

# Einstein-Cartan electrodynamics: physical four-potential, geometric photon mass, and axial spin coupling

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If the electromagnetic four-potential is a physical field rather than a gauge class, its irrotational component is physical as well, and the geometry of spacetime can respond to it directly. We develop the consequences of this reading in the context of Einstein-Cartan gravity at the classical level. Imposing metric compatibility and linear dependence of the affine connection on the potential, we derive a unique one-parameter extension of the Levi-Civita connection. The resulting torsion is purely axial, and its Ricci scalar contributes a term  $+6\lambda^2 A_\mu A^\mu$  that generates Proca electrodynamics with photon mass  $m_\gamma = \sqrt{3/(4\pi)} \lambda M_{\text{Pl}}$ , where  $\lambda$  is a dimensionless coupling. The same coupling also produces a universal spin-dependent force on matter and an axial-vector coupling to fermions. Because a single parameter controls all four observable sectors, the four sectors are correlated by construction, and the correlation is a signature of the geometric mechanism. We verify that the construction does not generate parity violation in the vacuum propagation of photons, and we confront the framework with laboratory, astrophysical, and cosmological bounds. The solar-wind bound on the photon mass forces  $\lambda \lesssim 2 \times 10^{-46}$ , which replaces the unexplained smallness of the photon mass by an unexplained smallness of the geometric coupling. We emphasize that the paper is a classical study; the quantization of the theory and its consequences for unitarity and for the cosmological abundance of longitudinal modes are left open. Three appendices contain the detailed derivations of the uniqueness of the connection, the Ricci scalar computation, and the spin-torsion force.

**Keywords:** Einstein-Cartan gravity; axial torsion; physical four-potential; Proca electrodynamics; photon mass; spin-torsion coupling

## I. INTRODUCTION

The standard formulation of electrodynamics treats the four-potential  $A_\mu$  as an auxiliary object whose only role is to generate the field strength  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ , and regards potentials related by a gauge transformation  $A_\mu \rightarrow A_\mu + \partial_\mu \chi$  as physically equivalent. This reading is economical, and it is sufficient for the computation of scattering amplitudes and for the classical dynamics of charged matter. It is not, however, the only reading compatible with the empirical evidence. The Aharonov-Bohm effect, predicted in 1959 and confirmed experimentally by Tonomura and collaborators in 1986 [1, 2], established that the four-potential produces measurable physical effects in regions where the field strength vanishes. The Maxwell-Lodge effect, replicated with modern instrumentation by Rousseaux and collaborators and by Daibo and collaborators [3, 4], extended this conclusion to the classical regime.

The standard interpretation of these experiments is well known and fully consistent with the gauge-invariant formulation [5]. The Maxwell-Lodge effect is explained by Faraday’s law: the time-varying magnetic field inside the solenoid induces an electric field outside it, and this electric field generates the electromotive force in the coil. The Aharonov-Bohm effect is explained by the holonomy  $\oint A_\mu dx^\mu$ , which is gauge-invariant and can be understood

in the fiber-bundle formulation of gauge theories [6]. Neither experiment forces the conclusion that the potential itself is physical. What they establish is weaker but still significant: the potential is the most economical variable in terms of which the physical effects can be described, and its irrotational component is not, in the standard formulation, a mere gauge artifact. In the reading we adopt in this paper, the four-potential is treated as a physical vector field. This reading is not forced by the experiments, but it is defensible and it offers a description in which the potential is the fundamental degree of freedom.

It is useful to place the construction we develop below within a broader classification of the mechanisms for generating mass in a gauge theory. The first mechanism is *parametric*: the mass is inserted directly into the Lagrangian as in Proca theory [7], or through a Higgs-like sector whose vacuum expectation value generates the mass dynamically [8, 9]. Both share the feature that the mass originates in the matter sector of the theory, and both leave the geometry of spacetime untouched. The second mechanism is *spontaneous*: a scalar field acquires a nonvanishing vacuum expectation value, and the gauge field acquires mass through the Higgs mechanism in the non-Abelian case [10]. The third mechanism, which is the one developed here, is *geometric*: the affine connection of spacetime is allowed to depend on the gauge potential, and the mass emerges as a curvature effect. In this last case, the Ricci scalar generates a mass term from the geometry rather than from a scalar condensate. The geometric mechanism has the advantage of introducing neither free parameters nor new fields, at the cost of

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abandoning the invariance of the connection under gauge transformations. Any mechanism that introduces a small mass scale relative to the Planck mass faces the hierarchy problem [11–13], and the present construction is no exception; we return to this point in the discussion.

The paper is a classical study. We do not address the quantization of the theory, and we do not claim to resolve the hierarchy problem or any problem of quantum gravity. The construction is presented as a classical geometric framework whose quantum completion is left open. The explicit breaking of  $U(1)$  invariance means that the theory is an effective field theory whose cutoff we do not specify; the consequences of this limitation are discussed in Sec. IV F.

A second consideration motivates the geometric approach. The affine connection of spacetime defines parallel transport and curvature [14]. In general relativity, it is taken to be the Levi-Civita connection, determined entirely by the metric. In Einstein-Cartan theory [15–17] and its generalizations, the connection may carry torsion beyond the Levi-Civita part, and torsion couples to the spin density of matter [18, 19]. A recurring theme in the literature, developed notably by Garcia de Andrade [20–22], has been the possibility that torsion and the electromagnetic potential might not be independent actors but two manifestations of the same geometric structure. The present work develops this proposal under a strict uniqueness criterion.

The three requirements we impose are strict metric compatibility, linear dependence on the electromagnetic potential  $A_\mu$ , and construction from the tensors already present in the theory without introducing new fields. Under these conditions, the space of admissible contortion tensors reduces to a two-parameter family, of which one parameter is eliminated by the requirement that spinless particles follow geodesics of the Levi-Civita connection. The surviving connection is

$$\Gamma_{\mu\nu}^\lambda = \Gamma_{\mu\nu}^\lambda(\text{LC}) + \lambda \epsilon^{\lambda\rho}_{\mu\nu} A_\rho, \quad (1)$$

where  $\Gamma_{\mu\nu}^\lambda(\text{LC})$  is the Levi-Civita connection,  $\epsilon^{\lambda\rho}_{\mu\nu}$  is the Levi-Civita pseudotensor, and  $\lambda$  is a dimensionless coupling. Equation (1) is the central geometric postulate of the paper, and the analysis consists of working out its consequences.

The construction has a distinctive structural feature. A single dimensionless parameter  $\lambda$  controls four apparently independent observables: the photon mass, the spin-torsion force, the axial-vector coupling to fermions, and the additional stress-energy contribution to the Einstein equations. The four sectors are correlated by construction, and the correlation is a signature of the geometric mechanism. This correlation is not shared by Proca electrodynamics [7], where the mass is an isolated parameter with no relation to any other observable, nor by an *ad hoc* combination of Proca theory with an axial-torsion coupling [18, 19], where the mass and the coupling are independent parameters. We stress, however, that the

correlation is a structural property, not a predictive advantage in the current empirical situation. Because the solar-wind bound forces  $\lambda \lesssim 2 \times 10^{-46}$ , all four observables are suppressed by the same factor, and none of them is currently measurable. The correlation would become testable only if some of the observables were partially enhanced by a mechanism that the paper does not model.

We also emphasize what the theory does *not* do. It does not generate parity violation in the vacuum propagation of photons, because the term  $A_\mu \tilde{F}^{\mu\nu} A_\nu$  that would appear in a Chern-Simons-like construction [23] vanishes identically:  $\tilde{F}^{\mu\nu}$  is antisymmetric while  $A_\mu A_\nu$  is symmetric, and their contraction is zero. This is a direct consequence of the algebraic structure of the ansatz. It does not explain the smallness of the photon mass; it replaces one unexplained parameter by another [11, 12]. It does not predict observable effects at currently accessible scales. The paper is therefore presented as a technical study of the geometric origin of Proca electrodynamics under the hypothesis that the four-potential is physical.

The paper is organized as follows. Section II establishes the geometric foundations, derives (1) from the stated principles, and computes the torsion tensor and its axial vector. Section III derives the field equations and examines the modification of the Einstein equations. Section IV develops the surviving phenomenological features and confronts them with current bounds, including a discussion of the effective field theory scope and of the cosmological consequences of longitudinal modes. Section V summarizes the results and discusses the hierarchy problem. Appendix A contains the proof of uniqueness of the connection, including a classification of the torsion tensor in four dimensions. Appendix B contains the full computation of the Ricci scalar. Appendix C contains the derivation of the spin-torsion force from the action.

Throughout, we work in natural units with  $\hbar = c = 1$ , adopt the metric signature  $(+ - - -)$ , and use the Levi-Civita pseudotensor normalized as  $\epsilon^{\alpha\beta\gamma\delta} = [\alpha\beta\gamma\delta]/\sqrt{-g}$  with  $[0123] = +1$ . Where SI units are needed for comparison with experiment, we restore factors of  $\hbar$  and  $c$  explicitly.

## II. GEOMETRIC FOUNDATIONS

The construction rests on three principles. First, the affine connection must be metric-compatible,  $\nabla_\lambda g_{\mu\nu} = 0$ , so that lengths and angles are preserved under parallel transport [24]. Second, the connection must depend linearly on the electromagnetic potential  $A_\mu$ , since quadratic or higher-order terms would correspond to non-minimal couplings not motivated by the requirement of a single geometric principle. Third, the connection must be constructed from the tensors already present in the theory, namely  $g_{\mu\nu}$ ,  $\epsilon_{\mu\nu\rho\sigma}$ , the Kronecker delta, and  $A_\mu$ , without introducing auxiliary fields. Before developing the consequences of these principles, we clarify the status of the potential.

### A. Physicality of the four-potential

In the standard formulation of electrodynamics [5], the potential is a gauge class: two potentials related by  $A_\mu \rightarrow A_\mu + \partial_\mu \chi$  are regarded as physically equivalent, and only the field strength  $F_{\mu\nu}$  is regarded as physical. This reading is economical, but it is not forced by the empirical evidence. The Aharonov-Bohm effect [1, 2] and the Maxwell-Lodge effect [3, 4] show that the potential is the most economical variable for describing physical effects that occur in regions where  $F_{\mu\nu}$  vanishes. In the standard interpretation, these effects are described by the holonomy  $\oint A_\mu dx^\mu$ , which is gauge-invariant [6], and no modification of the gauge-theoretic framework is required.

We adopt the reading in which the four-potential is a physical vector field. The choice is not based on a theorem but on a combination of empirical motivation and conceptual economy. The empirical motivation is the Aharonov-Bohm and Maxwell-Lodge effects, which make the potential the natural variable for describing the relevant physics. The conceptual economy is the observation that, if the potential is physical, the standard apparatus of gauge orbits is unnecessary: the description in terms of  $A_\mu$  is complete.

The reading can be formulated in the language of effective field theory [10]. In modern quantum field theory, the vacuum is not empty: it may contain condensates that break symmetries explicitly [25], and the structure of the vacuum is an empirical question. The explicit breaking of  $U(1)$  invariance at the level of the connection, which we develop below, is a statement about the vacuum state of the theory, analogous in structure to the explicit breaking of chiral symmetry by the quark condensate in quantum chromodynamics [26].

The consequence for the construction that follows is direct. If  $A_\mu$  is physical, then the geometry of spacetime can depend on it, just as it can depend on any other physical field. The connection (1) is not a gauge-dependent object in the standard sense, because there is no gauge symmetry to break. The dependence of the connection on  $A_\mu$  is the dependence of the geometry on a physical field.

The distinction between the present framework and the standard gauge-theoretic formulation is sharper than a choice about which redundancies to accept. In the standard formulation, the invariance of the action under  $A_\mu \rightarrow A_\mu + \partial_\mu \chi$  makes the potential a class of equivalence, and two potentials related by such a transformation describe the same physical configuration. In the present construction, the geometric mass term breaks this invariance explicitly, and the transformation  $A_\mu \rightarrow A_\mu + \partial_\mu \chi$  is not a symmetry of the action: it changes the Lagrangian, the equations of motion, and the curvature of spacetime. Consequently,  $A_\mu$  is not defined modulo a gauge transformation; it is a physical vector field uniquely determined by the source distribution once appropriate boundary conditions are imposed. The physicality of the potential is therefore not a choice about which redundancies to

accept, but a consequence of the explicit breaking of  $U(1)$  invariance at the level of the connection.

We note that the treatment of the potential as a physical field has antecedents in the literature on non-gauge-invariant formulations of electrodynamics, notably in the work of van Oosten [26], who uses the Fermi Lagrangian to obtain a theory in which the potential is uniquely determined and the Lorenz condition follows from charge conservation rather than being imposed as a gauge choice. As we discuss in Sec. IV F, the formulation of van Oosten differs from the present one in its treatment of the energy-momentum tensor, and we do not adopt his Lagrangian. But the two approaches share the feature that the Lorenz condition emerges dynamically rather than being imposed, and we cite van Oosten as the closest precedent for the treatment of  $A_\mu$  as a physical degree of freedom.

### B. Contortion ansatz

Metric compatibility decomposes the connection into a Levi-Civita part and a contortion tensor [17],

$$\Gamma_{\mu\nu}^\lambda = \Gamma_{\mu\nu}^\lambda(\text{LC}) + K_{\mu\nu}^\lambda, \quad (2)$$

with  $K_{\lambda\mu\nu} = -K_{\nu\mu\lambda}$ . The antisymmetry of the contortion tensor is equivalent to the statement that the torsion tensor  $T_{\mu\nu}^\lambda = \Gamma_{\mu\nu}^\lambda - \Gamma_{\nu\mu}^\lambda$  satisfies  $T_{\mu\nu}^\lambda = 2K_{[\mu\nu]}^\lambda$ . A complete classification of the irreducible components of the torsion tensor in four dimensions, and the role played by the totally antisymmetric part, is given in Appendix A.

To construct the most general linear ansatz for  $K_{\lambda\mu\nu}$  in terms of  $A_\rho$ , we combine the available tensorial objects with one free index and one factor of  $A$ , obtaining

$$K_{\lambda\mu\nu} = P g_{\lambda\mu} A_\nu + Q g_{\lambda\nu} A_\mu + R g_{\mu\nu} A_\lambda + S \epsilon_{\lambda\mu\nu\rho} A^\rho. \quad (3)$$

The requirement  $K_{\lambda\mu\nu} = -K_{\nu\mu\lambda}$  gives

$$(P + R)(g_{\lambda\mu} A_\nu + g_{\mu\nu} A_\lambda) + 2Q g_{\lambda\nu} A_\mu = 0. \quad (4)$$

Since the three tensor structures are independent, this forces  $Q = 0$  and  $R = -P$ . The surviving form is

$$K_{\lambda\mu\nu} = P(g_{\lambda\mu} A_\nu - g_{\mu\nu} A_\lambda) + S \epsilon_{\lambda\mu\nu\rho} A^\rho. \quad (5)$$

The first term is a vector-torsion contribution and the second is an axial-torsion contribution. Both satisfy the antisymmetry of the contortion tensor. To select between them, we use the requirement that spinless particles follow geodesics of the Levi-Civita connection, which is the minimal condition compatible with the classical tests of general relativity [27, 28].

The contortion contribution to the geodesic equation for a spinless particle is

$$K_{\alpha\beta}^\mu u^\alpha u^\beta = P[(A \cdot u)u^\mu - A^\mu(u \cdot u)] + S \epsilon^{\mu\rho}_{\alpha\beta} A_\rho u^\alpha u^\beta, \quad (6)$$

where the second term vanishes identically because  $\epsilon^{\mu\rho}_{\alpha\beta}$  is antisymmetric in  $\alpha, \beta$  while  $u^\alpha u^\beta$  is symmetric. The first

term does not vanish in general. For a timelike geodesic with  $u \cdot u = 1$  in our signature, it is  $P[(A \cdot u)u^\mu - A^\mu]$ , which would modify the motion of any charged or neutral particle in the presence of a nonvanishing potential. The classical tests of general relativity would be spoiled. Therefore  $P = 0$ , and the unique non-trivial contortion is the axial one. The detailed proof, including the classification of the irreducible components of torsion, is given in Appendix A.

After raising indices and relabeling  $S = \lambda$ , we obtain the connection (1). The coupling  $\lambda$  is dimensionless because  $A_\mu$  has mass dimension one in natural units and  $\Gamma$  has mass dimension one. The construction has no additional freedom beyond the value of  $\lambda$ , and every prediction that follows is a consequence of a single geometric parameter.

### C. Torsion tensor and axial vector

The torsion tensor associated with (1) is

$$T_{\mu\nu}^\lambda = 2\lambda \epsilon^{\lambda\rho}_{\mu\nu} A_\rho, \quad (7)$$

which is manifestly antisymmetric in the last two indices and traceless,  $T_{\mu\nu}^\mu = 0$ , because the contraction of the Levi-Civita pseudotensor over two indices vanishes. This property identifies the torsion as purely axial [18]. The associated axial vector, defined by  $\mathcal{A}^\mu = \epsilon^{\mu\nu\rho\sigma} T_{\nu\rho\sigma}$ , is

$$\mathcal{A}^\mu = 2\lambda \epsilon^{\mu\nu\rho\sigma} \epsilon_{\nu\rho\sigma\alpha} A^\alpha = -12\lambda A^\mu, \quad (8)$$

where we used the identity  $\epsilon^{\mu\nu\rho\sigma} \epsilon_{\alpha\nu\rho\sigma} = -6\delta^\mu_\alpha$ . If one adopts the alternative normalization  $S^\mu = \frac{1}{6} \epsilon^{\mu\nu\rho\sigma} T_{\nu\rho\sigma}$  common in the Einstein-Cartan literature [17], the relation becomes  $S^\mu = -2\lambda A^\mu$ .

## III. FIELD EQUATIONS

The dynamics is encoded in a minimal action containing the Ricci scalar computed from the full connection (1) and the standard electromagnetic kinetic term. This section derives the resulting field equations and examines the modification of the Einstein equations.

### A. Ricci scalar

For a metric-compatible connection with contortion  $K$ , the Ricci tensor satisfies [17, 24]

$$R_{\mu\sigma}(\Gamma) = R_{\mu\sigma}(\text{LC}) + \nabla_\lambda K_{\mu\sigma}^\lambda - \nabla_\sigma K_{\mu\lambda}^\lambda + K_{\alpha\lambda}^\lambda K_{\mu\sigma}^\alpha - K_{\alpha\sigma}^\lambda K_{\mu\lambda}^\alpha, \quad (9)$$

with all trace terms retained. For our ansatz, the traces vanish:  $K_{\mu\lambda}^\lambda = 0$  because  $\epsilon_{\alpha\mu\lambda\rho}$  is antisymmetric in  $\alpha, \lambda$  while  $g^{\lambda\alpha}$  is symmetric. The covariant divergence also drops out after contraction with  $g^{\mu\sigma}$ , since  $K^{\lambda\mu}_\mu = 0$ . The Ricci scalar reduces to

$$R(\Gamma) = R(\text{LC}) - g^{\mu\sigma} K_{\alpha\sigma}^\lambda K_{\mu\lambda}^\alpha. \quad (10)$$

Substituting the ansatz and contracting the Levi-Civita tensors, we obtain

$$g^{\mu\sigma} K_{\alpha\sigma}^\lambda K_{\mu\lambda}^\alpha = \lambda^2 C^{\rho\tau} A_\rho A_\tau, \quad (11)$$

where

$$C^{\rho\tau} \equiv g^{\mu\sigma} g^{\lambda\beta} g^{\alpha\gamma} \epsilon_{\beta\alpha\sigma\rho} \epsilon_{\gamma\mu\lambda\tau}. \quad (12)$$

The tensor  $C^{\rho\tau}$  is computed in Appendix B, where the Levi-Civita contraction identities are derived from first principles. The result is  $C^{\rho\tau} = -6g^{\rho\tau}$ , so that

$$g^{\mu\sigma} K_{\alpha\sigma}^\lambda K_{\mu\lambda}^\alpha = -6\lambda^2 A_\mu A^\mu, \quad (13)$$

and

$$R(\Gamma) = R(\text{LC}) + 6\lambda^2 A_\mu A^\mu. \quad (14)$$

The coefficient is  $+6\lambda^2$ , and its sign is fixed by the contraction identity  $C^{\rho\tau} = -6g^{\rho\tau}$  together with the vanishing of the trace terms in Eq. (9).

### B. Effective action and photon mass

We take as our starting point the action

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa_g} R(\Gamma) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right], \quad (15)$$

where  $\kappa_g = 8\pi G$  and  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ . Substituting (14), we obtain the effective action

$$S_{\text{eff}} = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa_g} R(\text{LC}) + \frac{3\lambda^2}{\kappa_g} A_\mu A^\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right]. \quad (16)$$

The second term is a mass term for the electromagnetic potential with coefficient  $3\lambda^2/\kappa_g$  in front of  $A^2$ , which corresponds to a Proca mass [7]

$$m_\gamma^2 = \frac{6\lambda^2}{\kappa_g} = \frac{3\lambda^2}{4\pi G} = \frac{3}{4\pi} \lambda^2 M_{\text{Pl}}^2, \quad (17)$$

where  $M_{\text{Pl}}^2 = 1/G$  is the unreduced Planck mass,  $M_{\text{Pl}} \approx 1.22 \times 10^{28}$  eV in natural units. Equivalently,

$$m_\gamma = \sqrt{\frac{3}{4\pi}} \lambda M_{\text{Pl}} \approx 0.49 \lambda M_{\text{Pl}}. \quad (18)$$

The geometric origin of the Proca mass is the central result of the present construction. The Ricci scalar, through the quadratic contribution generated by the axial contortion, generates a mass term from the geometry rather than from a fundamental scalar. The sign of the mass term is positive, and the energy density of the resulting Proca field is positive definite for all three physical polarizations (two transverse and one longitudinal). The theory therefore has a healthy spectrum at the classical level with no ghost or tachyon instabilities. We stress that this statement is classical. As discussed in Sec. IV F, the explicit breaking of  $U(1)$  invariance means that the theory is an effective field theory, and the quantum behavior of the longitudinal mode at high energies is not addressed here.

### C. Electromagnetic field equations

Variation of (16) with respect to  $A_\mu$  gives the Proca equation [7]

$$\partial_\nu F^{\nu\mu} + \frac{6\lambda^2}{\kappa_g} A^\mu = 0. \quad (19)$$

Expanding  $F^{\nu\mu}$  in terms of  $A^\mu$ , this becomes

$$\square A^\mu - \partial^\mu(\partial_\nu A^\nu) + m_\gamma^2 A^\mu = 0. \quad (20)$$

Taking the divergence and using the antisymmetry of  $F^{\nu\mu}$ , we obtain

$$m_\gamma^2 \partial_\mu A^\mu = 0, \quad (21)$$

which enforces the Lorenz condition  $\partial_\mu A^\mu = 0$  as a dynamical consequence for  $m_\gamma \neq 0$ , not as a gauge choice. The emergence of the Lorenz condition from the equations of motion is a characteristic feature of non-gauge-invariant formulations of massive vector fields, and it was already present in the treatment of the potential as a physical field by van Oosten [26]. As shown in those formulations, the potential is uniquely determined by the source distribution when supplemented by standard boundary conditions at infinity (for instance,  $A_\mu \rightarrow 0$  as  $r \rightarrow \infty$ ), which also ensure the dynamical emergence of the Lorenz condition. In our construction, the condition is not imposed by hand; it follows from the geometric mass term. With this condition, (20) reduces to the massive wave equation  $(\square + m_\gamma^2)A^\mu = 0$ , which propagates three physical polarizations [5]. The counting of degrees of freedom is identical to that of Proca theory.

### D. Modified Einstein equations and the potential as a source

Variation of (16) with respect to the metric yields

$$G_{\mu\nu}(\text{LC}) = \kappa_g \left( F_{\mu\rho} F_\nu{}^\rho - \frac{1}{4} g_{\mu\nu} F^2 \right) + 6\lambda^2 \left( A_\mu A_\nu - \frac{1}{2} g_{\mu\nu} A^2 \right). \quad (22)$$

The first term on the right-hand side is the standard electromagnetic stress-energy tensor [5]. The second term, which is absent in conventional Proca theory coupled to general relativity, arises from the geometric coupling and contributes even in regions where  $F_{\mu\nu} = 0$  but  $A_\mu \neq 0$ .

In the reading of the potential that we adopt, the appearance of  $A_\mu$  directly in the gravitational source is not a defect but a direct consequence of the hypothesis that  $A_\mu$  is physical. If the potential describes the state of a physical medium, then the gravitational field responds to it just as it responds to any other physical field. There is no gauge freedom to fix, and the configuration of the potential is part of the physical state of the system, on the

same footing as the metric and the matter distribution. The source term in Eq. (22) is therefore a physical source, not a gauge artifact.

The effective action (16) is invariant under diffeomorphisms, and therefore the contracted Bianchi identity  $\nabla_\mu G^{\mu\nu}(\text{LC}) = 0$  is automatically satisfied on shell [14]. The right-hand side of (22) is covariantly conserved by virtue of the Proca equation (19). The conservation is with respect to the Levi-Civita connection, because the contortion has been entirely absorbed into the effective action, and it holds for any solution of the equations of motion.

### E. Test particle motion

For a test particle of mass  $m$  and electric charge  $q$ , the geodesic equation in the full connection (1) reads

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta}(\text{LC}) \dot{x}^\alpha \dot{x}^\beta = \frac{q}{m} F^\mu{}_\nu \dot{x}^\nu + \lambda \epsilon^{\mu\rho}{}_{\alpha\beta} A_\rho \dot{x}^\alpha \dot{x}^\beta. \quad (23)$$

The torsion-mediated term vanishes identically for spinless particles, by the same antisymmetry argument that appears in Eq. (6). The force therefore acts only on matter with spin. This is the same structure as in the scalar-gradient contortion examined in Ref. [29], and it distinguishes the present construction from Proca electrodynamics, in which no spin coupling is present [7]. The force law differs from that of Ref. [26] because the gauge-breaking in our construction arises from the geometric mass term rather than from a modification of the minimal coupling in the matter Lagrangian.

## IV. PHENOMENOLOGY

The construction has three physical consequences that distinguish it from conventional Proca electrodynamics: a universal spin-dependent force on matter, an axial-vector coupling to fermions, and a contribution to the stress-energy tensor that is quadratic in the potential. We examine each of these and confront them with observational bounds, and we discuss the effective field theory scope of the construction and the cosmological consequences of longitudinal modes.

### A. Absence of vacuum birefringence

One might expect that the axial structure of the contortion would generate a Chern-Simons-like interaction between the electromagnetic potential and its dual field strength, and thereby induce cosmic birefringence. This expectation is not realized. The natural candidate for such a term is

$$S_{\text{CS}} = \frac{\lambda}{2} \int d^4x \sqrt{-g} A_\mu \tilde{F}^{\mu\nu} A_\nu, \quad (24)$$

where  $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$  is the dual field strength. This expression vanishes identically, because  $\tilde{F}^{\mu\nu}$  is antisymmetric while  $A_\mu A_\nu$  is symmetric:

$$A_\mu \tilde{F}^{\mu\nu} A_\nu = \frac{1}{2} A_\mu A_\nu (\tilde{F}^{\mu\nu} - \tilde{F}^{\nu\mu}) = 0. \quad (25)$$

The vanishing is an algebraic identity, not a dynamical result, and it can be traced to the same structure that allows the contortion to be axial: the contraction of a symmetric tensor with an antisymmetric one is always zero. The absence of birefringence in the vacuum propagation of photons therefore distinguishes the present construction from Chern-Simons electrodynamics with a scalar background field, where the interaction  $\phi F\tilde{F}$  is nonzero [23]. In the present theory, the axial structure of the contortion manifests itself only through the coupling to matter, not through the propagation of radiation.

### B. Spin-dependent force

For a spinning particle with spin tensor  $S^{\mu\nu}$ , the torsion contribution to the equations of motion can be derived from the action

$$S_{\text{torsion}} = \frac{\lambda}{2} \int d\tau S^{\mu\nu} \epsilon_{\mu\nu\rho\sigma} A^\sigma \dot{x}^\rho, \quad (26)$$

where  $\dot{x}^\rho = dx^\rho/d\tau$ . The full derivation, which uses only the Euler-Lagrange equation and does not require the Mathisson-Papapetrou-Dixon formalism, is given in Appendix C. The Mathisson-Papapetrou-Dixon formalism [30–32] is the standard framework for describing the motion of extended bodies with spin in curved spacetime, and our result agrees with it when specialized to the present contortion. The result is

$$F_{\text{torsion}}^\mu = \lambda S^{\rho\sigma} \epsilon_{\rho\sigma}^{\mu\alpha} \nabla_\alpha (A_\beta u^\beta) - \lambda S^{\rho\sigma} \epsilon_{\rho\sigma}^{\mu\alpha} A_\beta \nabla_\alpha u^\beta. \quad (27)$$

For a particle in geodesic motion,  $\nabla_\alpha u^\beta = 0$ , and the second term vanishes. The surviving term is the spin-torsion force

$$F_{\text{torsion}}^\mu = \lambda S^{\rho\sigma} \epsilon_{\rho\sigma}^{\mu\alpha} \nabla_\alpha (A_\beta u^\beta). \quad (28)$$

The force is linear in the spin and in the derivative of the potential, and vanishes identically for spinless particles.

The spin-dependent force is not constrained by composition-based tests of the equivalence principle, because those tests use unpolarized test bodies [33–35]. It is constrained by precision measurements of spin precession, by experiments with spin-polarized test masses, and by comagnetometer experiments that bound the spatial components of a background axial vector. The Gravity Probe B experiment measured geodetic precession with a fractional accuracy of  $3 \times 10^{-3}$  [36], but the corresponding constraint on  $\lambda$  is of order  $10^{13}$ , which does not compete with the photon mass bound. The bounds from comagnetometer experiments [37] are stronger by several orders

of magnitude, but they require an assumption about the magnitude and orientation of  $A_\mu$  in the Solar System. For the cosmological value  $A_0 \sim H_0$ , the comagnetometer bound on  $\lambda$  is of order  $10^{-40}$ , still weaker than the solar-wind bound by six orders of magnitude. The comparison between the two bounds is not a test of the model, because the two observables are sensitive to different combinations of  $A_\mu$  and its derivatives, and the paper does not attempt to model the Solar System configuration of  $A_\mu$  in detail.

### C. Axial-vector coupling to fermions

The axial torsion couples to fermions through the spin connection. For a Dirac field  $\psi$ , the interaction takes the form

$$\mathcal{L}_{\text{int}} = \lambda A_\mu \bar{\psi} \gamma^\mu \gamma^5 \psi, \quad (29)$$

which is a parity-odd axial-vector coupling. In a system of massless Dirac fermions, this coupling generates an effective chiral chemical potential  $\mu_5 = \lambda A_0$ , which in turn induces a chiral magnetic current along an applied magnetic field [38, 39].

The magnitude of the effect is controlled by  $\lambda A_0$ . For the fundamental value  $\lambda \lesssim 2 \times 10^{-46}$  and a cosmological value  $A_0 \sim H_0 \sim 10^{-33}$  eV, the effective chemical potential is  $\mu_5 \sim 10^{-79}$  eV, which is undetectable. The axial-vector coupling is therefore a formal feature of the theory that does not translate into observable consequences in realistic settings.

### D. Gravitational analog in Weyl semimetals

The mathematical structure of the geometric coupling is shared with the effective description of low-energy quasiparticles in Weyl semimetals with dislocation-induced torsion [40, 41]. In such systems, the effective contortion perceived by the quasiparticles is proportional to the dislocation density, and the effective coupling  $\lambda_{\text{eff}}$  is a property of the crystal lattice, not a fundamental constant of nature. The chiral magnetic effect observed in materials such as  $\text{Cd}_3\text{As}_2$ ,  $\text{Na}_3\text{Bi}$ , and  $\text{TaAs}$  shares the algebraic structure of the coupling (29), but with  $\lambda_{\text{eff}} \sim \mathcal{O}(1)$  rather than  $\lambda \sim 10^{-46}$ .

It is important to distinguish the two uses of the same mathematical structure. In the fundamental theory,  $\lambda$  is the dimensionless coupling that relates the affine connection to the electromagnetic potential, and it is constrained by the photon mass bound. In the condensed-matter analog,  $\lambda_{\text{eff}}$  is an emergent parameter describing the coupling of quasiparticles to the lattice geometry. The two parameters are not related by any known correspondence. The analog is therefore not a test of the fundamental theory, but a mathematical analogy that may help to clarify the physical content of the geometric coupling.

## E. Observational bounds

The single parameter of the theory is the dimensionless coupling  $\lambda$ , and the strongest bound comes from the photon mass. The current laboratory bound on the photon mass is  $m_\gamma < 10^{-14}$  eV from tests of Coulomb's law [42], while the solar-wind bound is  $m_\gamma < 10^{-18}$  eV [43]. Combining the solar-wind bound with Eq. (18) and using  $M_{\text{Pl}} = 1.22 \times 10^{28}$  eV, we obtain

$$\lambda < \frac{10^{-18} \text{ eV}}{0.49 \times 1.22 \times 10^{28} \text{ eV}} \approx 2 \times 10^{-46}. \quad (30)$$

This is the strongest constraint on the theory and dominates the phenomenological discussion.

Table I summarizes the bounds on  $\lambda$  from different observables, together with the corresponding physical assumptions. The photon mass bound is the strongest by many orders of magnitude. Bounds from Lorentz violation and from gyroscope precession are weaker but probe independent sectors of the theory, and they provide complementary constraints on the underlying geometric coupling.

## F. Effective field theory scope and cosmological consequences

The construction presented in this paper is classical. We have derived the field equations, verified the conservation of the stress-energy tensor through the Bianchi identities, and computed the classical spectrum. We have not quantized the theory, and we do not claim that the classical construction can be extended to arbitrarily high energies without modification. The explicit breaking of  $U(1)$  invariance by the geometric mass term means that the theory is an effective field theory. The scale at which the effective description breaks down is not specified by the classical analysis, and the paper does not attempt to identify it. In this sense, the construction is a classical geometric framework whose quantum completion is left open.

This limitation is not a defect of the construction. The classical theory already contains a rich structure, and the questions that the paper addresses (the uniqueness of the connection, the correct coefficient of the Ricci scalar, the existence of a spin-torsion force, the absence of vacuum birefringence) are well posed and well answered at the classical level. Whether the theory admits a consistent quantum completion, and what the consequences of that completion would be for the phenomenology, is a separate question that requires a separate analysis. We flag it here as an open problem.

One aspect of the quantum completion that deserves to be mentioned is the behavior of the longitudinal mode. The explicit breaking of  $U(1)$  makes the longitudinal mode a physical degree of freedom, and its classical energy density is positive. At the quantum level, the longitudinal

mode of a massive vector field is known to violate perturbative unitarity in the absence of a Higgs or Stueckelberg mechanism [10]. The present construction does not include such a mechanism, and we do not claim that it avoids the unitarity problem. The paper is a classical study, and the unitarity problem is one of the issues that a quantum completion would have to address.

A second aspect, closely related, concerns the cosmological abundance of longitudinal modes. In the classical theory, the longitudinal mode propagates freely and carries energy density proportional to  $A_\mu A^\mu$ . In any cosmological scenario, a stochastic background of such modes would be generated by the dynamics of the early universe, and it would contribute to the effective energy density of the universe through the coupling  $6\lambda^2 A^2$  in Eq. (22). The present paper does not attempt to model this background, and we do not compute the resulting cosmological constraints. We note, however, that the contribution is suppressed by  $\lambda^2$  and would be negligible for the value of  $\lambda$  implied by the solar-wind bound, unless the amplitude of the longitudinal background is many orders of magnitude larger than the cosmological values we have assumed. A detailed analysis is beyond the scope of the paper.

Finally, we note that the formulation of van Oosten [26], which uses the Fermi Lagrangian, differs from the present one in its treatment of the energy-momentum tensor. In van Oosten's formulation, the energy density of the field is not positive definite, and an electrostatic field exhibits a negative energy density. The present construction uses the Maxwell Lagrangian, not the Fermi Lagrangian, and the resulting energy density is positive definite for the Proca field. The two formulations are therefore not identical, and the present construction does not inherit the energy-density issues of the Fermi formulation. This distinction is important for the interpretation of the theory, and we make it explicit here.

## V. DISCUSSION AND CONCLUSIONS

We have developed a geometric framework in which the electromagnetic potential couples to the axial torsion of spacetime through a unique metric-compatible connection, under the hypothesis that the four-potential is a physical field. The construction is minimal in the sense that it introduces no new fields and no free parameters beyond a single dimensionless coupling  $\lambda$ . The resulting field equations reproduce Proca electrodynamics [7] for the potential, and the theory differs from Proca theory in three ways: it generates the mass geometrically rather than inserting it by hand, it couples universally to the spin content of matter, and it modifies the Einstein equations with a term that is quadratic in the potential. The paper is a classical study, and we have emphasized throughout that the quantization of the theory and its consequences for unitarity and for the cosmological abundance of longitudinal modes are left open.

The hypothesis that the four-potential is physical is

Table I. Bounds on the dimensionless coupling  $\lambda$  from different observables. The third column indicates the physical assumption required to translate the observable into a bound on  $\lambda$ . The photon mass bound is the strongest by many orders of magnitude and currently dominates the phenomenological discussion.

| Observable                             | Bound on $\lambda$    | Reference                 | Assumption                              |
|----------------------------------------|-----------------------|---------------------------|-----------------------------------------|
| Photon mass (solar wind)               | $< 2 \times 10^{-46}$ | Retinò et al. [43]        | $m_\gamma = 0.49 \lambda M_{\text{Pl}}$ |
| Photon mass (laboratory)               | $< 2 \times 10^{-42}$ | Williams et al. [42]      | Same                                    |
| Lorentz violation (comagnetometers)    | $< 10^{-40}$          | Kostelecký & Russell [37] | Solar-system $A_\mu$                    |
| Gyroscope precession (Gravity Probe B) | $< 10^{13}$           | Everitt et al. [36]       | Solar-system $A_\mu$                    |

the central conceptual commitment of the paper. The empirical case for this hypothesis rests on the observation that the potential is the most economical variable for describing the Aharonov-Bohm and Maxwell-Lodge effects [1–4]. The conceptual case rests on the observation that, if the potential is physical, the standard apparatus of gauge orbits is unnecessary. The choice is not forced, but it is defensible, and it has antecedents in the literature on non-gauge-invariant formulations of electrodynamics [26]. It is important to note that the physicality of the potential in the present framework does not arise from a choice about which redundancies to accept. It arises from the explicit breaking of  $U(1)$  invariance by the geometric mass term. In the standard formulation, the invariance of the action under  $A_\mu \rightarrow A_\mu + \partial_\mu \chi$  makes the potential a class of equivalence, and two potentials related by such a transformation describe the same physical configuration. In the present construction, the transformation  $A_\mu \rightarrow A_\mu + \partial_\mu \chi$  is not a symmetry of the action: it changes the Lagrangian, the equations of motion, and the curvature of spacetime. The potential is therefore not defined modulo a gauge transformation; it is a determinate physical field, uniquely fixed by the source distribution once appropriate boundary conditions are imposed.

The vacuum state of the theory is uniquely  $A_\mu = 0$ , the only configuration that preserves Lorentz invariance and minimizes the energy density. The explicit breaking of  $U(1)$  invariance does not make the vacuum degenerate; it promotes the longitudinal mode of the massive vector field from a gauge artifact to a genuine, propagating physical degree of freedom. Configurations of the form  $A_\mu = \partial_\mu \chi$  that satisfy the Proca equation are not alternative vacua but physical excitations of the longitudinal polarization, carrying energy and momentum. The classical energy density of these excitations is positive, as we have verified. At the quantum level, the longitudinal mode of a massive vector field without a Higgs or Stueckelberg mechanism is known to violate perturbative unitarity, and the present construction does not address this issue. We flag it as an open problem.

The principal structural feature of the geometric formulation is its rigidity. A single dimensionless parameter  $\lambda$  controls four apparently independent observables: the photon mass, the spin-torsion force, the axial-vector coupling to fermions, and the additional stress-energy contribution to the Einstein equations. This correlation is

not shared by Proca electrodynamics [7], where the mass is an isolated parameter, nor by an *ad hoc* combination of Proca theory with an axial-torsion coupling [18, 19], where the mass and the coupling are independent. In the geometric construction, the four observables are correlated by construction, and the correlation is a signature of the geometric mechanism. We stress, however, that the correlation is a structural property, not a predictive advantage in the current empirical situation. Because the solar-wind bound forces  $\lambda \lesssim 2 \times 10^{-46}$ , all four observables are suppressed by the same factor, and none of them is currently measurable. The correlation would become testable only if some of the observables were partially enhanced by a mechanism that the paper does not model.

The most serious conceptual issue raised by the theory is the hierarchy problem [11–13]. The solar-wind bound on the photon mass forces  $\lambda \lesssim 2 \times 10^{-46}$ , which is small even by the standards of Planck-suppressed operators. The geometric construction does not explain why  $\lambda$  should be small; it replaces one small number, the photon mass, by another, the coupling  $\lambda$ . This is an honest limitation of the present framework, and it deserves to be stated clearly. The advantage of the geometric formulation is not that it explains the smallness of the photon mass, but that it ties the smallness to a broader structure: the same small number controls the strength of the spin coupling, the axial-vector coupling to fermions, and the contribution of the potential to the stress-energy tensor.

The paper also clarifies what the theory does not predict. In particular, the construction does not generate parity violation in the vacuum propagation of photons, because the Chern-Simons-like term  $A_\mu \tilde{F}^{\mu\nu} A_\nu$  vanishes identically. The vanishing is an algebraic consequence of the structure of the ansatz, not an additional postulate, and it distinguishes the present framework from Chern-Simons electrodynamics with a scalar background field [23].

Several directions for future work suggest themselves. The most immediate is the quantization of the theory, which would determine whether the geometric coupling survives at the quantum level and would clarify the status of the longitudinal mode. We have noted that the explicit breaking of  $U(1)$  raises questions about unitarity that the classical analysis does not resolve, and a quantum completion would have to address these. A second direction is the construction of exact solutions, particularly in

spherically symmetric and cosmological settings, which would clarify the role of the potential in the gravitational sector and would allow the cosmological consequences of longitudinal modes to be computed [18]. A third direction is the extension to non-Abelian gauge fields [15], which would test whether the geometric mechanism is specific to electromagnetism or applies more broadly to the gauge sector of the Standard Model [25]. Finally, the detailed modeling of the gravitational analog in Weyl semimetals [40, 41] could provide a concrete laboratory realization of the mathematical structure of the coupling, even if the fundamental coupling  $\lambda$  remains inaccessible.

In summary, the theory proposed here offers a geometric perspective on the origin of photon mass and its relationship to the axial structure of spacetime, under the hypothesis that the four-potential is a physical field. It does not resolve the hierarchy problem, and it does not address the quantization of the theory or the unitarity of the longitudinal mode. It clarifies the geometric origin of Proca electrodynamics at the classical level and identifies the spin and axial-current structure of matter as the sector where the construction departs from standard Proca theory. The rigidity of the construction, with a single parameter controlling all observables, is a structural property that distinguishes the geometric mechanism from the standard parametric approach. The paper is presented as a classical technical study of this structure, and we have avoided claims about observable consequences or quantum completions that we have not calculated.

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**Renato Vieira dos Santos:** Conceptualization, Methodology, Formal analysis, Investigation, Writing – original draft, Writing – review and editing.

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## Appendix A: Classification of torsion and uniqueness of the connection

This appendix has two purposes. The first is to classify the irreducible components of the torsion tensor in four dimensions, so that the reader can see what is being discarded and why. The second is to prove that the connection (1) is unique under the three principles stated in Sec. II.

### 1. Irreducible components of the torsion tensor

In four dimensions, the torsion tensor  $T_{\mu\nu}^{\lambda}$  is antisymmetric in its last two indices and therefore has  $4 \times 6 = 24$  independent components. These components decompose into three irreducible pieces under the Lorentz group, classified by the vector, axial, and mixed representations [17, 18]:

- A *vector* (or trace) piece, defined by  $V_{\mu} = T_{\lambda\mu}^{\lambda}$ , which has 4 independent components.
- An *axial* (or pseudovector) piece, defined by  $A^{\mu} = \epsilon^{\mu\nu\rho\sigma} T_{\nu\rho\sigma}$ , which has 4 independent components.
- A *tensor* (or mixed) piece, which is traceless and has no axial component, with 16 independent components.

The three pieces are algebraically independent and can be combined into any torsion tensor. The tensor piece is the most general, and it is the piece that is usually neglected in simple treatments of torsion because it is

difficult to work with. For our purposes, the relevant observation is that the ansatz (3) contains only the vector and axial pieces at linear order in  $A_\mu$ , and the tensor piece is absent by construction. The three principles stated in Sec. II therefore restrict the construction to a two-parameter family, and the two parameters correspond to the vector and axial couplings  $P$  and  $S$ .

The vector piece is the one that would contribute to the geodesic equation for spinless particles and spoil the classical tests of general relativity. The axial piece is the one that survives, and it is the one that generates the mass term in the Ricci scalar. The tensor piece, which we have discarded by the requirement of linear dependence on  $A_\mu$ , would contribute at quadratic order and would complicate the analysis without changing the qualitative structure.

## 2. Proof of uniqueness

We now prove that the ansatz (3) admits a unique non-trivial solution satisfying the antisymmetry condition  $K_{\lambda\mu\nu} = -K_{\nu\mu\lambda}$  and the preservation of geodesics for spinless particles. The general framework for contortion tensors in metric-compatible theories is reviewed in Refs. [17, 24], and the specific form of the linear coupling to a vector field is discussed in the context of Einstein-Cartan theory in Refs. [15, 16]. Consider each term in (3) separately.

For the term  $Pg_{\lambda\mu}A_\nu$ , the antisymmetry condition requires

$$P(g_{\lambda\mu}A_\nu + g_{\nu\mu}A_\lambda) = 0. \quad (\text{A.1})$$

This must hold for all  $\lambda, \mu, \nu$ . Choosing  $\lambda = \nu \neq \mu$  gives  $2Pg_{\lambda\mu}A_\lambda = 0$ , and since  $g_{\lambda\mu}$  and  $A_\lambda$  are generically non-vanishing, we conclude  $P = 0$ . However, if we allow for a vector-torsion term, the general solution of the antisymmetry condition is  $P$  arbitrary, with  $R = -P$  and  $Q = 0$ , as shown in Eq. (5). The vector term is then eliminated by the requirement that spinless particles follow Levi-Civita geodesics, as argued in Sec. II B.

For the term  $Qg_{\lambda\nu}A_\mu$ , the antisymmetry condition reads

$$Q(g_{\lambda\nu}A_\mu - g_{\nu\lambda}A_\mu) = 0, \quad (\text{A.2})$$

which is automatically satisfied because  $g_{\lambda\nu} = g_{\nu\lambda}$ . Therefore the antisymmetry condition does not constrain  $Q$  directly, and the vanishing of  $Q$  follows from the independence of the tensor structures in Eq. (4). The term  $Qg_{\lambda\nu}A_\mu$  is symmetric in  $\lambda, \nu$ , and therefore the condition  $K_{\lambda\mu\nu} = -K_{\nu\mu\lambda}$  requires it to vanish independently of the other terms.

For the term  $Rg_{\mu\nu}A_\lambda$ , the antisymmetry condition reads

$$R(g_{\mu\nu}A_\lambda + g_{\mu\lambda}A_\nu) = 0, \quad (\text{A.3})$$

and the same argument as for  $P$  gives  $R = 0$  in the absence of the  $P$  term, or  $R = -P$  in its presence.

For the term  $S\epsilon_{\lambda\mu\nu\rho}A^\rho$ , the antisymmetry condition is automatically satisfied because the Levi-Civita pseudotensor changes sign under the exchange of any two indices, and in particular  $\epsilon_{\lambda\mu\nu\rho} = -\epsilon_{\nu\mu\lambda\rho}$ . This term provides the unique non-trivial solution, and after raising indices and relabeling  $S = \lambda$  we obtain the connection (1).

The proof shows that the axial piece is the only one compatible with all three principles, and that its coefficient  $\lambda$  is the only free parameter of the construction. The classification of torsion in four dimensions makes transparent why the axial piece is selected: it is the only piece that does not contribute to the geodesic equation at linear order and that has the appropriate transformation properties under parity to generate a mass term.

## Appendix B: Ricci scalar computation

This appendix contains the full computation of the Ricci scalar for the connection (1). The computation proceeds in three steps: the derivation of the Levi-Civita contraction identity, the evaluation of the quadratic contortion term, and the assembly of the final result.

### 1. Levi-Civita contraction identity

We derive the identity

$$\epsilon^{\alpha\beta\mu\nu}\epsilon_{\alpha\delta\mu\nu} = -6\delta^\beta_\delta \quad (\text{B.1})$$

in four dimensions with metric signature  $(+---)$  and normalization  $\epsilon^{0123} = +1/\sqrt{-g}$ . The derivation starts from the general contraction formula for two Levi-Civita symbols in  $d$  dimensions, contracting  $k$  indices:

$$\epsilon^{\mu_1 \dots \mu_k \alpha_1 \dots \alpha_{d-k}} \epsilon_{\mu_1 \dots \mu_k \beta_1 \dots \beta_{d-k}} = (-1)^{k(d-k)} k! (d-k)! \delta^{\alpha_1}_{\beta_1} \dots \delta^{\alpha_{d-k}}_{\beta_{d-k}}, \quad (\text{B.2})$$

where the antisymmetrization has weight one. The sign factor  $(-1)^{k(d-k)}$  follows from the signature of the metric and the ordering of the contracted indices.

For our case,  $d = 4$  and we are contracting  $k = 3$  indices (namely  $\alpha, \mu, \nu$ ), leaving one free index pair  $(\beta, \delta)$ .

Substituting these values into the general formula yields

$$\epsilon^{\alpha\beta\mu\nu}\epsilon_{\alpha\delta\mu\nu} = (-1)^{3\cdot 1} 3! 1! \delta^{[\beta}_{\delta} = -6 \delta^{\beta}_{\delta}. \quad (\text{B.3})$$

This direct application of the general formula avoids the

$$\epsilon_{\mu\nu\alpha\sigma}\epsilon^{\rho\sigma\lambda\tau} = -\left(\delta_{\mu}^{\rho}\delta_{\nu}^{\lambda}\delta_{\alpha}^{\tau} + \delta_{\nu}^{\rho}\delta_{\alpha}^{\lambda}\delta_{\mu}^{\tau} + \delta_{\alpha}^{\rho}\delta_{\mu}^{\lambda}\delta_{\nu}^{\tau} - \delta_{\mu}^{\rho}\delta_{\alpha}^{\lambda}\delta_{\nu}^{\tau} - \delta_{\nu}^{\rho}\delta_{\mu}^{\lambda}\delta_{\alpha}^{\tau} - \delta_{\alpha}^{\rho}\delta_{\nu}^{\lambda}\delta_{\mu}^{\tau}\right), \quad (\text{B.4})$$

which follows from the same general formula with  $k = 1$  and  $d = 4$  after antisymmetrizing the three free indices.

## 2. Evaluation of the quadratic contortion term

We now evaluate the tensor  $C^{\rho\tau}$  defined in Eq. (12). Using the metric to raise indices, we can write the Levi-Civita tensors in their fully covariant form and use the contraction identity (B.3). The computation in a local orthonormal frame gives

$$C^{00} = \eta^{\mu\sigma}\eta^{\lambda\beta}\eta^{\alpha\delta}\epsilon_{\beta 0\alpha\sigma}\epsilon_{\delta 0\mu\lambda}. \quad (\text{B.5})$$

The first epsilon is non-zero only when  $(\beta, \alpha, \sigma)$  is a permutation of  $(1, 2, 3)$ , and its value is the sign of that permutation. The same holds for the second epsilon with  $(\delta, \mu, \lambda)$ . The sum runs over the six permutations, and the product of the two signs is  $\text{sgn}(\lambda, \alpha, \mu)\text{sgn}(\alpha, \mu, \lambda) = +1$ , because  $(\alpha, \mu, \lambda)$  is a cyclic permutation of  $(\lambda, \alpha, \mu)$  and cyclic permutations of three elements have sign  $+1$ .

The metric prefactor requires separate care. The three inverse metric components  $\eta^{\mu\sigma}$ ,  $\eta^{\lambda\beta}$ , and  $\eta^{\alpha\delta}$  are non-vanishing only when their indices are equal, and in that case each equals  $-1$  for spatial indices. Since all three pairs  $(\mu, \sigma)$ ,  $(\lambda, \beta)$ , and  $(\alpha, \delta)$  are spatial, the metric prefactor contributes a factor  $(-1)^3 = -1$ . Combining the two contributions, the sum over the six permutations gives

$$C^{00} = (-1) \times (+1) \times 6 = -6. \quad (\text{B.6})$$

By Lorentz invariance,  $C^{\rho\tau} = -6g^{\rho\tau}$ , since  $g^{00} = +1$  and the relation is consistent with the sign of  $C^{00}$ . The remaining components can be checked by the same argument in a frame where the relevant direction is along one of the spatial axes, and the result is the same.

## 3. Assembly of the final result

Substituting  $C^{\rho\tau} = -6g^{\rho\tau}$  into Eq. (11), we obtain

$$g^{\mu\sigma}K_{\alpha\sigma}^{\lambda}K_{\mu\lambda}^{\alpha} = -6\lambda^2 A_{\mu}A^{\mu}, \quad (\text{B.7})$$

and therefore

$$R(\Gamma) = R(\text{LC}) + 6\lambda^2 A_{\mu}A^{\mu}. \quad (\text{B.8})$$

cumbersome explicit permutation sums and agrees with the standard identity quoted in the literature [14, 24]. The generalization to the contraction of one index pair, which is needed in Appendix C, is

The identity used in intermediate steps,  $\epsilon^{\lambda\rho}_{\mu\nu}\epsilon^{\mu\nu}_{\lambda\sigma} = -6\delta^{\rho}_{\sigma}$ , differs by a factor of three from the value  $-2\delta^{\rho}_{\sigma}$  that appears in some treatments of torsion. The correct factor is  $-6$ , and its derivation is given in this appendix. The result is consistent with the standard formula for the Ricci scalar in Riemann-Cartan spacetimes [17, 18, 24] and with the sign convention adopted in this paper.

## Appendix C: Derivation of the spin-torsion force

This appendix contains the derivation of the spin-torsion force from the action (26). The derivation uses only the Euler-Lagrange equation and tensorial notation; no forms or exterior calculus are required.

### 1. Setup and definition of the auxiliary vector

The torsion contribution to the action is

$$S_{\text{torsion}} = \frac{\lambda}{2} \int d\tau S^{\mu\nu} \epsilon_{\mu\nu\rho\sigma} A^{\sigma} \dot{x}^{\rho}, \quad (\text{C.1})$$

where  $\dot{x}^{\rho} = dx^{\rho}/d\tau$  and  $S^{\mu\nu}$  is the spin tensor, which we treat as an independent variable obeying  $DS^{\mu\nu}/D\tau = 0$  along the worldline. Define the auxiliary vector

$$B_{\alpha} \equiv \frac{\lambda}{2} S^{\mu\nu} \epsilon_{\mu\nu\alpha\sigma} A^{\sigma}, \quad (\text{C.2})$$

so that the action can be written in the compact form

$$S_{\text{torsion}} = \int d\tau B_{\alpha} \dot{x}^{\alpha}. \quad (\text{C.3})$$

The vector  $B_{\alpha}$  depends on the position  $x^{\beta}$  through the potential  $A^{\sigma}(x)$  and through the spin tensor  $S^{\mu\nu}(x)$  along the worldline.

### 2. Variation with respect to the trajectory

We vary the trajectory  $x^{\beta}(\tau) \rightarrow x^{\beta}(\tau) + \delta x^{\beta}(\tau)$  and compute the variation of the action. Two terms contribute: the explicit dependence of  $B_{\alpha}$  on the position, and the

dependence of  $\dot{x}^\alpha$  on the derivative of the variation. The variation is

$$\delta S_{\text{torsion}} = \int d\tau \left[ \partial_\beta B_\alpha \delta x^\beta \dot{x}^\alpha + B_\alpha \frac{d(\delta x^\alpha)}{d\tau} \right]. \quad (\text{C.4})$$

Integrating the second term by parts along the worldline and dropping the boundary term, we obtain

$$\delta S_{\text{torsion}} = \int d\tau \left[ \partial_\beta B_\alpha \dot{x}^\alpha - \frac{dB_\beta}{d\tau} \right] \delta x^\beta. \quad (\text{C.5})$$

The Euler-Lagrange equation is therefore

$$\frac{dB_\beta}{d\tau} = \partial_\beta B_\alpha \dot{x}^\alpha. \quad (\text{C.6})$$

### 3. Expansion of the total derivative

We expand the total derivative  $dB_\beta/d\tau$  using the chain rule. Since  $B_\beta$  depends explicitly on the position  $x^\alpha$  and on the spin tensor  $S^{\mu\nu}$  along the worldline, we have

$$\frac{dB_\beta}{d\tau} = \partial_\alpha B_\beta \dot{x}^\alpha + \partial_\tau B_\beta, \quad (\text{C.7})$$

where  $\partial_\tau B_\beta$  denotes the explicit dependence on  $\tau$  through the spin tensor. Substituting into Eq. (C.6), we obtain

$$(\partial_\alpha B_\beta - \partial_\beta B_\alpha) \dot{x}^\alpha + \partial_\tau B_\beta = 0. \quad (\text{C.8})$$

The first term is the antisymmetric part of the gradient of  $B$ , and the second term is the contribution from the explicit spin evolution.

### 4. Spin parallel transport and final expression

For a spin that is parallel-transported along the worldline,  $DS^{\mu\nu}/D\tau = 0$ , the explicit dependence of  $B_\beta$  on  $\tau$  is subleading, and the second term in Eq. (C.8) vanishes at leading order. The force is then dominated by the antisymmetric part of the gradient of  $B$ :

$$F_\beta = g_{\beta\mu} (\partial_\alpha B^\mu - \partial^\mu B_\alpha) \dot{x}^\alpha. \quad (\text{C.9})$$

Substituting the definition (C.2) of  $B_\alpha$  and using the contraction identity (B.4), we obtain after simplification

$$F_{\text{torsion}}^\mu = \lambda S^{\rho\sigma} \epsilon_{\rho\sigma}^{\mu\alpha} \nabla_\alpha (A_\beta u^\beta) - \lambda S^{\rho\sigma} \epsilon_{\rho\sigma}^{\mu\alpha} A_\beta \nabla_\alpha u^\beta, \quad (\text{C.10})$$

which is Eq. (27) of the main text. For geodesic motion,  $\nabla_\alpha u^\beta = 0$ , and the second term vanishes, leaving the reduced expression

$$F_{\text{torsion}}^\mu = \lambda S^{\rho\sigma} \epsilon_{\rho\sigma}^{\mu\alpha} \nabla_\alpha (A_\beta u^\beta). \quad (\text{C.11})$$

The derivation uses only the Euler-Lagrange equation and the algebra of the Levi-Civita tensor.

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