

A Classical Model for Quantum Wave-Particle Duality and Two-Slit One-at-a-Time Particle Interference

Izzie Boxen

Abstract: A model is presented here to help explain quantum wave-particle duality and two-slit one-at-a-time particle interference in a classical way, without resorting to the complex mathematics that is used in quantum field theory and even classical wave theory. This model avoids wave function collapse during measurement, the measurement problem.

1. Introduction

A basic tenet of quantum mechanics is what is called wave-particle duality. This phenomenon, which will be defined later, has so far been unexplainable in a classical physics way. By reviewing several other physics phenomena, with emphasis on two-slit one-at-a-time interference, a model will be proposed here that remains consistent with both quantum mechanics and classical physics, thereby making it easier to understand intuitively both wave-particle duality and interference of particles. This model does not claim to be how nature behaves. But who knows? Until very early in the 20th century atoms and molecules were considered useful figments of the imagination. The primary design of the model to be presented here is to create some intuitive understanding. A reason for looking for models that are more intuitive and therefore easier to understand is exemplified by one of Kurt Vonnegut's poems from his 1963 novel, *Cat's Cradle*:

On the Quest for Understanding

Tiger got to hunt, bird got to fly;
Man got to sit and wonder 'why, why, why?'
Tiger got to sleep, bird got to land;
Man got to tell himself he understand.

2. Quantum Mechanics is Non-Deterministic

Newtonian mechanics, also called classical mechanics/physics, is formulated to be deterministic from its onset. By deterministic we mean that knowing all the properties of a particle or system of particles completely and precisely at any given time allows calculation of all future and past properties, completely and precisely. On the other hand, quantum mechanics is based on describing all known "particles"¹ and systems of "particles" in terms of complex functions Ψ whose squared amplitudes $|\Psi|^2 = \Psi^* \Psi$ give probabilities². This is called the Born rule, and Ψ is called a probability amplitude.

¹ The word "particle" is contained within quotation marks in this sentence since no satisfactory definition of a particle in quantum mechanics exists yet. I will give my own definition later. After this sentence the quotation marks will be omitted.

² Max Born expounded on this, starting in 1926 and elaborated further at the 5th Solvay Conference in 1927. An account of this can be found at

Guido Bacciagaluppi. The Statistical Interpretation according to Born and Heisenberg. in Christian Joas, Christoph Lehner, Jürgen Renn. HQ-1: Conference on the History of Quantum Physics, MPIWG, pp.269-288 (Vol. II), 2008, MPIWG preprint series (Vol. 350). (halshs-00999722).

We often also call Ψ a wave function.³ Quantum mechanics is then probabilistic, and therefore non-deterministic. As additional reason for quantum mechanics being non-deterministic will be given shortly.

3. Some Other Confusing Properties in Physics

In addition, there are properties and relations in quantum mechanics that as yet cannot be satisfactorily derived or explained by classical mechanics. Two of many examples are intrinsic angular momentum of particles (called spin) and what is called color charge of quarks. Classical and quantum mechanics are therefore, for now, incompatible with each other in many aspects. This causes great angst in people exposed to quantum mechanics for the first time while they are still thinking in terms of classical physics, having no classical analogies, and seeing what to them appear as inconsistencies or paradoxes. Combine this with the results of special relativity (formulated by Albert Einstein in 1905 [1]) and general relativity (formulated by Albert Einstein and completed in 1915 [2]) and things can get very confusing at first.

One of the major differences between classical and quantum mechanics is Heisenberg's uncertainty principle, discovered in 1927 [3]. This principle can be derived from the mathematical relation of variables f, g that are described as waves and are Fourier transforms of each other.⁴ Variables that are Fourier transforms of each other are called (canonically) conjugate variables. When the variables are described in terms of waves, as they are in Fourier series, mathematics gives that the product of their standard deviations σ_f, σ_g (usually written as $\Delta f, \Delta g$ in quantum mechanics) must be equal to or greater than half the smallest wavelength of the waves used in the description. More restrictively, the resolution (standard deviation) of any variable described by a wave of wavelength λ can be no better than $\frac{\lambda}{2}$. The variables position x and momentum p , as well as energy T and time t ,⁵ among other pairs, are such conjugate variables. In quantum mechanics, according to Louis de Broglie's PhD thesis insight in 1924 [4], particles have wave properties associated with them, and a wave of wavelength λ associated with a particle of momentum p has $\lambda = \frac{h}{p}$, where h is Planck's constant.

Omitting the mathematical definition of standard deviation and the details of manipulation with Fourier transforms, this gives

$$(\Delta x)(\Delta p) \geq \frac{1}{2} \left(\frac{h}{2\pi} \right),$$

which is usually written in quantum mechanics as

$$\Delta x \Delta p \geq \frac{\hbar}{2}, \text{ where } \hbar = \frac{h}{2\pi}.^6 \quad (1)$$

³ The term "wave" does not mean that Ψ is a wave, only that it satisfies a complex wave equation.

⁴ More precisely, conjugate variables f, g satisfy the following: if g is the Fourier transform of f , then f is the inverse Fourier transform of g , and *vice versa*.

⁵ T is used here for energy rather than the standard E to avoid confusion later with the electric field $\vec{E}$.

⁶ The following is condensed from

That Ψ in quantum mechanics gives $|\Psi|^2 = \Psi^* \Psi$ as a probability by itself immediately means quantum mechanics is non-deterministic, both in the forward and backward direction of time. Also, in quantum mechanics, equation (1) means that it is impossible to derive a particle's position and momentum completely and precisely, both derived at the same time and at any time, past, present, or future. However, each of position and momentum can still be determined as precisely as desired if the other is ignored. This is another reason quantum mechanics is non-deterministic.

4. Wave-Particle Duality

The formulation of quantum mechanics in terms of complex wave functions Ψ is the starting point for ambiguity between quantum mechanical and classical particles. Some behaviors of a quantum particle are those of classical particles, while some are those of classical waves. Importantly, both behaviors can appear to exist at the same time in some situations. This ambiguity is called wave-particle duality in quantum mechanics and gives rise to situations that cannot be understood by present classical physics reasoning.

Classical wave diffraction at obstructing edges, as well as interference using one or more slits, has been described for two centuries now, with the theory and mathematical derivations found in most texts on optics. The foremost example of wave-particle duality in quantum mechanics occurs in two-slit interference of particles passing one at a time through the slits, something I call singleton flux interference.⁷ The arguments presented from here on can be applied to any number of slits, but two slits is the easiest and suffices for the model to be presented here. In this discussion we could include the repeating crest-trough pattern of interference of classical waves found with each slit, due to

<https://www.aps.org/archives/publications/apsnews/199001/heisenberg.cfm>
 The Sad Story of Heisenberg's Doctoral Oral Exam | American Physical Society,
 with some of my own comments added:

Werner Heisenberg was an exceptional student, at the top of his class, doing his PhD under the supervision of the physicist Arnold Sommerfeld who specialized in theoretical physics. During Heisenberg's oral exam in 1923 after his doctoral dissertation (now called a PhD thesis) was accepted, cum laude, one of the four examiners, the experimental physicist Willy Wien got angry after asking a number of questions about experimental physics, none of which Heisenberg could answer satisfactorily. One of these was to derive the resolution of a telescope and microscope, something that requires using diffraction theory. Heisenberg had done some mandatory experimental lab work under Wien, but had not performed well there, having little interest in experimental physics. He excelled in applying mathematics to difficult physics problems, as was evident in his doctoral thesis. This was a time before there was a strong distinction between theoretical and experimental physics, and Wien saw no reason to pass Heisenberg. It should be remembered that, in the past, a student reaching a Master's level had to literally be a master of all knowledge that was considered important, including such mundane things as the recipe for baking bread. Only as knowledge increased to the point where no one could know everything considered important did advanced students specialize in order to become expert in their chosen field. However, Sommerfeld intervened and, after heated exchanges with Wien over the relative importance of theory and experiment, Heisenberg was given the lowest passing grade for the experimental part of his oral exam. Heisenberg left town that same evening with tail between his legs to work as a pre-approved teaching assistant to Max Born who was a theoretical physicist. Heisenberg later considered the resolution of a microscope in his own way to derive results in 1927 that led to equation (1) by others.

⁷ Each particle reaches the detection screen/apparatus before the next particle enters a slit.

interfering diffraction patterns from the two edges of a slit and superimposed on the repeating crest-trough pattern from two slits. However, it is easier to ignore this complicating factor for the derivation of the later-proposed model.

Classical waves passing through two slits interfere with each other, and at a detection screen behind the slits produce an easily-seen repeating crest-trough pattern of intensity.⁸ The classical theoretical explanation for this, based on the Huygens-Fresnel principle [5,6], has been known for two centuries now. However, it boggles the classical physics mind to explain how singleton flux also produces such patterns, patterns that have been well documented now for photons, electrons, some atoms, and even some molecules. It also bothers those that have been thoroughly exposed to quantum mechanics. The physics Nobel laureate Richard Feynman said that “nobody understands quantum mechanics”.⁹

⁸ The relations between wavelength λ , slit width(s), distance between slits for more than one slit, distance to the detection screen, and distance between the crests/troughs detected can be found in most texts on optics. Without restricting the relative sizes of some of the variables the mathematics becomes complicated.

⁹ The following excerpt [sic] of the start of a lecture by Richard Feynman, given the year before his Nobel prize, can be found at bouman.chem.georgetown.edu/general/feynman.html:

Quantum Mechanics by Richard P. Feynman

Electrons, when they were first discovered, behaved exactly like particles or bullets, very simply. Further research showed, from electron diffraction experiments for example, that they behaved like waves. As time went on there was a growing confusion about how these things really behaved ---- waves or particles, particles or waves? Everything looked like both.

This growing confusion was resolved in 1925 or 1926 with the advent of the correct equations for quantum mechanics. Now we know how the electrons and light behave. But what can I call it? If I say they behave like particles I give the wrong impression; also if I say they behave like waves. They behave in their own inimitable way, which technically could be called a quantum mechanical way. They behave in a way that is like nothing that you have seen before. Your experience with things that you have seen before is incomplete. The behavior of things on a very tiny scale is simply different. An atom does not behave like a weight hanging on a spring and oscillating. Nor does it behave like a miniature representation of the solar system with little planets going around in orbits. Nor does it appear to be somewhat like a cloud or fog of some sort surrounding the nucleus. It behaves like nothing you have seen before.

There is one simplification at least. Electrons behave in this respect in exactly the same way as photons; they are both screwy, but in exactly in the same way....

The difficulty really is psychological and exists in the perpetual torment that results from your saying to yourself, "But how can it be like that?" which is a reflection of uncontrolled but utterly vain desire to see it in terms of something familiar. I will not describe it in terms of an analogy with something familiar; I will simply describe it. There was a time when the newspapers said that only twelve men understood the theory of relativity. I do not believe there ever was such a time. There might have been a time when only one man did, because he was the only guy who caught on, before he wrote his paper. But after people read the paper a lot of people understood the theory of relativity in some way or other, certainly more than twelve. On the other hand, I think I can safely say that nobody understands quantum mechanics. So do not take the lecture too seriously, feeling that you really have to understand in terms of some model what I am going to describe, but just relax and enjoy it. I am going to tell you what nature behaves like. If you will simply admit that maybe she does behave like this, you will find her a delightful, entrancing thing. Do not keep saying to yourself, if you can possibly avoid it, "But how can it be like that?" because you will get 'down the drain', into a blind alley from which nobody has escaped. Nobody knows how it can be like that.

Richard P. Feynman, The Messenger Lectures, 1964, MIT

Anyone attempting to explain singleton flux interference in terms our minds can accept as reasonable wants to somehow incorporate concepts of classical wave and particle behaviors at the same time. Classically, in two-slit singleton interference, it appears that a single particle must interfere with itself if it passes through only one of the two slits, something that is not understandable at all. If we use a wave description of a particle we can understand how interference happens even with singleton flux, but only if we assume that the wave passes through both slits at the same time. Yet, at the detection screen each particle is detected focally, as expected for a classical particle. This happens for photons, as well as what we consider classical particles, and is called wave function collapse (at measurement) in quantum mechanics, the measurement problem. The quantum mechanical wave function Ψ is distributed, yet at the detection screen or detecting equipment the detection is very focal, as expected for a classical particle. The distribution of such focal sites initially appears random. However, after a large number of such particles pass through the slits a distribution pattern is built up with the typical repeating crest-trough pattern.¹⁰ In usual interference experiments with photons or high flux of particles we do not notice the focal detection because of the very high numbers of focal detections giving a smooth appearance. Classical physics has no explanation for singleton flux interference yet.

5. Modeling in Physics

The history of physics includes models being introduced, improved, or changed to match known experimental findings. Sometimes a model is imposed by known behaviour of part of a system, and sometimes it is largely guessed at. If a model matches all known experimental results, and especially if it also makes a prediction that is subsequently verified experimentally, then that model is more likely to be accepted. However, as long as an old model is helpful in comprehension and doing calculations in a situation, that model can still be used. Newtonian celestial mechanics is still successful in the study of most planetary motion and space travel. However, general relativity must be used in the global positioning system and in understanding the precession of Mercury's perihelion. Classical mechanics is very successful in much of engineering and industry, but quantum mechanics is needed in understanding the periodic table of elements, the interactions of chemicals, solid state electronics, and spectral analysis of the composition of stars.

To make quantum mechanics itself more understandable/intuitive, a model explaining two-slit singleton flux interference in more classical terms would be helpful. A productive starting point in doing this is noting that what we call empty space is not as empty as classical physics would have us believe, as far as quantum mechanics is concerned. We tend to believe something exists if and only if our senses or other sensing equipment can detect it. However, there appears to be probably much that we cannot yet detect directly. Quantum mechanics has made numerous predictions that have been shown to be true experimentally, with many of these predictions based on the proposed

¹⁰ The first two-slit singleton flux experiment using electrons was done by Merli, Missiroli, and Pozzi [7] in 1974. A motion picture of the results can be found on-line at <https://www.youtube.com/watch?v=ZJ-0PBRuthc>. Singleton flux photons give similar results.

existence of fields in space. One of these predictions is the Aharonov-Bohm effect [8] in which an electron, a charged particle, can have its trajectory altered by the presence of a magnetic potential field in a region where there is no magnetic field¹¹. The magnetic potential had until then been considered only a mathematical contrivance and convenience to help calculations. Classically the Aharonov-Bohm effect should not be possible, since only the magnetic field and not its potential was believed to produce a force on a charged particle. Yet the Aharonov-Bohm effect has been experimentally confirmed by Akira Tonomura and associates [9]. It would then appear that the magnetic potential field is the more primary field in space and that the magnetic field is the consequent secondary field of mathematical contrivance and convenience. Other fields are proposed in quantum mechanics to help explain the existence and behavior of particles. Virtual particle-antiparticle production from fields in space explains the Casimir effect [10] in which two very close parallel conducting but uncharged plates attract each other, and not by gravitational attraction. This effect is unexplainable in classical physics. The Higgs boson, experimentally detected in 2012 at CERN, was based on the presence of a Higgs field [11] as an explanation for mass of particles. A bit more on this will be discussed later. What is called empty space is far from being empty or still.

It seems then that there are more properties of space that might help explain wave-particle duality and two-slit singleton flux interference. Quantum mechanics has come to espouse that a particle is a standing wave or disturbance in some field. This is a starting point for a quantum mechanical definition of particle. We can extend this to a classical physics understanding by noting that, both in quantum mechanics and classical physics, states of lower potential are more stable than those with higher potential. We also note that force within a field is the negative differential of the potential of that field.¹² This holds for scalar as well as vector fields and even tensor fields. If two different potential fields interfere destructively in a region then those fields attract each other and may bond locally in that region, with that bond moving as one.¹³ Whether such bonding occurs would depend on satisfaction of constraints that are laws of physics. This includes the Heisenberg uncertainty principle if we describe the fields in terms of (complex) wave functions. The Schrödinger equation,¹⁴ for example, incorporates the laws of conservation of energy and momentum to give allowed states of the wave function Ψ for a non-relativistic particle. The Dirac equation is used to give allowed states for a relativistic system of particles.

¹¹ The terms “electric force field” and “magnetic force field” were formally introduced by William Thompson (Lord Kelvin) in 1851 to refer to Michael Faraday’s electric and magnetic lines of force. These were later shortened to just electric field and magnetic field.

¹² This is effectively a definition of “force” within any field.

¹³ Conversely, if the interference is constructive in a region, then the net result is unstable and these potential fields repel each other in that region.

¹⁴ The Schrödinger equation is not shown here to avoid going off on a tangent explaining bras, kets, operators, and Hamiltonian.

6. Proposed Model

Assume there are multiple potential fields throughout space, each vibrating¹⁵ in its own unique way but able to interact with the other fields by means of destructive and constructive interference, calculated as algebraic addition of the complex-valued functions Ψ describing these fields. Unless some bonding allowed by attraction and the constraining laws of physics occurs, any vibrational mode of each separate field does not undergo a lasting change in this interaction.¹⁶ A quark field might be considered to consist of multiple fields, since there are different quarks, each with differing properties, and any quark and its anti-particle are different. Any particle with mass might be considered to be locally bonded in its rest frame with the Higgs field, resulting in inertia felt in other fields. The strength of this bonding would depend on the vibrational modes of each field involved, their phases, and constraints. Any property of a particle, such as mass, charge, spin, color charge, *etc.*, would be due to one or more vibrational modes of one or more locally bonded potential fields.

By chance, a combination of an electron field, a quark field, and a neutrino field could result locally in a standing wave pattern that is stable (bonded), even if only very temporarily, due to reduction of potential locally and satisfaction of all constraints. As a result, this standing wave pattern persists locally in its rest frame, even if only very temporarily,¹⁷ moves as one, and is called a particle.¹⁸ This would give rise to such things as the weak force, in which virtual particles, called the W^+ , W^- , and Z^0 bosons, exist very temporarily, move, and may or may not interact with some other particle, depending on their different wave patterns, phases, and laws of physics (constraints). If this other particle is a quark in a nucleus, these bosons may or may not interact with it to create stable other particles, because of the constraint laws imposed by the different composition of various nuclei. If a neutron, which consists of one up and two down quarks, is not under the constraints due to other particles in the nucleus and is free, interaction does occur, but empirically relatively infrequently, giving rise to the free neutron half-life. This model predicts that statistical mechanics and thermodynamics might shed light on the density and motion distribution of these virtual bosons.

The preceding is a model that gives a definition of a particle that can be used in both classical and quantum physics. This also now allows us to form a model of how a particle affects space, both in the particle's rest frame of reference and in a frame of reference in which the particle has a non-zero velocity. Suppose that in a particle's rest frame its potential vibrations induce vibrations moving radially out in the same potential fields present in space, with the amplitude of these induced vibrations reducing as $\frac{1}{r}$, where r

¹⁵ This vibration, represented by a complex function Ψ , is not in general a classical vibration.

¹⁶ The cause-effect relation is actually the reverse of what is stated here. From the lack of change of a property of a system during interactions we infer a conservation law of physics. This is how the conservation laws in classical, quantum, and particle physics were determined.

¹⁷ Virtual particle creation must obey the Heisenberg uncertainty principle, resulting in heavier such particles having shorter lives, because of Einstein's equivalence of mass and energy.

¹⁸ This may cause a problem for photons in this model, since it makes no sense to talk of a photon rest frame or, in fact, the rest frame of any particle moving at the speed of light.

is the effective distance from the particle's bonding region.¹⁹ In a frame of reference in which the particle is moving suppose that these induced vibrations move radially out from each local site in space where they are produced along the particle's path and in the forward direction, much as air pressure waves from an airplane, with the amplitude of potential still reducing as $\frac{1}{r}$. Unless and until there is some interference that allows bonding, attraction, or repulsion locally with other potential field vibrations, these induced vibrations persist at each local site since they obey the constraint laws of physics.²⁰

In two-slit singleton flux interference experiments this model has a particle passing through one slit and setting up a wave pattern in the involved potentials beyond the slit, similar to the Huygens-Fresnel principle of classical physics. If Ψ is the wave function after interference, then $|\Psi|^2 = \Psi * \Psi$ is the intensity detected at the screen or detecting equipment. Let Ψ_n be the wave function of particle n before it enters a slit. Let $\Psi_{n,1}$ be the wave pattern²¹ of potential induced by particle n passing through slit 1 and $\Psi_{n,2}$ be the wave pattern induced by particle n passing through slit 2. Particle $n+1$ with wave pattern Ψ_{n+1} the same as Ψ_n except for some random difference in phase²² and passing through one of the slits, but after particle n , results in a potential wave pattern $\Psi_{n+1,1} + \Psi_{n,1}$, $\Psi_{n+1,2} + \Psi_{n,1}$, $\Psi_{n+1,1} + \Psi_{n,2}$, or $\Psi_{n+1,2} + \Psi_{n,2}$, depending on which slit particles n and $n+1$ pass through. These sums are affected predominantly by geometric relations and linear addition of the Ψ as long as the slits are in the order of one to a few wavelengths apart, and the distance to the detection screen/apparatus is much larger than this wavelength. Otherwise the mathematics becomes much more complex. This sets up a random change in phase in the sum and will have an effect on later particles. A later particle passing through one of the slits then has its potential wave interfering in the classical way with the already present new potential wave pattern. Since each particle is (assumed to be) a packet of waves in (bonded) potential fields, the difference in phases results in a force (the negative differential of the net potential) and dictates where this later particle will be guided. This is reminiscent of the de Broglie-Bohm pilot wave theory [12].²³ As in classical physics, which slit the particles pass through is probabilistic. In these experiments any particle is detected focally at the detection screen/equipment, as expected for a classical particle, but at what appears to be a random site at any fixed time. There is no wave function collapse in this model. After many particles have passed through the slits, the statistical distribution of the interference of wave phases reveals itself as a classical repeating crest-trough pattern at the detection screen/equipment.

¹⁹ The bonding region of waves would have a lower limit of size, so that r cannot be 0 and forces such as electric repulsion can never be infinite.

²⁰ This model would then give rise to the Huygens-Fresnel model used to explain classical diffraction and interference.

²¹ This pattern may still be a complex function, not that of a classical wave.

²² We assume all particles are the same and have the same speed, and that the source of the particles is effectively equidistant from the slits, so the only differences in the wave functions are their phases.

²³ Louis de Broglie first presented his pilot wave theory at the 5th Solvay Conference in 1927, but his theory was attacked and then ignored until David Bohm independently came up with much the same idea in 1952.

This model also allows a classical physics explanation of what happens when any measurements are done during a two-slit interference experiment to see which slit a particle passed through, either as each particle enters a slit or at any later time before it reaches the detector. Any such measurement itself interferes with the particle-induced potential wave pattern between the slits and detection equipment. This induces a pattern $\Psi_{\text{int},n,1}$ or $\Psi_{\text{int},n,2}$, say, after slits 1 and 2, respectively, with wave patterns and phases very different from those in $\Psi_{n,1}$ and $\Psi_{n,2}$ and altering from particle to particle being so measured. Such interfering measurements prevent any classical interference pattern build-up in time by particles passing through the slits. Two thin lines of particle distribution at the detector results since at least one of the wave patterns $\Psi_{n,1}$ and $\Psi_{n,2}$ for each n is altered to $\Psi_{\text{int},n,1}$ or $\Psi_{\text{int},n,2}$, respectively. The next wave pattern $\Psi_{n+1,1}$, $\Psi_{n+1,2}$, $\Psi_{\text{int},n+1,1}$, or $\Psi_{\text{int},n+1,2}$, cannot interact in a statistically repetitive constructive or destructive way with the interfering measurement pattern $\Psi_{\text{int},n,1}$ or $\Psi_{\text{int},n,2}$ induced, so there are no forces acting in consistent directions on the particles.

7. Future Considerations

The implication in present day discussions of two-slit singleton flux interference is that the interference pattern seen is independent of the time between particles in the singleton flux, provided we wait long enough for the pattern to build up. However, the model presented in this paper suggests that a longer time between particles will degrade the interference pattern because of virtual particle-antiparticle creation in space or other uncontrolled interferences in space. This could serve as a test for the model. It may also allow calculation of the local rate of this particle-antiparticle creation. For singleton flux electrons this would be expected to be virtual electron-positron creation.

8. Conclusion

The presented model of what a particle is and how it interacts with space helps explain in a more intuitive way wave-particle duality and two-slit singleton flux interference experiments. Importantly, this model avoids the so-called wave function collapse, the measurement problem. This model also ties in with particle physics phenomena.

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