

Mathematical Implications of Non-Homogenous Para-Topological Convergences Between Orthogonal Unbridged n-Spaces I

A minimal mathematical framework for dynamical crossings, transient tachyonic modes and entropy-mediated inter-universe coupling

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Abstract:

Investigated is a hypothetical mathematical framework describing a dynamical crossing between two otherwise orthogonal and causally unbridged (n)-dimensional spaces or spacetimes, interpreted here as distinct universes. The crossing is represented by a localized, time-dependent kernel that temporarily modifies the effective mode spectrum of the coupled system.

The central object is a non-homogeneous para-topological crossing kernel which induces a transient mixing between the two spaces without requiring a permanent geometric bridge. In an effective field description, the crossing may generate a temporarily negative effective squared frequency,

$[\omega_k^2(t) < 0]$, thereby producing a transient tachyonic regime. Shown is, that this does not necessarily imply a persistent tachyonic degree of freedom after the crossing. Instead, the process can be represented by a Bogoliubov transformation characterized by coefficients (α_k) and (β_k) .

Subsequently introduced ergo is an effective entropy/information sector and investigated is its feedback on the crossing dynamics. The resulting system admits stabilizing, destabilizing and potentially symmetry-breaking regimes. In particular, entropy transport between the two universes can be distinguished mathematically from entropy production, allowing the possibility of asymmetric post-crossing states even when the initial configuration is symmetric. May be, that the phenomenon of matter transportation or of radiation energy could occur between the two space manifolds over the crossing zone ("skip effect") but this special situation is discussed in a later paper. Two similar crossing zones between the same universes then will be defined as a „congruence“.

The present work however concentrates on and constitutes a phenomenological mathematical framework rather than a claim of an established physical theory. Its principal purpose is to identify the necessary dynamical structures, stability criteria and falsifiable mathematical consequences of such a crossing scenario. Some more treatises to this theme discussing intensively these sort of phenomena, will follow.

Key-words: Double-universe; manifold-crossing; paratopology; n-dimensional spacetime; crossing-zone; tachyon-state; crossing kernel; dynamical bistructure; field-propagation; entropy thermodynamics; tachyonic instability, Bogoliubov mixing, brane collisions und entropy-to-curvature conversion.

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1. Introduction:

The possibility of describing two otherwise disconnected physical spaces by a transient common dynamical structure raises a number of questions concerning topology, causality, field propagation and thermodynamics [1.],[2.]. May be, that this description here later can be reduced to two metrical and topological similiar zones of the same universe. But for now two different manifolds are assumed.

Considered are two spaces

$$[U_1; U_2] \tag{1.}$$

which, prior to a crossing event, possess no direct propagating bridge between them.

They are refered as two **orthogonal unbridged spaces** in the present framework.

The fundamental assumption is not that a permanent wormhole or geometric tunnel exists. Instead, assumed is, that for a finite interval

$$t \in (t_-; t_+) \tag{2.}$$

the dynamical equations admit a coupling between otherwise independent sectors. The central question therefore is:

Can a transient mathematical convergence of two otherwise orthogonal spaces produce a physically meaningful transfer of field excitations without requiring a permanent bridge between the spaces?

Investigated is this question through an effective crossing kernel.

2. Geometric and kinematic setup:

Let the two spaces be represented by

$$[U_1=(M_1, g_1); U_2=(M_2, g_2)] \quad (3.)$$

with the restriction of

$$[\dim M_1 = \dim M_2 = n] \quad (4.)$$

because assumed is, that after a form of a minimal principle two manifolds $(M_1; M_2)$ with the same dimension-number crosses with maximal probability.

Prior to the crossing,

$$[U_1 \cap U_2 = \emptyset] \quad (5a.)$$

in the ordinary geometric sense.

Nevertheless, introduced is an abstract crossing space

$$[C] , \quad (5b.)$$

which does not necessarily constitute a conventional submanifold embedded in either universe.

Instead, (C) represents the domain in which the dynamical evolution permits mode conversion between the two sectors.

Thus,

$$[C \neq \text{permanent geometric bridge}] .$$

This distinction is fundamental.

3. Para-topological crossing:

Introduced now is a crossing operator

$$[K_C(t); H_1 \oplus H_2 \rightarrow H_1 \oplus H_2] \quad (6.)$$

where (H_i) denotes the relevant field-state space associated with (U_i) .

The adjective *para-topological* is used here to indicate that the coupling modifies the admissible dynamical relations between the spaces without requiring a conventional change of their global topology.

The crossing is non-homogeneous when

$$[K_C(x_1; x_2; t) \neq K_C(x_1'; x_2'; t)] \quad (7.)$$

in general.

This allows spatial, temporal and mode-dependent crossing strengths.

4. Minimal crossing kernel:

For the first model choosen is

$$[f_C(t)=e^{-t^2/\tau_c^2}] \quad (8.)$$

and defined is:

$$[\Delta_C f_C(t)] \quad (9.)$$

as the effective strength of the crossing.

The kernel is maximal at

$$[t=0] \quad (10a.)$$

and asymptotically vanishes:

$$\lim_{|t| \rightarrow \infty} f_C(t) = 0 \quad (10b.)$$

Thus the system satisfies

$$uncoupled\ state \rightarrow crossing \rightarrow uncoupled\ state \quad .$$

This property will be referred to as **transient closure**.

5. Relative and common modes:

Defined is:

$$\phi_+ = \frac{\phi_1 + \phi_2}{\sqrt{2}} \quad (11a.)$$

and

$$\phi_- = \frac{\phi_1 - \phi_2}{\sqrt{2}} \quad (11b.)$$

The (+) mode represents the common component, whereas the (-) mode represents the relative component between the two universes.

The inverse transformation is

$$\phi_1 = \frac{\phi_+ + \phi_-}{\sqrt{2}} \quad (12a.)$$

$$\phi_2 = \frac{\phi_+ - \phi_-}{\sqrt{2}} \quad (12b.)$$

The crossing may therefore be diagonal in the (+,-) basis even when it is nontrivial in the universe basis.

6. Effective mode equation:

For a Fourier mode (k) , the minimal crossing equation is

$$\ddot{\phi}_k + [k^2 + m_0^2 - \Delta_C f_C(t)] \phi_k = 0 \quad (13a.)$$

Define:

$$\omega_k^2(t) = k^2 + m_0^2 - \Delta_C f_C(t) \quad (13b.)$$

The system becomes temporarily tachyonic whenever

$$\omega_k^2(t) < 0 \quad (13c.)$$

For the zero mode this requires

$$\Delta_C > m_0^2 \quad (13d.)$$

More generally,

$$k^2 < \Delta_C - m_0^2. \quad (13e.)$$

Thus the tachyonic band is finite in momentum space.

7. Turning points and tachyonic interval:

The boundaries of the tachyonic interval satisfy

$$k^2 + m_0^2 = \Delta_C e^{-t_k^2 / \tau_C^2} \quad (14a.)$$

Consequently there is:

$$t_k = \tau_C \sqrt{\ln \left(\frac{\Delta_C}{k^2 + m_0^2} \right)} \quad (14b.)$$

The tachyonic interval then is

$$[-t_k < t < t_k] \quad (14c.)$$

The corresponding instantaneous growth rate is

$$\Omega_k(t) = \sqrt{-\omega_k^2(t)} . \quad (14d.)$$

8. Tachyonic amplification:

Define:

$$X_k = \int_{-t_k}^{t_k} \Omega_k(t) dt \quad . \quad (15a.)$$

In the semiclassical regime, the dominant amplification behaves as

$$N_k \sim \sinh^2 X_k \quad (15b.)$$

For

$$X_k \gg 1 \quad (15c.)$$

obtained is

$$N_k \sim \frac{1}{4} e^{2X_k} \quad (15d.)$$

Thus a temporary negative effective frequency can produce a finite population of post-crossing excitations.

9. Bogoliubov description:

Before the crossing defined is:

$$u_k^{(into)} = \frac{1}{\sqrt{2} \omega_0} e^{-i \omega_0 t} \quad (16a.)$$

where

$$\omega_0 = \sqrt{k^2 + m_0^2} \quad (16b.)$$

After the crossing there is the situation of:

$$u_k^{into} = \alpha_k u_k^{out} + (\beta_k u_k)^{out *} \quad (16c.)$$

Canonical normalization requires

$$|\alpha_k|^2 - |\beta_k|^2 = 1 \quad . \quad (16d.)$$

The post-crossing excitation number is

$$N_k = |\beta_k|^2 \quad (16e.)$$

The transient tachyon is therefore not identified with a permanent tachyonic particle species. It is dying out, so to speak.

10. Stability after the crossing:

Since

$$f_C(t) \rightarrow 0 \quad (17a.)$$

for large $(|t|)$, obtained is

$$\omega_{k,out}^2 = k^2 + m_0^2 \quad (17b.)$$

For

$$[m_0^2 > 0] \quad (17c.)$$

all post-crossing modes are stable:

$$\omega_{k,out}^2 > 0 \quad . \quad (17d.)$$

The crossing therefore produces

transient instability + permanent excitation rather than a permanent tachyon.

This distinction is central to the framework.

11. Quadratic crossing action:

A minimal quadratic action is

$$S_C = \frac{1}{2} \int dt d^{nk} [|\dot{\phi}_k|^2 \omega_k^2(t) |\phi_k|^2] \quad (18a.)$$

Substitution gives:

$$S_C = \frac{1}{2} \int dt d^{nk} [|\dot{\phi}_k|^2 (k^2 + m_0^2) |\phi_k|^2 + \Delta_C f_C(t) |\phi_k|^2] \quad (18b.)$$

The crossing therefore enters as a time-localized quadratic interaction.

12. Introduction of an effective entropy sector:

Now introduced is:

$$[\sigma_1(t); \sigma_2(t)] \quad (19a.)$$

representing effective information/entropy variables associated with the two universes.

The effective masses become

$$m_1^2 = M_0^2 - \Delta_C f_C(t) + \lambda_S \sigma_1 \quad (19b.)$$

and

$$m_2^2 = M_0^2 - \Delta_C f_C(t) + \lambda_S \sigma_2 \quad (19c.)$$

The sign of (λ_S) determines the direct feedback.

13. Entropy production versus entropy transport:

Defined now is

$$[P_i] \quad (20a.)$$

as local entropy production and

$$[J_S] \quad (20b.)$$

as entropy transport.

A minimal exchange law is

$$J_S = \Gamma_S (\sigma_1 - \sigma_2) \quad (20c.)$$

Then there is:

$$\dot{\sigma}_1 = P_1 - J_S \quad (20d.)$$

$$\dot{\sigma}_2 = P_2 + J_S \quad (20e.)$$

Consequently

$$\dot{\sigma}_1 + \dot{\sigma}_2 = P_1 + P_2 \quad (20f.)$$

The exchange term cancels.

This establishes a clean distinction between transport and production of entropy processes.

14. Symmetric and antisymmetric entropy sectors:

Define:

$$\sigma_+ = \frac{\sigma_1 + \sigma_2}{\sqrt{2}} \quad (21a.)$$

$$\sigma_- = \frac{\sigma_1 - \sigma_2}{\sqrt{2}} . \quad (21b.)$$

Then

$$\dot{\sigma}_- = \frac{P_1 - P_2}{\sqrt{2}} 2\Gamma_s \sigma_- \quad (21c.)$$

The exchange coefficient therefore determines the relaxation rate of:

$$\tau_{mix} = \frac{1}{2\Gamma_s} \quad (22.)$$

15. Entropy-induced mass splitting:

Obtained now is

$$m_1^2 - m_2^2 = \lambda_s (\sigma_1 - \sigma_2) \quad (23a.)$$

Thus

$$\Delta m^2 = \sqrt{2} \lambda_s \sigma_- \quad (23b.)$$

An entropy asymmetry therefore produces a mass asymmetry in the effective crossing theory. This creates a possible route from information asymmetry to dynamical symmetry breaking.

16. One-universe tachyonic regime:

The condition

$$[m_1^2 < 0 < m_2^2] \quad (23c.)$$

can be written as

$$\Delta_C f_C(t) - m_0^2 \lambda_s \sigma_1 \quad (23d.)$$

while simultaneously

$$\Delta_C f_C(t) - m_0^2 < \lambda_s \sigma_2 \quad (23e.)$$

Hence an asymmetric tachyonic phase is mathematically possible whenever the entropy-induced mass splitting exceeds the distance of the symmetric state from the stability boundary.

17. Linearized symmetry analysis:

Around a symmetric configuration there is written:

$$\left[\phi_- = \frac{\phi_1 - \phi_2}{\sqrt{2}} \right] \quad (24a.)$$

$$\sigma_- = \frac{\sigma_1 - \sigma_2}{\sqrt{2}} \quad (24b.)$$

A minimal linearized system takes the form of:

$$\phi''_- + m_C^2 \phi_- + g_s \sigma_- = 0 \quad (24c.)$$

$$\dot{\sigma}_- = A \phi_- - 2\Gamma_s \sigma_- \quad (24d.)$$

With

$$\phi_- \sigma_- \propto e^{st} \quad (24e.)$$

obtained is the characteristic equation

$$(s^2 + m_C^2)(s + 2\Gamma_s) + g_s A = 0 \quad (24f.)$$

This equation determines the stability of the symmetric crossing state.

18. Symmetry-breaking criterion:

The symmetric solution is stable if

$$\Re(s) < 0 \quad (25a.)$$

for all characteristic roots.

A symmetry-breaking instability occurs when at least one mode satisfies

$$\Re(s) > 0 \quad (25b.)$$

The threshold therefore is determined by

$$\Re(s) = 0 \quad (25c.)$$

This gives a mathematically well-defined critical surface in the parameter space of:

$$(\Delta_C; \lambda_s; \Gamma_s; g_s; A) \quad (25d.)$$

19. Interpretation of the feedback:

For positive (λ_s) there is:

$$\sigma_i \uparrow \rightarrow m_i^2 \uparrow \quad (26a.)$$

Entropy therefore tends to suppress the tachyon.

For negative (λ_s) ,

$$\sigma_i \uparrow \rightarrow m_i^2 \downarrow \quad (26b.)$$

Entropy then can reinforce the tachyonic instability.

Thus

$\lambda_s > 0$: *stabilizing feedback*

whereas

$\lambda_s < 0$: *destabilizing feedback* .

20. Quenching condition:

The tachyonic phase terminates when

$$m_{eff}^2(t_*) = 0 \quad . \quad (27a.)$$

For the symmetric system,

$$\lambda_s \sigma_* = \Delta_C f_C(t_*) - m_0^2 \quad (27b.)$$

This condition defines the **entropy-mediated quenching surface**.

If the entropy variable reaches this surface before the geometric crossing ends, the tachyonic growth can be terminated dynamically.

21. Competing timescales:

Three characteristic timescales appear:

$$[\tau_C] \quad (28a.)$$

for the crossing,

$$\tau_T \sim |m_{eff}^2|^{-1/2} \quad (28b.)$$

for tachyonic growth, and

$$\tau_{mix} = (2\Gamma_s)^{-1} \quad (28c.)$$

for entropy equilibration.

The dimensionless ratios

$$R_T = \frac{\tau_C}{\tau_T} \quad (28d.)$$

and

$$R_s = \frac{\tau_{mix}}{\tau_C} \quad (28e.)$$

provide natural control parameters for entropy flux situations during the crossing existence.

22. Qualitative dynamical regimes:

The model suggests four principal regimes in its possibilities.

I. Weak crossing:

$$[\Delta_C \leq m_0^2] \quad (29a.)$$

No tachyonic interval occurs.

II. Transient tachyon:

$$[\Delta_C > m_0^2] \quad (29b.)$$

but entropy feedback is negligible.

Then [*tachyonic growth* $\rightarrow$ *Bogoliubov excitation* $\rightarrow$ *stable final state* .]

III. Entropic quenching:

The crossing initially satisfies

$$[m_{eff}^2 < 0] \quad (29c.)$$

but entropy feedback drives

$$[m_{eff}^2 \rightarrow 0^+] \quad (29d.)$$

The instability becomes self-limited.

IV. Symmetry-breaking crossing:

A relative entropy fluctuation grows,

$$[\sigma_- \neq 0] \quad (30a.)$$

and produces

$$[m_1^2 \neq m_2^2] \quad (30b.)$$

In the strongest case,

$$[m_1^2 < 0 < m_2^2] \quad (30c.)$$

One universe then enters the tachyonic branch while the other remains stable.

23. Energy generated by the crossing:

The post-crossing excitation energy is schematically

$$\rho_C = \int \frac{d^{nk}}{(2\pi)^n} \omega_{k,out} |\beta_k|^2 \quad (31.)$$

The low $-(k)$ modes are preferentially amplified because the tachyonic condition is easiest to satisfy in the infrared. This provides a potentially observable signature **within any future physical realization of the model**, although no observational prediction is claimed at this stage.

24. Entropy and quantum purity:

A crucial conceptual distinction must be maintained.

The full state of the combined system may remain pure:

$$[S_{total}=0] \quad (32a.)$$

in the idealized unitary description, while a reduced state of one universe has nonzero entropy:

$$S_1 = -Tr(\rho_1 \ln \rho_1) \quad (32b.)$$

Consequently, entropy associated with the crossing may represent entanglement entropy rather than fundamental non-unitarity.

This distinction will become important in a future microscopic formulation.

25. Current interpretation of the crossing:

The complete process can now be represented schematically as

$$U_1 \perp U_2 \quad (33a.)$$

followed by

$$U_1 \times \leftrightarrow K_C U_2 \quad (33b.)$$

during a finite interval, followed by

$$U_1 \perp U_2 \quad (33a./c.)$$

The crossing therefore need not produce a permanent connection.

Instead, it can leave behind:

field excitations, mode correlations, entanglement, possibly entropy asymmetry .

26. Central conjecture:

The framework suggests the following conjecture:

Crossing Stability Conjecture.

A transient coupling between two initially orthogonal unbridged (n)-spaces can produce a finite tachyonic interval without producing a persistent tachyon, provided that the post-crossing effective mass spectrum is positive. If the crossing is coupled to a dynamical entropy/information sector, the resulting feedback can either quench the instability or, for an appropriate coupling sign and transport rate, destabilize the symmetric state and generate an asymmetric tachyonic branch.

This conjecture presently is a statement about the mathematical model, not an established physical result.

27. Open problems:

Several questions remain deliberately unresolved.

27.1. Microscopic origin of the crossing kernel.

What fundamental geometry or field theory produces $[K_c(t)]$?

27.2. Definition of para-topology.

Can the proposed crossing be formulated using established mathematical structures such as fiber bundles, category-theoretic morphisms, noncommutative geometry or another generalized topology?

27.3. Unitarity.

Under what conditions does

$$\left[|\alpha_k|^2 - |\beta_k|^2 = 1 \right] \tag{34a.}$$

remain valid for the full two-universe system?

27.4. Backreaction.

The crossing field should ultimately modify the background geometry:

$$g_{\mu\nu} \rightarrow g_{\mu\nu} + \delta g_{\mu\nu} \tag{34.b}$$

This has not yet been included.

27.5 Nonlinear tachyon dynamics.

The present treatment is predominantly quadratic. A complete theory requires self-interactions such as

$$\left[V(\phi) = \frac{1}{2} m^2 \phi^2 + \frac{\lambda_\phi}{4} \phi^4 + \dots \right] \quad (35.)$$

27.6 Entropy flow.

Most importantly, there is so far treated entropy transport phenomenologically. A microscopic definition of

$$[J_s] \quad (36.)$$

remains to be constructed.

28. Planned next stage:

The next mathematical development should solve the coupled system of:

$$\begin{aligned} \ddot{\phi}_1 + [k^2 + m_0^2 - \Delta_c f_C(t) + \lambda_s \sigma_1] \phi_1 &= 0 \\ \ddot{\phi}_2 + [k^2 + m_0^2 - \Delta_c f_C(t) + \lambda_s \sigma_2] \phi_2 &= 0 \\ \dot{\sigma}_1 &= P_1 - \Gamma_s(\sigma_1 - \sigma_2) \\ \dot{\sigma}_2 &= P_2 - \Gamma_s(\sigma_1 - \sigma_2) \end{aligned} \quad (37a.-d.)$$

This is a coupled system for two equations of motion of $(\phi_1; \phi_2)$ and two differential equations for $(\sigma_1; \sigma_2)$.

The next objective is to determine analytically the critical surface of:

$$\Re(s) = 0 \quad (38.)$$

and thereby establish the boundary between

[*stable crossing*], [*transient tachyon*], [*entropically quenched tachyon*], [*symmetry – breaking runaway*].]
Only after this stability analysis should the entropy flux be interpreted thermodynamically.

29. Conclusion/Summary:

Constructed is a minimal mathematical model of a transient crossing between two otherwise orthogonal and unbridged (n)-spaces. This crossing is represented by a localized kernel

$$[K_C(t)] \quad (39.)$$

which temporarily modifies the effective frequency spectrum. For sufficiently strong coupling, a finite tachyonic interval appears. The resulting instability does not necessarily survive the crossing; instead it may produce finite Bogoliubov excitation characterized by

$$[N_k = |\beta_k|^2] \quad (40.)$$

The addition of an effective entropy/information sector introduces a new feedback channel. Positive coupling can produce dynamical quenching, whereas negative coupling can enhance the instability. When two entropy variables are assigned to the two universes separately, entropy asymmetry produces an effective mass splitting and can, in principle, generate a state satisfying

$$[m_1^2 < 0 < m_2^2] \quad (41.)$$

The mathematically decisive object is therefore not the entropy itself but the stability spectrum of the coupled crossing-entropic dynamical system [3.].

The framework remains speculative. Its value at this stage is that it converts the original qualitative idea of a “crossing of two universes” into a sequence of explicit mathematical questions with identifiable parameters, stability criteria and limiting cases similar to [4.],[5.]. A localized, non-homogeneous crossing kernel generates a finite tachyonic instability region and, following its disappearance, leaves behind an interesting Bogoliubov-transformed mode structure; a coupled entropy/information sector can subsequently dynamically alter the symmetry of the post-crossing state. After all, this approach isn't simply "tachyon cosmology" or a brane collision; the most interesting connections lie in time-dependent mode instabilities, Bogoliubov mixing, tachyonic windows, entropy/isocurvature modes, and transitions between distinct geometric sectors.

30. Outlook to the next presentation:

In the next treatise II (working paper 0.2) not only the model is described but the central equation will be really completely solved

$$(s^2 + m_C^2)(s + 2\Gamma_s) + g_{SA} = 0 \quad (42.)$$

From these solutions then the exact limit of stability of the symmetric crossing-state can be determined. Afterwards the non-linear area can be examined and eventually can be built in the real flux of entropy between the both universes with its balance of entropy, direction and possible back-reaction to the crossing-kernel. Then the three niveaux of crossing, tachyon and entropy transport can be coupled together.

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32. Non-scientific comment:

--- Every crossing tells a story.---

--- Crossing lines, not boundaries. ---

33. Acknowledgements:

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34. Verification:

This paper definitely is written without support from an AI, LLM or chatbot like Grok or Chat GPT 4 or other artificial tools. It is fully, purely human work in every universe.

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