against the collective topological resistance of the entire surrounding network.

Consequently, the energy required to excite these internal tensor modes does not scale with the DMI attraction $D_{LL,inter} \approx E_{Fermi}$, but rather with the collective rigidity of the isospin lattice, parameterized approximately by $N \times E_{Fermi}$. Because N is immensely large, this collective locking mechanism shifts the threshold for internal cell deformations, and thus the scale for new physics, to a large intermediate value between the electroweak scale E_F and the Planck scale E_P . This effect provides a robust physical justification for the purity of the Standard Model at current collider energies. The massive energy gap explains why the vacuum remains geometrically rigid up to the multi-TeV regime, perfectly hiding any composite quadrupole or higher-tensor resonances from current experimental reach

(B3) The U(1) Fields

The emergent massless photon represents the true 'zero-mode' of the internal $SO(4)$

rotational space. Within this structure, the electromagnetic generator is a very specific linear combination of $SU(2)_L$ and $SU(2)_R$: it corresponds exactly to a shared rotational axis (a shared internal dimension) between the R_L^3 and R_R^3 representation spaces. This specific synchronous rotation leaves the relative anti-aligned geometry of the isospin vectors completely undisturbed, merely shifting their global rotational phase.

While the Higgs is an oscillation of the thickness or the coupling strength of the inter-cell wall and alters the scalar value of the bond between cell A and B, the photon is not blind to directions in isospin space because it modulates the phase (but not the intensity) of the isospin transmission. The photon is thus a non-bending, purely kinematic oscillation. In other words, the Higgs is an amplitude wave of the 2-cell interface; the photon is a resistance-free phase wave.

This means that, unlike the W bosons, which attempt to twist the left-handed tetrahedral vertices, it does not correspond to a direction in isospin space. While the W bosons actively bend the tetrahedral axes and thereby become heavy, the photon oscillates only by phase, i.e. without a direction in isospin space. It is a simultaneous, symmetric oscillation that leaves the isospin vectors of neighboring cells phase-synchronized, without prying them out of their all-out rest position. Because it does not bend the geometry protected by the DMI binding energy $D_{LL,inter} \approx E_F$, it encounters no restoring force. Lacking any topological resistance from the DMI couplings, the photon remains strictly massless.

In order to explain the mixings of the photon, we consider (31). This relation shows that the photon is composed of 3 contributions: the SM W_L^3 field, an unphysical W_R^3 and a $B - L$ contribution – the latter referring to the X-photon of the fundamental 7+1D QED.

If, in a Gedanken experiment, we ignore the $B - L$ contribution and use the formula $Q = T_3^R + T_3^L$, the transformation induced by a 3D photon refers to a shared rotational axis between the $\mathbb{R}_L^3$ and $\mathbb{R}_R^3$ representation spaces. In the extra 4 dimensions, a left-isoclinic rotation (T_3^L) and a right-isoclinic rotation (T_3^R) twist two orthogonal planes simultaneously. (For more details on the geometry see the discussion in Footnote 6.) When perfectly synchronized in the linear combination $T_3^R + T_3^L$, their actions exactly cancel each other out in the (ϕ_0, ϕ_3) plane (which leaves the

vacuum expectation value $(0, 0, 0, v)$ untouched) while constructively doubling their rotation within the orthogonal (ϕ_1, ϕ_2) plane. Thus, geometrically, the photon acts as a simple 2D rotation that perfectly bypasses the condensed vacuum axis.

Now, if we include the $B-L$ contribution, the electromagnetic $U(1)_Q$ symmetry must be understood through a cascade of gauge field mixings originating in a fundamental 7+1D quantum electrodynamics. At the microscopic tetron level, the fundamental $U(1)_X$ symmetry is represented mathematically as a pure phase rotation of the individual tetron operators, $f \rightarrow e^{i\alpha} f$. When this symmetry is locally gauged, it yields a massless gauge boson – the X field.

In the monolayer world, there are no tetron- and X -fields, only quarks and leptons appear as collective wave excitations composed of these tetrans. To understand how the fundamental tetron phase translates to our macroscopic observations, we must look at the effective $SU(4)$ symmetry which arises from Klein’s four-group V_4 acting on the corners of a tetrahedron, and actually include the concept of color in the tetron model. As explained in the main text, a lepton emerges as the totally symmetric singlet mode of V_4 , where all four tetrans oscillate entirely in phase $(++++)$. Conversely, the three quark colors correspond to the three orthogonal, mixed-sign modes of the V_4 group $(++--)$, $(+-+-)$, and $(+--+)$, where two nodes always oscillate out of phase with the other two.

When the phase α of the X -field penetrates the monolayer, it does not act on the vertices independently; rather, it acts as a wave that must couple to these specific V_4 eigenstates. Because the 4-tetron cell is a rigidly bound system, any local gauge transformation propagating through it must preserve the overall structural ‘volume’ of the cell. Mathematically, this dictates that the corresponding $SU(4)$ generator must remain strictly traceless.

This geometric boundary condition forces the fundamental phase α to be distributed unequally among the modes to maintain a net-zero sum across the cell’s excitation basis. Since there are three balanced quark modes and only one totally symmetric lepton mode, the traceless constraint demands that $3 \times c_{quark} + 1 \times c_{lepton} = 0$. Consequently, if the gauge interaction assigns a phase of -1 to the lepton mode $(++++)$, the rigid V_4 lattice geometry automatically forces the three quark modes to absorb a compensating fractional phase of $+1/3$. Thus, the fractional $B-L$ weighting

vector $\text{diag}(1/3, 1/3, 1/3, -1)$ (corresponding to the 15th generator of $SU(4)$) is not a native property of the fundamental 7+1D QED, but rather the inevitable kinematic response of the V_4 symmetry functions distributing a single phase across the monolayer.

Consequently, at the quark and lepton level, this fundamental tetron phase shift manifests mathematically exactly as the $B - L$ charge, making the X field the pure $U(1)_{B-L}$ gauge boson. This strict geometric requirement also perfectly explains the absence of the orthogonal $B + L$ force. A gauge field coupling to $B + L$ would require a generator with a non-zero trace, which is structurally forbidden by the V_4 symmetry of the tetron cell. (In passing, it also explains the absence of the ‘9th gluon’ $\sum_\alpha |q_\alpha\rangle\langle\bar{q}_\alpha|$, cf. (14).)

However, we do not observe this X field as an independent particle because the isospin lattice geometry forces it into an immediate mixing process. Before it starts to propagate, it fuses with the W_R^3 field (a field that has been discussed to be inherently ghostly in our framework). This initial linear combination generates two new fields: the SM B -field corresponding to the hypercharge

$$Y = T_3^R + \frac{B - L}{2} \quad (39)$$

and a Z' boson, the latter emerging as the orthogonal counterpart to the B -field. Geometrically, the Z' attempts to induce an elastic wave across the right-handed isospin lattice. But since, according to (A4), the vacuum has crystallized strictly into the left-handed $|T_{L,out}\rangle \otimes |T_{L,in}\rangle$ state, the right-handed sector is empty, making the Z' an unobservable phantom field at accessible energies (or in the case (A3) of the $|T_{L,out}\rangle \otimes |T_{R,out}\rangle$ vacuum making the Z' a state that violates the coherence conditions at the cell wall).

The surviving B -field then undergoes a second mixing with the W_L^3 field (the left-handed isospin rotation) to produce the massive Z boson and the massless $U(1)_Q$ 3D-photon, usually called A .

To better understand the interplay, we must recognize that when the macroscopic photon field imposes a local phase transformation by an angle α , two synchronized mechanisms are triggered simultaneously:

- (i) The photon accesses the fundamental 7+1D QED, applying a phase shift of

$\alpha \frac{B-L}{2}$ to each constituent tetron operator, effectively ticking the internal clock of the excitation.

(ii) In addition, it enforces a spatial torsion of $\alpha(T_3^L + T_3^R)$ and thus a simultaneous oscillation of the 2 tetrahedrons involved.

This synchronization is the microscopic essence of charge conservation, operating as a strict kinematic constraint. While the pure $|T_{L,out}\rangle \otimes |T_{L,in}\rangle$ vacuum remains completely invariant under Q (since $B-L=0$ and the spatial rotations cancel out), a charged particle like an electron sitting at the interface breaks this balance locally ($B-L=-1$). For the photon to propagate this excitation without tearing the strong D_{LL} vacuum bonds, the spatial rotation of the wave at the boundary must interlock flawlessly with both the absolute spatial rotation and the abstract tetron phase shift.

Ultimately, the $U(1)_Q$ symmetry in the tetron lattice is not a primary force, but the emergent result of a perfect kinematic coupling. The photon is the only wave where the ‘gear’ of the internal tetron phase and the ‘gears’ of the isospin lattice geometry mesh with absolute, slip-free precision. This allows the photon to push the dynamic excitation through the interface without applying any elastic stress to the surrounding vacuum. In contrast, the Z boson represents an asymmetric, abrasive rotation. When it attempts to twist the condensed tetrahedra out of alignment, the lattice resists via the elastic restoring forces of the D_{LL} bonds – a friction that manifests macroscopically as the Z boson’s heavy mass.

One may worry whether the fundamental X-photon could have undesirable phenomenological side effects. However, this is not the case. Imagine our universe as a discrete monolayer floating in an empty 7D space (the ‘bulk’). In this empty bulk, the X-photon (corresponding to the tetron phase) travels freely as a massless wave of ‘light’.

When X hits the monolayer, it cannot just pass through. In solid-state physics, when a photon enters a crystal, it instantly couples with the mechanical vibrations of the crystal (i.e. the phonons) to form a new, hybrid quasi-particle called a polariton. In my model, the hypercharge (i.e. the B -field) is exactly this polariton. The moment the 7+1D photon enters the monolayer, it is forced to lock gears with the geometric spatial torsion of the lattice (W_R^3). It ceases to be a free wave and

becomes a hybrid 3+1D field.

If particles could freely cross between the lattice and the bulk, we would see dramatic anomalies in particle physics and cosmology – like, for example, missing energy in high-energy particle collisions, where a X-particle could be radiated out of our monolayer into the bulk. However, there is an effect which prevents such things to happen at accessible energies: the geometry of the lattice naturally traps everything inside, requiring no extra mathematical tricks.

(B4) Back to the Gluons

If the vacuum is maximally chiral (purely $SU(2)_L$), how can the strong interaction preserve parity and treat left- and right-handed quarks on an entirely equal footing (i.e., remain vector-like)?

This apparent paradox is resolved once we clarify which distinct spaces gluons act upon compared to W bosons. The key lies in the separation between the rotation of the entire tetrahedron and the permutation of the internal vertices.

(i) The W_L boson is a fluctuation of the inter-cell bond D_{LL} . It describes the geometric twist of the entire tetrahedron as a rigid body within isospin space. Because the vacuum consists exclusively of T_L tetrahedra, this rigid rotation exists solely in the L -sector. Consequently, the W boson couples only to δQ_L and remains maximally chiral.

(ii) Gluons in QCD perform a completely different operation. Rather than rotating the entire tetrahedron in space, they permute its internal vertices. As defined in (13), the three quarks are mixed-symmetry superpositions of the four vertices. A gluon (as an $SU(3)_c$ generator) is mathematically simply a permutation or rotation matrix that exchanges, for example, vertex 1 with vertex 2.

A physical quark excitation over the vacuum is composed of both chiralities: $q \sim \delta(Q_L) + \delta(Q_R)$. When a gluon is emitted or absorbed, it gauges only the 3 color indices $\alpha = (+ + --)$, $(+ - +-)$ and $(+ - -+)$. The operation of swapping vertex 1 and vertex 2 is completely agnostic to whether this permutation occurs in the L -sector (acting on δQ_L) or in the R -sector (acting on δQ_R). The gluon operates strictly within the vertex space (resp the color indices), which is orthogonal to the charge conjugation space (L/R with $8_R^c = 8_L$).

Color symmetry as the symmetry under vertex permutation is an internal topological property of the tetrahedron. Whether a tetrahedron resides in the L -space or R -space, and whether it is out or in, it always possesses four vertices. Because quark excitations travel through the lattice as the combination $\delta Q_L + \delta Q_R$, and vertex permutations act identically on δQ_L and δQ_R , the strong interaction manifests macroscopically as a vector-like, parity-conserving gauge force. The spontaneous symmetry breaking during the cooling of the vacuum freezes the isospin network to L , while leaving the internal permutation symmetry of the vertices, i.e. the color dof, completely intact, all in all matching the observed phenomenology of the Standard Model.

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