

On the purported “projection-operator” in a specific modified Epstein-Glaser normalization subtraction scheme

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Abstract. We show that a modified Epstein-Glaser (EG) normalization truncation operator T_w presented a couple of years ago is, due to an automatically emerging cascaded sub vectorspace structure, not a an automorphism on the vectorspace $\mathcal{S}(\mathbb{R}^m)$ of Schwartzfunctions, in contrast to the original EG-operator W . Apart from the (rather simple) proof itself, we hint on the origin of the difficulties with using T_w instead of W .

1 Introduction

In 1975 Epstein and Glaser (EG) developed a mathematically rigorous causal approach to QED [1], focussing on certain so far neglected mathematical peculiarities. Especially the operator valued distributional character, instead of the usual operator character, of the field operators in position- and momentum-space (e.g. $\varphi^\pm(x)$ and $\hat{\varphi}^\pm(p)$ respectively), was treated in a strict manner. As a consequence, so called disitribution splitting emerged as a necessary tool and within that framework a special operator W in the so called Schwartzspace of testfunctions $\mathcal{S}(\mathbb{R}^m)$ has been introduced.

Some years ago, a modified version [2] of the previously mentioned EG approach was introduced and found usage. It differs from the original EG-version in the operator defined as wavefunction truncation operator. This altered operator seems to appear for the first time in [3] but was not considered there with the EG background in mind. Then within the framework of [2], various features are shown, which are common with regular renormalizationschemes outside of the EG approach. Unfortunately, the mathematical basis on which these comparisons in [2] are deduced, seem to need some further work.

In our paper we will only focus on the operator modification prescription, where the resultinf entity os wrongly treated as an operator in Schwartzspace of testfunctions. No further interpretational questions, especially concerning further conclusions drawn in [2], are discussed. This paper is structured as follows: First, some notational and mathematical preliminaries (section 2) are given. Then, as a simplified first step in our reasoning, we will examine (section 3) a basic truncation operator T_n that simply subtracts the first n terms of a Taylorseries of a given testfunction. This T_n is shown to not be an operator in the Schwartzspace of testfunctions. The section afterwards (section 4) then finally shows this as well for the “operator” T_w introduced in [2] (please note that in our paper T_w will be denoted by T_w^n). Readers familiar with the overall topic might directly jump to section 4. A final and concluding look at the original operator W , as introduced in the EG frame work, is provided in section 5.

2 Preliminaries

To start with, we recall some fundamental definitions to clarify notational issues and to hint on subtleties which could otherwise lead to confusion and misunderstandings. With $\mathcal{S}(\mathbb{R}^m)$ we denote

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the Schwartz-space of testfunctions, fulfilling

$$\begin{aligned} & \mathcal{S}(\mathbb{R}^m) \\ = & \{ \phi \in \mathcal{C}^\infty(\mathbb{R}^m, \mathbb{C}) \mid \forall \alpha, \beta \in \mathbb{N}_0^m, \exists C \in \mathbb{R}, \\ & C \geq 0 : \sup_{x \in \mathbb{R}^m} |x^\alpha D^\beta \phi(x)| \leq C \} \end{aligned} \quad (1)$$

with $\mathbb{N}_0^m$ denoting that α and β being a multi-index including zero as natural number, $\mathcal{C}^\infty(\mathbb{R}^m, \mathbb{C})$ the infinitely often continuously differentiable function from $\mathbb{R}^m$ to the complex plane $\mathbb{C}$. Regarding multi indices the author notes that in all subsequent sections, the multi index notation is omitted for the sake of brevity. None of the following proofs relies on multi index notation, hence it can be added at will, without altering any of this paper's results. Given that $\mathcal{S}(\mathbb{R}^m)$ is a normed vector space, it's dual space of linear and continuous functionals is denoted by $\mathcal{S}'(\mathbb{R}^m)$. For further considerations we make an important remark on the linearity of the dual space's elements. Given $f \in \mathcal{S}'(\mathbb{R}^m)$, $\varphi \in \mathcal{S}(\mathbb{R}^m)$, φ is often decomposed in terms of a Taylorseries

$$\varphi(x) = \sum_{k=0}^{\infty} \frac{x^k}{k!} \varphi^{(k)}(x) \Big|_{x=0}$$

around $x = 0$ with $\varphi^{(k)}$ as the k -th derivative with respect to x . Rewriting it as

$$\sum_{k=0}^n \frac{x^k}{k!} \varphi^{(k)}(x) \Big|_{x=0} + \sum_{k=n+1}^{\infty} \frac{x^k}{k!} \varphi^{(k)}(x) \Big|_{x=0}$$

one might be tempted to mistake linearity of f as

$$\begin{aligned} \langle f, \varphi \rangle &= \left\langle f(x), \sum_{k=0}^n \frac{x^k}{k!} \varphi^{(k)}(x) \Big|_0 \right\rangle \\ &+ \left\langle f(x), \sum_{k=n+1}^{\infty} \frac{x^k}{k!} \varphi^{(k)}(x) \Big|_0 \right\rangle. \end{aligned}$$

This is wrong for the simple reason that linearity of f is true for linear combinations of testfunctions only but not necessarily for sums of arbitrary functions. But expanding φ in a Taylorseries is an expansion in sums of (arbitrary) functions which are neither basefunctions of the Schwartzspace of testfunctions nor testfunctions at all. Hence linearity of f does not apply to the single terms in a Taylorseries but to the whole series as one inseparable,

symbolically written term only.

As Taylorexansion plays a crucial role in distributionsplitting of distributions with singular order $\omega \geq 0$ (see e.g. [4]), we recall a later-on used representation of Taylorseries in terms of Taylorpolynomials. Let φ be the function to be expanded, then:

$$\begin{aligned} \varphi(x) &= \sum_{k=0}^n \frac{(x-a)^k}{k!} \varphi^{(k)}(x) \Big|_a \\ &+ \int_a^x \frac{(x-t)^n}{n!} \varphi^{(n+1)}(t) \Big|_a dt, \end{aligned} \quad (2)$$

where the first sum is called the “jet” and written as $j_0^n(\varphi(x))$. The integral is called the “remainder-term” and denoted by $R_n(\varphi(x))$. Integrating $R_n(x)$ by parts yields, after infinitely many steps, the common Taylorseries.

3 The T_n -operator

To get the main idea of our later overall reasoning, we first concentrate on a simple example which has some vital properties in common with our later “operator” of [2]. Let us take a look at the easy operation of truncating a Taylorseries' first $n+1$ (i.e. the constant and the polynomials up to the degree n) terms and denote it by T_n . Then we have for $\varphi \in \mathcal{S}(\mathbb{R}^m)$:

$$\begin{aligned} T_n : \mathcal{S}(\mathbb{R}^m) &\rightarrow \mathcal{S}_n^T, \\ \varphi \mapsto T_n(\varphi) &= T_n \left(\sum_{k=0}^{\infty} \frac{x^k}{k!} \varphi^{(k)}(x) \Big|_0 \right) \\ &= \sum_{k=n+1}^{\infty} \frac{x^k}{k!} \varphi^{(k)}(x) \Big|_0, \end{aligned}$$

with $n \in \mathbb{N}$. $\mathcal{S}_n^T$ is defined as the image of $\mathcal{S}(\mathbb{R}^m)$ under the action of T_n , i.e. $T_n(\mathcal{S}(\mathbb{R}^m)) =: \mathcal{S}_n^T$. No assumption so far is made whether $\mathcal{S}_n^T \subseteq \mathcal{S}(\mathbb{R}^m)$ (and we will later see that, indeed, T_n does not map to Schwartz test functions). For T_n being a projection operator, besides other requirements, we need $\mathcal{S}_n^T \subseteq \mathcal{S}_n$ with $\mathcal{S}_n$ being the vectorspace of functions vanishing at the origin of order n , $\mathbb{N} \ni n \geq 0$ (i.e. the function and its derivatives up to the n -th order are equal to zero at $x=0$). However, we will show that

Proposition.

i.) $\mathcal{S}_n^T$ is not a subvectorspace of $\mathcal{S}(\mathbb{R}^m) \forall n > 0$.

ii.) Furthermore T_n cannot be a linear operator in Schwartzspace, especially not a projection operator onto a certain subvectorspace of $\mathcal{S}(\mathbb{R}^m)$.

Proof 3.1 (of i.).

Let $\varphi \in \mathcal{S}(\mathbb{R}^m)$, $T_n : \mathcal{S}(\mathbb{R}^m) \rightarrow \mathcal{S}_n^T$, $T_{n'} : \mathcal{S}(\mathbb{R}^m) \rightarrow \mathcal{S}_{n'}^T$, $n, n' \in \mathbb{N}$, $n' \geq n > 0$. $\mathcal{S}_n(\mathbb{R}^m), \mathcal{S}_{n'}(\mathbb{R}^m)$ the space of functions vanishing at the origin at the order of n, n' , respectively. Clearly $\mathcal{S}_n \supset \mathcal{S}_{n'}$.

Now assume $\mathcal{S}_n^T$ is a subset of $\mathcal{S}_n$ (that would be to assume T_n is an operator in Schwartzspace), then addition of elements of $\mathcal{S}_n^T$ would be allowed and the respective sum also $\in \mathcal{S}_n$.

As n was chosen arbitrarily, the same reasoning then is valid for $n' \neq n$, i.e. $\mathcal{S}_{n'}, \mathcal{S}_n^T$.

With $\mathcal{S}_{n'} \subset \mathcal{S}_n$, addition of elements of these two subvectorspaces is allowed, too (elements of $\mathcal{S}_{n'}$ are also elements of $\mathcal{S}_n$). Therefore

$$T_n(\varphi) - T_{n'}(\varphi) \in \mathcal{S}_n(\mathbb{R}^m) \subset \mathcal{S}(\mathbb{R}^m).$$

But

$$\begin{aligned} & T_n(\varphi) - T_n(T_{n'}(\varphi)) \\ &= \sum_{k=n+1}^{\infty} \frac{x^k}{k!} \varphi^{(k)}(x) \Big|_0 - \sum_{k=n'+1}^{\infty} \frac{x^k}{k!} \varphi^{(k)}(x) \Big|_0 \\ &= \sum_{k=n+1}^{n'} \frac{x^k}{k!} \varphi^{(k)}(x) \Big|_0 \notin \mathcal{S}_n(\mathbb{R}^m). \end{aligned} \quad (3)$$

Eq.(3) is a polynomial in x of finite degree and thus not $\mathcal{S}_n(\mathbb{R}^m)$. Especially it is not a testfunction fulfilling the condition of eq.(1). As a result we get that the mapping T_n maps outside of $\mathcal{S}(\mathbb{R}^m)$. But by definition into $\mathcal{S}_n^T$. That means the assumption of $\mathcal{S}_n^T$ being a subset of $\mathcal{S}_n$ leads to a contradiction. $\square$

It is helpful trying to weaken the assumptions and looking whether our proof breaks down :

a.) One might try to only allow additions of elements of a certain subspace $\mathcal{S}_n$ and not of $\mathcal{S}_n$ and other subspaces $\mathcal{S}_{n'}$ (with $n' \geq n$). As the proof shows, this is impossible because for $n' \geq n$ one always has $\mathcal{S}_{n'} \subset \mathcal{S}_n$. Here “ $\subset$ ” is meant in the sense of “is a subvectorspace” which requires the possibility of addition of two such elements of $\mathcal{S}_n$ and $\mathcal{S}_{n'}$ as per the vector space axioms.

b.) Trying to define $\mathcal{S}_{=n}$ as subspace of testfunctions vanishing only at the order n at zero and not for higher or lesser orders, leads at once to contradictions if we assume $\mathcal{S}_{=n}$ to be equipped with a (sub)vectorspace-structure : Let $\varphi \in \mathcal{S}_{=n}$. Then $-\varphi \in \mathcal{S}_{=n}$, but $\varphi + (-\varphi) = 0 \notin \mathcal{S}_{=n}$.

c.) Additionally one might argue, expanding $\varphi \in \mathcal{S}(\mathbb{R}^m)$ in a series is only valid if expansion is done in basefunctions of $\mathcal{S}(\mathbb{R}^m)$, so monomials in x as in the Taylor-case are illegitimate and the single sums in eq.(3) cannot be subtracted. But as mentioned in the introductory part, this is an allowed procedure.

Especially above’s remarks a and b are important, as they underline the importance of the **automatically emerging cascaded sub vector space structures**.

So far we proved by algebraic reasoning that T_n is not in general an operator in the Schwartz-space of testfunctions. It is instructive to prove the same directly for any single element φ_n of $\mathcal{S}_n^T$ ($n > 0$) - that these $\notin \mathcal{S}(\mathbb{R}^m)$:

Proof 3.2 (of ii.).

With eq.(2) and $\varphi \in \mathcal{S}(\mathbb{R})^m$, $T_n : \mathcal{S} \rightarrow \mathcal{S}_n$ we get

$$T_n(\varphi) = R_n(\varphi) = \int_a^x \frac{(x-t)^n}{n!} \varphi^{(n+1)}(t) \Big|_a dt.$$

Integrating by parts we trivially have :

$$\begin{aligned} T_n(\varphi(x)) &= -\frac{x^n}{n!} \varphi^{(n)}(x) \Big|_0 \\ &\quad + \int_a^x \frac{(x-t)^{n-1}}{(n-1)!} \varphi^{(n)}(x) \Big|_0 dt \\ &= -j_n^n(\varphi(x)) + T_{n-1}(\varphi(x)) \\ &= -j_{n-1}^n(\varphi(x)) + T_{n-2}(\varphi(x)) \\ &\quad \vdots \\ &= -j_0^n(\varphi(x)) + \varphi(x). \end{aligned} \quad (4)$$

If now $T_n(\varphi(x))$ were a proper testfunction, then $\lim_{x \rightarrow \pm\infty} T_n(\varphi(x)) = 0$. But here we have the sum of a bounded function ($\varphi(x)$) and an unbounded polynomial in x , $j_0^n(\varphi(x))$; all in all an unbounded function and therefore $T_n(\varphi) \in \mathcal{S}_n^T \not\subset \mathcal{S}(\mathbb{R}^m)$. $\square$

It is important to note that no reordering of the Taylorsum has been done, as this in our situation

is not allowed.

Interestingly, we see that truncation of the jet $j_0^n(\varphi(x))$ from $\varphi(x)$ by T_n , the resulting function $T_n(\varphi(x))$ is for $x \rightarrow \pm\infty$ divergent with (in the worst case, see next sentence) **the subtracted jet as the divergence**. The above sum in eq.(4) might, under certain circumstances, be written in another way. For example if $T_n(\varphi(x)) = j_0^l(\varphi(x)) + \psi(x)$ for some $n > l > 0$ and $\psi(x) \in \mathcal{S}(\mathbb{R}^m)$ as well. Nevertheless, the proof and divergence still hold even for that situation.

We conclude that in a subtraction scheme where it is possible to rewrite a function, which is an element of an operator's range or spectrum, in a decomposed way as in eq.(4) as a testfunction and some jet (i.e. some unbounded polynomial), one has to carefully check whether such a linear operator really is an endomorphism in $\mathcal{S}$ or not.

4 The T_w^n -operator

As mentioned in the beginning, our operator T_w^n is the same as T_w used in [2], where here for the sake of clarity the index n is also mentioned in the operator's name. Explicitely, T_w^n is defined as :

$$\begin{aligned} T_w^n : \mathcal{S}(\mathbb{R}^m) &\rightarrow \mathcal{S}_n^{T_w}, \quad \text{where} \\ \varphi &\mapsto T_w^n(\varphi) \\ &= T_w^n \left(\sum_{k=0}^{\infty} \frac{x^k}{k!} \varphi^{(k)}(x) \Big|_0 \right) \\ &:= \varphi(x) - j_0^{n-1}(\varphi(x)) - w \frac{x^n}{n!} \varphi^{(n)}(x) \Big|_0 \\ &= R_{n-1}(\varphi(x)) - w \frac{x^n}{n!} \varphi^{(n)}(x) \Big|_0, \end{aligned} \quad (5)$$

with $\mathcal{S}_n^{T_w}$ being the image of $\mathcal{S}$ under the action of T_w^n . w is a so called auxilliary function complying to $w(0) = 1$ (both in the original EG frame work as well as in [2]). Different than in the original Epstein-Glaser approach, according to [2] w does not need to be in $\mathcal{S}$ but "what is needed is that w decay at infinity in the weak sense that it sport momenta of sufficiently high order" (see [2]). The following steps will be similar to those in the preceding section. First we will derive that $T_w^n(\mathcal{S})$ is not a subvectorspace of $\mathcal{S}$ and afterwards that some elements of $T_w^n(\mathcal{S})$ are $\notin \mathcal{S}$.

With $\mathcal{S}_n^{T_w} := T_w^n(\mathcal{S})$ we claim:

Proposition.

- iii.) $\mathcal{S}_n^{T_w}$ is not a subvectorspace of $\mathcal{S}(\mathbb{R}^m) \forall n > 0$,
- iv.) $\mathcal{S}_n^{T_w} \cap \mathcal{S} \neq \mathcal{S}_n^{T_w}$, i.e. there exist elements of $\mathcal{S}_n^{T_w}$ which are not testfunctions.

Proof 4.1 (of iii.).

Let $\varphi \in \mathcal{S}(\mathbb{R}^m)$, $T_w^n : \mathcal{S}(\mathbb{R}^m) \rightarrow \mathcal{S}_n^{T_w}$, $T_w^{n'} : \mathcal{S}(\mathbb{R}^m) \rightarrow \mathcal{S}_{n'}^{T_w}$, $n, n' \in \mathbb{N}$, $n' \geq n > 0$, w an auxilliary function as above. $\mathcal{S}_n(\mathbb{R}^m)$ is the space of functions vanishing at the origin at the order of n . Obviously $\mathcal{S}_n \supset \mathcal{S}_{n'}$.

If $\mathcal{S}_n^{T_w}$ is a subset of $\mathcal{S}_n$, then it trivially follows that addition of elements of $\mathcal{S}_n^{T_w}$ is allowed in $\mathcal{S}_n$. As n was arbitrarily chosen, the same is then valid for $\mathcal{S}_{n'}$, $\mathcal{S}_{n'}^{T_w}$ and elements thereof as well - for some other $n' \neq n$.

With $\mathcal{S}_{n'}$ being a subset of $\mathcal{S}_n$, addition of elements of these two subvectorspaces is allowed, too. Automatically as a consequence $\mathcal{S}_{n'}^{T_w} \subseteq \mathcal{S}_{n'} \subseteq \mathcal{S}_n \subseteq \mathcal{S}$ (" $\subseteq$ " is here only meant as subset, not as subvectorspace). Therefore

$$T_w^n(\varphi) - T_w^{n'}(\varphi) \in \mathcal{S}_n(\mathbb{R}^m) \subset \mathcal{S}(\mathbb{R}^m),$$

and choosing $n' = n+1$ leads to:

$$\begin{aligned} &T_w^n(\varphi) - T_w^{n+1}(\varphi) \\ &= \frac{x^n}{n!} \varphi^{(n)}(x) \Big|_0 \\ &\quad - w \left(\frac{x^n}{n!} \varphi^{(n)}(x) \Big|_0 - \frac{x^{n+1}}{(n+1)!} \varphi^{(n+1)}(x) \Big|_0 \right) \end{aligned} \quad (6)$$

Eq.(6) requires as a necessary (but not sufficient) condition on T_w^n that, as $x \rightarrow \pm\infty$, w compensates not only in a multiplicative way the expression in brackets but also the added polynomial in x of (in this special case) degree n . For every pair (n, n') an adjusted w is needed, cancelling the last polynomial in (6) exactly. This is in contradiction with our initial, uniform definition of T_w^n . $\square$

At the end of proof 3.1 we tried to avoid contradictions by modifying our definitions and assumptions but were again led to the same result as before. The same applies here also. We remark :

- d.) A possible objection to our proof attacks the assumption of the auxilliary function w in T_w^n

being the same for all n . Indeed another definition is possible. But it results in the only difference that for $n' > n$ and corresponding w, w' one has:

$$\begin{aligned} & T_w^n(\varphi) - T_{w'}^{n'}(\varphi) \\ &= w' \frac{x^{n'}}{n'!} \varphi^{(n')}(x) \Big|_0 - w \frac{x^n}{n!} \varphi^{(n)}(x) \Big|_0 \\ &+ \sum_{k=n}^{n'-1} \frac{x^k}{k!} \varphi^{(k)}(x) \Big|_0, \end{aligned}$$

where we encounter similar trouble as above.

- e.) Some could argue that in physically meaningful calculations only distributions with a uniform singular order are examined and added/subtracted but not distributions with different ones (as we made heavily use of in proof 4.1). Abolishing those additions/subtractions we could doubt the relevance of proposition iii. as this situation would not happen.

However, we are not allowed to neglect addition/subtraction of distributions of different singular order. For instance, the vertex-function comprises a sum of distributions of different singular order where testfunctionsubtractions via the W -operator are to be performed individually within that sum, each according to the distribution's singular order (see [4], chapter 3.9). We see, again it does not help only allowing for addition of elements of $T_w^n(\mathcal{S})$ but not for those of $T_w^n(\mathcal{S})$ and $T_w^{n'}(\mathcal{S})$ for $n \neq n'$ (the essence of above's remark a and b).

We still have to prove (iv.), which can already be seen by the definition of T_w^n and is straight forward:

Proof 4.2 (of iv.).

Let $\varphi \in \mathcal{S}(\mathbb{R}^m)$, $T_w^n : \mathcal{S}(\mathbb{R}^m) \rightarrow \mathcal{S}_n^{T_w}$ as above. From eq.(5) we have:

$$\begin{aligned} T_w^n(\varphi(x)) &= \varphi(x) \\ &- \underbrace{j_0^{n-1}(\varphi(x))}_I - \underbrace{w \frac{x^n}{n!} \varphi^{(n)}(x) \Big|_0}_{II}, \end{aligned}$$

which is again a sum of a test function ($\varphi(x)$) and an unknown term ($-I-II$). To fulfill the test function requirement of $T_w^n(\varphi(x)) \rightarrow 0$ for $x \rightarrow \pm\infty$, the

unknown term has to $\rightarrow 0$, too, for large x . Thus an approximately cancellation of I by II , at least for large x , is needed. This is possible for a specific φ but not $\forall \varphi$ at the same time. For example one recognizes this by trying to let $-I-II = \varepsilon$ with some $\mathbb{R} \ni \varepsilon > 0$ for specific φ , n and then taking the limit $\varepsilon \rightarrow 0$. w then needs to depend on φ 's first n Taylor-terms and on n itself. This might work for many φ but certainly not for all. $T_w^n(\varphi(x))$ is not a test function in Schwartz space. $\square$

As in eq.(4), problems arose from the polynomial $j_0^{n-1}(\varphi(x))$ that forced us to search for compensating terms which do not exist in general.

Additionally we discuss the fact that, contrary to the stated requirements in [2] for w , where is is mentioned that " T_w is also a projector",

Proposition.

- v.) T_w^n is a projectionoperator (on whatever space) only if $w^{(k)}(0) = 0$ for $1 \leq k \leq n$.

Proof 4.3 (of v.).

If T_w^n were a projector, then idem potency is required, i.e. $T_w^n \circ T_w^n = T_w^n$. With

$$T_w^n \varphi(x) = R_{n-1} \varphi(x) - w \frac{x^n}{n!} \varphi^{(n)}(x) \Big|_0$$

we have

$$\begin{aligned} & T_w^n \circ \{T_w^n \varphi(x)\} \\ &= R_{n-1} \circ T_w^n \varphi(x) - w \frac{x^n}{n!} (T_w^n \varphi)^{(n)}(x) \Big|_0 \\ &= T_w^n \varphi(x) - w \frac{x^n}{n!} (T_w^n \varphi)^{(n)}(x) \Big|_0 \\ &= T_w^n \varphi(x) \\ &- w \frac{x^n}{n!} \left((R_{n-1} \varphi)^{(n)}(x) \Big|_0 \right) \\ &+ w \frac{x^n}{n!} \left\{ \left(w \frac{x^n}{n!} \varphi^{(n)}(x) \Big|_0 \right)^{(n)}(x) \Big|_0 \right\} \\ &= T_w^n \varphi(x) - \underbrace{w \frac{x^n}{n!} \varphi^{(n)}(0)}_I \\ &+ \underbrace{w \frac{x^n}{n!} \left(\sum_{l=0}^n \binom{n}{l} \partial_x^l w \cdot \partial_x^{n-l} \frac{x^n}{n!} \right) \Big|_0}_{II}. \end{aligned}$$

To get $T_w^n \circ T_w^n = T_w^n$, the two terms I and II have to cancel each other. This is identically (i.e. for

all x) achieved only if in the Bernoulli-sum (II) all those parts with $w^{(k)} = 0$, $1 \leq k \leq n$ vanish. In such a case we would have:

$$\begin{aligned} & \dots \\ &= T_w^n \varphi(x) - w \frac{x^n}{n!} \varphi^{(n)}(0) \left[1 - \left(w \cdot \partial^n \frac{x^n}{n!} \right) \Big|_0 \right] \\ &= T_w^n. \end{aligned}$$

But such constraints on w are not only not given in [2], they are explicitly relaxed and it is stated “Just $w(0) = 1$ is now required in principle from the weight function” (see [2]). Hence, T_n^w is not idem potent and thus not a projection operator in the general case. $\square$

5 The W-operator

The operator W , as it is defined in [1], p.236, reads as

$$\mathcal{S}_n \ni (W\varphi)(x) = \varphi(x) - w(x) \sum_{k=0}^{n-1} \frac{x^k}{k!} \varphi^{(k)}(x) \Big|_0,$$

with $w \in \mathcal{S}$, $w(0)=1$ and $w^{(k)}(0)=0$ for $1 \leq k \leq \omega$. Short calculations show that no difficulties arise with W like those we encountered with T_n and T_w^n . Propositions for W , similar to i. and ii. for T_n or iii. and iv. for T_w^n do not hold. This is mainly due to the w accompanying all terms of the subtracted jet $j_0^{n-1}\varphi$. Even the projectionoperator-condition is met, owing to the specific definition of W . The range of W is a valid subvectorspace of $\mathcal{S}$.

In the previous section we examined whether it was important to define the operator’s w in dependence of the vanishing-order n or not. We note that, if the w in W is independent of n , then the requirement

$$w^{(k)}(0) = 0 \quad \forall k > 0$$

is to be met. This can be seen by considering v., replacing T_w^n by W and recalling the nested subvectorspace-structure-arguments from proofs of i. and iii..

References

- [1] H.Epstein and V.Glaser, *The role of locality*, Ann. Inst. Henri Poincare A **XIX** (1973) 211

- [2] J.M.Gracia-Bondia, *Improved Epstein-Glaser renormalization in coordinate space I.Euclidean framework*, arXiv:hep-th/0202023v1

- [3] ANKuznetsov, F.V.Tkachov and V.V.Vlasov, *Techniques of distributions in perturbative quantum field theory I*, arXiv:hep-th/9612037v1

- [4] G.Scharf, *Finite Quantum Electrodynamics - The causal approach*, Springer, Berlin, 1995