
Hypothetical Dialogue on the Physics of the Higgs Sector

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Abstract. This essay stages a hypothetical dialogue between a critic of the Standard Model (EG) and a mainstream particle physicist (MP) on the foundations of the Higgs sector. The exchange starts from textbook ground: whether electroweak symmetry breaking preserves unitarity and energy–momentum conservation. It then moves, step by step, into contested territory: vacuum entanglement, microcausality, the triviality of four-dimensional scalar field theory, the metastability of the electroweak vacuum, and the fine-tuning of the Higgs mass.

The dialogue argues that triviality and fine-tuning are not cosmetic defects but signals that the continuum, strictly four-dimensional formulation of the Higgs sector cannot be fundamental. Fractional Field Theory (FFT), built on a minimally fractal spacetime of dimension $D = 4 - \varepsilon$, is presented as a candidate way out. Its central conceptual claim is that self-similarity merges locality and

nonlocality into a single-scale dependent notion. The open problems this raises, above all for unitarity and causality, are stated plainly in the closing section.

Keywords: Higgs mechanism; electroweak symmetry breaking; unitarity; triviality; hierarchy problem; vacuum metastability; fractional calculus; Fractional Field Theory; dimensional regularization; self-similarity.

1. Introduction

The discovery of a 125 GeV scalar at the Large Hadron Collider in 2012 confirmed the central prediction of the Brout–Englert–Higgs mechanism [1–5]. Yet the Higgs sector remains the least understood part of the Standard Model (SM). It is the only sector built on an elementary scalar, the only one whose mass parameter is not protected by a symmetry, and the only one whose quartic self-coupling is believed to lack a consistent continuum limit in four dimensions.

The dialogue form is chosen deliberately. Foundational disputes in physics are rarely settled by a single calculation; they are settled, or at least clarified, by pressing a position until its hidden assumptions surface. Two voices carry the exchange:

- **EG**, a critic who regards triviality, fine-tuning and vacuum metastability as symptoms of a deeper inconsistency, and who advocates Fractional Field Theory (FFT) as a remedy [36–43].
- **MP**, a mainstream particle physicist who defends the standard effective-field-theory (EFT) reading of the SM, concedes what must be conceded, and presses EG on the costs of the alternative.

The argument proceeds in the order the original exchange followed. Sections 2–4 settle what is not in dispute: unitarity, conservation laws, entanglement and microcausality. Sections 5–7 move to triviality, metastability and fine-tuning, where the standard picture is genuinely strained. Sections 8–9 present FFT and its central claim about self-similarity. Section 10 separates what has been argued

from what remains to be shown. Natural units ($\hbar = c = 1$) are used throughout, with the Higgs potential written as

$$V(H) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2, \quad m_h^2 = 2\lambda v^2, \quad v \simeq 246 \text{ GeV.} \quad (1)$$

2. Unitarity and Energy–Momentum Conservation

The opening question is elementary but not idle: does spontaneous symmetry breaking in the electroweak sector respect the two most basic constraints of relativistic quantum theory? Both participants agree that it does, but for reasons worth stating carefully.

EG: Is the Higgs mechanism of electroweak symmetry breaking strictly unitary? Does it conserve energy and momentum?

MP: Within quantum field theory, yes on both counts. The Higgs mechanism is not a process that unfolds in time and changes the laws of physics. It is a statement about the theory's ground state: the Lagrangian is invariant under $SU(2)_L \times U(1)_Y$, while the vacuum that minimizes the energy is not. The equations of motion never

change, so nothing in the mechanism can switch the conservation laws off [1–3].

EG: Take unitarity first. What exactly is being preserved?

MP: Two things. The first is the unitarity of the S-matrix, $S^\dagger S = \mathbb{1}$, which encodes conservation of probability. Without a Higgs boson, the amplitude for scattering longitudinally polarized W and Z bosons grows with the centre-of-mass energy:

$$\mathcal{A}(W_L W_L \rightarrow W_L W_L) \sim \frac{s}{v^2} \quad (\text{no Higgs}), \quad \mathcal{A} \sim \frac{m_h^2}{v^2} \quad (\text{with Higgs}). (2)$$

The partial-wave bound is violated at roughly 1–2 TeV unless something intervenes. Higgs exchange cancels the growing terms exactly; this is the content of the Lee–Quigg–Thacker bound and of the earlier tree-unitarity analysis of Cornwall, Levin and Tiktopoulos [6, 7]. The second is the gauge-theoretic structure. Three of the four real components of the Higgs doublet become the longitudinal modes of $W^\pm$ and Z . In the unitary gauge only physical states appear; in covariant gauges, BRST symmetry and the Kugo–

Ojima quartet mechanism guarantee that ghosts and unphysical polarizations decouple from the physical subspace [8].

EG: And energy and momentum?

MP: Those follow from spacetime symmetry, not from the internal gauge symmetry. By Noether’s theorem, invariance of the action under spacetime translations yields a conserved energy–momentum tensor [9, 10]. The SM Lagrangian is translation invariant before and after symmetry breaking, because the vacuum expectation value (VEV) is a constant. A constant background does not single out a point or a moment, so it cannot inject or remove energy.

EG: Then how do particles “gain” mass without drawing energy from somewhere?

MP: Mass is not acquired in a dynamical event; it is a parameter of the quadratic part of the action once fields are expanded around the true vacuum. For a fermion with Yukawa coupling y_f , the term $y_f v \bar{\psi}\psi/\sqrt{2}$ is simply a mass term. A popular image speaks of particles experiencing “drag” from the Higgs field, but that image

misleads. A Lorentz-scalar background exerts no friction on uniform motion; it changes the dispersion relation to $E^2 = \mathbf{p}^2 + m^2$. Energy is conserved in every process, with rest energy and kinetic energy trading off exactly as relativity requires.

3. Vacuum Entanglement and Unitarity

The second exchange brings quantum information into the discussion. EG's worry is that the vacuum is not a quiet backdrop but a highly entangled state, and that mass generation might inherit some of that disorder.

EG: The quantum vacuum carries a large entanglement entropy. How can mass generation stay unitary in the presence of large, nonlocal entropy fluctuations?

MP: The key is to distinguish the entropy of a subsystem from the state of the whole. Partition space into a region A and its complement B . The global vacuum $|\Omega\rangle$ is a pure state, with zero von Neumann entropy. Tracing out B gives a mixed reduced state for A :

$$\rho_A = \text{Tr}_B |\Omega\rangle\langle\Omega|, \quad S_A = -\text{Tr}(\rho_A \ln \rho_A). \quad (3)$$

In $3 + 1$ dimensions S_A is ultraviolet divergent and, at leading order, scales as the area of the boundary divided by the square of the short-distance cutoff [12, 13]. That entropy measures correlation between A and B , not thermal disorder and not a loss of information. The global state still evolves as $U(t) = e^{-iHt}$, and the vacuum, being an eigenstate of H , does not evolve at all except for a phase [14].

EG: You are describing entanglement as if it were a property of a region. It is not. Entanglement is not a localized property of the vacuum.

MP: That correction is right, and the wording matters. The entropy S_A is assigned to a region, but entanglement itself is a relation between regions. The Reeh–Schlieder theorem makes this precise: acting on the vacuum with operators localized in any open region, however small, generates a dense set of states in the whole Hilbert

space [11, 14]. The vacuum is correlated across arbitrarily large spacelike separations.

EG: Then the question sharpens. If the vacuum is a nonlocal web of correlations, why does a local process, such as a W boson propagating through the Higgs condensate, not get scrambled by it?

MP: Because correlation is not interaction. The vacuum is stationary, so its entanglement structure does not fluctuate in time; there is no time-dependent “entropy noise” for a propagating particle to absorb. What the vacuum does contain are quantum fluctuations of the field operators around their mean. Those fluctuations are fully accounted for by loop corrections. They renormalize masses and couplings, but they do so through a unitary Hamiltonian and in a way constrained by the Ward–Takahashi and Slavnov–Taylor identities of the gauge theory [8, 9].

4. Locality, Microcausality and the Status of the Higgs VEV

Having granted that the vacuum is nonlocally correlated, the dialogue turns to the principles that keep those correlations from becoming nonlocal influences.

MP: Two structural principles do the work. The first is cluster decomposition. For operators localized near x and y , vacuum correlations factorize at large separation:

$$\lim_{|x-y| \rightarrow \infty} \langle \Omega | \mathcal{O}_1(x) \mathcal{O}_2(y) | \Omega \rangle = \langle \Omega | \mathcal{O}_1(x) | \Omega \rangle \langle \Omega | \mathcal{O}_2(y) | \Omega \rangle. \quad (4)$$

The connected part decays exponentially when there is a mass gap and as a power law when massless fields are present [9, 15]. The second is microcausality: local observables at spacelike separation commute,

$$[\phi(x), \phi(y)] = 0 \quad \text{for} \quad (x - y)^2 < 0, \quad (5)$$

so no operation at x can change measurement statistics at y outside its light cone. Correlations exist, but they cannot carry signals or alter parameters remotely. In the axiomatic framework, locality, the

spectrum condition and Lorentz covariance together underwrite a well-defined, unitary scattering theory [15].

EG: You keep speaking of the VEV as if it were a physical object filling space. Suppose the Higgs sector is nontrivial and interactions propagate at finite speed. By the time an influence from the Higgs vacuum reaches a massless W or Z , the scalar sector could already have collapsed because of its triviality in four dimensions.

MP: That argument treats the VEV as a signal emitted from somewhere. It is not. The Higgs field and the gauge fields are defined at every spacetime point, and the mass term for the W boson, $\frac{1}{4}g^2v^2 W_\mu^+ W^{-\mu}$, is local: it couples fields at the same point. No influence has to travel, so there is no lag to exploit.

There is also a subtler point. In a gauge theory, the statement “ $\langle H \rangle = v$ ” is gauge dependent. Elitzur’s theorem forbids the spontaneous breaking of a local gauge symmetry, so a gauge-variant VEV can be nonzero only after gauge fixing [16]. The physical content is captured by gauge-invariant composite operators, as shown by

Fröhlich, Morchio and Strocchi [17]. “The VEV fills space” is shorthand for a gauge-fixed description of a gauge-invariant ground state, not a claim about a material medium. Whether the scalar sector “collapses” is a separate question, which is triviality, and it deserves its own discussion.

5. Triviality of the Scalar Sector in Four Dimensions

Triviality is where the dialogue first reaches a genuine fault line. The participants agree on mathematics but disagree sharply on what it implies.

MP: Let us state the result precisely. In four-dimensional $\lambda\phi^4$ theory, if one insists on removing the ultraviolet cutoff ($\Lambda \rightarrow \infty$) while holding low-energy physics fixed, the renormalized coupling is driven to zero. The continuum limit is a free field. This was established rigorously for $D > 4$ and strongly supported in $D = 4$ by Aizenman and Fröhlich [18, 19], quantified on the lattice by Lüscher and Weisz [20], and proven for the four-dimensional Ising and ϕ^4 scaling limits by Aizenman and Duminil-Copin [21].

The one-loop running of the pure scalar coupling already shows the mechanism. With $V = \lambda(H^\dagger H)^2$, the coupling grows logarithmically and hits a Landau pole:

$$\lambda(\mu) = \frac{\lambda(\mu_0)}{1 - \frac{3\lambda(\mu_0)}{2\pi^2} \ln(\mu/\mu_0)}. \quad (6)$$

Fixing λ at low energy and pushing the pole to infinity forces $\lambda(\mu_0) \rightarrow 0$.

EG: Then triviality means the stable minimum disappears when $\lambda = 0$. The Higgs scalar turns into a massless particle that cannot interact with the other massless particles. Mass generation fails.

MP: If λ were literally zero with $\mu^2 > 0$, the potential would be unbounded below. I agree that is not a viable theory. But two steps in the argument do not follow. First, triviality concerns the limit $\Lambda \rightarrow \infty$. At any finite cutoff the theory is interacting, and the measured coupling is $\lambda \approx 0.13$ at the electroweak scale, giving $m_h \approx 125 \text{ GeV}$ [4, 5, 50]. Second, renormalization-group “flow” is a change of resolution scale, not an evolution in time. Nothing

happens “instantaneously”; a different description applies at a different energy.

The practical consequence is the triviality bound: the larger m_h , the lower the scale Λ at which new physics must appear [20, 22]. For a 125 GeV Higgs, the pure-scalar Landau pole lies far above the Planck scale.

EG: You are quoting the pure-scalar result. The Higgs is not isolated.

MP: Correct, and the full one-loop SM beta function shows how much the gauge and Yukawa couplings change the running [23]:

$$16\pi^2\beta_\lambda = 24\lambda^2 + 12\lambda y_t^2 - 6y_t^4 - 9\lambda g^2 - 3\lambda g'^2 + \frac{9}{8}g^4 + \frac{3}{4}g^2 g'^2 +$$

The top-quark term $-6y_t^4$ dominates and drives λ downward rather than toward a Landau pole. In fairness to your position, this does not prove the SM has a continuum limit either. The $U(1)_Y$ coupling has its own Landau pole at an enormous scale, so the SM, taken alone, is also not known to exist as a rigorous continuum theory. Its

accepted status is that of an effective field theory valid below some finite cutoff [29, 53].

6. Metastability, the Limits of Perturbative RG, Decoherence and the Ether

EG now widens the attack from triviality to the reliability of the whole perturbative picture of the Higgs vacuum.

EG: Your line of argument is faulty. The electroweak minimum is unstable under quantum corrections. One can only speculate that the Higgs potential flattens at high scales, that perturbative RG remains valid there, and that decoherence does not undermine quantum physics altogether. And the premise that the Higgs is everywhere at every moment, as a global background, is as wrong as the ether hypothesis.

MP: There are four claims here, and they deserve separate answers.

Stability. You are right that the minimum is not absolutely stable.

With $m_h \approx 125 \text{ GeV}$ and the measured top mass, next-to-next-to-

leading-order analyses find that λ turns negative near 10^{10} – 10^{11} GeV [24, 25]. A deeper minimum then exists at large field values, and our vacuum is a false vacuum. The precise word is *metastable* rather than unstable: the computed tunnelling rate gives a lifetime vastly longer than the age of the universe [25–27]. The conclusion is sensitive to the top-quark mass and is modified by any new physics or gravitational effects below the instability scale. That the measured parameters sit so close to the stability boundary is itself a striking, unexplained fact.

Perturbative RG. Granted as a methodological point: extrapolating the SM to 10^{19} GeV assumes no new degrees of freedom on the way, which is an assumption, not a result. Within the SM itself the extrapolation is well controlled, because all couplings stay small, so perturbation theory does not break down internally [25]. What fails is our knowledge of what lies above the tested range, not the arithmetic.

Decoherence. Here I disagree. Decoherence is not a departure from quantum mechanics. It is unitary evolution of a system together with its environment, followed by tracing out the environment [28]. It explains why superpositions become unobservable, but it cannot “destroy” quantum theory, because it is derived from it.

The ether. The analogy has a sound kernel and a misleading part. The misleading part: the luminiferous ether defined a preferred rest frame, which is exactly what Michelson and Morley tested and relativity removed [52]. A constant Lorentz-scalar VEV defines no rest frame; it looks the same to every inertial observer, so no drift experiment can detect motion relative to it. The sound kernel: as Section 4 noted, the VEV is a gauge-fixed construct [16, 17], and its contribution to vacuum energy, of order $(100 \text{ GeV})^4$, conflicts by many orders of magnitude with the observed cosmological constant. The “background that fills space” is therefore not as innocent as textbooks suggest.

EG: So, the standard picture rests on working hypotheses.

MP: At high energy, yes. Absolute stability, unlimited extrapolation and a literal reading of the VEV are working hypotheses. The measured low-energy physics is not.

7. Incomplete or Inconsistent? Triviality and Fine-Tuning

The pivot of the dialogue is a single word. MP is willing to call the Standard Model incomplete; EG insists it is inconsistent.

EG: The Standard Model is not merely incomplete. It is inconsistent, because of the triviality problem and the fine-tuning problem.

MP: The distinction is worth drawing carefully. A theory is incomplete if it omits phenomena: dark matter, neutrino masses, gravity. It is inconsistent if its own axioms contradict each other. On triviality, I accept a precise version of your claim. A strictly four-dimensional, interacting scalar sector valid at all scales does not exist as a continuum quantum field theory [18–21]. A Standard Model claimed to be fundamental at arbitrarily high energy is therefore untenable.

EG: And fine-tuning?

MP: With a hard momentum cutoff Λ , the Higgs mass parameter receives corrections dominated by the top quark:

$$\delta m_h^2 \simeq -\frac{3y_t^2}{8\pi^2} \Lambda^2 + \dots \quad (8)$$

Fermion masses are protected by chiral symmetry and gauge-boson masses by gauge symmetry, but no symmetry of the SM protects an elementary scalar mass [30, 31]. If Λ is the Planck scale, the bare parameter must cancel the correction to about one part in 10^{34} , since $(125 \text{ GeV}/10^{19} \text{ GeV})^2 \approx 10^{-34}$. I would still call this a failure of naturalness rather than of consistency. Naturalness is a criterion for a good explanation, not a theorem [31, 33]. In dimensional regularization the quadratic term does not even appear; the sensitivity resurfaces only through heavy particles that couple to the Higgs, as Bardeen emphasized [32, 35].

EG: That observation is the heart of the matter. The quadratic divergence disappears when the dimension of spacetime is

continued to $D = 4 - \varepsilon$. In conventional practice ε is a calculational device, sent to zero at the end. I take the lesson the other way. If treating the dimension as continuous tames the divergences, perhaps the dimension of spacetime is physically continuous, and departs slightly from four in the deep ultraviolet [38, 42].

MP: Then the burden shifts to you. The effective-field-theory reading of the SM is consistent at every energy where it has been tested [29, 53]. Your proposal must reproduce all of that and also resolve the problems I have conceded. Let's hear it.

8. Fractional Field Theory as a Way Out

EG now sets out the alternative. Fractional Field Theory starts from the premise that spacetime is not a smooth four-dimensional manifold at all scales, but a minimally fractal one whose dimension deviates slightly from four in the deep ultraviolet [36–42].

EG: Fractional Field Theory is a promising way out. Its starting point is that the dimension of spacetime is a running, scale-dependent quantity, $D = 4 - \varepsilon(\mu)$, with $\varepsilon \ll 1$ and $\varepsilon \rightarrow 0$ in the

infrared. Ordinary derivatives are replaced by fractional operators, which are the natural calculus on such a space [44, 45]. In the low-energy limit the theory reduces to the SM; in the deep ultraviolet it does not.

MP: Begin with triviality. How does a small ε help?

EG: Because below four dimensions the scalar quartic interaction is relevant rather than marginal, and an interacting fixed point appears. For the Higgs doublet, with four real components, the one-loop flow in $D = 4 - \varepsilon$ reads

$$\beta_\lambda = -\varepsilon\lambda + \frac{3\lambda^2}{2\pi^2}, \quad \lambda^* = \frac{2\pi^2\varepsilon}{3}. \quad (9)$$

This is the Wilson–Fisher fixed point [34, 54]. The coupling is no longer driven to zero; the scalar sector remains interacting in the ultraviolet. Triviality, in this view, is an artifact of insisting on exactly four dimensions at all scales [40].

MP: And fine-tuning?

EG: The quadratic sensitivity is a property of four-dimensional integrals with a hard cutoff. With fractional kinetic operators, propagators scale as $(p^2)^{-\alpha}$ with α tied to ε , which softens ultraviolet behaviour. Dimensional regularization already shows that continuous dimension removes power-law divergences [35]; FFT treats that continuity as physical rather than formal [38, 42]. I also argue that the electroweak scale and the pattern of SM parameters may emerge as attractors of nonlinear dynamics, through bifurcation and period-doubling scenarios, rather than as finely tuned inputs [37, 43].

MP: Then let me state the price, because it is steep. First, unitarity and causality. Fractional operators are nonlocal: they integrate over a neighbourhood with a power-law kernel. Locality and microcausality are precisely what guarantee a unitary, causal S-matrix in standard QFT [9, 15]. Nonlocal kinetic terms can introduce ghosts or acausal propagation; prescriptions such as Anselmi's fakeons aim to preserve unitarity in related higher-derivative settings, but no general result covers fractional operators [49].

Second, Lorentz invariance must be recovered in the infrared to extraordinary precision. Third, experiment constrains any deviation of the dimension at accessible scales: precision quantum electrodynamics bounds ε to below about 10^{-11} at atomic and electron-scale energies [55]. FFT must be built so that ε is negligible wherever we have looked.

EG: I accept the bounds; they are why ε must be minimal and scale dependent. I would add that FFT is not isolated. Multifractional spacetimes, causal dynamical triangulations and asymptotic safety all find that the effective dimension of spacetime changes in the ultraviolet [46–48]. Dimensional flow appears to be a common feature of attempts to go beyond the smooth continuum.

9. Self-Similarity and the Merging of Locality and Nonlocality

The final exchange concerns MP's strongest objection, the threat that nonlocal operators pose to unitarity. EG's reply is conceptual: in a self-similar framework, the very distinction between local and nonlocal changes meaning.

EG: Locality and nonlocality blend into a single concept once one realizes that Fractional Field Theory is self-similar. On a smooth manifold, “local” means a point and “nonlocal” means action at a distance. On a self-similar space, zooming into a point reveals structure that mirrors the larger scale [51]. A fractional derivative integrates over a neighbourhood with a power-law kernel, and power laws are the signature of scale invariance. What looks nonlocal at one resolution is the definition of local at the next. The nonlocality is not arbitrary; it is fixed by the fractal dimension, and so it is a geometric constraint rather than a source of uncontrolled influence.

MP: There is real substance here, and a precedent you can lean on. Conformal field theories have power-law correlations at all distances, yet they are local, causal and unitary. Power-law structure and nonlocality of correlations are fully compatible with a healthy quantum theory. Scale invariance also turns the renormalization group from a calculational tool into a statement about geometry, which matches your reading of RG flow as dimensional flow.

But the decisive question is not about correlations; it is about dynamics. Unitarity of a propagator can be tested directly. By the Källén–Lehmann representation, a propagator compatible with positive probabilities has a non-negative spectral density [9]. For scale-invariant fields with propagator $\propto (p^2)^{-\alpha}$, positivity holds only when the field’s scaling dimension respects the unitarity bound, which corresponds to $\alpha \leq 1$ [56, 57]:

$$\Delta_\phi \geq \frac{D-2}{2} \quad \Leftrightarrow \quad \alpha = \frac{D}{2} - \Delta_\phi \leq 1. \quad (10)$$

A unitary propagator therefore cannot fall faster than $1/p^2$ at high momentum. The ultraviolet softening that FFT needs for fine-tuning seems to require $\alpha > 1$, which is exactly where ghosts appear.

EG: That bound is derived on a smooth background with an integer-dimensional momentum measure. In FFT the dimension itself flows. The suppression of ultraviolet sensitivity comes primarily from the reduced number of modes in $D = 4 - \varepsilon$, not from a propagator steeper than $1/p^2$. Self-similarity is what ties the two together: the

scaling of the measure and the scaling of the kernel are set by the same ε .

MP: That is a coherent strategy, and it turns the debate into a well-posed calculation. Write down the fractional Higgs–gauge action, derive its spectral representation on the multifractal measure, and show the density is positive and the S-matrix unitary as $\varepsilon \rightarrow 0$. If that works, the claim that self-similar nonlocality conserves probability becomes a theorem rather than an image.

EG: Agreed. Self-similarity does not exempt FFT from unitarity; it is the structure through which I expect unitarity to be shown.

10. Concluding Remarks and Open Problems

The dialogue ends with a narrower but sharper disagreement than it began with. Both participants accept that the Higgs mechanism is unitary and conserves energy and momentum, and that vacuum entanglement is compatible with both. They also accept that a strictly four-dimensional scalar sector cannot be fundamental, and that the electroweak vacuum is metastable. They differ on whether

these facts call for an effective-field-theory cutoff or for a new conception of spacetime. Table 1 summarizes where the exchange leaves each issue.

Table 1. Where the dialogue leaves each issue.

Issue	Common ground	Where EG and MP part ways
Unitarity, energy– momentum	Preserved by electroweak symmetry breaking	None
Vacuum entanglement	Nonlocal correlations; no nonlocal influence	None, after the wording is corrected
Status of the VEV	Gauge-fixed description; no preferred frame	Whether a uniform background is physically meaningful
Triviality	No interacting continuum limit for 4D ϕ^4	EFT cutoff versus physical dimensional flow
Metastability	$\lambda < 0$ near 10^{10} – 10^{11} GeV	Whether perturbative extrapolation is meaningful

Fine-tuning	1 part in 10^{34} with a Planck-scale cutoff	Failure of naturalness versus inconsistency
Fractional Field Theory	Evades triviality via a Wilson–Fisher fixed point	Whether unitarity and causality survive

The case for Fractional Field Theory rests on one structural observation. Dimensional regularization, the Wilson–Fisher fixed point and the dimensional flow seen in several approaches to quantum gravity all point to the same idea: a spacetime dimension that is not exactly four in the ultraviolet removes the pathologies of the scalar sector. Self-similarity is proposed as the principle that makes the resulting nonlocality controlled rather than arbitrary.

For that case to be complete, the following problems must be resolved:

1. **Unitarity.** Derive the spectral representation of the fractional Higgs–gauge propagators on a multifractal measure and show

- that the spectral density is non-negative, reconciling FFT with the Källén–Lehmann and unitarity bounds [9, 56, 57].
2. **Causality and Lorentz invariance.** Show that microcausality and Lorentz symmetry are recovered as $\varepsilon \rightarrow 0$ to the precision required by experiment.
 3. **Gauge consistency.** Construct fractional covariant derivatives and the corresponding BRST structure so that unphysical states decouple [8].
 4. **Dynamics of ε .** Specify how $\varepsilon(\mu)$ is determined, rather than fixed by hand, and how it vanishes in the infrared within the bounds of precision QED [55].
 5. **Predictions.** Identify observable signatures at accessible energies, such as deviations in the Higgs self-coupling or in longitudinal vector-boson scattering, that distinguish FFT from a generic EFT.

The Standard Model has earned its status through decades of precision tests. Its Higgs sector, however, carries clear signs that the smooth four-dimensional continuum is an approximation. Whether

Fractional Field Theory is the right replacement will be decided by whether it can meet the unitarity test that this dialogue has set for it.

References

- [1] F. Englert and R. Brout, “Broken symmetry and the mass of gauge vector mesons,” *Phys. Rev. Lett.* **13**, 321 (1964).
- [2] P. W. Higgs, “Broken symmetries and the masses of gauge bosons,” *Phys. Rev. Lett.* **13**, 508 (1964).
- [3] G. S. Guralnik, C. R. Hagen and T. W. B. Kibble, “Global conservation laws and massless particles,” *Phys. Rev. Lett.* **13**, 585 (1964).
- [4] ATLAS Collaboration, “Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC,” *Phys. Lett. B* **716**, 1 (2012).
- [5] CMS Collaboration, “Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC,” *Phys. Lett. B* **716**, 30 (2012).

- [6] J. M. Cornwall, D. N. Levin and G. Tiktopoulos, "Derivation of gauge invariance from high-energy unitarity bounds on the S matrix," *Phys. Rev. D* **10**, 1145 (1974).
- [7] B. W. Lee, C. Quigg and H. B. Thacker, "Weak interactions at very high energies: the role of the Higgs-boson mass," *Phys. Rev. D* **16**, 1519 (1977).
- [8] C. Becchi, A. Rouet and R. Stora, *Ann. Phys.* **98**, 287 (1976);
T. Kugo and I. Ojima, *Prog. Theor. Phys. Suppl.* **66**, 1 (1979).
- [9] S. Weinberg, *The Quantum Theory of Fields, Vol. I: Foundations* (Cambridge University Press, 1995).
- [10] E. Noether, "Invariante Variationsprobleme," *Nachr. Ges. Wiss. Göttingen, Math.-Phys. Kl.*, 235 (1918).
- [11] H. Reeh and S. Schlieder, "Bemerkungen zur Unitäräquivalenz von Lorentzinvarianten Feldern," *Nuovo Cimento* **22**, 1051 (1961).
- [12] L. Bombelli, R. K. Koul, J. Lee and R. D. Sorkin, "Quantum source of entropy for black holes," *Phys. Rev. D* **34**, 373 (1986).

- [13] M. Srednicki, “Entropy and area,” *Phys. Rev. Lett.* **71**, 666 (1993).
- [14] E. Witten, “Notes on some entanglement properties of quantum field theory,” *Rev. Mod. Phys.* **90**, 045003 (2018).
- [15] R. F. Streater and A. S. Wightman, *PCT, Spin and Statistics, and All That* (Benjamin, New York, 1964).
- [16] S. Elitzur, “Impossibility of spontaneously breaking local symmetries,” *Phys. Rev. D* **12**, 3978 (1975).
- [17] J. Fröhlich, G. Morchio and F. Strocchi, “Higgs phenomenon without symmetry breaking order parameter,” *Nucl. Phys. B* **190**, 553 (1981).
- [18] M. Aizenman, “Proof of the triviality of ϕ_d^4 field theory and some mean-field features of Ising models for $d > 4$,” *Phys. Rev. Lett.* **47**, 1 (1981).
- [19] J. Fröhlich, “On the triviality of $\lambda\phi_d^4$ theories and the approach to the critical point in $d \geq 4$ dimensions,” *Nucl. Phys. B* **200**, 281 (1982).

- [20] M. Lüscher and P. Weisz, “Scaling laws and triviality bounds in the lattice ϕ^4 theory,” *Nucl. Phys. B* **290**, 25 (1987).
- [21] M. Aizenman and H. Duminil-Copin, “Marginal triviality of the scaling limits of critical 4D Ising and ϕ_4^4 models,” *Ann. Math.* **194**, 163 (2021).
- [22] N. Cabibbo, L. Maiani, G. Parisi and R. Petronzio, “Bounds on the fermions and Higgs boson masses in grand unified theories,” *Nucl. Phys. B* **158**, 295 (1979).
- [23] M. E. Machacek and M. T. Vaughn, “Two-loop renormalization group equations in a general quantum field theory (III). Scalar quartic couplings,” *Nucl. Phys. B* **249**, 70 (1985).
- [24] G. Degrand et al., “Higgs mass and vacuum stability in the Standard Model at NNLO,” *JHEP* **08**, 098 (2012).
- [25] D. Buttazzo et al., “Investigating the near-criticality of the Higgs boson,” *JHEP* **12**, 089 (2013).

- [26] S. Coleman, “Fate of the false vacuum: semiclassical theory,” *Phys. Rev. D* **15**, 2929 (1977).
- [27] C. G. Callan and S. Coleman, “Fate of the false vacuum. II. First quantum corrections,” *Phys. Rev. D* **16**, 1762 (1977).
- [28] W. H. Zurek, “Decoherence, einselection, and the quantum origins of the classical,” *Rev. Mod. Phys.* **75**, 715 (2003).
- [29] K. G. Wilson and J. Kogut, “The renormalization group and the ϵ expansion,” *Phys. Rep.* **12**, 75 (1974).
- [30] L. Susskind, “Dynamics of spontaneous symmetry breaking in the Weinberg–Salam theory,” *Phys. Rev. D* **20**, 2619 (1979).
- [31] G. ’t Hooft, “Naturalness, chiral symmetry, and spontaneous chiral symmetry breaking,” in *Recent Developments in Gauge Theories*, NATO ASI Ser. B **59**, 135 (Plenum, New York, 1980).
- [32] W. A. Bardeen, “On naturalness in the standard model,” FERMILAB-CONF-95-391-T (1995).

- [33] G. F. Giudice, “Naturally speaking: the naturalness criterion and physics at the LHC,” arXiv:0801.2562 (2008).
- [34] K. G. Wilson and M. E. Fisher, “Critical exponents in 3.99 dimensions,” *Phys. Rev. Lett.* **28**, 240 (1972).
- [35] G. ’t Hooft and M. Veltman, “Regularization and renormalization of gauge fields,” *Nucl. Phys. B* **44**, 189 (1972).
- [36] E. Goldfain, “Complexity in quantum field theory and physics beyond the standard model,” *Chaos, Solitons & Fractals* **28**, 913 (2006).
- [37] E. Goldfain, “Feigenbaum scaling, Cantorian space–time and the hierarchical structure of standard model parameters,” *Chaos, Solitons & Fractals* **30**, 324 (2006).
- [38] E. Goldfain, “Fractional dynamics and the TeV regime of field theory,” *Commun. Nonlinear Sci. Numer. Simul.* **13**, 666 (2008), doi:[10.1016/j.cnsns.2006.06.001](https://doi.org/10.1016/j.cnsns.2006.06.001).

- [39] E. Goldfain, "Fractional dynamics and the Standard Model for particle physics," *Commun. Nonlinear Sci. Numer. Simul.* **13**, 1397 (2008).
- [40] E. Goldfain, "Critical behavior in continuous dimension, $\varepsilon^{(\infty)}$ theory and particle physics," *Chaos, Solitons & Fractals* **38**, 928 (2008).
- [41] E. Goldfain, *Introduction to Fractional Field Theory* (Aracne Editrice, Rome, 2015).
- [42] E. Goldfain, "Fractional dynamics and physics beyond effective field theory (Part 1)," *Qeios* (preprint), <https://www.qeios.com/read/G8F1IG>.
- [43] E. Goldfain, "On the bifurcation structure of particle physics," *Qeios* (preprint), <https://www.qeios.com/read/O04HOC>.
- [44] V. E. Tarasov, *Fractional Dynamics: Applications of Fractional Calculus to Dynamics of Particles, Fields and Media* (Springer and Higher Education Press, 2010).

- [45] R. Herrmann, *Fractional Calculus: An Introduction for Physicists*, 3rd ed. (World Scientific, Singapore, 2018).
- [46] G. Calcagni, “Geometry and field theory in multi-fractional spacetime,” *JHEP* **01**, 065 (2012), arXiv:1107.5041.
- [47] J. Ambjørn, J. Jurkiewicz and R. Loll, “Spectral dimension of the universe is scale dependent,” *Phys. Rev. Lett.* **95**, 171301 (2005).
- [48] O. Lauscher and M. Reuter, “Fractal space-time structure in asymptotically safe gravity,” *JHEP* **10**, 050 (2005).
- [49] D. Anselmi, “Fakeons and Lee–Wick models,” *JHEP* **02**, 141 (2018).
- [50] Particle Data Group, S. Navas *et al.*, “Review of Particle Physics,” *Phys. Rev. D* **110**, 030001 (2024).
- [51] B. B. Mandelbrot, *The Fractal Geometry of Nature* (W. H. Freeman, San Francisco, 1982).

- [52] A. A. Michelson and E. W. Morley, *Am. J. Sci.* **34**, 333 (1887);
A. Einstein, “Zur Elektrodynamik bewegter Körper,”
Ann. Phys. (Leipzig) **17**, 891 (1905).
- [53] J. Polchinski, “Renormalization and effective Lagrangians,”
Nucl. Phys. B **231**, 269 (1984).
- [54] J. Zinn-Justin, *Quantum Field Theory and Critical Phenomena*, 4th
ed. (Oxford University Press, 2002).
- [55] A. Zeilinger and K. Svozil, “Measuring the dimension of space-
time,” *Phys. Rev. Lett.* **54**, 2553 (1985); B. Müller and A. Schäfer,
“Improved bounds on the dimension of space-time,”
Phys. Rev. Lett. **56**, 1215 (1986).
- [56] G. Mack, “All unitary ray representations of the conformal
group $SU(2,2)$ with positive energy,” *Commun. Math. Phys.* **55**, 1
(1977).

- [57] H. Georgi, “Unparticle physics,” *Phys. Rev. Lett.* **98**, 221601 (2007); B. Grinstein, K. Intriligator and I. Z. Rothstein, “Comments on unparticles,” *Phys. Lett. B* **662**, 367 (2008).