

Deriving the Standard Model of Particle Physics from Geometry of Space-time Structure and Establishing the Full-Spectrum Equation for High-Energy Particle Transient States

Taiwei Song

Shanghai Luyi New Energy Co., Ltd., Shanghai Riyue New Energy Co., Ltd.

August 11, 2026

Abstract

This paper briefly introduces The Geometry of Spacetime Structure. Starting from the fundamental logic of the Geometry of Spacetime Structure, it rigorously proves that the Standard Model of particle physics is a necessary corollary of this theory, establishes the characterizing equations for the full spectrum of high-energy particle transient states, and accurately derives the energy levels of all known high-energy particle transient states. Moreover, using only fundamental natural constants, it precisely determines such important natural parameters as the neutron–proton mass difference (1.2933 MeV) and the neutron decay lifetime (879 s). Unlike Quantum Field Theory, the Geometry of Spacetime Structure takes the visible and measurable stable proton and electron as the indivisible fundamental particles of nature with nonzero mass, whose mass, charge, and spin are all intrinsic quantum invariants of the spacetime structure. By employing an intuitive and concise quasi-4-dimensional spacetime motion geometry to reveal the laws of natural change, it not only unifies the fundamental interactions of nature but also unifies the related theoretical systems of physics and mathematics created for the purpose of revealing the laws of nature.

Keywords: Geometry of Space-Time Structure; space-time-matter integration; symmetry; energy conservation; proton and electron; neutron lifetime.

1. Introduction

The Standard Model of particle physics takes the gauge symmetry group $SU(3)_c \times SU(2)_L \times U(1)_Y$ as its skeleton and describes the microscopic laws of the strong, weak, and electromagnetic interactions. Its construction relies on a presupposed globally static flat Minkowski space-time, rather than the real curved natural space-time; it is a phenomenological effective field theory [1,2]. Weinberg pointed out that the Standard Model is essentially a low-energy effective field theory [3]; Wilczek criticized its symmetry structure as lacking geometric logic [4]; G. 't Hooft pointed out that its scalar particle mass faces a naturalness crisis [5]; Gross and Politzer admitted that the choice of the $SU(3)$ color group is empirical [6,7].

The author has been creating and perfecting Geometry of Space-Time Structure [8–11], a kinematics geometry built on natural space-time space, aiming to dynamically reveal and characterize the laws of motion and change of the natural world from the most fundamental and universal perspective. Starting from the basic logic of Geometry of Space-time Structure, this paper rigorously proves and derives the gauge group and basic field equations of the Standard Model of particle physics, revealing that the Standard Model is merely a phenomenological representation of the logic and conclusions contained in Geometry of Space-Time Structure.

2. Basic Assumptions and Important Conclusions of Geometry of Space-Time Structure [8–11]

The natural world is precisely the objectively existing space-time geometry, it is the topological total set and geometric manifestation of various material forms in three-dimensional space evolving with time. Analyzing and characterizing natural laws with a dynamic analytic-geometric logic can achieve the most fundamental and most extensive grand unification of physics and mathematics theories aimed at revealing such laws; this is precisely the goal of the Geometry of Space-time Structure.

2.1 Basic Assumptions of Geometry of Space-Time Structure

Geometry of Space-Time Structure is a kinematics geometry established on natural space-time space. Natural space-time space is defined as the real space of nature. Its mathematical form is a quasi-four-dimensional space composed of a static three-dimensional natural space plus a quasi-one-dimensional time dimension, with coordinate form (x,y,z,ict) or abbreviated as (r,ict) . Time t flies unidirectionally at the speed of light c , acting only at the present moment on any point of three-dimensional natural space (for the same observer). There is no independently existing time space; the time dimension is “imaginary” and naturally takes an imaginary coordinate. The time variable is an independent intrinsic independent variable, never stopping, reflecting the essential property of motion and change in nature.

Axiom 1 (Axiom of Time-Light): Time is an independent variable advancing unidirectionally at the speed of light, and the speed of light is the upper limit of speed.

The time variable of natural space-time space is an independent variable flying unidirectionally at the speed of light. No object can surpass time, so the speed of light being maximal is nature’s choice. Time and light are integrated; microscopically, the time quantum is the (average) light quantum.

Axiom 2 (Definition of Distance Differential):

The distance differential dS of quasi-four-dimensional natural space-time space is defined as:

$$dS^2 = n_x^2 dx^2 + n_y^2 dy^2 + n_z^2 dz^2 - c^2 dt^2 \quad (1)$$

where $n(x,y,z)$ is the generalized refractive index tensor (metric tensor) of the spatial medium. When $dt=0$, Eq. (1) becomes the optical path differential of the isochronous three-dimensional natural space. Eq. (1) implies that straight lines and planes generally do not exist in natural space; space is naturally curved. On an infinitely small space-time unit ($\Delta r \rightarrow 0, \Delta t \rightarrow 0$), Eq. (1) approximates Minkowski space, and the Lorentz transformation holds.

Under ideal near-vacuum conditions, $n_{x,y,z}=1$, and the space-time distance ΔS between any two points simplifies to:

$$\Delta S^2 = \Delta r^2 - c^2 \Delta t^2 \quad (2)$$

The space-time coordinate system (r,ict) and the momentum-energy coordinate system $(P,iE/c)$ are in a Fourier-transform relationship. The invariance of the space-time interval in Eq. (2) is equivalent, in the $(P,iE/c)$ coordinate system, to the invariance relation of the momentum-energy interval of a free particle:

$$E^2 = (Pc)^2 + (m_0 c^2)^2 \quad (3)$$

The metric distance differential near any point in an arbitrary n -dimensional geometric space as defined by Riemann B. [12], $dS^2 = \sum g_{ij} dx_i dx_j$, is only an abstract mathematical form. However, near any point in real natural space-time space, the orthogonality relations among the four dimensions hold, so Eq. (1) has only components with $i=j$ (inner-product form). Eq. (1) actually determines the first- and second-order derivative (rate-of-change) relations of the spatial structure (or manifold), i.e., it determines the intrinsic change relations of the geometric manifold. This is precisely the content of differential geometry.

Points in natural space-time space have size; they are open subdomains (subsets) of space. (Coordinate points are only labels for spatial positions and have no size.). Curves and surfaces are composed of points, curves have “width,” and surfaces have “thickness”. Natural space-time space is a topological space. Open local subdomains have relations such as intersection and union, which correspond to the unity of natural material states, i.e., they are all manifestations of energy. Therefore, there must exist a unified state-structure description and common fundamental energy quanta (elementary particles).

Axiom 3 (Geometric Unit, Dimension, Structural Form, and Corresponding Material Properties):

Points in natural space have size. The movement of a spatial geometric point has only three mutually perpendicular degrees of freedom: “front-back, left-right, up-down”. Points, and structures composed of points, are all open subdomains containing certain physical contents. Length, surface area, morphological structure, and the spatial volume they enclose are all measures and manifestations of material energy. Points and structures in space can carry a binary material property of whether they carry positive or negative charge; points (particles) in space can also carry a binary selective material-form property of whether they carry positive or negative spin, which is also a microscopic quantum property. These two opposite binary selections are additional countable variables (independent countable variable dimensions) of the changes of spatial points and structures. In the limits of subdomains, $\Delta r \rightarrow 0$ (infinitely small) and $r \rightarrow \infty$ (infinitely far), time, space, and matter are integrated.

The capacity of any subdomain of geometric space (length of a line segment, area of a surface, volume of a closed body enclosed by surfaces) can be divided arbitrarily small, but never to zero; that is, a limit exists. At the infinitesimal scale, any arbitrarily small further subdivision is still arbitrarily small; the rate of change (derivative) is itself, this is the natural exponential relation. So the geometric logic naturally contains Lie group and Lie algebra relations. In 3D natural space, this kind of infinitesimal quantity is precisely the materialized energy quantum (the mass quantum is the proton, the charge quantum is the electron, and the time-light quantum is the photon). Moreover, such microscopic arbitrarily small quantities in space cannot be precisely recorded on the “coordinate axes”, their positions are uncertain. This is precisely the logical foundation of quantum mechanics. Macroscopic relatively stable geometric structures (states) are composed of a large number of microscopic quanta with uncertainty, this naturally contains the theoretical logic of quantum statistics.

All material structures in nature are composed of elementary particles. An elementary particle is defined as: the smallest energy quantum unit that is stable in an isolated state, cannot be further divided, and represents the fundamental form and properties of material energy. The elementary particles with mass $m \neq 0$ are the proton and the electron; the one with mass $m = 0$ is the photon. The 3D motion and change form of an elementary particle is characterized by the independent variable coordinate space (x, y, z) ; the intrinsic invariant energy of an elementary particle is represented by the inertial mass m ; the charge property of an elementary particle is represented by the binary positive and negative charge quantum $\pm e$; the spin property of the elementary particle structure form is represented by the binary positive and negative spin quantum $\pm \hbar$ (divided into fermions with $m \neq 0$ and $\pm \hbar/2$, and bosons with $m = 0$ and $\pm \hbar$, corresponding to the SU(2) group). Mass, charge, and spin are the intrinsic natural properties of elementary particles. The mass of one charge quantum is the electron mass m_e . Obviously, $m = 0$ bosons are all electrically neutral; electrically neutral particles with $m \neq 0$ are not elementary particles.

In the infinitely small spatial subdomain $\Delta r \rightarrow 0$, time, space, and matter are integrated, $\Delta r, \Delta t, \Delta m, \Delta E$, etc., must have definite corresponding relations, namely:

$$\Delta E = \Delta mc^2, \Delta E = \frac{h}{\Delta t} = h\nu = \frac{hc}{\Delta r} \quad (4)$$

This shows that the near-neighbor strong interaction relation and relativistic relations such as the mass-energy equation hold. For the charge quantum—the electron—under “confinement” in the infinitely small space, the electromagnetic energy corresponding to one charge quantum ($e^\pm$) is $m_e c^2$. The energy level of a photon with frequency ν or angular frequency $\omega=2\pi\nu$ is $h\nu(\hbar\omega)$. The energy level of $\hbar$ or $\hbar/2$ spin is $1\hbar = 1h/2\pi$ for $\omega = 1$, which can be neglected. This is precisely the essential meaning of Planck’s constant $\hbar$ or h .

In Geometry of Space-time Structure, the state of an object (or system) is represented by a space-time structural state. The space-time structural state of an object or system is defined as the dynamic holographic spatial structure information set of the object or system (which may be called the “space-time total information tensor”), including, at any given time t , the spatial structural form relations at various levels inside the object and the spatial form relations of its external interactions with other matter. Simply and intuitively speaking, it is the time-varying spatial holographic construction diagram presented by the object or system itself. For non-contact long-range interactions such as universal gravitation and Coulomb force, the space-time structural state is global. According to the complexity and main features of the research object, the holographic content of the space-time structural state can be simplified and selected.

Let $\Psi(t)$ denote the space-time structural state of an object at time t , $\Psi(x, y, z, t)$ denote the sub-space-time structural state of the object at point (x, y, z) at time t , and $\Psi_i(x, y, z, t)$ denote the space-time structural state of the i -th material property of $\Psi(x, y, z, t)$, where $i = m, \pm e, \pm \hbar$, etc. Let:

$$\Psi(t) \equiv \{\Psi(x, y, z, t)\}_{xyz} \equiv \{\{\Psi_i(x, y, z, t), i = m, \pm e, \pm \hbar, \text{etc.}\}\}_{xyz} \quad (5)$$

For example, for an isolated fermion, its $\Psi(t)$ is $\{r(x, y, z, t), m, \pm e, \pm \hbar/2\}$; its spatial curvature (second derivative) corresponds to force. If the state function directly represents energy, it is the Hamiltonian plus spin representation (this is the traditional mechanics method); the spatial rate of change of the energy structure, i.e., the spatial gradient, is the force. Of course, the state function in the (r, ict) coordinate system can, through Fourier transformation, become an equivalent state function in the $(P, iE/c)$ or $(K, i\omega/c)$ coordinate system.

Axiom 4 (Interactions of Matter, Energy Conservation, Overall Stability): The space-time structural state of an object is essentially the material-energy structural form and density distribution of the object; its total capacity corresponds to the total energy of the object. The interaction between objects is proportional to the mutual overlap of their space-time structural states; the closer the distance, the greater the overlap. Objects with positive and negative material properties (charge, spin, etc.) repel those of the same kind and attract those of the opposite kind. Different motion states of the same object have different space-time structural states, while the rest mass remains unchanged. For an isolated natural system, total energy is conserved; the degree of superposition of the space-time structural states among different internal particles (units) determines the overall stability of the system. The calculation of overlap starts from the naturally isolated space-time structural state and energy level of each particle (unit); the interaction state between every two particles is calculated separately. The superposition of the interaction states of a given particle with all other particles is the sub-space-time structural state of this particle in the system.

The space-time structural states and numerical structures of the same particle with positive/negative charge and positive/negative spin are completely identical, only the directions and signs are opposite. They can completely overlap and cancel to 0, the original material form annihilates and becomes

$m=0$ photons. This is precisely what happens in electron-positron pair annihilation.

For universal gravitation or Coulomb force, because of the 3D spatial symmetry around a point and because there are only the distance r and the inertial mass m or charge q variables, the force is obviously $\propto m_1 m_2 / r^2$ or $q_1 q_2 / r^2$, this is precisely the relation that the closer the distance, the greater the overlap. If the distance between two objects at a certain time t is r_t , then only the spatial region $r \geq r_t$ can be the region where the space-time structural states of the two objects overlap and couple. For a spherically symmetric $\propto 1/r^2$ distribution of force, the energy integral from ∞ to r_t is precisely the interaction energy $\propto 1/r_t$. The repulsive case is the same, except that superposition becomes “separation,” and “-” becomes “+”; this is why the “number of electromagnetic lines” remains unchanged no matter how they transform.

For the overall space-time structural state (set) of an isolated system, its internal morphological structure and density may both be changing, but the total integral (i.e., the total energy) remains unchanged; therefore it exhibits periodicity of phase change. Obviously, overall electrical neutrality is also the basis for the relative stability of material states.

From the above, it can be seen that in the natural world, forces are completely unified. Geometrically, curvature greater than 0 belongs to Riemannian space; physically, it is constrained by conservative potential energy (total kinetic energy less than 0), and the whole exhibits a self-similar condensed-state characteristic of clustering of similar things.

2.2 Important Conclusions of the Geometry of Space-Time Structure

① The stable, indivisible elementary particles in nature are only the proton and electron with mass $m \neq 0$, and the photon with mass $m = 0$. The neutron is not an elementary particle but a composite topological configuration of a proton and an electron, the most elementary composite body. Other high-energy particles are all transient composites composed of protons and positive/negative electrons. Relatively light composite particles are all transient states of matter-energy in infinitely small space with structural forms such as $(e^- e^+ e^-)$, $(e^+ e^- e^+)$ or $\Sigma(e^-, e^+)$, $e^\mp \Sigma(e^-, e^+)$, etc. There are no independent neutrons in atomic nuclei; protons in the nucleus form strongly mutually attractive pep quantum-pair structures by exchanging electrons with near neighbors. The mass m_p or m_e , charge $\pm e$, and spin $\hbar/2$ of the proton and electron are intrinsic quantum invariants of the space-time structure and are natural constants. There is no electrically neutral elementary particle with mass $m \neq 0$; proton number and charge number are naturally conserved. The energy level of the near-neighbor strong interaction between protons is the exchanged material energy $\Delta E = \Delta m c^2 = e^D m_e c^2$, where e^D is the number of near-neighbor pep quantum pairs that one proton can form, and D is the geometric dimension of the nuclear topological structure (fractional). All nucleons (protons) in the nucleus have equal status, with no distinction of inner and outer; it is a high-energy holographic fully symmetric statistical topological structural state, and there exist multi-nucleon (odd-number) angular-spin quantum states. Except for a few light nuclei such as H_e , decay is universal. The strong interaction is the electromagnetic interaction of exchanged electrons, and so-called “weak interaction” is also electromagnetic interaction.

② The $0 \rightarrow 1$ transition of number sets from algebra to geometry changes the computational logic from addition and multiplication to exponential operations, physical quantities change from definite simple scalars to uncertain multidimensional-choice vectors (tensors), this is the establishment of geometric space and dimensions, naturally producing logic relations such as vectors, matrices, calculus, Fourier transforms, and Lie group transformations. Physically, the creation of $0 \rightarrow 1$ of an infinitely small space ($\Delta r \rightarrow 0$) is the creation of material entity quantum

units (atomic nuclei, atoms), and the spatial motion state of a quantum unit exhibits e^D uncertain degenerate states; at the nucleon scale, e^D is the energy-level parameter (space-time-matter integration, compact structure, $D < 3$); at the atomic scale, the energy-level parameter is $D = \ln e^D$; at the same time, the corresponding natural logical relations of relativity and quantum mechanics are established. All high-energy composite-particle transient states at the nucleon scale are symmetric state structures of non-compact space with $D = 3$.

③ The natural universe is a topological spatial structure with no boundary but with “size”. For an observer at any point in cosmic space, it is at the center. In the $r \rightarrow \infty$ infinitely far region, the spatial curvature is ∞ (stellar mass-energy also $\rightarrow \infty$), space is naturally curved, redshift naturally exists, and all are “visual images” brought by the propagation of light information, there is no “Big Bang”.

④ Energy and matter are conserved and indestructible. Time is the absolute main variable flying unidirectionally at the speed of light, the time quantum is the photon. Space-time, matter, and energy are integrated, all forces (forms) are unified in the properties of energy and matter, merely manifest with different primary–secondary roles from different viewpoints, such as different spacetime scales, different energy levels, and different observed objects.

⑤ Space dimension is degree of freedom, determining the main symmetry of material states. Three-dimensional natural space is the arena for material motion and change, the time dimension is the main driving independent variable, space and time dimensions are continuous independent variable dimensions. Mass is the main energy carrier of an object (the main parameter of object stability); positive and negative electricity are dual material properties attached to massive objects; spin is merely a weak near-neighbor statistical binary selective material property carried by elementary particles. Neither charge nor spin is an independent space dimension. The energy level of an object or system is proportional to the number of degenerate states of the space-time structure; degenerate states are symmetric states, which naturally connects with symmetry group theory. At visible natural micro- and macroscopic scales, the structural dimension D of an object or the motional degrees of freedom of its internal particle units satisfies $0 \leq D \leq 3$, and the dimension is mostly fractional. Fractional dimensions correspond to fractal structures and self-similarity, and space-time variation is a self-organizing process under statistical correlation.

3. Deriving the Gauge Group and Field Equations of the Standard Model from Geometry of Space-Time Structure, Establishing the Full-Spectrum Equations for High-Energy Particles

3.1 Gauge Group

From the $0 \rightarrow 1$ transition from number to geometry (“spatial size” from 0 to nonzero), addition and multiplication become exponential operations, and the discrete becomes continuous — Naturally possesses a Lie group and Lie algebra structure. Axiom 3 of Geometry of Space-time Structure establishes that there exist only two elementary fermions in nature — the proton (+e, spin $\hbar/2$) and the electron (-e, spin $\hbar/2$), proton number conservation holds naturally. Charge is uniquely determined by the binary selective property (+e or -e). Mass, charge, and spin are the intrinsic, invariant, smallest quantum units of elementary particles, naturally corresponding to the unitary groups.

(1) Charge Conservation of the Quantum Unit and U(1)

The proton carries one charge quantum unit +e, the electron carries one charge quantum unit -e. and the charge quantum is an intrinsic invariant of the space-time structure of elementary fermions. The phase transformation $\Psi \rightarrow e^{i\theta}\Psi$ does not change the real-valued distance element of Eq. (1), its continuous one-parameter unitary group is precisely $U(1) = \{e^{i\theta} | \theta \in \mathbb{R}\}$. Its covariant

derivative naturally introduces the gauge field A_μ , and the coupling strength is the charge e .

(2) 1/2 Spin and SU(2)

The intrinsic spins of the proton and electron are both $\hbar/2$. Spin is not an externally added degree of freedom but an intrinsic manifestation of the rotational symmetry of the three-dimensional spatial manifold at the $r \rightarrow 0$ infinitesimal quantum level, it takes only two spin quantum unit values, $\pm\hbar/2$, so its mathematical representation is naturally the discrete-parameter unitary group $SU(2) = \{U \in GL(2, \mathbb{C}) | U^\dagger U = I_2, \det U = 1\}$ in a 2-dimensional complex space $\mathbb{C}^2$, whose generators are the Pauli matrices:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The spin operator is $S_i = \hbar\sigma_i/2$, satisfying the commutation relation $[S_i, S_j] = i\hbar\epsilon_{ijk}S_k$. The $SU(2)_L$ subgroup of the weak interaction in the Standard Model is precisely the geometric projection of this spin invariance onto the left-handed chiral component.

The spatial part of Eq. (1) ($dr^2 = dx^2 + dy^2 + dz^2$) is invariant under three-dimensional spatial rotations, corresponding to the special orthogonal group $SO(3)$ Lie group, describing all continuous rotation operations in three-dimensional space. $SU(2)$ is the universal covering group of $SO(3)$, ($SU(2)/\{+I_2\} \cong SO(3)$), the smallest nontrivial 2-dimensional representation of $SU(2)$ is the mathematical representation of the 1/2 spin of the proton and electron.

(3) The Intrinsic Symmetry Implicit in the Infinitesimal Space as $r \rightarrow 0$ and SU(3)

From the geometry of space-time structure, at the infinitely small spatial scale ($r \rightarrow 0, \Delta V \rightarrow 0$), the $0 \rightarrow 1$ creation of a geometric structure point (e.g., a composite particle) uniquely establishes the form of the transient particle (i.e., the topological dimension D of the arbitrarily small manifold, also the particle's motional degrees of freedom) and the corresponding energy level. The particle energy level is determined by the zero-distance topological winding-knot spin variables on the arbitrarily small 3D manifold. When the volume element $\Delta V \rightarrow 0$, the macroscopic flat metric becomes invalid, and real coordinates are no longer sufficient to describe the phase and amplitude variation relations of the manifold's "imaginary" topological transformation. The "imaginary" topological winding-knot structure of the infinitesimal manifold can naturally be characterized by introducing a three-dimensional complex vector space unit sphere that describes phase changes. Under continuous rotational symmetry transformations, points on the complex unit sphere keep the distance element invariant, with its value being 1, this is precisely the continuous transformation group $SU(3) = \{U \in GL(3, \mathbb{C}) | U^\dagger U = I_3, \det U = 1\}$, its generators $T^a (a=1, \dots, 8)$ satisfy the Lie algebra $[T^a, T^b] = if_{abc} T^c$. The T^a matrices can be obtained entirely by extending the 2×2 Pauli matrices to three dimensions, this precisely embodies the essence of the zero-distance electromagnetic topological winding-spin coupling in the transient state of high-energy particles. The so-called intrinsic gauge symmetry represented by $SU(3)$ is, in essence, still a possible topological symmetry attached to the instantaneous state of a particle created from $0 \rightarrow 1$ in natural space-time at $\Delta V \rightarrow 0$; it is an inseparable, implicit "intrinsic property" of a high-energy particle transient state, whose macroscopic manifestation is the discrete energy level, and this energy level can be fully represented by combinations of electromagnetic winding-spin quantum numbers. QCD wants to use "smaller-than-proton" quarks, etc., to describe this "virtual topological structure" change, thus "quarks" are naturally unobservable.

Therefore, $SU(3)_c \times SU(2)_L \times U(1)_Y$ is merely a mathematical representation of three intrinsic symmetries in the geometry of space-time structure.

3.2 Derivation, Extension, and Analysis of Field Equations—Establishing the Full-Spectrum Characterization Equations for High-Energy Particles Transient States

The Geometry of Space-time Structure holds that, except for the proton and electron, all other high-energy particle transient states are composite particles formed from positive-negative electron pairs, or from positive-negative electron pairs together with one electron or proton, their structural forms are respectively: neutral spin-0 (mesons) $\sum(e^-e^+)$, charged $e^\mp$ (leptons) $e^\mp \sum(e^-,e^+)$, charged $e^\mp$ (baryons) $P^\mp \sum(e^-,e^+)$, etc. They are all transient states of space-time topological winding-knot of matter-energy quanta in the infinitely small space $r \rightarrow 0$. This section will give new physical meaning to Klein-Gordon equation and Dirac equation of the Standard Model, create a generalized Dirac equation for the proton, and, by analyzing these equations, unify the high-energy particle spectrum.

To unify the notation, the quasi-four-dimensional space-time coordinates (x,y,z,ict) or (r,ict) are written below as (ct,x,y,z) or abbreviated as (ct,r), the corresponding momentum-energy coordinate system (P,iE/c) is changed to (E/c,P), etc.

3.2.1 Klein-Gordon Equation and the Spectrum of 0-Charge, 0-Spin High-Energy Particles

(1) Natural Derivation of the Covariant Equation

The Geometry of Space-time Structure holds that, the space-time creation of the $0 \rightarrow 1$ topological transition in an infinitesimal topological neighborhood at the limit $r \rightarrow 0$, time, space, and matter are integrated, space time change relations are equivalent and all correspond to material energy $\Delta E = \Delta m c^2$. The quasi-4D space-time space at this time approximates Minkowski space, the distance differential of the 4D momentum-energy space corresponding to Eq. (1), expressed by

operators, is the four-dimensional d'Alembertian: $\square = \partial^\mu \partial_\mu = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2$; the distance invariant of the 4D momentum-energy space is $-m^2 c^2$, where m is the rest mass. Multiplying both sides by the state function Ψ and rearranging naturally gives the Klein-Gordon equation:

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 + m^2 \right) \Psi = (\square + m^2) \Psi = 0 \quad (6)$$

Eq. (6) can be obtained directly by substituting Eq. (3) into the operator relations $E = i\hbar \frac{\partial}{\partial t}$ and $p = -i\hbar \nabla$; this is precisely the traditional derivation method ($\hbar = c = 1$). If $m=0$, the equation becomes the free photon wave equation $\square \Psi = 0$. Klein-Gordon equation is directly given from the distance invariance of particles in ideal Minkowski space through operator substitution and contains a second-order time derivative, both its physical assumptions and mathematical form are somewhat unsatisfactory, and its extraordinary solutions and corresponding physical picture are not very clear. In the infinitely small topological neighborhood $r \rightarrow 0$, time, space, and matter are integrated, photons and positive-negative electron pairs (e^-,e^+) transform into each other; electromagnetic and mass energy transform into each other. This is the physical picture presented by the high-energy $\sum(e^-,e^+)$ forming a transient 0-charge, 0-spin particle state (meson). This section will give KG equation its true meaning for analyzing this physical picture.

(2) The Essence of Transient Particle Mass, Topological Operator Eigenvalues and Energy-Level Analysis of the Extended KG Equation

In quantum field theory, the KG equation is used to describe 0-spin particles (mesons) and is defined as the equation of motion of a scalar field. In fact, the strong interaction between protons exchanges high-energy electrons, not so-called “mesons”; at the same time, for the space-time creation of the $0 \rightarrow 1$ topological transition at the infinitely small $r \rightarrow 0$, such as a transient

particle state, the spatial position coordinate variable r is no longer important and is uncertain; or rather, Δr or ΔV is the smallest quantum unit of the particle as a spatial structural quantum, an invariant of “size”; the space-time winding spinor of the high-energy positive-negative electron pair (e^-, e^+) is the main variable determining the 0-charge, 0-spin particle state; this is not a scalar. The Geometry of Space-time Structure holds that the space-time (particle) creation of the $0 \rightarrow 1$ topological transition at the infinitely small $r \rightarrow 0$, creates the “topological form” of the spatial particle (with holographic space-time symmetry, and no distinction between inner and outer), a specific space-time quantum, its motional degrees of freedom D in natural space (i.e., its morphological structure dimension), and the energy level E in one-to-one correspondence with D . For an isolated particle, its size is one whole quantum unit, and its number of motional states is e^D . The particle mass (energy) is characterized by the intrinsic topological degrees of freedom of its spatial manifold. The unchanging rest mass of protons is the main body of the rest mass of a composite particle body (object); the electromagnetic structure energy constrained and “solidified” in the infinitely small space $r \rightarrow 0$ is the source of “newly added” rest mass. This is the essence of rest mass (variation). For a transient composite particle, apart from the net charge quantum unit (proton or electron), its mass is the entire energy endowed by the transient high-energy positive-negative electron pairs $(\sum(e^-, e^+))$ winding-knot at zero distance in the infinitely small space $r \rightarrow 0$, and its spatial form or “distribution” has a symmetric spin-topological manifold structure characteristic and can be represented by a fourth-order matrix tensor $M_{4 \times 4}$. For transient particle states formed only by high-energy positive-negative electron pairs (e^-, e^+) with 0 charge and 0 spin, the ultimate decay particles are only photons, with no charged proton or electron, and the rest mass is zero. Considering the unity of the complete particle spectrum, assume that the fourth-order diagonal matrix tensor of this particle spectrum is $M = m_e \text{diag}\{1, -1, i, -i\}$. Substituting into (6) gives:

$$(\square + M^2) \psi = (\partial^\mu \partial_\mu + M^2) \psi = 0 \quad (7)$$

This is the Klein-Gordon equation extended by the geometry of space-time structure; it is the variation equation characterizing 0-charge, 0-spin high-energy particles (transient states) formed by positive-negative electron pairs $\sum(e^-, e^+)$. Its solution is different from the radial-potential logic of traditional quantum mechanics.

In Eq. (7), the trace of the mass matrix M , $\text{Tr}(M) = m_e(1 - 1 + i - i) = 0$, indicates that the global net rest mass of the particle transient state formed by $\sum(e^-, e^+)$ is strictly zero; its transient mass is entirely composed of local electromagnetic topological winding energy. At the same time, the tensor square trace of M , $M^2 = m_e^2 \text{diag}\{1, 1, -1, -1\}$, $\text{Tr}(M^2) = 0$, $\det M^2 = 1$, characterizes the equal-weight, fully symmetric superposition and mutual cancellation of particles and antiparticles, with no global mass shift and no residual-mass ground state; the transient ultimately annihilates completely into pure photons. $M \neq 0$, the tensor is not empty, characterizes the existence of local electromagnetic topological coupling inside the infinitely small neighborhood, contributing transient mass energy. This is precisely the origin of the transient mass of the particle spectrum.

Next, solve the extended Klein-Gordon equation (7). Here a simplified method for obtaining energy levels without accurately solving the equation is adopted:

a. Degeneration of the topological-neighborhood operator: In the neighborhood of the topological singularity ($r \rightarrow 0$, $\Delta t \rightarrow 0$), the radial coordinate is frozen, the radial differential degree of freedom disappears, and the space-time differential structure degenerates into an angular-momentum topological algebraic structure, the Laplacian retains only the angular topological rotational degrees of freedom. Let Λ denote the characteristic action-strength quantity, and let r_0 be the

characteristic scale of the infinitesimal topological neighborhood. From (4), $\Lambda \propto 1/r_0$. Then:

$$\nabla^2 \rightarrow -\Lambda^2 \hat{L}^2 \quad (8)$$

b. Quasi-stationary resonance approximation: The particle body formed by $\Sigma(e^-, e^+)$ is a short-lived space-time topological transient, it has no globally stable state, but its energy levels (resonance peaks) are definite and determined solely by the angular-spin quantum topological winding-knot coupling. ψ is approximated by a locally decaying wave packet as a quasi-stationary state:

$$\psi \sim e^{-iEt} Y_{lm}(\theta, \varphi) \quad (9)$$

Thus, the eigenvalue of angular momentum is: $\hat{L}^2 \psi = l(l+1)\psi$, $l \geq 1$.

c. The mass tensor matrix M^2 term can cancel out: Here ψ is not the state function of the mass-tensor coupling effect in the transient process. Due to the full symmetry of the particle state, $M^2 = m_e^2 \text{diag}\{1, 1, -1, -1\}$ represents particles and antiparticles appearing with equal weight in the particle-state space; therefore, the four components of $\psi^\dagger M^2 \psi$ have equal absolute values and cancel in sign. That is:

$$\psi^\dagger M^2 \psi = 0 \quad (10)$$

Substituting (9) into equation (7) and multiplying from the left by $\psi^\dagger$, we obtain the general formula for the topological eigen-energy levels of the extended KG equation:

$$E_l = \Lambda \sqrt{l(l+1)} \quad (11)$$

$\Lambda \propto 1/r_0$, corresponding to the surface curvature on the topological neighborhood, here represents the characteristic interaction strength of the positive-negative electron pair (e^-, e^+) . The ground-state energy level of one (e^-, e^+) pair is $2m_e c^2$. According to the geometry of space-time structure, for the $0 \rightarrow 1$ topological creation transition, the number of topologically equivalent states of this ground state in the infinitely small D-dimensional symmetric space is e^D . Therefore, the characteristic interaction strength of the positive-negative electron pair (e^-, e^+) is $2m_e c^2 e^D$. In addition, high-energy particle transient states are diffusive topological symmetric structures in 3D natural space, so $D = 3$. Therefore, $\Lambda = 2m_e c^2 e^3 = 20.5274 \text{ MeV}$. The precise discrete energy levels of the extended KG equation are:

$$E = E_l = 2m_e c^2 \cdot e^3 \cdot \sqrt{l(l+1)} \quad (12)$$

From Eq. (12), the energy spectrum of 0-charge, 0-spin high-energy particle transient states characterized by the KG equation can be precisely calculated.

In addition, according to the relevant fractal theory in Geometry of Space-Time Structure, the spatial structure dimension D of a high-energy particle transient can be presented in two structural ways: a fully spatially uniform symmetric distribution form (relatively low energy) and an axially extended winding symmetric form (relatively high energy). The energy levels of the two obviously have an intrinsic difference Δ in the geometric sense, this is also the reason why some high-energy particle transient states have energy-level bifurcation. When $r \rightarrow 0$, for the high-energy transient formed by winding-knot of electron pairs (e^-, e^+) , the defining quantity of its closed spatial manifold is the spherical solid angle 4π [13,14], therefore, the intrinsic energy-level difference caused by the difference in geometric presentation of structural symmetry is $\Delta = 4\pi * 2m_e c^2 = 12.8428 \text{ MeV}$. From the perspective of $0 \rightarrow 1$ creation, the correction of Δ to the energy level E in formula (9) is more likely to appear at smaller values such as $l = 1$. Taking E_l as the ground state, the bifurcated energy levels are $E_l \pm \Delta/2 = E_l \pm 6.4214 \text{ MeV}$.

(3) Simplified Energy-Spectrum Calculation Table and Its Correspondence with the 0-Charge, 0-Spin High-Energy Particle Spectrum

In the table below, E_l is determined by Eq. (12), with l from 1 to 99, $D = 3$, $2m_e c^2 e^3 = 20.52 \text{ MeV}$. Experimental discovered energy spectra come from PDG-2024 experimental masses [15].

Table 1. Discrete energy-level table for 0-charge, 0-spin high-energy particle states (unit: MeV)

l	E_l	l	E_l	l	E_l	Remarks
1	29.03	34	707.93	67	1385.26	$E_1 - 6.42 = 29.03$ $-6.42 = 22.61$ $E_5 + 22.61 = 135.04$ $E_6 + 6.42 = 139.45$
2	50.28	35	728.46	68	1405.78	
3	71.11	36	748.98	69	1426.31	
4	91.80	37	769.51	70	1446.83	
5	112.43	38	790.03	71	1467.36	
6	133.03	39	810.56	72	1487.88	$E_{38} - 790.03 = 8.09$
7	153.62	40	831.08	73	1508.41	$E_{73} - 1508.41 = 3.61$ $E_{46} - 954.23 = -3.55$ $E_{49} - 1015.81 = -3.65$
8	174.19	41	851.61	74	1528.93	
9	194.75	42	872.13	75	1549.46	
10	215.30	43	892.66	76	1569.98	
11	235.84	44	913.18	77	1590.51	
12	256.37	45	933.71	78	1611.03	
13	276.90	46	954.23	79	1631.56	
14	297.43	47	974.76	80	1652.08	
15	317.96	48	995.28	81	1672.61	
16	338.48	49	1015.81	82	1693.13	
17	359.01	50	1036.33	83	1713.66	$E_{24} - 502.68 = 5.07$ $E_{26} - 543.73 = -4.13$
18	379.53	51	1056.86	84	1734.18	
19	400.06	52	1077.38	85	1754.71	
20	420.58	53	1097.91	86	1775.23	
21	441.11	54	1118.43	87	1795.76	
22	461.63	55	1138.96	88	1816.28	
23	482.16	56	1159.48	89	1836.81	
24	502.68	57	1180.01	90	1857.33	
25	523.21	58	1200.53	91	1877.86	
26	543.73	59	1221.06	92	1898.38	
27	564.26	60	1241.58	93	1918.91	
28	584.78	61	1262.11	94	1939.43	
29	605.31	62	1282.63	95	1959.96	
30	625.83	63	1303.16	96	1980.48	

l	E_l	l	E_l	l	E_l	Remarks
31	646.36	64	1323.68	97	2001.01	
32	666.88	65	1344.21	98	2021.53	
33	687.41	66	1364.73	99	2042.06	

Notes:

- ① In Table 1, only the corresponding positions of some well-known meson energy levels are marked in yellow.
- ② $l + 0.41 \leq \sqrt{l(l+1)} \leq l + 0.50$, as l increases, E_l tends toward an arithmetic relation (~ 20), which is a major feature of the high-energy meson spectrum.
- ③ The energy levels marked in blue are positions where new particles are likely to be discovered with high probability.
- ④ $E_1=29.03\text{MeV}$ is the base value for the regularly energy-level separations occurring in the same family.

Since the eigenenergy spectrum is a stationary-state approximate solution, the energy levels of different particle transients are not specifically determined, it is therefore necessary to use Table 1 and PDG-2024 experimental mass data for matching. The comparison table between the experimental data and the theoretical energy levels (including composite states) is as follows:

Table 2. Comparison of PDG-2024 experimental masses of mesons with theoretical results (MeV)

Particle	Experimental mass	Theoretical topological direct correspondence or possible superposition combination state	Theoretical mass	Absolute error
π^0	134.98	E_6	133.03	1.95
π^0	134.98	$E_5+E_1-\Delta/2$ (112.43+29.03- 6.42)	135.04	0.06
$\pi^\mp$	139.57	$E_6+\Delta/2$ (133.03+6.42)	139.45	0.12
K^0	497.61	E_{24}	502.68	5.07
K^0	497.61	$E_{24}-\Delta/2$	496.26	1.35
η^0	547.86	E_{26}	543.73	4.13
η^0	547.86	$\{ (E_5+E_1-\Delta/2) + (E_6+\Delta/2) \} * 2$	548.96	1.10
ω	781.94	E_{38}	790.03	8.09
ω	781.94	$E_{38}-\Delta/2$ (790.03-6.42)	783.61	1.67
η'	957.78	E_{46}	954.23	3.55
η'	957.78	$E_{20}+E_{26}-\Delta/2$ (420.58+543.73-6.42)	957.89	0.11
ϕ	1019.46	E_{49}	1015.81	3.65
ρ^0	1504.8	E_{73}	1508.41	3.61

Note: For direct correspondence values (yellow-marked), excluding the large value 8.09, the mean absolute error is 3.66; the mean absolute error of superposition combination states is 0.73.

(4) Discussion

- ① From the energy-spectrum comparison, the experimental masses of π^0 、 K^0 、 η 、 ω 、 η' 、 ϕ 、 ρ^0 are highly consistent with the theoretical values.
- ② The direct-correspondence error value 3.66 is relatively large, while the error value using superposition combination states is only 0.73. This indicates that for the $0 \rightarrow 1$ creation topological transition, for the same structure dimension D , the particle state of low-energy-level

spectrum indeed has two different spatial symmetry forms, causing an intrinsic energy-level separation deviation $\Delta/2$. In particular, for the structural identification of the π meson transient state, if the combination form $(E_5 + E_1 - \Delta/2)$ or $(E_6 + \Delta/2)$ is chosen, and higher energy-level spectral lines are characterized by superposition combination states, the accuracy will be better.

③ $E_1=29.03$ MeV is the ground-state energy level, it naturally appears with high probability at the boundary regions between neighboring sub-spectra in the meson spectrum, or as the spacing gap (multiple) between low-lying energy levels.

④ Extended Klein-Gordon equation, its eigen-solutions accurately describe the 0-charge, 0-spin high-energy particle transient states—the neutral meson particle spectrum. Near 420 MeV, or in the 110–120 MeV region, there is a high probability of discovering new particle transient states.

3.2.2 Dirac Equation and the Spectrum of High-Energy Particles Composed of Electrons and Positive-Negative Electron Pairs (Leptons)

(1) Derivation of the Dirac Equation, and the Corresponding Topological Mass Tensor

According to Geometry of Space-time Structure, the energy of a single electron in a high-energy state consists of rest mass energy, relativistic kinetic energy, and spin-electromagnetic-field topological coupling energy. The electron carries intrinsic spin $\hbar/2$, and its quantum state is completely characterized by a four-component spinor; its dynamic equation can naturally be expressed in matrix-operation form.

High-energy particle motion can be described by 4D Minkowski space, vectors are four-component, and operators are naturally 4×4 matrices. Linearize the global energy term in spinor space: the α matrix characterizes the spin magnetic moment and realizes spin-electromagnetic field coupling; the β matrix describes the mass-annihilation duality. Including the electromagnetic scalar potential ϕ and vector potential $\mathbf{A}$, the electron Hamiltonian is:

$$H = c\boldsymbol{\alpha} \cdot (\mathbf{p} + e\mathbf{A}) + \beta m_e c^2 - e\phi \quad (13)$$

where α is a fourth-order spinor matrix characterizing the $\pm\hbar/2$ spin property, naturally constructed from 2×2 Pauli matrices and satisfying spin-vector commutation relations; the β matrix is the constant coefficient of mass energy, naturally a real constant unitary matrix, and paired with the three α_i to form a complete 4D vector. Electrons have positive and negative electron states, so $\beta = \text{diag}\{1, 1, -1, -1\}$ is the natural choice; the two $+1$ correspond to positive-energy states, and the two -1 correspond to negative-energy states. Define the γ^μ matrices ($\gamma^0 = \beta, \gamma^i = \beta\alpha_i, i = 1, 2, 3$), introduce the covariant derivative $D_\mu = \partial_\mu + ieA_\mu$, and obtain the covariant Dirac equation describing the high-energy motion of a single electron:

$$(i\gamma^\mu D_\mu - m_e)\psi = 0 \quad (14)$$

The premise of the classical Dirac equation analysis is that a single electron is taken as an independent unit and mass is a scalar. This can solve the interaction problem between the electron and other matter, and also includes the production and annihilation of positive-negative electron pairs, but it certainly cannot explain many high-energy particle transient states with $\pm e$, because these transients with $e^\pm$, constructed in the form $(e^-e^+e^-)$ or $(e^+e^-e^+)$, and presented by $e^\pm$ and $\Sigma(e^-e^+)$ pairs through zero-distance electromagnetic energy winding, are a “new” particle with large mass, a quantum whole corresponding to a “new mass” coupled with electromagnetic spinor energy; the single electron no longer exists.

The geometry of space-time structure introduces a topological mass tensor, giving the Dirac equation an entirely new physical meaning: it can describe a charged light particle spectrum—a high-energy transient composite particle state formed by a single net charge e^- and one or more

positive-negative electron pairs $\sum(e^-, e^+)$ winding-knot in the $r \rightarrow 0$ topological space. This point-like quantum structure has no center and no boundary; it is a holographic fully symmetric manifold, transiently produced and annihilated. This not only explains the source of new mass but also reveals the essence of the diagonal, unitary, zero-trace matrix structural form of the mass tensor. For this kind of high-energy particle transient state with e^- , after annihilation e^- remains unchanged; therefore there is one definite static electron mass energy m_e , and charge is conserved. In the equation, m_e is represented by $m_e I_4$, where I_4 is the 4×4 identity matrix; all other additional mass energy comes entirely from electromagnetic topological winding coupling contributions, the same form as in Section 3.2.1, i.e., the mass tensor $M = m_e \text{diag}\{1, -1, i, -i\}$. Substituting into (14) gives the generalized covariant Dirac equation:

$$(i\gamma^\mu D_\mu - m_e I_4 - M)\psi = 0 \quad (15)$$

(2) Solving the Generalized Dirac Equation—Topological Operator Eigenvalues and Particle Transient Energy-Level Spectrum

For particle transients constructed from e^- and $\sum(e^-, e^+)$ pairs, there is spin $S = \pm\hbar/2$. Electromagnetic topological winding-knot involves two types of coupling: angular momentum L with angular momentum L , and angular momentum L with spin S . Under the constraint of the $0 \rightarrow 1$ creation topological transition in the infinitely small space $r \rightarrow 0$, the two couplings independently contribute mass energy. Let E_l denote the angular-momentum coupling contribution, and E_{lj} denote the contribution of angular momentum and spin coupling, $j=l \pm 1/2$. For an isolated transient particle state, total momentum ~ 0 , scalar potential $\phi = 0$, vector potential $A = 0$. $M = m_e \text{diag}\{1, -1, i, -i\}$.

A notable feature of the mathematical analysis and derivation of the Geometry of Space-time Structure is that it uses the most essential natural laws and the simplest intuitive geometric logic to analyze and characterize complex phenomenological relations. Below, the same simplification method as in the previous section's analysis of the extended KG equation is adopted. Under the constraint of the $0 \rightarrow 1$ creation topological transition in the infinitely small space $r \rightarrow 0$, the r dimension is frozen, leaving only the characteristic scale quantity r_0 ; the space-time differential structure degenerates into an angular-momentum topological algebraic structure; the Laplacian retains only the angular topological rotation degree of freedom; there is only the overall topological spin $S = \pm\hbar/2$; spin coupling with its own angular momentum $\hat{L} \cdot \hat{S}$ and $\hat{L}^2$ are two independent degrees of freedom; the system is integrated and has holographic, inner-outer-free full symmetry (including all topological rotational symmetries and topological dimension D , i.e., topological degrees of freedom symmetry, here $D = 3$). The unit strength of different couplings is equal, $\Lambda \propto 1/r_0$; here $\Lambda = 2m_e c^2 \times e^3 = 20.5274 \text{ MeV}$. Under such a specific physical picture, the main energy-level contribution terms $\hat{L}^2$ and $\hat{L} \cdot \hat{S}$ are relatively independent and both commute with the total energy operator. Therefore, the general topological energy level E of the generalized covariant Dirac equation is:

$$E = m_e c^2 + E_l + E_{lj} = m_e c^2 + \Lambda \frac{\hat{L}^2}{\hbar^2} + \Lambda \frac{\hat{L} \cdot \hat{S}}{\hbar^2} \quad (16)$$

where $\hat{L}^2 = l(l+1)\hbar^2$, $\hat{L} \cdot \hat{S} = (J^2 - L^2 - S^2)/2 = \frac{\hbar^2}{2} [j(j+1) - l(l+1) - s(s+1)]$, $l \geq 1$,

$s=1/2$, $j=l \pm 1/2$, $E_l = \Lambda \hat{L}^2 / \hbar^2$, $E_{lj} = \Lambda \hat{L} \cdot \hat{S} / \hbar^2$. Substituting into (16), the precise and specific topological eigenvalue energy level E of the generalized Dirac equation is obtained:

$$E = m_e c^2 + \Lambda l(l+1) + \frac{\Lambda}{2} [j(j+1) - l(l+1) - s(s+1)] \quad (17)$$

Substituting $j = l \pm 1/2$ into Eq. (17) and letting $E_{l+}=E_{l,j=l+1/2}$, $E_{l-}=E_{l,j=l-1/2}$, the specific calculation formulas for the topological eigenvalue energy-level spectrum are obtained after rearrangement:

$$\begin{cases} E_+ = m_e c^2 + E_l + E_{l+} = m_e c^2 + \Lambda l(l+1) + \frac{\Lambda}{2} l, & \text{当 } j = l + 1/2 \\ E_- = m_e c^2 + E_l + E_{l-} = m_e c^2 + \Lambda l(l+1) - \frac{\Lambda}{2} (l+1), & \text{当 } j = l - 1/2 \end{cases} \quad (18)$$

where $l \geq 1$, $\Lambda = 2m_e c^2 e^3 \approx 20.527 \text{ MeV}$, $m_e c^2 = 0.511 \text{ MeV}$.

(3) Simplified Energy-Spectrum Calculation Table and Its Correspondence with the Energy Spectra of Leptons and Other Related High-Energy Particle Transients

Using Eq. (18), Tables 3 and 4 below are calculated, where $l = 1 \sim 15$, and calculated values retain 2 decimal places. If considering that the low-energy-level spectrum, due to the same structure dimension D , has two different spatial symmetry forms, causing an intrinsic energy-level separation deviation $\Delta = 4\pi \cdot 2m_e c^2$, the separated energy-level correction value is taken as $\pm \Delta/2 \approx \pm 6.421 \text{ MeV}$. Experimental values come from PDG-2024 [15].

Table 3. Discrete energy-level calculation table for the high-energy particle spectrum composed of electrons and positive-negative electron pairs (unit: MeV)

l	E_l	E_{l+}	E_l+E_{l+}	$E_+=E_l+E_{l+}+0.51$	E_{l-}	E_l+E_{l-}	$E_-=E_l+E_{l-}+0.51$
1	41.05	10.26	51.31	51.82	-20.53	20.52	21.03
2	123.16	20.53	143.69	144.20	-30.79	92.37	92.88
3	246.33	30.79	277.12	277.63	-41.05	205.28	205.79
4	410.55	41.05	451.60	452.11	-51.32	359.23	359.74
5	615.82	51.32	667.14	667.65	-61.58	554.24	554.75
6	862.15	61.58	923.73	924.24	-71.85	790.30	790.81
7	1149.53	71.85	1221.38	1221.89	-82.11	1067.42	1067.93
8	1477.97	82.11	1560.08	1560.59	-92.37	1385.60	1386.11
9	1847.47	92.37	1939.84	1940.35	-102.64	1744.83	1745.34
10	2258.01	102.64	2360.65	2361.16	-112.90	2145.11	2145.62
11	2709.62	112.90	2822.52	2823.03	-123.16	2586.46	2586.97
12	3202.27	123.16	3325.43	3325.94	-133.43	3068.84	3069.35
13	3735.99	133.43	3869.42	3869.93	-143.69	3592.30	3592.81
14	4310.75	143.69	4454.44	4454.95	-153.96	4156.79	4157.30
15	4926.58	153.96	5080.54	5081.05	-164.22	4762.36	4762.87

Note: ① E_l is the angular-momentum coupling; E_+ and E_- , and E_{l+} and E_{l-} , are respectively the total energy levels and spin-angular-momentum couplings for $j = l + 1/2$ and $j = l - 1/2$.

② Yellow-marked values are related to the energy levels of well-known leptons; 0.51 MeV is the electron rest mass.

③ The energy-level difference of E_l increases with increasing l , as $2l\Lambda$; for adjacent values, the energy-level differences of E_{l+} , E_{l-} are all the constant $\Lambda/2 = 10.263 \text{ MeV}$, and $E_{l+} + E_{l-} = \Lambda/2$.

Using Table 3 and PDG-2024 experimental mass data, the comparison table of lepton and other particle energy levels with theoretical energy levels, Table 4, is obtained:

Table 4. Comparison of PDG-2024 experimental masses of well-known leptons and other particles with theoretical calculation results (unit: MeV)

Particle type	Experimental mass	Possible superposition combination state of theoretical topological construction	Theoretical mass	Deviation	Note
$e^\mp$	m_e	$l=0$, The electron itself	m_e	0	identical
$\mu^\pm$	105.66	$1E_-*5 + 0.51, (1E_-=21.03)$	105.66	0	identical
$\pi^\pm$	139.57	$E_2+E_{1+}+6.42, (E_2=123.16, E_{1+}=10.26)$	139.84	0.27	Exact match
$K^\pm$	493.68	$3E_+ 277.63 + 3E_- 205.79 + E_{1+}10.26$	493.68	0	identical
$\rho^\pm$	775.5	$4E_+452.11+205.79+123.16-6.42+0.51$	775.15	-0.35	Exact match
$\tau^\pm$	1776.86	$7E_+ 1221.89 +5E_- 554.75$	1776.64	-0.22	Exact match
$D^\pm$	1869.6	$7E_+ 1221.89 +5E_- 554.75 +2E_-92.88$	1869.52	-0.08	Exact match

(4) Discussion

- ① The theoretical value and experimental value of $\mu^\pm$ are both 105.66MeV, and the theoretical value and experimental value of $K^\pm$ are both 493.68MeV, both with zero deviation; the energy-level combination structures are small multi-energy-level composite structures in which $\hat{L}^2$ coupling and $\hat{L} \cdot \hat{S}$ coupling appear simultaneously; the equal-status holographic symmetric coupling mechanism of one net-charged $e^\pm$ electron and $\Sigma(e^-, e^+)$ pairs dominates.
- ② The theoretical value of $\tau^\pm$ is 1776.64MeV, and the experimental value is 1776.86MeV, with a deviation of only 0.22MeV, a precise match. The possible energy-level combination is $E_{7+} 1221.89 + E_{5-} 554.75$, also with $\hat{L}^2$ coupling and $\hat{L} \cdot \hat{S}$ coupling appearing simultaneously; the mechanism is similar to ①.
- ③ The theoretical value of $\pi^\pm$ is 139.84MeV, only 0.27MeV different from the experimental value 139.57MeV. The combination state is dominated by $E_2 = 123.16$, with slightly lower precision than the theoretical value 139.45MeV in Table 2 (entirely from $\hat{L}^2$ coupling). This precisely indicates that it has both $e^\pm$ charge and possibly 0-spin physical properties.
- ④ The generalized Dirac equation given by the Geometry of Space-time Structure, under the constraint of the $0 \rightarrow 1$ creation topological transition in the infinitely small space-time $r \rightarrow 0$, has eigen-solutions that can accurately describe the particle transient spectrum whose decay final states with net charge $e^\pm$ are all electron $e^\pm$ and photons. It is estimated that near 64MeV, 206MeV, and 360MeV, there is a relatively high probability of discovering new charged particle transient states.

3.2.3 Generalized Dirac Equation for the Proton and the Spectrum of High-Energy Particle Transients Composed of Protons and Positive-Negative Electron Pairs (Baryons)

(1) Creating the Generalized Dirac Equation for the Proton

The Geometry of Space-time Structure holds that composite particles are all transient states of matter-energy quanta formed by topological winding-knot of electromagnetic energy under the constraint of infinitely small space-time. For a composite particle transient (baryon) composed of a proton P^+ carrying e^+ and positive-negative electron pairs $\Sigma(e^-, e^+)$, apart from the fact that at low energy levels the proton becomes the de facto center of inertial mass, the physical basis, dominated by strong electromagnetic energy action, is no different from the particle transient spectrum constructed from $e^\mp$ and $\Sigma(e^-, e^+)$ pairs demonstrated in Section 3.2.2. The spin of the proton is also $\hbar/2$. Replacing m_e in the Hamiltonian of Eq. (13) with m_p gives the Hamiltonian for the high-energy motion of a proton in an electromagnetic field. Therefore,

replacing m_e in the electron's generalized covariant Dirac equation (15) with m_p , while the mass tensor $M = m_e \text{diag}\{1, -1, i, -i\}$ remains unchanged, gives the proton's generalized covariant Dirac equation:

$$(i\gamma^\mu D_\mu - m_p I_4 - M)\psi = 0 \quad (19)$$

Equation (19) is the relativistic dynamic equation describing the composite particle transient (baryon spectrum) composed of a proton P^+ carrying e^+ and positive-negative electron pairs $\Sigma(e^-, e^+)$. Thus, this paper has unified the entire high-energy particle transient spectrum within a concise framework characterized by one set of dynamic equations.

(2) Solving the Proton's Generalized Dirac Equation and Calculation and Analysis of Its Energy Level Spectrum

For the composite particle transient (baryon) composed of a proton P^+ and positive-negative electron pairs $\Sigma(e^-, e^+)$, the proton actually becomes the inertial center of this space-time transient topological structure; the topological symmetry is mainly characterized by center symmetry. As a quantum whole, the mass energy of the composite particle transient from electromagnetic spinor topological winding-knot coupling is mainly dominated by the total angular momentum ($J = L + S$) coupling energy level; the total angular momentum $\hat{J}^2$ commutes with the Hamiltonian operator, and its eigenvalue energy level is $E_j \propto \hat{J}^2/\hbar^2$. At the same time, the proton carries $\hbar/2$ spin, and there exists an independent mass-energy contribution from the spin-angular momentum coupling $\hat{L} \cdot \hat{S}$ of the proton's intrinsic spin and the positive-negative electron pairs' angular momentum; this part is completely the same as the related content in Section 3.2.2 and also commutes with the Hamiltonian operator; the corresponding mass energy can be denoted by E_{lj} , $E_{lj} \propto \hat{L} \cdot \hat{S}/\hbar^2$. The unit strength of different couplings is likewise equal, $\Lambda = 2m_e c^2 \times e^3 = 20.5274 \text{ MeV}$. Using logic similar to Section 3.2.2, the topological eigenvalue energy level E of the proton's generalized Dirac equation (19) is:

$$E = m_p c^2 + E_j + E_{lj} = m_p c^2 + \Lambda \frac{\hat{J}^2}{\hbar^2} + \Lambda \frac{\hat{L} \cdot \hat{S}}{\hbar^2} \quad (20)$$

where $\hat{J}^2 = j(j+1)\hbar^2$, $\hat{L} \cdot \hat{S} = (J^2 - L^2 - S^2)/2 = \frac{\hbar^2}{2} [j(j+1) - l(l+1) - s(s+1)]$, $l \geq 1$,

$s=1/2$, $j=l \pm 1/2$, $E_j = \Lambda \hat{J}^2/\hbar^2$, $E_{lj} = \Lambda \hat{L} \cdot \hat{S}/\hbar^2$. Substituting into (20), the precise and specific topological eigenvalue energy level E of the proton's generalized Dirac equation is obtained:

$$E = m_p c^2 + \Lambda j(j+1) + \frac{\Lambda}{2} [j(j+1) - l(l+1) - 3/4] \quad (21)$$

Substituting $j = l \pm 1/2$ into Eq. (21) and letting $E_{j+} = E_{j=l+1/2}$, $E_{j-} = E_{j=l-1/2}$, $E_{l+} = E_{l,j=l+1/2}$, $E_{l-} = E_{l,j=l-1/2}$, the specific calculation formulas for the topological eigenvalue energy-level spectrum are obtained after rearrangement:

$$\begin{cases} E_+ = m_p c^2 + E_{j+} + E_{l+} = m_p c^2 + \Lambda j(j+1) + \frac{\Lambda}{2} l, & \text{当 } j = l + 1/2 \\ E_- = m_p c^2 + E_{j-} + E_{l-} = m_p c^2 + \Lambda j(j+1) - \frac{\Lambda}{2} (l+1), & \text{当 } j = l - 1/2 \end{cases} \quad (22)$$

where $l \geq 1$, $\Lambda = 2m_e c^2 e^3 \approx 20.527 \text{ MeV}$, $m_p c^2 = 938.272 \text{ MeV}$.

Using Eq. (22), Tables 5 and 6 below are calculated, where $l = 1 \sim 15$, and calculated values retain 2 decimal places. If considering that the same structural dimension D , due to differences in spatial symmetry manifestation, causes intrinsic energy-level splitting, the energy-level correction value is taken as $\pm \Delta/2 = \pm 4\pi \cdot 2m_e c^2/2 \approx \pm 6.421 \text{ MeV}$. Experimental values come from PDG-2024 [15].

Table 5. Discrete energy-level calculation table for the high-energy particle spectrum composed of protons and positive-negative electron pairs (unit: MeV)

l	E_{j+}	E_{l+}	$E_+ = m_p c^2 + E_{j+}$	E_{j-}	E_{l-}	$E_- = m_p c^2 + E_{j-}$
1	76.98	10.26	1015.25	15.40	-20.53	953.67
2	179.61	20.53	1117.88	76.98	-30.79	1015.25
3	323.30	30.79	1261.57	179.61	-41.05	1117.88
4	508.04	41.05	1446.32	323.30	-51.32	1261.57
5	733.84	51.32	1672.11	508.04	-61.58	1446.32
6	1000.69	61.58	1938.96	733.84	-71.85	1672.11
7	1308.60	71.85	2246.87	1000.69	-82.11	1938.96
8	1657.56	82.11	2595.83	1308.60	-92.37	2246.87
9	2047.57	92.37	2985.84	1657.56	-102.64	2595.83
10	2478.64	102.64	3416.91	2047.57	-112.90	2985.84
11	2950.76	112.90	3889.03	2478.64	-123.16	3416.91
12	3463.93	123.16	4402.20	2950.76	-133.43	3889.03
13	4018.16	133.43	4956.43	3463.93	-143.69	4402.20
14	4613.44	143.69	5551.72	4018.16	-153.96	4956.43
15	5249.78	153.96	6188.05	4613.44	-164.22	5551.72

- Notes: ① E_+ and E_- , E_{j+} and E_{j-} , E_{l+} and E_{l-} are respectively the total energy levels, total angular momentum couplings, and spin-angular-momentum couplings corresponding to $j = l + 1/2$ and $j = l - 1/2$. Here, $lE_{j+} = (l + 1)E_{j-}$, i.e. for the same l , E_{j-} is one level lower than E_{j+} .
- ② The yellow markings in the table are values related to the energy levels of some well-known baryons and other particles; $m_p c^2 = 938.272$ MeV.
- ③ For adjacent values of l , the energy-level differences of E_{l+} and E_{l-} are both the constant $\Lambda/2 = 10.263$ MeV; moreover, $E_{l+} + E_{l-} = -\Lambda/2$.

Using Table 5 and the PDG 2024 experimental mass data, we obtain Table 6, a comparison between the experimental energy levels and theoretical values for baryons and other particles:

Table 6. Comparison of PDG-2024 experimental masses of well-known baryons and other particles with theoretical calculation results (unit: MeV)

Baryon type	Experimental mass	Theoretical configuration	Theoretical mass	Error	Note
$\Delta(1232)$	1232 ± 2	$3E_+ 1261.57 + 2E_{l-} - 30.79$	1230.78	< 1	$m_p c^2 + 293$
$\Lambda^0(1115)$	1115.68	$2E_+ 1117.88$	1117.88	2.20	
$\Sigma^+(1189)$	1189.37	$2E_+ + E_{l+} 71.85, 3E_+ + 6E_{l-} - 71.85$	1189.72	0.35	Exact match
$\Sigma^0(1192)$	1192.64	$3E_+ + 5E_{l-} - 61.58 - 6.42$	1193.57	0.93	
$\Sigma^-(1197)$	1197.45	$\Sigma^0 + 6.42$	1199.06	1.61	
$\Xi^0(1315)$	1314.8	$3E_+ 1261.57 + 5E_{l+} 51.32$	1312.89	< 2	
$\Xi^-(1321)$	1321.3	$\Xi^0 + 6.42$	1321.22	0.1	Exact match
$N(1440)$	1430-70	$4E_+ 1446.32 - 6.42$	1439.90	0.1	Roper Resonance
$N(1520)$	1515-25	$4E_+ 1446.32 + 1E_{j+} 76.98$	1523.30	3.30	
$\Omega^-(1672)$	1672.45	$5E_+ 1672.11, 6E_{l-} 1672.11$	1672.11	0.34	Exact match

Baryon type	Experimental mass	Theoretical configuration	Theoretical mass	Error	Note
Λ_c^+	2286.46	$7E_+2246.87 + 4E_{l+}41.05$	2287.92	1.46	
$\eta_c(1S)$	2983.9 ± 0.5	$9E_+2985.84, 106E_{l+}2985.84$	2985.84	<2	
$\chi_{c2}(1P)$	3510.67	$10E_+3416.91 + 9E_{l+}92.37$	3509.28	1.39	
$\psi(4040)$	4039 ± 1	$11E_+3889.03 + 14E_{l+}143.69 + 6.42$	4029.14	0.14	Exact match

(3) Discussion

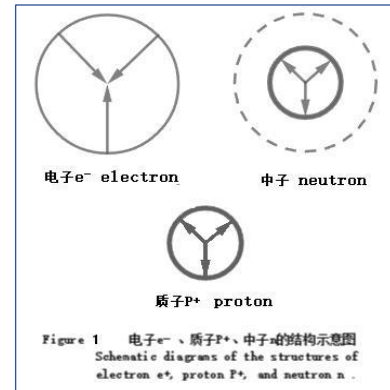
- ① From Table 6, it can be seen that the known baryon energy levels are all in the combined form of total angular-momentum coupling as the main energy levels and spin-angular-momentum coupling contributing the fine structure; the theoretical values match the experimental values precisely.
- ② In the baryon spectrum, multiple sub-spectra exhibit equal energy-level spacing or linear increasing relationships, which are mainly determined by the linear increase (decrease) characteristic of spin-angular-momentum coupling energy levels. Low-level baryons take the proton as the center of mass, with positive-negative electron pairs' topological winding-knot at near-zero distance, and there is a high probability of an approximately spherical shell-layer configuration.
- ③ For transients of particles with three states—positive charge, negative charge, and electrical neutrality—the energy-level differences often exhibit the relation $\pm\Delta/2 \approx \pm 6.421 \text{ MeV}$, which precisely indicates that the mass-energy of high-energy particle transients is entirely constructed from the electromagnetic-energy topological winding combination; being essentially of the same origin.
- ④ Higher-level baryon transients are shorter-lived and their decays are more complex, with unclear energy-level separation; in addition, the spin property is relatively weak, independent (e^-, e^+) angular-momentum coupling may occur, that is, secondary spectral lines similar to $\Lambda\sqrt{l(l+1)}$ in the meson spectrum with relatively large values of l .

4. Neutron Structure, and Precise Calculation of the Neutron Energy Level and Lifetime

The neutron is a unique and also the most important composite particle; it is completely different from the high-energy particle transient states analyzed above. The action and structure of the charge properties of e^- and P^+ completely overlap and cancel each other (presenting pure matter energy), forming a huge binding force on both, especially on the electron e^- . Only in this way is the unique high-energy stable state of the neutron, composed of positive and negative charges, formed. The neutron energy level is higher than the proton energy level by $\Delta E = (m_n - m_p)c^2 = +1.2933\text{MeV} = 2.531m_e c^2$. The neutron's high energy level ΔE and decay period τ are both among the most important space-time material structure constants, can be obtained from the analysis of neutron dynamics equations

4.1 Structure of the Neutron

The neutron is composed of one proton and one high-energy electron. According to Geometry of Space-time Structure, the structure relationship among the neutron, proton, and electron is shown in schematic Figure 1. The mass centers and positive negative charge centers of the proton and electron completely coincide; the external electrical property disappears, giving 0 charge; the electron is constrained by a centrally symmetric high-energy potential barrier, and the proton is wrapped near the center



of the space-time structure. The neutron magnetic moment is opposite to the spin direction (similar to the electron). The energy level of the neutron constructed by electron topological winding can be approximated as mainly composed of ground-state spin-angular-momentum coupling (+) and ground-state electron spin-proton spin coupling (-), Using the method in Section 3.2.2, respectively approximated as $4\pi m_e c^2$ and $-3\pi m_e c^2$, $\Delta E \approx 4\pi m_e c^2 - 3\pi m_e c^2 = \pi m_e c^2 > 2.531 m_e c^2$. Below, a precise algorithm based on Geometry of Space-time Structure is given.

4.2 Neutron-Proton Mass Difference

As the most basic composite configuration of a proton and electron, the neutron's energy level and lifetime are fundamental material parameters and must be uniquely determined by the most basic natural constants. Constructing a neutron belongs to the basic $0 \rightarrow 1$ creation, the spatial scale size is $\Delta r \rightarrow 0$, the internal spatial coordinates (x,y,z) are neglected; what is created is "form," and what is determined is the "form" of the structure dimension D (i.e., the motional degrees of freedom of internal elementary particles). The neutron's "form" is mainly manifested by the electron. At this time, the structural form can only be characterized by how many equivalent electrons there are. The topological winding form of the electron in the neutron in three-dimensional space is a compact convex closed surface body, the determining quantity is one spherical solid angle 4π ; this is the number of structurally equivalent electron states (symmetric states). Its relation to the electron degrees of freedom D is $e^D = 4\pi$, so $D = \ln(4\pi) = 2.53102425$. Each degree of freedom contributes one electron mass energy, therefore:

$$(m_n - m_p)c^2 = D \cdot m_e c^2 = \ln(4\pi) \cdot m_e c^2 = 2.531 m_e c^2 = 1.2933 \text{ MeV} \quad (23)$$

This is completely consistent with the experimental value.

4.3 Neutron Lifetime

The neutron lifetime is likewise uniquely determined by the basic natural constants. As the most elementary, most essential, and most direct composite material particle, the neutron decay process should naturally embody the basic laws of energy-matter interaction and change. The final state of

neutron decay is a proton, an electron, and photons, with the interactions $\frac{e^2}{4\pi\epsilon_0 r}$ and $\frac{Gm_p m_e}{r}$, the

"transition probability" should naturally reflect such secondary effects. From Eq. (4), the neutron lifetime τ is inversely proportional to its high energy level $\Delta E = (m_n - m_p)c^2$; the electron moves in the full space-time centered on the proton, its stability is obviously proportional to $\pi m_p/m_e$, and at the same time should embody the secondary correction of the non-contact

long-distance interaction, namely $\sqrt{\frac{e^2}{4\pi\epsilon_0 r} / \frac{Gm_p m_e}{r}}$. Thus:

$$\tau = \frac{\pi h}{(m_n - m_p)c^2} \cdot \frac{m_p}{m_e} \cdot \sqrt{\frac{e^2}{4\pi\epsilon_0 Gm_p m_e}} \quad (24)$$

Substituting the constant values gives: $\tau = 879 \text{ s}$, which is highly consistent with the 2021 measured value $(877.75 \pm 0.28 \text{ s})$ of Gonzalez et al. [16]. This formula perfectly unifies the basic interactions of nature. Neutron decay is considered the most basic "weak interaction" reaction, but this formula contains no characteristic quantity of weak interaction; weak interaction is not a basic material interaction force, it is actually a form of electromagnetic interaction, and electroweak unification "naturally" holds. This formula was first discovered by Yixing D.Z. of China [17].

5. Summary

Starting from the basic logic of Geometry of Space-Time Structure, this paper rigorously derives

the core architectural relations of the Standard Model of particle physics. At the same time, it briefly demonstrates that Geometry of Space-Time Structure can completely unify the basic logical relations of physics and mathematics created for the purpose of revealing natural laws. It establishes a complete set of characterization equations for the high-energy particle transient spectrum, and the eigen-energy-level combinations of the equations precisely match experimental masses. In addition, using only the most basic natural constants, it precisely derives the neutron–proton mass difference, neutron lifetime, etc. However, there is little point in studying too many high-energy particles (transients). The most important thing is the fusion of light nuclei.⁹

Finally, special thanks are due to the author’s family for their long-term understanding, support, hard work, and dedication!

References

- [1] M. E. Peskin, D. V. Schroeder. An Introduction to Quantum Field Theory[M]. Westview Press, 1995.
- [2] M. D. Schwartz. Quantum Field Theory and the Standard Model[M]. Cambridge University Press, 2014.
- [3] S. Weinberg. Effective Field Theory, Past and Future [J]. PoS CD09, 2009: 001.
- [4] F. Wilczek. Origins of Mass [J]. Nuclear Physics B-Proceedings Supplements, 2004, 134: 3–13.
- [5] G. 't Hooft. Naturalness, Chiral Symmetry, and Spontaneous Chiral Symmetry Breaking [J]. NATO Sci. Ser. B, 1980, 59: 135.
- [6] D. J. Gross, F. Wilczek. Ultraviolet Behavior of Non-Abelian Gauge Theories [J]. Physical Review Letters, 1973, 30(26): 1343–1346.
- [7] H. D. Politzer. Reliable Perturbative Results for Strong Interactions [J]. Physical Review Letters, 1973, 30(26): 1346–1349.
- [8] Tai Wei Song. Physical Foundations and Mathematical Logic of the Natural World. viXra.org/abs/2512.0009, 2025.
- [9] Taiwei Song. Logical Foundations of Geometric Space and Natural Space-Time Space, Int Nat Sci Int Rese, 1(2), 01–04, 2026. <https://www.opastpublishers.com/search-results.php>; <https://vixra.org/abs/2605.0113>
- [10] Taiwei Song. Elementary Particles, Atomic Nuclei, and Strong Interactions: The Application of the Geometry of Space-Time Structures at Extremely Small Spatial Scale. viXra.org/abs/2510.0091, 2025.
- [11] Taiwei Song. Space Warp, Space-Time Transformation, and Cosmic Redshift: The Application of the Geometry of Space-Time Structures on Large Spatial Scales. viXra.org/abs/2510.0087, 2025.
- [12] Riemann B., On the Hypotheses which lie at the Bases of Geometry, Nature, Vol. VIII. Nos. 183, 184, pp. 14–17, 36, 37. <http://www.emis.de/classics/Riemann/WKCGeom.pdf>, 1873.
- [13] S. S. Chern, A simple intrinsic proof of the Gauss-Bonnet formula for closed Riemannian manifolds, Annals of Mathematics, 1944, 45(4): 747–752.
- [14] N. Manton, P. Sutcliffe, Topological Solitons. Cambridge University Press, 2004.
- [15] S. Navas et al. (PDG), Review of Particle Physics, Phys. Rev. D 110, 030001, 2024.
- [16] F. M. Gonzalez et al., Precision Measurement of the Neutron Lifetime, Physical Review Letters, 2021, 127(16): 162501.
- [17] Yixing D.Z., Neutron Lifetime, zhuanlan.zhihu.com/p/462596878, 2022.