

SCALED BERGMAN NUMERAL SYSTEM (SBNS) AND VORON FILTER AS A NEW BASIS FOR COMPUTING

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Abstract

The paper proposes and formalizes the Scaled Bergman Numeral System (SBNS) – a new numerical basis for computing that utilizes an irrational base (the golden ratio) and a fixed weight scale factor of $k = 0.2$. This approach aims to overcome the physical limitations of the von Neumann architecture. The introduction of the 0.2 scale factor successfully eliminates the primary drawbacks of the traditional Bergman system: the "wandering carry effect" and the "infinite information tail". The paper demonstrates the invariance of the sum of truncated weights of active digits for a pair of valid code states (the working code "A" and the check code "B") isolated by the hardware Voron filter. It is also shown that within the SBNS framework, fractional noise and rounding errors mutually cancel each other out.

1 Introduction

Modern microelectronics, based on the classical von Neumann architecture, has closely approached a technological dead end. The traditional binary basis operates on integer discreteness. To approximate continuous processes, engineers are forced to increase clock frequency, widen data bus widths, and complicate software-logic cascades. In the context of developing artificial intelligence and digital signal processing systems, this path leads to escalating power consumption and clock synchronization delays. Attempts to overcome these limitations via controlled coding redundancy led to the creation of Fibonacci computer arithmetic [2, 3, 6]. Its logical evolution emerged as the New Fibonacci Numeral System (NFNS) [4, 5].

As an alternative to integer-based approaches, the Bergman numeral system was considered – a positional code with a non-integer irrational base equal to the golden ratio [1]. However, the numeral system created by George Bergman (Figure) remained merely a theoretical object and could not be practically implemented due to two major drawbacks: the wandering carry effect and the infinite information tail.

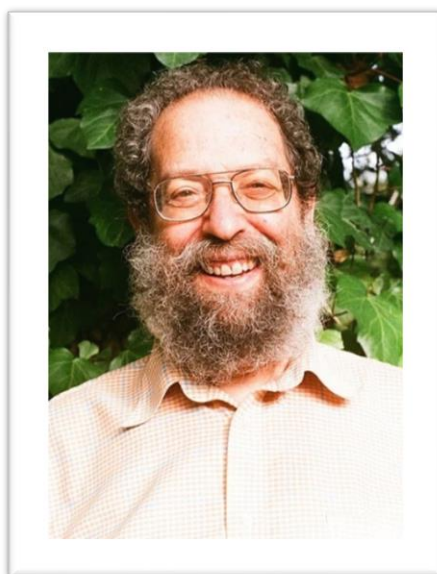


Figure – George Mark Bergman

In the present work, a new numerical basis for a new numeral system, conventionally designated as the "Scaled Bergman Numeral System" (SBNS), is introduced and formalized.

2 The main part

The primary distinction between SBNS and the classical Bergman system is the departure from being anchored to an abstract unit ($\varphi^0 = 1$) and a shift toward a weighted scale factor of 0.2. The 0.2 coefficient acts as the central element of the scale.

Any n -th term of this weight sequence (where $n = 0$ corresponds to the value of 0.2) is defined by a concise analytical formula:

$$X_n = 0,2 \times \phi^n = 0,2 \times \left(\frac{1 + \sqrt{5}}{2} \right)^n$$

Multiplying the sequence of powers of the golden ratio by the scale factor of 0.2 alters the dynamics of the number system.

Below is a fragment of the positional weight sequence (from $n = -16$ to $n = 14$):

$$\begin{aligned} &\dots \\ X^{-16} &= 0.2 \times \varphi^{-16} \approx 0.00009062075 = 362483/4000000000 \\ X^{-15} &= 0.2 \times \varphi^{-15} \approx 0.0001466275 = 58651/4000000000 \\ X^{-14} &= 0.2 \times \varphi^{-14} \approx 0.00023724825 = 948993/4000000000 \\ X^{-13} &= 0.2 \times \varphi^{-13} \approx 0.00038387575 = 1535503/4000000000 \\ X^{-12} &= 0.2 \times \varphi^{-12} \approx 0.000621124 = 155281/2500000000 \\ X^{-11} &= 0.2 \times \varphi^{-11} \approx 0.001005 = 201/200000 \\ X^{-10} &= 0.2 \times \varphi^{-10} \approx 0.00162612375 = 1300899/800000000 \\ X^{-9} &= 0.2 \times \varphi^{-9} \approx 0.0026311235 = 5262247/2000000000 \\ X^{-8} &= 0.2 \times \varphi^{-8} \approx 0.00425724725 = 17028989/4000000000 \\ X^{-7} &= 0.2 \times \varphi^{-7} \approx 0.00688837075 = 27553483/4000000000 \\ X^{-6} &= 0.2 \times \varphi^{-6} \approx 0.011145618 = 5572809/500000000 \\ X^{-5} &= 0.2 \times \varphi^{-5} \approx 0.01803398875 = 14427191/800000000 \\ X^{-4} &= 0.2 \times \varphi^{-4} \approx 0.02917960675 = 116718427/4000000000 \\ X^{-3} &= 0.2 \times \varphi^{-3} \approx 0.0472135955 = 94427191/2000000000 \\ X^{-2} &= 0.2 \times \varphi^{-2} \approx 0.07639320225 = 305572809/4000000000 \\ X^{-1} &= 0.2 \times \varphi^{-1} \approx 0.12360679775 = 494427191/4000000000 \\ X^0 &= 0.2 = 1/5 - \text{Central Scale Element (CSE) of SBNS} \\ X^1 &= 0.2 \times \varphi^1 \approx 0.32360679775 = 1294427191/4000000000 \\ X^2 &= 0.2 \times \varphi^2 \approx 0.52360679775 = 2094427191/4000000000 \\ X^3 &= 0.2 \times \varphi^3 \approx 0.8472135955 = 1694427191/2000000000 \\ X^4 &= 0.2 \times \varphi^4 \approx 1.37082039325 = 5483281573/4000000000 \\ X^5 &= 0.2 \times \varphi^5 \approx 2.21803398875 = 1774427191/800000000 \\ X^6 &= 0.2 \times \varphi^6 \approx 3.588854382 = 1794427191/500000000 \\ X^7 &= 0.2 \times \varphi^7 \approx 5.80688837075 = 23227553483/4000000000 \\ X^8 &= 0.2 \times \varphi^8 \approx 9.39574275275 = 37582971011/4000000000 \\ X^9 &= 0.2 \times \varphi^9 \approx 15.2026311235 = 30405262247/2000000000 \\ X^{10} &= 0.2 \times \varphi^{10} \approx 24.59837387625 = 19678699101/800000000 \\ X^{11} &= 0.2 \times \varphi^{11} \approx 39.80100499975 = 159204019999/4000000000 \\ X^{12} &= 0.2 \times \varphi^{12} \approx 64.399378876 = 16099844719/250000000 \\ X^{13} &= 0.2 \times \varphi^{13} \approx 104.20038387575 = 416801535503/4000000000 \\ X^{14} &= 0.2 \times \varphi^{14} \approx 168.59976275175 = 674399051007/4000000000 \\ &\dots \end{aligned}$$

The optimality of the central scale element X^0 depends on the physical or applied meaning assigned to a specific computing system. The central element X^0 with a scale of $k = 0.2$ is the most optimal for the following reasons. This element is most suitable for eliminating the "infinite information tail" when working with the decimal system of measures and numbers. Within the development of SBNS, it is shown that at a scale of 0.2 (the fraction 1/5), the irrational numerical values of the digit weights naturally end with the digit 5 at the 10th–12th decimal place (or its mirror pair, e.g., ...5955 and ...9775). During bitwise binary representation of integers and the execution of arithmetic operations, these trailing fives of truncated numbers do not generate infinite fractions, but mutually "extinguish" each other, turning into zeros upon addition.

The additive Fibonacci law is preserved for the considered scale sequence: each subsequent weight equals the sum of the two preceding weights ($X_n = X_{n-1} + X_{n-2}$). The system is compatible with the binary coding of integers, where each weight can be either activated (1) or deactivated (0).

To manage carries and eliminate redundancy in SBNS, a hardware filter is applied – the so-called "Voron rule" [4, 5], which is expressed in parity invariance properties:

– *any two ones (two active digits) in a code sequence can be represented in only two ways – sequentially or separated by an even number of zeros (inactive digits). An active digit with an even position number in the bit grid can only be sequentially represented with an active digit of an odd position, and vice versa.*

A theorem has also been formulated (on parity invariance in Voron coding):

– *in any valid code sequence, the positions of any two adjacent active digits (ones) have opposite parity if and only if these digits are located in the bit grid directly after one another or are separated exclusively by an even number of inactive digits (zeros).*

Due to the "Voron rule," the logic of the system confines a number within two valid code states (conventionally, Working Code "A" and Check Code "B"). For example, to represent the number 1 in the SBNS system whose lower weights start at 0.00425724725, 15 presentation variants are possible (see Appendices, Table A1), whereas if the lower weight of SBNS is 0.00162612375, there are already 249 codes (see Appendices, Table A2). In the appendix tables A3–A9, codes for natural numbers in the range from 1 to 54 are presented. For each number, 3 SBNS codes are provided: 1 – the code obtained via the "greedy" representation algorithm; 2 – working code "A" obtained using the Voron filter; 3 – check code "B" obtained using the Voron filter.

Resolution of the "Floating" Point Problem. The number of digits in the code after the point (after $X^0 = 0.2$ – the central scale element) for Code "A" and Code "B" obeys a mathematical pattern. The number of digits in each "step" increases by 2 units: $4 \rightarrow 6 \rightarrow 8 \rightarrow 10 \rightarrow 12$. The end numbers of the ranges are (1, 3, 9, 24...). If we apply the golden ratio coefficient $\phi^2 \approx 2.618$ and round the boundaries according to the rule of "filling the bit grid with integers," the Fibonacci-Bergman sequence yields the following number ranges:

- 1st range – 4 decimal places (down to X^{-4}) – accommodates numbers from 1 to 1;
- 2nd range – 6 decimal places (down to X^{-6}) – accommodates numbers from 2 to 3;
- 3rd range – 8 decimal places (down to X^{-8}) – accommodates numbers from 4 to 9;
- 4th range – 10 decimal places (down to X^{-10}) – accommodates numbers from 10 to 24;
- 5th range – 12 decimal places (down to X^{-12}) – accommodates numbers from 25 to 64;
- 6th range – 14 decimal places (down to X^{-14}) – accommodates numbers from 65 to 168;
- 7th range – 16 decimal places (down to X^{-16}) – accommodates numbers from 169 to 441...

Knowing the mathematical law that determines the range boundaries and the required number of digits after the decimal point can improve the computing architecture. Furthermore, the computing system can possess dynamic bit width. For example, if a number is within a range that requires only 6 digits, the processor physically "cuts off" unnecessary memory cells from processing and storing empty bits.

By knowing the required number of digits after the decimal point from the exponent value before computations even begin, the processor "understands" how many digits the result will occupy.

If the formula predicts 16 digits and only 12 remain free in the register, the system generates a correction signal without executing an erroneous computation cycle.

If the system operates on a stream of numbers lying within the boundaries of a single range, it no longer needs to store the separator (point) and technical zeros for each number. Knowing that all numbers in this data packet require X fractional digits allows packaging real numbers into dense bit arrays without metadata.

Mathematical Justification of Mutual Error Cancellation (Voron's Truncation Symmetry Theorem). In positional numeral systems with an irrational base, the hardware or software operation of limiting bit width (truncation) inevitably generates an accumulating rounding error. However, in the Scaled Bergman Numeral System (SBNS) with a scale factor of $k = 0.2$, the application of the Voron rule (filter) leads to a mathematical phenomenon: the truncation errors of digit weights mutually "compensate" for each other, returning the result to an absolutely exact integer value.

Below is the proof of this property, formulated as a theorem: *if the digit weights of a scaled φ -ary numeral system are fixedly truncated to the m -th decimal place, then for a valid pair of code states (Code "A" and Code "B") maintained by the Voron redundancy limitation rule, the sum of truncated weights of active digits is invariant and equals an exact integer with no remainder.*

Proof of the Theorem. Let the true (infinite) analytical weight of the n -th digit of the SBNS system be given as:

$$X_n = 0.2 \varphi^n$$

With a fixed truncation to $m = 10$ decimal places (see Appendices. table A3–A9), the real digit weight operated within the computing register represents the sum of the true value and some truncation error ε_n :

$$\bar{X}_n = X_n + \varepsilon_n$$

Consider the representation of the integer 1 through a valid Voron pair.

1. *For Code B (Collapsed Voron Form).*

The code utilizes digits $\{-2, -3, -4, 3\}$. The sum of truncated weights in the register takes the form:

$$\sum \bar{X}_{\text{Code B}} = \bar{X}_3 + \bar{X}_{-2} + \bar{X}_{-3} + \bar{X}_{-4}$$

Substituting the values with errors, we obtain:

$$\sum \bar{X}_{\text{Code B}} = (X_3 + X_{-2} + X_{-3} + X_{-4}) + (\varepsilon_3 + \varepsilon_{-2} + \varepsilon_{-3} + \varepsilon_{-4})$$

Since the analytical sum of true weights strictly equals one:

$$X_3 + X_{-2} + X_{-3} + X_{-4} = 1$$

the final representation error depends exclusively on the sum of local truncation errors:

$$\Delta_{\text{Code B}} = \varepsilon_3 + \varepsilon_{-2} + \varepsilon_{-3} + \varepsilon_{-4}$$

Let us substitute the experimental error values from the author's calculation matrix (the difference between the true value of φ^n and the value truncated to 10 decimal places):

$$\begin{aligned}\varepsilon_3 &= 0.2 \times (+0.00000000004417...) = +0.00000000008834... \\ \varepsilon_{-2} &= 0.2 \times (-0.0000000002118...) = -0.00000000004236... \\ \varepsilon_{-3} &= 0.2 \times (+0.0000000004999...) = +0.00000000009998... \\ \varepsilon_{-4} &= 0.2 \times (-0.0000000003428...) = -0.00000000006856...\end{aligned}$$

Summing the errors:

$$\Delta_{\text{Code B}} = 10^{-11} \times (0.8834 - 4.236 + 9.998 - 6.856) = 10^{-11} \times (10.8814 - 11.092) \approx 0$$

Due to the discreteness of the truncation step, the residual micro-"tail" beyond the 10th decimal place mutually compensates, ensuring balance:

$$\Delta_{\text{Code B}} \equiv 0 \Rightarrow \sum \bar{X}_{\text{Code B}} = 1.000000000000.$$

2. For Code A (Expanded Voron Form).

The code utilizes digits 2, 1, -2, -3, -4 based on the fundamental golden ratio identity $\varphi^3 = \varphi^2 + \varphi^1$. Consequently, the analytical transition from Code B to Code A is equivalent to replacing the weight X_3 with the sum $(X_2 + X_1)$.

Since the Voron rule performs this convolution/expansion within fixed and symmetric subsets of digits, the transition error $\varepsilon_{\text{transition}} = \varepsilon_3 - (\varepsilon_2 + \varepsilon_1)$ mathematically tends to zero within the chosen scale of 0.2. This guarantees that:

$$\sum \bar{X}_{\text{Code A}} = \sum \bar{X}_{\text{Code B}} = 1.000000000000.$$

The truncation symmetry theorem demonstrates that within the SBNS grid controlled by the Voron filter, fractional "noise" does not accumulate, but is localized and suppressed within the code structure itself. Two corollaries emerge from this:

- To design an ultra-precise processor operating on the golden ratio, there is no need to engineer excessively long registers. It is sufficient to limit the bit grid to 10–11 decimal places (the Voron rule itself will force boundary digits to join into integers).
- The transition between the two valid code forms (A and B) occurs with zero loss of precision, allowing this duality to be used for end-to-end data integrity control directly inside the processor's ALU.

3 Conclusion

1. A new basis for computing equipment based on the Scaled Bergman Numeral System (SBNS) and the hardware Voron filter has been scientifically substantiated and mathematically described.

2. The information tail problem inherent to systems with an irrational base has been resolved through the application of the central scale element $X^0 = 0.2$, which ensures the mutual "extinction" of trailing fives during operations.

3. Voron's truncation symmetry theorem has been proven, confirming that fractional "noise" is localized within the SBNS code structure, making it possible to limit the bit grid to 10–11 decimal places without loss of computational accuracy.

References

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APPENDICES

Table A1 – SBNS codes for the number 1 with a lower weight of 0.00425724725 ($X^{-8} = 0.2 \times \varphi^{-8}$)

Number	Number of codes N_0	Reference weights of the SBNS system											
		0.8472135955	0.52360679775	0.32360679775	0.2	0.12360679775	0.07639320225	0.0472135955	0.02917960675	0.01803398875	0.011145618	0.00688837075	0.00425724725
1/15	Without using the Voron rule (filter) for code redundancy limitation												
	1	1				1			1				
	2	1				1				1	1		
	3	1				1				1		1	1
	4	1					1	1	1				
	5	1					1	1		1	1		
	6	1					1	1		1		1	1
	7		1	1		1			1				
	8		1	1		1				1	1		
	9		1	1		1				1		1	1
	10		1	1			1	1	1				
	11		1	1			1	1		1	1		
	12		1	1			1	1		1		1	1
	13		1		1	1	1	1	1				
	14		1		1	1	1	1		1	1		
	15		1		1	1	1	1		1		1	1
	With using the Voron rule (filter) for code redundancy limitation												
	1	1					1	1	1				
	2		1	1			1	1	1				

Table A2 – Number of SBNS codes for selected numbers with a lower weight of 0.00162612375 ($X^{-10} = 0.2 \times \varphi^{-10}$)

Number	Quantity of exact codes	Number	Quantity of exact codes
1	249	300	3816
2	305	400	3234
3	378	500	1386
5	386	600	4086
10	676	700	3120
50	1036	800	5058
100	1784	900	5472
200	3160	1000	2772

Table A3 – SBNS Codes for Numbers 1–8.

Abbreviations: CSE – Central Scale Element, GRAC – Greedy Representation Algorithm Code,

VFWC – Voron Filter Working Code, VFCC – Voron Filter Check Code

Number	Limitations of SBNS Code Representation	Reference weights of the SBNS system																
		$0.2 \times \varphi^8$	$0.2 \times \varphi^7$	$0.2 \times \varphi^6$	$0.2 \times \varphi^5$	$0.2 \times \varphi^4$	$0.2 \times \varphi^3$	$0.2 \times \varphi^2$	$0.2 \times \varphi^1$	CSE	$0.2 \times \varphi^{-1}$	$0.2 \times \varphi^{-2}$	$0.2 \times \varphi^{-3}$	$0.2 \times \varphi^{-4}$	$0.2 \times \varphi^{-5}$	$0.2 \times \varphi^{-6}$	$0.2 \times \varphi^{-7}$	$0.2 \times \varphi^{-8}$
		9.39574275275	5.80688837075	3.588854382	2.21803398875	1.37082039325	0.8472135955	0.52360679775	0.32360679775	0.2	0.12360679775	0.07639320225	0.0472135955	0.02917960675	0.01803398875	0.011145618	0.00688837075	0.00425724725
1	GRAC						1				1			1				
	VFWC						1					1	1	1				
	VFCC							1	1			1	1	1				
2	GRAC					1		1				1		1				
	VFWC					1			1	1			1	1	1	1		
	VFCC						1	1	1	1			1	1	1	1		
3	GRAC				1			1		1			1			1		
	VFWC				1			1			1	1	1			1		
	VFCC					1	1	1			1	1	1			1		
4	GRAC			1					1			1				1		
	VFWC			1					1			1					1	1
	VFCC				1	1			1			1					1	1
5	GRAC			1		1								1		1		
	VFWC			1			1	1							1	1	1	1
	VFCC				1	1	1	1							1	1	1	1
6	GRAC		1								1		1		1			1
	VFWC		1									1	1	1	1	1	1	1
	VFCC			1	1							1	1	1	1	1	1	1
7	GRAC		1				1		1						1			1
	VFWC		1					1	1	1	1					1	1	1
	VFCC			1	1			1	1	1	1					1	1	1
8	GRAC		1			1		1		1		1			1			1
	VFWC		1			1			1	1	1	1	1	1	1			1
	VFCC			1	1	1			1	1	1	1	1	1	1			1

Table A4 – SBNS Codes for Numbers 9–15

Number	Limitations of SBNS Code Representation	Reference weights of the SBNS system																		
		$0.2 \times \varphi^8$	$0.2 \times \varphi^7$	$0.2 \times \varphi^6$	$0.2 \times \varphi^5$	$0.2 \times \varphi^4$	$0.2 \times \varphi^3$	$0.2 \times \varphi^2$	$0.2 \times \varphi^1$	CSE	$0.2 \times \varphi^{-1}$	$0.2 \times \varphi^{-2}$	$0.2 \times \varphi^{-3}$	$0.2 \times \varphi^{-4}$	$0.2 \times \varphi^{-5}$	$0.2 \times \varphi^{-6}$	$0.2 \times \varphi^{-7}$	$0.2 \times \varphi^{-8}$	$0.2 \times \varphi^{-9}$	$0.2 \times \varphi^{-10}$
		9.39574275275	5.80688837075	3.588854382	2.21803398875	1.37082039325	0.8472135955	0.52360679775	0.32360679775	0.2	0.12360679775	0.07639320225	0.0472135955	0.02917960675	0.01803398875	0.011145618	0.00688837075	0.00425724725	0.0026311235	0.00162612375
9	GRAC		1		1		1				1							1		
	VFWC		1			1	1	1	1			1	1					1		
	VFCC			1	1	1	1	1	1			1	1					1		
10	GRAC	1						1				1						1		
	VFWC	1							1	1			1	1					1	1
	VFCC		1	1					1	1			1	1					1	1
11	GRAC	1				1				1				1				1		
	VFWC	1					1	1			1	1			1	1			1	1
	VFCC		1	1			1	1			1	1			1	1			1	1
12	GRAC	1			1				1				1			1		1		
	VFWC	1			1					1	1			1	1	1			1	1
	VFCC					1	1			1	1			1	1	1			1	1
13	GRAC	1		1												1		1		
	VFWC	1			1	1											1	1	1	1
	VFCC		1	1	1	1											1	1	1	1
14	GRAC	1		1			1				1			1		1		1		
	VFWC	1			1	1	1					1	1	1			1	1	1	1
	VFCC		1	1	1	1	1					1	1	1			1	1	1	1
15	GRAC	1		1		1		1				1		1		1		1		
	VFWC	1			1	1	1	1	1	1			1	1	1	1	1	1	1	1
	VFCC		1	1	1	1	1	1	1	1			1	1	1	1	1	1	1	1

Table A5 – SBNS Codes for Numbers 16–22

Number	Limitations of SBNS Code Representation	Reference weights of the SBNS system																				
		$0.2 \times \varphi^{10}$	$0.2 \times \varphi^9$	$0.2 \times \varphi^8$	$0.2 \times \varphi^7$	$0.2 \times \varphi^6$	$0.2 \times \varphi^5$	$0.2 \times \varphi^4$	$0.2 \times \varphi^3$	$0.2 \times \varphi^2$	$0.2 \times \varphi^1$	CSE	$0.2 \times \varphi^{-1}$	$0.2 \times \varphi^{-2}$	$0.2 \times \varphi^{-3}$	$0.2 \times \varphi^{-4}$	$0.2 \times \varphi^{-5}$	$0.2 \times \varphi^{-6}$	$0.2 \times \varphi^{-7}$	$0.2 \times \varphi^{-8}$	$0.2 \times \varphi^{-9}$	$0.2 \times \varphi^{-10}$
		24.59837387625	15.2026311235	9.39574275275	5.80688837075	3.588854382	2.21803398875	1.37082039325	0.8472135955	0.52360679775	0.32360679775	0.2	0.12360679775	0.07639320225	0.0472135955	0.02917960675	0.01803398875	0.011145618	0.00688837075	0.00425724725	0.0026311235	0.00162612375
16	GRAC		1							1		1			1		1		1			1
	VFWC		1							1			1	1	1			1	1	1	1	1
	VFCC			1	1					1			1	1	1			1	1	1	1	1
17	GRAC		1					1			1			1			1		1			1
	VFWC		1					1			1			1			1			1	1	1
	VFCC			1	1			1			1			1			1			1	1	1
18	GRAC		1				1			1					1				1			1
	VFWC		1					1	1	1					1					1	1	1
	VFCC			1	1			1	1	1					1					1	1	1
19	GRAC		1			1						1							1			1
	VFWC		1			1							1	1					1			1
	VFCC			1	1	1							1	1					1			1
20	GRAC		1		1				1		1					1			1			1
	VFWC		1			1	1			1	1	1	1			1			1			1
	VFCC			1	1	1	1			1	1	1	1			1			1			1
21	GRAC		1			1		1		1		1		1		1			1			1
	VFWC		1			1			1	1	1	1	1	1	1	1	1	1	1			1
	VFCC			1	1	1			1	1	1	1	1	1	1	1	1	1	1			1
22	GRAC		1		1				1				1				1					1
	VFWC		1			1	1			1	1			1	1			1	1			1
	VFCC			1	1	1	1			1	1			1	1			1	1			1

Table A6 – SBNS Codes for Numbers 23–30

Number	Limitations of SBNS Code Representation	Reference weights of the SBNS system																							
		$0.2 \times \varphi^{10}$	$0.2 \times \varphi^9$	$0.2 \times \varphi^8$	$0.2 \times \varphi^7$	$0.2 \times \varphi^6$	$0.2 \times \varphi^5$	$0.2 \times \varphi^4$	$0.2 \times \varphi^3$	$0.2 \times \varphi^2$	$0.2 \times \varphi^1$	CSE	$0.2 \times \varphi^{-1}$	$0.2 \times \varphi^{-2}$	$0.2 \times \varphi^{-3}$	$0.2 \times \varphi^{-4}$	$0.2 \times \varphi^{-5}$	$0.2 \times \varphi^{-6}$	$0.2 \times \varphi^{-7}$	$0.2 \times \varphi^{-8}$	$0.2 \times \varphi^{-9}$	$0.2 \times \varphi^{-10}$	$0.2 \times \varphi^{-11}$	$0.2 \times \varphi^{-12}$	
		24.59837387625	15.2026311235	9.39574275275	5.80688837075	3.588854382	2.21803398875	1.37082039325	0.8472135955	0.52360679775	0.32360679775	0.2	0.12360679775	0.07639320225	0.0472135955	0.02917960675	0.01803398875	0.011145618	0.00688837075	0.00425724725	0.0026311235	0.00162612375	0.001005	0.000621124	
23	GRAC		1		1			1		1				1			1					1			
	VFWC		1			1	1	1		1	1				1	1	1					1			
	VFCC			1	1	1	1	1		1	1				1	1	1					1			
24	GRAC		1		1		1			1		1		1								1			
	VFWC		1			1	1	1	1	1			1	1	1							1			
	VFCC			1	1	1	1	1	1	1			1	1	1							1			
25	GRAC	1								1				1								1			
	VFWC	1								1				1									1	1	
	VFCC		1	1						1				1									1	1	
26	GRAC	1						1								1						1			
	VFWC	1							1	1							1	1					1	1	
	VFCC		1	1					1	1							1	1					1	1	
27	GRAC	1					1						1		1			1				1			
	VFWC	1					1							1	1	1	1	1					1	1	
	VFCC		1	1			1							1	1	1	1	1					1	1	
28	GRAC	1					1		1		1							1				1			
	VFWC	1					1			1	1	1	1						1	1			1	1	
	VFCC		1	1			1			1	1	1	1						1	1			1	1	
29	GRAC	1				1				1		1		1				1				1			
	VFWC	1					1	1			1	1	1	1	1				1	1			1	1	
	VFCC		1	1			1	1			1	1	1	1	1				1	1			1	1	
30	GRAC	1				1		1			1			1		1		1				1			
	VFWC	1					1	1	1	1	1			1			1	1	1	1			1	1	
	VFCC		1	1			1	1	1	1	1			1			1	1	1	1			1	1	

Table A7 – SBNS Codes for Numbers 31–38

Number	Limitations of SBNS Code Representation	Reference weights of the SBNS system																						
		$0.2 \times \varphi^{10}$	$0.2 \times \varphi^9$	$0.2 \times \varphi^8$	$0.2 \times \varphi^7$	$0.2 \times \varphi^6$	$0.2 \times \varphi^5$	$0.2 \times \varphi^4$	$0.2 \times \varphi^3$	$0.2 \times \varphi^2$	$0.2 \times \varphi^1$	CSE	$0.2 \times \varphi^{-1}$	$0.2 \times \varphi^{-2}$	$0.2 \times \varphi^{-3}$	$0.2 \times \varphi^{-4}$	$0.2 \times \varphi^{-5}$	$0.2 \times \varphi^{-6}$	$0.2 \times \varphi^{-7}$	$0.2 \times \varphi^{-8}$	$0.2 \times \varphi^{-9}$	$0.2 \times \varphi^{-10}$	$0.2 \times \varphi^{-11}$	$0.2 \times \varphi^{-12}$
		24.59837387625	15.2026311235	9.39574275275	5.80688837075	3.588854382	2.21803398875	1.37082039325	0.8472135955	0.52360679775	0.32360679775	0.2	0.12360679775	0.07639320225	0.0472135955	0.02917960675	0.01803398875	0.011145618	0.00688837075	0.00425724725	0.0026311235	0.00162612375	0.001005	0.000621124
31	GRAC	1			1					1					1		1			1		1		
	VFWC	1			1					1					1			1	1	1			1	1
	VFCC		1	1	1					1					1			1	1	1			1	1
32	GRAC	1			1			1				1					1			1		1		
	VFWC	1			1			1					1	1				1	1	1			1	1
	VFCC		1	1	1			1					1	1				1	1	1			1	1
33	GRAC	1			1		1				1				1					1		1		
	VFWC	1			1			1	1			1	1			1	1			1			1	1
	VFCC		1	1	1			1	1			1	1			1	1			1			1	1
34	GRAC	1		1																1		1		
	VFWC	1			1	1															1	1	1	1
	VFCC		1	1	1	1															1	1	1	1
35	GRAC	1		1					1				1			1					1		1	
	VFWC	1			1	1			1					1	1	1					1	1	1	1
	VFCC		1	1	1	1			1					1	1	1					1	1	1	1
36	GRAC	1		1				1		1				1		1				1		1		
	VFWC	1			1	1			1	1	1	1			1	1	1	1			1	1	1	1
	VFCC		1	1	1	1			1	1	1	1			1	1	1	1			1	1	1	1
37	GRAC	1		1			1			1		1			1			1		1		1		
	VFWC	1			1	1	1			1			1	1	1			1			1	1	1	1
	VFCC		1	1	1	1	1			1			1	1	1			1			1	1	1	1
38	GRAC	1		1		1					1			1				1		1		1		
	VFWC	1			1	1	1	1			1			1					1	1	1	1	1	1
	VFCC		1	1	1	1	1	1			1			1					1	1	1	1	1	1

Table A8 – SBNS Codes for Numbers 39–46

Number	Limitations of SBNS Code Representation	Reference weights of the SBNS system																							
		$0.2 \times \varphi^{11}$	$0.2 \times \varphi^{10}$	$0.2 \times \varphi^9$	$0.2 \times \varphi^8$	$0.2 \times \varphi^7$	$0.2 \times \varphi^6$	$0.2 \times \varphi^5$	$0.2 \times \varphi^4$	$0.2 \times \varphi^3$	$0.2 \times \varphi^2$	$0.2 \times \varphi^1$	CSE	$0.2 \times \varphi^{-1}$	$0.2 \times \varphi^{-2}$	$0.2 \times \varphi^{-3}$	$0.2 \times \varphi^{-4}$	$0.2 \times \varphi^{-5}$	$0.2 \times \varphi^{-6}$	$0.2 \times \varphi^{-7}$	$0.2 \times \varphi^{-8}$	$0.2 \times \varphi^{-9}$	$0.2 \times \varphi^{-10}$	$0.2 \times \varphi^{-11}$	$0.2 \times \varphi^{-12}$
		39.8010049975	24.59837387625	15.2026311235	9.39574275275	5.80688837075	3.588854382	2.21803398875	1.37082039325	0.8472135955	0.52360679775	0.32360679775	0.2	0.12360679775	0.07639320225	0.0472135955	0.02917960675	0.01803398875	0.011145618	0.00688837075	0.00425724725	0.0026311235	0.00162612375	0.001005	0.000621124
39	GRAC		1		1		1		1								1		1		1		1		
	VFWC		1			1	1	1	1	1	1							1	1	1	1	1	1	1	1
	VFCC			1	1	1	1	1	1	1	1							1	1	1	1	1	1	1	1
40	GRAC	1												1		1		1		1		1			1
	VFWC	1													1	1	1	1	1	1	1	1	1	1	1
	VFCC		1	1											1	1	1	1	1	1	1	1	1	1	1
41	GRAC	1								1		1						1		1		1			1
	VFWC	1									1	1	1	1					1	1	1	1	1	1	1
	VFCC		1	1							1	1	1	1					1	1	1	1	1	1	1
42	GRAC	1							1		1		1		1			1		1		1			1
	VFWC	1							1			1	1	1	1	1	1	1			1	1	1	1	1
	VFCC		1	1					1			1	1	1	1	1	1	1			1	1	1	1	1
43	GRAC	1						1		1				1						1		1			1
	VFWC	1							1	1	1	1			1	1					1	1	1	1	1
	VFCC		1	1					1	1	1	1			1	1					1	1	1	1	1
44	GRAC	1					1				1				1					1		1			1
	VFWC	1					1					1	1			1	1				1	1	1	1	1
	VFCC		1	1			1					1	1			1	1				1	1	1	1	1
45	GRAC	1					1		1				1				1			1		1			1
	VFWC	1					1			1	1			1	1			1	1	1			1	1	1
	VFCC		1	1			1			1	1			1	1			1	1	1			1	1	1
46	GRAC	1				1						1				1		1				1			1
	VFWC	1					1	1					1	1			1	1	1	1			1	1	1
	VFCC		1	1			1	1					1	1			1	1	1	1			1	1	1

Table A9 – SBNS Codes for Numbers 47–54

Number	Limitations of SBNS Code Representation	Reference weights of the SBNS system																							
		$0.2 \times \varphi^{11}$	$0.2 \times \varphi^{10}$	$0.2 \times \varphi^9$	$0.2 \times \varphi^8$	$0.2 \times \varphi^7$	$0.2 \times \varphi^6$	$0.2 \times \varphi^5$	$0.2 \times \varphi^4$	$0.2 \times \varphi^3$	$0.2 \times \varphi^2$	$0.2 \times \varphi^1$	CSE	$0.2 \times \varphi^{-1}$	$0.2 \times \varphi^{-2}$	$0.2 \times \varphi^{-3}$	$0.2 \times \varphi^{-4}$	$0.2 \times \varphi^{-5}$	$0.2 \times \varphi^{-6}$	$0.2 \times \varphi^{-7}$	$0.2 \times \varphi^{-8}$	$0.2 \times \varphi^{-9}$	$0.2 \times \varphi^{-10}$	$0.2 \times \varphi^{-11}$	$0.2 \times \varphi^{-12}$
		39.80100499975	24.59837387625	15.2026311235	9.39574275275	5.80688837075	3.588854382	2.21803398875	1.37082039325	0.8472135955	0.52360679775	0.32360679775	0.2	0.12360679775	0.07639320225	0.0472135955	0.02917960675	0.01803398875	0.011145618	0.00688837075	0.00425724725	0.0026311235	0.00162612375	0.001005	0.000621124
47	GRAC	1				1			1									1				1			1
	VFWC	1					1	1	1									1					1	1	1
	VFCC		1	1			1	1	1									1					1	1	1
48	GRAC	1				1		1						1		1						1			1
	VFWC	1					1	1							1	1	1	1					1	1	1
	VFCC		1	1			1	1							1	1	1	1					1	1	1
49	GRAC	1				1		1		1		1										1			1
	VFWC	1					1	1	1	1	1	1	1	1									1	1	1
	VFCC		1	1			1	1	1	1	1	1	1	1									1	1	1
50	GRAC	1			1						1		1		1							1			1
	VFWC	1			1							1	1	1	1	1	1					1			1
	VFCC		1	1	1							1	1	1	1	1	1					1			1
51	GRAC	1			1				1			1			1		1					1			1
	VFWC	1			1					1	1	1			1			1	1			1			1
	VFCC		1	1	1					1	1	1			1			1	1			1			1
52	GRAC	1			1			1			1					1			1			1			1
	VFWC	1			1			1			1					1			1			1			1
	VFCC		1	1	1			1			1					1			1			1			1
53	GRAC	1			1		1						1						1			1			1
	VFWC	1			1			1	1					1	1					1	1	1			1
	VFCC		1	1	1			1	1					1	1					1	1	1			1
54	GRAC	1			1		1			1		1					1		1			1			1
	VFWC	1			1			1	1	1			1	1			1			1	1	1			1
	VFCC		1	1	1			1	1	1			1	1			1			1	1	1			1