Dark matter is the negative entropy force brought by the cosmic expansion

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Abstract

Particles in quantum mechanics have the property of quantum wave entropy. Similar to stretching a spring, under the specific conditions of cosmic expansion, particle wave entropy exhibits a negative entropy force that resists expansion. This negative entropy force provides particles with an attractive effect different from gravity. Like a spring constant, the negative entropy force has a proportional coefficient. The negative entropy force is proportional to the Hubble constant and also proportional to gravitational rotational speed. The effect of this negative entropy force is roughly the same as that of dark matter, serving as the physical origin of dark matter effects. The negative entropy force leads to a result where the rotation speed of a galaxy is proportional to the fourth root of the distance, showing an approximately flat and slowly rising curve. The negative entropy force also can explain the Tully-Fisher relation. Galaxies have two characteristic radiuses: the flat radius and the cutoff radius. These two radiuses form a dark matter region. The dark matter effect becomes noticeable starting from the flat radius and begins to decline noticeably from the cutoff radius. The flat radius and cutoff radius of a galaxy are determined by its mass. Galaxies of different masses and sizes will show different behaviors of dark matter. The negative entropy force also applies to photons and can explain the dark matter effects in gravitational lensing. The negative entropy force can easily derive the results of MOND. MOND comes from negative entropy force, and has no relation to gravity. Therefore, without assuming the existence of dark matter or modifying gravity theory, we can still normally explain the observed dark matter phenomena in galaxies. Dark matter doesn't exist.

Keywords

Dark matter, MOND, quantum wave entropy, cosmic expansion, negative entropy force, Tully-Fisher relation, flat radius, cutoff radius, gravitational lensing, proportional coefficient.

Introduction

In physics and astronomy, the physical nature of dark matter is an important question that still needs to be answered [1][2][3]. Currently, there are mainly two types of theoretical explanations. The first is the dark matter theory. This theory assumes that there is a special type of particle in the universe. These types of particles are involved in gravitational interactions but don't take part in electromagnetic interactions, so current astronomical techniques can't detect them. These special particles are called dark matter. Dark matter particles cannot be detected by conventional experimental techniques. Galaxies are filled with dark matter, forming dark matter halos that provide extra gravity. However, experiments have so far failed to detect these dark matter particles, which makes this explanation quite questionable. The other is the MOND theory [2][3][4][5], which suggests that under very low

gravitational accelerations, gravity deviates from Newton's law and general relativity. MOND theory requires modifications to the fundamental theory of gravity, posing a serious challenge to the entire foundation of physics. Explaining the effects of dark matter is a huge problem. Do dark matter particles really exist? Does gravity really need to be modified under ultra-low accelerations? Or is there some unknown property of physics that could explain dark matter phenomena?

In a previous paper, the author introduced the concept of quantum wave entropy [6]. In quantum mechanics, particles have a special entropy property where the entropy density is inversely proportional to the particle's wavelength. This special entropy property was named quantum wave entropy. The formula for quantum wave entropy is given in (1.1).

$$dS = D\kappa_B \alpha \frac{dr}{\lambda} \tag{1.1}$$

Here, D is the intrinsic degree of freedom of the particle, and α is a probability constant. The constant κ_B is the Boltzmann constant. We define a new constant to simplify the formula.

$$\kappa_p = D\alpha\kappa_B \tag{1.2}$$

So get formula (1.3).

$$dS = \kappa_p \frac{dr}{\lambda} \tag{1.3}$$

There's also a relationship (1.4) between a particle's wavelength and its momentum.

$$\lambda = \frac{h}{P} \tag{1.4}$$

So (1.3) can be changed into (1.5).

$$dS = \frac{\kappa_p}{h} P dr \tag{1.5}$$

After in-depth research, the author found an astonishing result. Based on the concept of quantum wave entropy, we can normally explain dark matter phenomena without having to assume the existence of special dark matter particles or modify existing gravity theory. This paper is a detailed account of this theoretical discovery.

1. Dark matter is the negative entropy force brought by the cosmic expansion

We know that for thermodynamic entropy, there is a kind of entropic force. Changes in entropy bring about a force called an entropic force [7]. For example, pressure can be understood as a type of entropic force. Entropic force can be expressed using formula (2.1).

$$F = T\frac{dS}{dr} \tag{2.1}$$

Entropy force can also manifest as a kind of attraction, which can be called negative pressure or negative entropy force. For example, a stretched spring has a force pulling inward, and this force is an example of negative entropy force [7].

Quantum wave entropy is also a type of thermodynamic entropy, so it also has the properties of entropy force. For quantum wave entropy, the temperature satisfies formula (2.2).

$$T = \frac{dE}{dS} = \frac{d\left(\frac{P^2}{2m}\right)}{\frac{\kappa_p}{h}Pdr} = \frac{h}{m\kappa_p}\frac{dP}{dr} \quad (2.2)$$

So formula (2.1) changes to (2.3).

$$F = T\frac{dS}{dr} = \frac{h}{m\kappa_p}\frac{dP}{dr}\frac{\frac{\kappa_p}{h}Pdr}{dr} = \frac{P}{m}\frac{dP}{dr} = P\frac{dV}{dr} = mV\frac{dV}{dr} \quad (2.3)$$

According to formula (1.3), because the universe is expanding [6], the wavelength of particles increases with the expansion of the universe, so the density of quantum wave entropy decreases. As a result, particles experience an entropic force effect. This entropic force pulls inward, resisting the expansion of the universe. This entropic force comes from the expansion of the universe. It's similar to the effect of stretching a spring. A particle wave is like a spring, and the universe's expansion stretches the particle wave, making its wavelength longer. So, the particle wave generates a contracting force, and the particle experiences a negative entropic force. This negative entropic force follows formula (2.3).

In formula (2.3), V is the particle's velocity. But in the special case of cosmic expansion, $\frac{dV}{dr}$ is no longer just the usual change in velocity. It's the velocity change caused by the expansion of the universe. Cosmic expansion follows Hubble's law [2]. So can get formula (2.4) here.

$$\frac{dV}{dr} = \mu H \quad (2.4)$$

The constant μ here is the proportional coefficient of the negative entropy force, somewhat similar to a spring's elastic coefficient. This proportional coefficient needs to be calculated based on actual measurement data. So formula (2.3) changes to (2.5).

$$F = \mu HmV \quad (2.5)$$

Because this force is a negative entropy force brought about by the expansion of the universe, it acts in the opposite direction of the universe's expansion, so it manifests as an attractive force. For a galaxy, the negative entropy force experienced by a star toward the center of the galaxy. For a planetary system, the negative entropy force acting on a planet points toward the central star.

Note that the velocity V in formula (2.3) is not the star's cosmic expansion velocity. Since momentum P=mV, the velocity here corresponds to the speed related to the particle's wavelength. Only the entropy change caused by cosmic expansion generates entropy force, so only $\frac{dV}{dr}$ is related to cosmic expansion. Please make sure to distinguish it.

There's another thing to pay attention to. The velocity in formula (2.5) might not be the particle's total speed; it could just be the velocity component in a certain direction. The exact nature of the velocity in formula (2.5) depends on the specific problem.

For the motion of stars and planets, the velocity in formula (2.5) is their rotational speed around the center. This velocity satisfies formula (2.6).

$$V = \sqrt{\frac{GM}{r}} \quad (2.6)$$

So, because the universe is expanding, stars will generate a negative entropy force. This negative entropy force will provide an extra pull on the stars. This pull follows formula (2.7).

$$F = \mu HmV = \mu Hm\sqrt{\frac{GM}{r}} \quad (2.7)$$

So, a star experiences two forces of attraction. One is gravity, and the other is negative entropy force. The total attraction follows formula (2.8).

$$F = \frac{GMm}{r^2} + \mu Hm\sqrt{\frac{GM}{r}} \quad (2.8)$$

So, the centripetal force of a rotating star actually satisfies formula (2.9).

$$\frac{mV^2}{r} = \frac{GMm}{r^2} + \mu Hm\sqrt{\frac{GM}{r}} \quad (2.9)$$

So, the rotation speed of the star actually satisfies formula (2.10).

$$\frac{V^2}{r} = \frac{GM}{r^2} + \mu H\sqrt{\frac{GM}{r}} \quad (2.10)$$

$$V^2 = \frac{GM}{r} + \mu H\sqrt{GMr} \quad (2.11)$$

$$V = \sqrt{\frac{GM}{r} + \mu H\sqrt{GMr}} \quad (2.12)$$

Because the Hubble constant is very small, we can see that when the distance r is very short, the negative entropy force is also very small and can be ignored. But as the distance increases, the negative entropy force gradually grows, its effects become noticeable, and it can no longer be ignored, even surpassing gravity. This property actually matches the phenomenon of dark matter.

According to the rotation speed formula (2.12), at large distances, the gravitational item can be roughly ignored, leaving only the speed caused by negative entropy forces. The rotation speed of a galaxy is roughly proportional to the fourth root of the distance. When the distance isn't particularly huge, this can be seen as approximately flat. Formula (2.12) basically fits part of the galaxy rotation curves [1] [2] [3]. Formula (2.12) is calculated based on a point-mass model where the galaxy's mass is concentrated at its center, and (2.12) is also a static equilibrium equation. Actual galaxies aren't point masses, but they have a matter density distribution. Plus, the actual galactic disk is in a dynamically changing state. All these factors affect how flat the rotation curve is. So, for galaxy rotation curves, further in-depth research is still needed.

From formula (2.10), we can get a result. The centripetal acceleration of a star is actually formula (2.13).

$$a = \frac{V^2}{r} = \frac{GM}{r^2} + \mu H\sqrt{\frac{GM}{r}} = \frac{GM}{r^2}\left(1 + \frac{\mu H}{\sqrt{GM}}r^{\frac{3}{2}}\right) = a_N\left(1 + \frac{\mu H}{\sqrt{GM}}r^{\frac{3}{2}}\right) \quad (2.13)$$

Here, a_N is the Newtonian gravitational acceleration.

From formula (2.11), we can see that as the distance r increases, the galaxy's rotation speed first decreases and then increases, creating a turning point in the speed. At this turning point, the rotation speed will reach an extreme. Beyond this turning point, the galaxy's rotation speed curve becomes slowly rising or flat. This turning point marks the distance where the effect of dark matter becomes noticeable. We call this distance the flat radius. So, the flat radius satisfies the following formula.

$$\frac{dV^2}{dr} = \frac{d\left(\frac{GM}{r} + \mu H\sqrt{GMr}\right)}{dr} = 0$$

So we can calculate that the formula for the flat radius is (2.14).

$$R_d = (\frac{4GM}{\mu^2 H^2})^{\frac{1}{3}} \quad (2.14)$$

A galaxy's dark matter effects only start to show up when distances get close to or exceed the flat radius.

Based on data from the Milky Way, we can calculate the value of the proportionality constant μ. The mass of the Milky Way is about 200 to 400 billion times that of the Sun. The Sun's mass is roughly 10^{30} kg, so the Milky Way's mass is around 10^{41} kg. The Milky Way starts showing a flat rotation curve at about 5 kpc, which is when the effect of dark matter becomes noticeable. So the flat radius of the Milky Way is roughly 3×10^{19} meters. By taking these numbers into formula (2.14), we can calculate that the proportionality constant μ is about 100, $\mu \approx 100$.

After knowing the value of the proportionality constant μ, we can calculate the Sun's flat radius. The Sun's mass is about 10^{30} kg, and taking it into formula (2.14), we get that the Sun's flat radius is roughly 10^{16} meters. So, within the solar system, there's no significant dark matter effect. At the Sun's flat radius, the Sun's gravitational acceleration is about $10^{-12}\, m/s^2$. This is 100 times smaller than the $10^{-10}\, m/s^2$ predicted by MOND. So, MOND does not apply to the solar system. This is just a rough estimate, not the final conclusion. The exact calculation of the Sun's flat radius depends on the precise value of the proportional coefficient μ. The exact value of μ still needs to be determined through experiments.

According to formula (2.14), we can see that a galaxy's flat radius is closely related to its mass. The larger the galaxy's mass, the bigger its flat radius. Based on this result, experiments can be conducted. We measure the distance at which a galaxy's rotation speed starts to flatten, then measure the galaxy's mass, and check the relationship between the two. By this way, we can verify whether this theory matches the dark matter phenomena in actual galaxies.

From formula (2.14), we can also see that different galaxies, if they have different masses, will have different flat radius. Different galaxies will show different manifestations of dark matter. Even if two galaxies have the same mass, if the density distribution of matter differs, the dark matter effect may also appear different at the same radius location. If a galaxy has a very high stellar density and its radius is smaller than the flat radius, this galaxy will not show obvious dark matter phenomena. The amount of galaxy mass, the size of the galaxy radius, and the density of the galaxy all affect how dark matter appears. Small, high-density dwarf galaxies, where most stars are concentrated in a relatively small area and the galaxy radius is small, will have a flat radius larger than the galaxy radius, so dark matter effects are even less likely to appear. On the other hand, for larger galaxies, although the flat radius is bigger, the stars are more sparsely distributed, the galaxy radius is larger, and dark matter effects are more likely to show up. This property roughly matches experimental results on the dark matter boundary in galaxies [8].

From formula (2.14), we can get following result.

$$\mu^2 H^2 = \frac{4GM}{R_d^3}$$

Taking above result into formula (2.11), we can get formula (2.15).

$$V^2 = \frac{GM}{r} + \frac{2GM}{r}\sqrt{\frac{r^3}{R_d^3}} \quad (2.15)$$

When the distance is greater than the flat radius, the rotational speed caused by gravity gradually decreases. We'll ignore that part and only calculate the rotational speed due to the negative entropy force. From this, we can calculate the equivalent mass caused by the negative entropy force. Let's calculate the equivalent mass at distances that are integer multiples of the flat radius.

$$r = nR_d$$

Taking the above results into formula (2.15), ignore the first item, and keep only the second item, we can get following formula.

$$V^2 = \frac{2GM}{nR_d}\sqrt{\frac{(nR_d)^3}{R_d^3}} = \frac{2\sqrt{n}GM}{R_d}$$

This speed is equal to the gravitational rotation speed caused by the equivalent mass at the flat radius location. So we can get following formula.

$$V^2 = \frac{2\sqrt{n}GM}{R_d} = V_d^2 = \frac{GM_d}{R_d}$$

So the equivalent mass brought by the negative entropy force is the result below.

$$M_d = 2\sqrt{n}M \qquad (2.16)$$

So, at the distance of $r = R_d$, the equivalent mass caused by the negative entropy force is twice that of normal matter. At the distance of $r = 5R_d$, the equivalent mass caused by the negative entropy force is 4.5 times that of normal matter. At the distance of $r = 10R_d$, the equivalent mass caused by the negative entropy force is 6.3 times that of normal matter. This result roughly matches the estimated mass of dark matter.

By plugging the flat radius formula (2.14) into (2.11), we can calculate the speed at the flat radius location.

$$V^2 = \frac{GM}{R_d} + \mu H\sqrt{GMR_d} = \frac{3GM}{R_d} \qquad (2.17)$$

Taking (2.14) into (2.17), at the flat radius location, we can get the relationship between rotational velocity and galaxy mass.

$$V^3 = \frac{\sqrt{27}}{2}\mu HGM \qquad (2.18)$$

The cube of the flat rotation speed of a galaxy is proportional to its mass. However, actual measurements show that the fourth power of the flat rotation speed is proportional to the galaxy's mass. But let's take another look at formula (2.11). When the radius exceeds the flat radius, the rotational speed caused by gravity can be neglected, and the rotation speed is mainly due to the negative entropy force. If we pick roughly the same radius R to examine different galaxies, we can get a rough result that the fourth power of the rotation speed is proportional to the galaxy's mass. Doing this calculation matches the Tully-Fisher relation [9]. This issue still needs further in-depth study.

$$V^4 = \mu^2 H^2 GRM \qquad (2.19)$$

For the case of gravitational lensing, according to general relativity, the gravitational deflection of light is equivalent to calculating it using twice the Newtonian gravitational acceleration [10]. Another Planck gravitation theory also gives the same result [11][12]. So, for the case of gravitational lensing, the acceleration formula (2.13) needs to be changed to formula (2.20).

$$a = 2a_N\left(1 + \frac{\mu H}{\sqrt{GM}}r^{\frac{3}{2}}\right) = \frac{2GM}{r^2}\left(1 + \frac{\mu H}{\sqrt{GM}}r^{\frac{3}{2}}\right) \qquad (2.20)$$

Obviously, the negative entropy force would increase the gravitational deflection angle of light. Since photons also have particle-wave properties, they also possess the attribute of quantum wave entropy, and the expansion of the universe would give photons a negative entropy force. Using formula (2.20), we can similarly calculate the additional deflection angle caused by the negative entropy force without assuming the existence of dark matter. This is just a rough result. The negative entropy force on photons is still a topic that needs to be studied further.

If we carefully observe the rotational speed formula (2.11), we can see that as the radius increases, the rotational speed also increases, without any distance limit. This clearly does not match reality. Therefore, the negative entropy force must have a limiting range and cannot remain valid at infinite distances. This is also similar to a spring. When a spring is stretched beyond a certain length, it will lose its elasticity. Based on the properties of entropic force, it can be reasonably inferred that the rotational speed must be greater than the cosmic expansion speed at that location in order for negative entropic force to be effective. If the rotational speed is less than the cosmic expansion speed, the cosmic expansion speed gradually takes over, and the entropic force effect brought by cosmic expansion will gradually weaken. So, we can reasonably assume that the radius where the rotational speed equals the cosmic expansion speed is a turning point for negative entropic force. For the radius of this turning point, we call it the cutoff radius R_c. Beyond this cutoff radius, negative entropic force will sharply decrease, and the effects of dark matter will disappear accordingly. The negative entropic force decreases exponentially, which is a reasonable assumption. Thus, the negative entropic force formula becomes (2.21), the galaxy rotation speed formula becomes (2.22), and the acceleration formula experienced by photons in gravitational lensing becomes (2.23).

$$F = \mu e^{-\frac{r}{R_c}} Hm\sqrt{\frac{GM}{r}} \tag{2.21}$$

$$V^2 = \frac{GM}{r} + \mu e^{-\frac{r}{R_c}} H\sqrt{GMr} \tag{2.22}$$

$$a = \frac{2GM}{r^2}\left(1 + \frac{\mu H}{\sqrt{GM}} e^{-\frac{r}{R_c}} r^{\frac{3}{2}}\right) \tag{2.23}$$

We can understand that the proportionality coefficient of the negative entropy force is not a fixed constant, but has an exponentially decreasing nature. When the distance is much smaller than the cutoff radius, the coefficient can be approximately considered constant. When the distance approaches or exceeds the cutoff radius, the coefficient decreases rapidly in an exponential manner.

At the cutoff radius, the rotational speed caused by gravity is already very small, so to simplify calculations and derivations, it is neglected, and only the rotational speed caused by the negative entropy force is considered. From this, we can calculate the value of the cutoff radius. According to formula (2.11), the expansion speed of the universe equals Hr, so the following formula exists.

$$V^2 = \mu H\sqrt{GMR_c} = H^2 R_c^2$$

So we can get the cutoff radius.

$$R_c = \left(\frac{\mu^2 GM}{H^2}\right)^{\frac{1}{3}} \tag{2.24}$$

By comparing with the flat radius formula (2.14), we can see the difference between the

two radiuses.

$$\frac{R_c}{R_d} = (\frac{\mu}{\sqrt{2}})^{\frac{4}{3}}$$

The cutoff radius is about 322 times the flat radius.

Of course, formula (2.24) is just the author's theoretical guess. The actual cutoff radius of galaxies might not follow formula (2.24) but be represented in some other way. This topic is still worth further study.

So, the dark matter effect of galaxies exists within an effective range. If distance smaller than the flat radius, the dark matter effect isn't obvious. If distance beyond the cutoff radius, the dark matter effect also isn't obvious. Only distance within the effective range does the dark matter effect become noticeable. This property roughly matches the dark matter phenomena observed in reality.

2. Derive MOND

It can be seen that the proportional coefficient in negative entropy force can bring flexible changes to the theory. If we assume that the proportional coefficient decreases with the square root of distance, then we can very easily derive the result of MOND.

We assume the following formula for the proportional coefficient (3.1).

$$\mu = \frac{\mu_0}{\sqrt{r}} \tag{3.1}$$

The negative entropy force formula (2.7) changes to (3.2).

$$F = \frac{\mu_0}{\sqrt{r}} Hm\sqrt{\frac{GM}{r}} = \frac{\mu_0 Hm\sqrt{GM}}{r} \tag{3.2}$$

The rotation speed of the star changes to formula (3.3).

$$\frac{V^2}{r} = \frac{GM}{r^2} + \frac{\mu_0 H\sqrt{GM}}{r} \tag{3.3}$$

$$V^2 = \frac{GM}{r} + \mu_0 H\sqrt{GM} \tag{3.4}$$

When the distance r is very large, the first term becomes very small and can be ignored, leaving only the second term. The rotation speed of the star will approach a constant.

$$V^4 = GM\mu_0^2 H^2 \tag{3.5}$$

Comparing the results with MOND, we can see that the characteristic acceleration a_0 assumed in MOND satisfies formula (3.6).

$$a_0 = \mu_0^2 H^2 \tag{3.6}$$

The constant μ_0 can be calculated using formula (3.6).

The actual centripetal acceleration and Newtonian gravitational acceleration satisfy formula (3.7).

$$a = a_N + \sqrt{a_0 a_N} \tag{3.7}$$

So, based on the theory of negative entropy force, we can easily derive the results of MOND. The negative entropy force theory gives MOND another physical explanation. We can interpret MOND results from the perspective of negative entropy force, which decouples MOND from gravity theory. Before this, MOND required modifications to gravity theory. But gravity theory is a fundamental part of physics. Changing gravity theory would affect overall physics, bringing huge theoretical challenges. Now we can find a new physical explanation

for MOND through negative entropy force, without needing to modify gravity theory. Negative entropy force only appears in special cases, so we can handle this theory separately, unrelated to other physics theories.

Negative entropy force has a proportional coefficient, which is a big theoretical advantage. We can change the proportional coefficient depending on the situation, making the theory very adaptable and applicable to different scenarios. The proportional coefficient might have other forms. For example, in MOND, the proportional coefficient μ_0 might have different values in galaxies of different masses, so a_0 could vary in different galaxy. This allows MOND to be more flexible and adapt to a wider range of situations. This topic is worth exploring further.

Whether negative entropy force can solve other dark matter problems still needs further study. For example, whether negative entropy force is effective for the CMB problem or the large-scale structure of the universe. Because dark matter only affects the CMB and the large-scale structure of the universe through gravity, and the negative entropy force is equivalent to the gravitational effects brought by dark matter, so the negative entropy force should have the same effect on the CMB and the large-scale structure of the universe as dark matter. These issues all need to be studied in more depth.

From the explanation above, we can see the advantages of this negative entropy force theory. There's no need to modify Newton's law of gravity, no need to modify general relativity, and no need to assume some special kind of dark matter. Completely within the existing framework of physics, based on the properties of entropy and considering the special case of cosmic expansion, introducing a negative entropy force, we can solve the problems related to dark matter.

The universe is vast, and our current astronomical observation technology is still limited, so we know very little about it. So even if experiments prove that negative entropy forces are real and effective, they definitely can't solve all the dark matter problems. In galaxies, there's probably still some normal matter that hasn't been detected. Part of the dark matter issue should be explainable by this missing normal matter.

Conclusion

The author introduces a theoretical model of negative entropy force, aiming to solve the dark matter problem. Based on the quantum wave entropy properties of particles, under the special conditions of cosmic expansion, a negative entropy force will be generated. This negative entropy force is the physical origin of the dark matter effect. The negative entropy force produces effects similar to dark matter. With this negative entropy force, we don't need to modify existing gravity theories or assume the existence of some special dark matter to properly explain galaxy rotation curves and gravitational lensing phenomena. This avoids a series of problems brought by dark matter. So, dark matter doesn't exist.

On the contrary, based on the theory of negative entropy force, we can easily derive the results of MOND. We need to take another look at MOND theory. MOND comes from negative entropy forces. MOND might have nothing to do with gravity at all.

This theory still needs experimental verification. The exact value of the proportionality constant μ still needs to be determined through experiments. If the theory is indeed valid, it

could have a huge impact on astronomy and cosmology. There are still many properties of the negative entropy force theory that need to be further studied and refined. We look forward to more researchers joining the study of negative entropy force.

This theory originates from quantum wave entropy. Quantum wave entropy has shown enormous potential in many issues. It's a new concept worth exploring in depth.

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