

A two-dimensional soft-logarithmic potential in disk galaxies: testing against MOND and dark-matter halos with SPARC rotation curves

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(Dated:)

We consider an effective gravitational potential whose extra term is a regularised two-dimensional logarithmic potential — the logarithmic Green’s function of the two-dimensional Poisson equation. We derive its source surface density, $\Sigma(R) \propto r_c^2/(R^2 + r_c^2)^2$ (a two-dimensional Plummer-like distribution), and show that its total mass satisfies $M = V_{\text{flat}}^2/G_2$ exactly, so that the model has a well-defined physical normalisation. The circular-velocity contribution is $V_{2\text{D}}^2(R) = V_{\text{flat}}^2 R^2/(R^2 + r_c^2)$, which differs from a pseudo-isothermal (PI) halo in the approach to the flat asymptote ($1/R^2$ vs. $1/R$ corrections). We fit this model, MOND (two interpolating functions), NFW, PI and Burkert halos to the SPARC sample of 175 disk galaxies (3391 velocity measurements), with the stellar mass-to-light ratio free per galaxy and a systematic error floor scanned over 0–15%. At the reference 10% floor, the cored models give lower total χ^2 than NFW and both MOND forms: $\chi^2/N = 0.57$ (2D), 0.57 (Burkert), 0.60 (PI), versus 0.79 (NFW), 1.38 (MOND-simple) and 1.66 (MOND-standard). The three cored models (2D, Burkert, PI) have similar goodness of fit, but the information criteria moderately favour the 2D model over PI ($\Delta\text{AIC} \simeq 93$) and provide only a modest preference over Burkert ($\Delta\text{AIC} \simeq 13$); the present data do not establish a decisive physical distinction among them. The discriminating observable is the shape of the transition to the flat asymptote: $1/R^2$ for the 2D potential, $1/R$ for PI, and a peak followed by a slow decline for Burkert. We show that SPARC covers the relevant radial range only partially (23% of data points in the discrimination zone). Against MOND the result depends on the model-selection criterion, because MOND has substantially fewer global parameters. Among the models evaluated at each error-floor value, the 2D model gives the lowest total χ^2 ; MOND-standard is evaluated only at the reference 10MOND-simple in the reference fit because of its much smaller parameter count. We conclude that the soft-logarithmic potential is a viable, normalised member of the cored-model family, with a specific falsifiable prediction for the transition region.

I. INTRODUCTION

A. Galaxy rotation curves and the mass discrepancy

Disk galaxy rotation curves remain flat far beyond the stellar disk [1, 2], implying either dark matter halos or a breakdown of Newtonian dynamics [3]. Collisionless cold dark matter simulations produce approximately cuspy, NFW-like profiles [4], while observed rotation curves are often better described by cored phenomenological profiles; the extent to which baryonic physics reconciles this difference remains under active investigation [5, 6].

B. MOND and dark-matter alternatives

In the modified-gravity picture, MOND [3] reproduces the radial acceleration relation (RAR) with a single acceleration scale $a_0 \simeq 1.2 \times 10^{-10} \text{ m s}^{-2}$ [7], but the origin of a_0 — and its apparent proximity to cH_0 — has remained unexplained [8].

C. Motivation for a two-dimensional logarithmic potential

The two-dimensional Poisson equation $\nabla^2\Phi = 2\pi G_2\Sigma$ has the logarithmic Green’s function $\Phi \propto \ln R$ for a point source (contrast the $1/R$ potential in three dimensions). A regularised version of this potential — the finite- r_c form derived in Section II — generates a rotation-curve contribution

$$V_{2\text{D}}^2(R) = V_{\text{flat}}^2 \frac{R^2}{R^2 + r_c^2}, \quad (1)$$

which rises linearly near the centre and saturates to a flat asymptote. In this work, the two-dimensional Poisson equation is used as a phenomenological field equation defining an effective gravitational potential in the galactic plane. We do not assume that the full four-dimensional spacetime dynamics is literally two-dimensional; the circular-speed relation is defined by the usual radial equation of motion in the effective potential, $V_c^2 = R d\Phi/dR$.

D. Scope of this work

We investigate whether the regularised two-dimensional logarithmic potential can provide a quantitatively viable and falsifiable description of the

additional gravitational component inferred from disk-galaxy rotation curves. We derive its surface-density distribution and exact mass normalisation (Section II), compare it on equal footing with MOND, NFW, PI and Burkert halos over the SPARC sample (Sections III–V), and identify the observable — the asymptotic approach to the flat velocity — that discriminates it from cored dark-matter halos (Section VI). The data test the model at the level of an effective, phenomenological description; they do not by themselves establish the underlying physics as genuinely two-dimensional (Section VII).

II. THEORETICAL MODEL: FIELD-EQUATION DERIVATION

A. Two-dimensional Poisson equation

We consider the two-dimensional Poisson equation

$$\nabla^2 \Phi = 2\pi G_2 \Sigma, \quad (2)$$

where Σ is an effective source surface density and G_2 is the corresponding effective two-dimensional gravitational coupling, with dimensions $L^2 M^{-1} T^{-2}$ (equivalently V^2/M).

B. Regularised logarithmic potential

Consider the regularised logarithmic potential

$$\Phi(R) = \frac{V_{\text{flat}}^2}{2} \ln(R^2 + r_c^2) + \text{const.} \quad (3)$$

C. Surface-density distribution

Using $\nabla^2 \ln(R^2 + r_c^2) = 4r_c^2/(R^2 + r_c^2)^2$, the effective source surface density implied by Eq. (3) is

$$\Sigma(R) = \frac{1}{2\pi G_2} \nabla^2 \Phi = \frac{V_{\text{flat}}^2}{\pi G_2} \frac{r_c^2}{(R^2 + r_c^2)^2}. \quad (4)$$

This is a two-dimensional Plummer-like surface-density profile (not the three-dimensional Plummer halo, whose density falls as r^{-5}): a flat core with $\Sigma(0) = V_{\text{flat}}^2/(\pi G_2 r_c^2)$, falling as $\Sigma \propto R^{-4}$ at large radius. The source is a specific, finite-mass distribution — not a constant-density disk.

D. Exact mass normalisation

The total mass is

$$M_{2D} = 2\pi \int_0^\infty R \Sigma(R) dR = \frac{V_{\text{flat}}^2}{G_2}, \quad (5)$$

i.e. $V_{\text{flat}}^2 = G_2 M_{2D}$ exactly, where M_{2D} denotes the *effective two-dimensional source mass* (not a three-dimensional halo mass). This is an exact mass-asymptotic-velocity relation for the adopted 2D source distribution, not a general virial theorem: it follows from the specific form of Eq. (3) and its normalisation. V_{flat} therefore carries the total-source mass information, and r_c the source scale; the $\Sigma \sim R^{-4}$ fall-off guarantees finite total mass.

E. Analytic rotation curve

The circular velocity follows as

$$V_c^2(R) = R \frac{d\Phi}{dR} = V_{\text{flat}}^2 \frac{R^2}{R^2 + r_c^2}, \quad (6)$$

which is the central result of the model.

F. Inner and outer asymptotic limits

$$V_c(R) \propto R \quad (R \ll r_c), \quad V_c(R) \rightarrow V_{\text{flat}} \quad (R \gg r_c), \quad (7)$$

with the large-radius expansion

$$V^2(R) = V_{\text{flat}}^2 \left(1 - \frac{r_c^2}{R^2} + \dots \right). \quad (8)$$

Here V_{flat} denotes the asymptotic velocity *parameter* of the model, not necessarily the observed velocity at the outermost measured radius; for galaxies whose data do not reach $R \gg r_c$ it is an extrapolated quantity. The inner linear rise corresponds to the constant-density core of Σ ; the outer approach to flatness scales as $1/R^2$ — the signature that discriminates the model from cored halos (Section VI).

III. COMPARISON MODELS

For each galaxy, the model rotation velocity satisfies

$$V_{\text{mod}}^2(R) = V_{\text{gas}}^2(R) + \Upsilon V_{\text{disk}}^2(R) + V_{\text{bul}}^2(R) + V_{\text{extra}}^2(R), \quad (9)$$

where Υ is the stellar mass-to-light ratio in the $3.6 \mu\text{m}$ band, assumed radially constant and free per galaxy; the bulge component V_{bul} uses the fixed SPARC value $\Upsilon_{\text{bul}} = 0.7$ [9]. Table I lists the extra components and free parameters of all models.

A. Pseudo-isothermal halo

The 3D profile $\rho(r) = \rho_0/[1 + (r/r_c)^2]$ gives

$$V_{\text{PI}}^2(R) = V_{\text{flat}}^2 \left[1 - \frac{r_c}{R} \arctan \frac{R}{r_c} \right], \quad (10)$$

TABLE I. Model definitions. “Galaxy params” are the galaxy-specific free parameters of each model. Total parameter counts are $k = 525$ for the two-parameter extra-component models (175×3) and $k = 176$ for MOND ($175 \Upsilon_i + 1$ global a_0).

Model	Extra component	Galaxy params
2D soft-log	V_{flat}, r_c	$\Upsilon, V_{\text{flat}}, r_c$
PI	V_{flat}, r_c	$\Upsilon, V_{\text{flat}}, r_c$
NFW	M_{vir}, c	$\Upsilon, M_{\text{vir}}, c$
Burkert	V_{peak}, r_c	$\Upsilon, V_{\text{peak}}, r_c$
MOND-simple	global a_0	Υ, a_0 (global)
MOND-standard	global a_0	Υ, a_0 (global)

where V_{flat} is a shorthand for the asymptotic normalisation $V_{\text{flat}}^2 = 4\pi G\rho_0 r_c^2$. The large-radius expansion is

$$V_{\text{PI}}^2(R) = V_{\text{flat}}^2 \left(1 - \frac{\pi r_c}{2R} + \dots \right). \quad (11)$$

The key difference from Eq. (8) is the power of the correction: $1/R^2$ for the 2D potential versus $1/R$ for the PI halo.

B. Burkert halo

The Burkert profile [10] $\rho(r) = \rho_0 r_c^3 / [(r + r_c)(r^2 + r_c^2)]$ gives

$$V_{\text{B}}^2(R) = \frac{2\pi G\rho_0 r_c^2}{x} \left[\ln(1+x) - \arctan x + \frac{1}{2} \ln(1+x^2) \right], \quad (12)$$

Unlike the 2D and PI models, the Burkert rotation curve reaches a peak at $x \simeq 3.2$ and declines slowly at larger radii ($V^2 \propto \ln x/x$ asymptotically). In the fits we use the equivalent peak-velocity parameterisation ($\log V_{\text{peak}}, \log r_c$), where V_{peak} is the maximum circular velocity of the halo profile; this avoids the strong (ρ_0, r_c) degeneracy of the raw form. The two are related by $V_{\text{peak}}^2 = h_{\text{max}} (2\pi G\rho_0 r_c^2) \simeq 2.70 G\rho_0 r_c^2$ with $h_{\text{max}} = 0.4303$ at $x \simeq 3.24$ (computed by the fitting pipeline from the analytic maximum of Eq. 12).

C. NFW halo

The NFW density profile [4] follows

$$V_{\text{NFW}}^2(R) = V_{\text{vir}}^2 \frac{\ln(1+x) - x/(1+x)}{\ln(1+c) - c/(1+c)}, \quad x \equiv R/r_s, \quad r_s \equiv R_{\text{vir}}/c, \quad (13)$$

where $V_{\text{vir}}^2 = GM_{\text{vir}}/R_{\text{vir}}$, with free $(\log_{10} M_{\text{vir}}, \log_{10} c)$, and M_{vir} defined at $200\rho_c$. We do not impose a cosmological mass-concentration prior.

D. MOND

We use the simple interpolating function [11], $a_{\text{obs}} = a_{\text{bar}} \nu(a_{\text{bar}}/a_0)$ with $\nu(y) = (1 + \sqrt{1 + 4/y})/2$, and the

standard function $\mu(x) = x/\sqrt{1+x^2}$, for which the MOND acceleration is obtained by solving the implicit relation $\mu(g/a_0)g = g_{\text{bar}}$ at each radius (analytic solution: $g^2 = g_{\text{bar}}^2 [1 + \sqrt{1 + 4a_0^2/g_{\text{bar}}^2}]/2$). The acceleration scale a_0 is a single global free parameter in both cases. Throughout, we refer to the adopted interpolation functions explicitly (“MOND-simple”, “MOND-standard”) rather than to MOND as a theory.

IV. DATA AND STATISTICAL METHODOLOGY

A. SPARC sample

We use the public SPARC database [9], which provides rotation curves $V_{\text{obs}}(R)$ with errors, together with the baryonic components $V_{\text{gas}}, V_{\text{disk}}$ and V_{bul} for 175 disk galaxies. After quality cuts ($\sigma_V > 0$, $V_{\text{obs}} > 0$, at least 4 points per galaxy) we retain 3391 velocity measurements.

B. Stellar mass-to-light ratio

Υ is free per galaxy for all models (including MOND), as is standard in SPARC mass modelling [12]. The bulge uses the fixed SPARC value $\Upsilon_{\text{bul}} = 0.7$. For the dwarf sample definition and the mass-velocity analysis we use fixed reference values $\Upsilon_{\text{disk}} = 0.5$ and $\Upsilon_{\text{bul}} = 0.7$ applied separately, i.e. $M_{\star} = 0.5 L_{\text{disk}} + 0.7 L_{\text{bul}}$.

C. Systematic-error treatment

Because the quoted errors do not include galaxy-level systematics (inclination, distance), we add an error floor and scan its amplitude:

$$\sigma_{\text{eff}}^2 = \sigma_V^2 + (f_{\text{sys}} V_{\text{obs}})^2, \quad f_{\text{sys}} = 0, 0.03, 0.05, 0.10, 0.15. \quad (14)$$

We adopt $f_{\text{sys}} = 0.10$ only as a reference sensitivity case; no physical interpretation is assigned to this value. The floor is intended as a fractional error-floor sensitivity analysis, not as a complete model of observational covariance.

D. Model comparison statistics

For each galaxy we fit $\sqrt{V_{\text{mod}}^2}$ to V_{obs} by nonlinear least squares with multiple initialisations. Model comparison uses total χ^2 , $\text{AIC} = \chi^2 + 2k$, $\text{BIC} = \chi^2 + k \ln N$ ($N = 3391$), and the BIC-based approximate log-Bayes factor [13]

$$\ln \text{BF}_{2\text{D}/M} = \frac{\text{BIC}_M - \text{BIC}_{2\text{D}}}{2}, \quad (15)$$

so that positive values favour the 2D model. We note explicitly that Eq. (15) is the BIC approximation to the log evidence ratio, not an exact Bayesian evidence integral. Because the 3391 measurements are not fully independent (radial correlations within a galaxy, galaxy-level systematics), the information criteria should be read as relative rather than absolute; all reported quantities are computed from unrounded likelihoods.

V. RESULTS

A. Global rotation-curve fits

Figure 1 shows representative fits. At the reference $f_{\text{sys}} = 0.10$, the cored models give lower total χ^2 than NFW and both MOND forms (Table II, with values auto-generated by the analysis pipeline): $\chi^2/N = 0.57$ (2D), 0.57 (Burkert), 0.60 (PI), versus 0.79 (NFW), 1.38 (MOND-simple) and 1.66 (MOND-standard). Among the cored models, the 2D potential has the lowest total χ^2 and the lowest AIC, but the differences are small ($\Delta\text{AIC} \lesssim 100$); this is the central result of the comparison and is analysed further in Section VI.

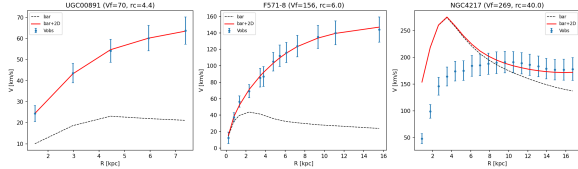


FIG. 1. Representative rotation-curve fits with the 2D model (red) and baryonic component (dashed), selected by lowest (UGC02023), median (NGC4157) and highest (UGC02916) per-galaxy χ^2/N of the 2D model.

B. Systematic-error scan

Table III shows χ^2/N as a function of the systematic error floor. The 2D model gives the lowest total χ^2 at every error-floor value evaluated; MOND-standard is evaluated only at the reference 10% floor. The ordering among 2D, PI and Burkert is stable across the shared error-floor values, with all three within $\sim 8\%$ in χ^2/N (the largest spread, 7%, occurs at $f_{\text{sys}} = 0$). The preference over

NFW and MOND-simple is large and consistent across the scan. This does not imply that the 2D model has the lowest BIC against MOND, whose global parameterisation has substantially fewer free parameters (Section VII).

C. Sub-samples

Table IV splits the sample by surface brightness ($\Sigma_{\text{disk}} < 10 L_{\odot} \text{pc}^{-2}$, 92 low-surface-brightness, LSB, galaxies; the SPARC disk central surface brightness, not a mean or effective value) and by a model-independent dwarf definition: $M_{\star} < 10^9 M_{\odot}$ computed from the SPARC photometry with fixed $\Upsilon_{\text{disk}} = 0.5$ and $\Upsilon_{\text{bul}} = 0.7$ applied separately. This classification is used solely for sample partitioning and is not obtained from the fitted mass-to-light ratios. The 2D model has the lowest χ^2/N among the models shown in every sub-sample, and the advantage is largest in systems generally associated with larger mass discrepancies: LSB ($\chi^2/N = 0.343$ vs. 0.502 NFW, 1.152 MOND-simple) and dwarfs (0.121 vs. 0.310 NFW, 1.229 MOND-simple). The low χ^2/N values in the dwarf sub-sample indicate that the adopted error floor is conservative there and should not be interpreted as a direct estimate of model adequacy.

D. MOND acceleration scale

The MOND-simple fit converges to $a_0^{\text{fit}} = (1.16 \pm 0.05) \times 10^{-10} \text{ m s}^{-2}$, consistent with the canonical value $1.2 \times 10^{-10} \text{ m s}^{-2}$ [7]. The fitted value happens to be numerically close to $cH_0/(2\pi) \simeq 1.08 \times 10^{-10} \text{ m s}^{-2}$ for the adopted value of H_0 ; we do not interpret this numerical proximity as evidence for a theoretical relation. We report the value as a fit result and an empirical coincidence.

E. Stellar mass-asymptotic-velocity relation

Using the SPARC photometry, we compute $M_{\star} = \Upsilon_{\text{disk}} L_{\text{disk}} + \Upsilon_{\text{bul}} L_{\text{bul}}$ with the fitted Υ_{disk} and fixed $\Upsilon_{\text{bul}} = 0.7$; we do not add a gas-mass estimate from V_{gas} because the relation $M_{\text{gas}} \simeq 1.4 V_{\text{gas}}^2 R/G$ is not valid for an extended gas disk, and the SPARC rotation-curve files do not provide direct HI masses. We therefore report the *stellar* mass-asymptotic-velocity relation as an exploratory correlation (not a full baryonic Tully-Fisher relation, BTFR), and caution that gas-dominated dwarfs are incomplete in $M_{\star}$. Regressing $\log V_{\text{flat}}$ on $\log M_{\star}$ (forward) and the inverse regression give

$$\text{forward: } \alpha = 0.10, \quad \text{inverse: } \alpha = 0.54, \quad \text{rms} \simeq 0.30 \text{ dex.} \quad (16)$$

The large spread between the two regression directions (regression dilution, and the covariance between Υ and

TABLE II. Reference fit at $f_{\text{sys}} = 0.10$: parameter counts, reduced χ^2 , AIC, BIC, ΔAIC relative to the 2D model, and the BIC-based approximate log-Bayes factor $\ln\text{BF}_{2\text{D}/M}$ (positive values favour the 2D model).

Model	k	χ^2/N	AIC	BIC	ΔAIC	$\ln\text{BF}_{2\text{D}/M}$
2D log potential	525	0.57	2976	6194	0	+0
PI halo	525	0.60	3069	6287	93	+47
NFW halo	525	0.79	3715	6933	740	+370
Burkert halo	525	0.57	2989	6207	13	+7
MOND-simple	176	1.38	5022	6100	2046	-47
MOND-standard	176	1.66	5970	7049	2994	+428

TABLE III. Reduced χ^2/N as a function of the systematic error floor f_{sys} . MOND-standard is evaluated only at the reference $f_{\text{sys}} = 0.10$.

f_{sys}	2D	PI	NFW	Burkert	MOND-s	MOND-t
0.00	7.32	7.85	11.22	7.67	26.93	—
0.03	2.04	2.16	2.86	2.08	5.85	—
0.05	1.27	1.34	1.77	1.29	3.37	—
0.10	0.57	0.60	0.79	0.57	1.38	1.66
0.15	0.32	0.34	0.45	0.32	0.77	—

V_{flat} , both being fit parameters of the same fit) prevents a precise test of $V^4 \propto M$; we therefore do not claim a BTFR prediction. An exploratory figure is provided in the appendix.

VI. MODEL DISCRIMINATION: CORED MODELS AND THE TRANSITION REGION

A. The cored-model family

The three cored models — 2D soft-log, PI and Burkert — have similar goodness of fit ($\chi^2/N = 0.57\text{--}0.60$ at the reference floor), with the information criteria moderately favouring the 2D model over PI ($\Delta\text{AIC} \simeq 93$) and giving only a modest preference over Burkert ($\Delta\text{AIC} \simeq 13$). This is the honest summary of the comparison: the present data do not establish a decisive preference among the cored interpretations. What the data do establish is that cored extra components fit substantially better than NFW and MOND forms.

B. Transition-region prediction

The cleanest difference among the cored models is the approach to the asymptotic velocity. The normalised curves are

$$\frac{V_{2\text{D}}^2}{V_{\text{flat}}^2} = \frac{x^2}{1+x^2}, \quad x \equiv R/r_c, \quad (17)$$

$$\frac{V_{\text{PI}}^2}{V_{\text{flat}}^2} = 1 - \frac{\arctan x}{x}. \quad (18)$$

with

$$1 - \frac{V_{2\text{D}}^2}{V_{\text{flat}}^2} \sim x^{-2}, \quad (19)$$

$$1 - \frac{V_{\text{PI}}^2}{V_{\text{flat}}^2} \sim \frac{\pi}{2x}. \quad (20)$$

A 2D logarithmic potential requires an R^{-2} approach; a pseudo-isothermal halo requires an R^{-1} approach. The Burkert curve differs qualitatively: it peaks at $x \simeq 3.2$ and declines at larger radii, so it predicts a *downturn* in the outer rotation curve that neither the 2D nor the PI model produces.

C. SPARC radial coverage

Figure 2 shows the two monotonic curves (2D and PI) and the SPARC coverage. The data cover R/r_c with median 1.2 and 5–95% range 0.08–8.9, where r_c is the fitted 2D core radius — the coverage statistic is therefore model-dependent, and we state this explicitly. 23% of the data points fall inside the discrimination zone $x \in [2, 5]$, where the two profiles differ appreciably and where the present data provide useful leverage. The interval $x \in [2, 5]$ is an operational discrimination region covering the radii where the profiles differ appreciably; we do not claim it is the unique mathematical optimum. The discriminating measurement is a rotation curve with many high-quality points across $R/r_c \sim 1\text{--}5$, and for Burkert, extended data beyond the peak that would reveal the outer downturn.

TABLE IV. Sub-samples at $f_{\text{sys}} = 0.10$. Dwarf galaxies are defined by $M_* < 10^9 M_\odot$ with fixed $\Upsilon = 0.5$; LSB by $\Sigma_{\text{disk}} < 10 L_\odot \text{pc}^{-2}$.

Sample	N_{gal}	2D	NFW	MOND-simple
All	175	0.568	0.786	1.377
LSB	92	0.343	0.502	1.152
HSB	83	0.729	0.990	1.539
Dwarf	69	0.121	0.310	1.229
Big	106	0.722	0.949	1.428

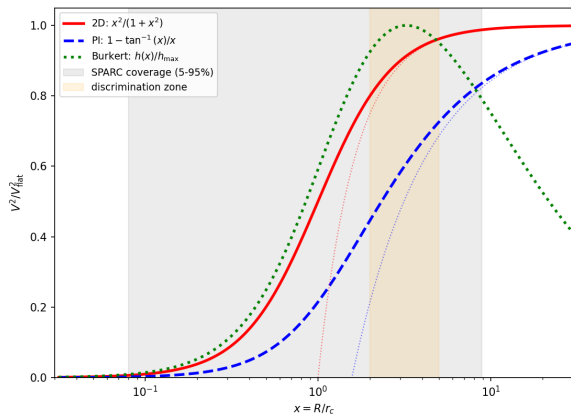


FIG. 2. Transition-shape comparison. Red solid: 2D soft-log potential $V^2/V_{\text{flat}}^2 = x^2/(1+x^2)$; blue dashed: PI halo $1 - \arctan(x)/x$; green dotted: Burkert (peak at $x \simeq 3.2$, outer decline). Grey band: 5–95% of the SPARC R/r_c coverage (2D fitted r_c); orange band: discrimination zone $x \in [2, 5]$.

D. Robustness tests

We checked the robustness of the comparison in two further ways: (i) removing the bulge component ($V_{\text{bul}} = 0$) and re-fitting all models preserves the ranking (2D $\chi^2/N = 0.164$ vs. 1.272 NFW, 1.989 MOND-simple); (ii) fixing $\Upsilon = 0.5$ preserves the ranking (2D 0.864 vs. 1.570 NFW, 1.369 MOND-simple). We did not perform a full bootstrap or cross-validation; given the radial correlations within each galaxy, a proper treatment would require a galaxy-level covariance model, which we leave to future work.

VII. DISCUSSION

A. What the data actually establish

Four results stand out. (i) The 2D soft-logarithmic potential has a finite source distribution and exact normalisation: the corresponding surface density (Eq. 4) is a two-dimensional Plummer-like distribution with total mass $M = V_{\text{flat}}^2/G_2$ exactly (Eq. 5). (ii) It provides a good phenomenological fit: together with the PI and Burkert cored models it fits SPARC rotation curves sub-

stantially better than NFW and both adopted MOND interpolating functions at every systematic-error floor, with the largest advantage in LSB and dwarf galaxies. (iii) Its strongest observational discriminator is the transition shape: $1/R^2$ approach to flatness, versus $1/R$ for PI and an outer downturn for Burkert. (iv) The present data do not establish a fundamental 2D gravitational theory; the model is an effective potential motivated by the 2D logarithmic Green’s function.

B. Effective potential versus fundamental 2D gravity

Whether the underlying physics is genuinely two-dimensional or an effective description of a cored halo is not settled by rotation curves alone. Equation (4)–(5) are exact mathematical properties of the adopted potential; the data test the potential as an effective description. The present analysis is phenomenological and does not derive the coupling between the ordinary Newtonian baryonic potential and the effective 2D component from a unified field equation — we state this caveat explicitly. The logarithmic potential is mathematically reminiscent of dimensional-reduction scenarios [14, 15], but no microscopic derivation connecting those scenarios to the present galaxy-scale model is assumed.

C. MOND parameter-count issue

The MOND comparison must be read with care. MOND-simple yields a worse fit per point ($\chi^2/N = 1.38$ vs. 0.57) but has substantially fewer parameters ($k = 176$ vs. 525); BIC therefore favors MOND-simple in the overall comparison because of its parameter-count advantage, despite its larger χ^2 ($\ln \text{BF}_{2\text{D}/\text{M}} = -47$). Because a_0 is a global parameter shared by all galaxies, we do not assign a separate per-galaxy information-criterion penalty to MOND; MOND is therefore excluded from the per-galaxy AIC comparison below.

Among the two-parameter per-galaxy models (2D, PI, Burkert, NFW), the 2D model has the lowest per-galaxy AIC in 76 of 175 galaxies (43%), Burkert in 68 (39%), PI in 23 (13%) and NFW in 8 (5%) — consistent with the total χ^2 comparison, in which the 2D and Burkert models are nearly indistinguishable. The χ^2 difference between

2D and MOND is large ($2.4\times$ per point, 2.7×10^3 in total), and whether the BIC penalty is appropriate depends on whether the per-galaxy core parameters are genuine degrees of freedom or a parametrisation of a universal profile. We state the parameter-count degeneracy explicitly rather than claim a decisive MOND exclusion.

D. Relation to cored dark-matter halos

The 2D potential, PI and Burkert halos have very similar goodness of fit, with χ^2/N differing by only about 5–6% at the reference floor, although the information criteria moderately favour the 2D model over PI ($\Delta\text{AIC} \simeq 93$) and provide only a modest preference over Burkert ($\Delta\text{AIC} \simeq 13$); the present analysis does not by itself establish that these modest information-criterion differences reflect a physically meaningful distinction. The data weakly but systematically favour the 2D model in total χ^2 ; the physically falsifiable difference is the transition shape and, for Burkert, the outer downturn. We identify $R/r_c \sim 1\text{--}5$ as the region where future extended rotation curves will settle the question. Our comparison is with the selected halo profiles (PI, NFW, Burkert) under our fitting assumptions; we do not claim a comparison with the full range of published SPARC halo-model catalogues [6], which include additional profiles (Einasto, DC14, cored-NFW, Lucky13) and marginalise over distance and inclination.

E. Limitations

The diagonal error-floor treatment does not model radial covariance within galaxies; the per-galaxy Υ is degenerate with V_{flat} in the mass-velocity analysis; the absence of direct HI masses prevents a complete baryonic Tully–Fisher test; the coverage statistic R/r_c is model-dependent; and the connection to holographic and

dimensional-reduction ideas is a motivation, not a claim of this work.

VIII. CONCLUSIONS

We derived the surface-density distribution and total mass of a regularised two-dimensional logarithmic potential ($\Sigma \propto r_c^2/(R^2 + r_c^2)^2$; $M = V_{\text{flat}}^2/G_2$) and tested the resulting rotation-curve model against MOND (two interpolating functions), NFW, PI and Burkert halos on the SPARC sample of 175 galaxies. At the reference 10% error floor, the cored models give lower total χ^2 : $\chi^2/N = 0.57$ (2D and Burkert), 0.60 (PI), versus 0.79 (NFW), 1.38 (MOND-simple) and 1.66 (MOND-standard). Among the models evaluated at each error-floor value, the 2D model gives the lowest total χ^2 ; it performs best in LSB and dwarf galaxies, but the three cored models are statistically close; the distinguishing observable is the transition shape — the $1/R^2$ versus $1/R$ approach to flatness (and the Burkert outer downturn) — which we identify as the falsifiable prediction of the model family. The MOND fit returns $a_0 = (1.16 \pm 0.05) \times 10^{-10} \text{ m s}^{-2}$. We conclude that the soft-logarithmic potential is a viable, normalised and falsifiable member of the cored-model family describing disk-galaxy rotation curves, and we identify the radial range and precision required to discriminate it from cored halos.

DATA AVAILABILITY

The SPARC rotation-curve data used in this work are publicly available from the SPARC database [9]. The analysis pipeline and reproducibility scripts used to generate all tables and figures will be deposited in a public repository upon publication.

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