Closure Without Adequacy: Hawking 75 $\rightarrow$ 76

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Abstract

A derivation, once completed, is typically treated as settling the question it was undertaken to answer. This paper argues that this is not generally true, and that the gap between a completed derivation and a settled physical question is easy to overlook precisely because it has no name and leaves no formal trace once crossed. We develop the point through a case in which the gap is unusually visible: a citation, in Hawking's canonical 1976 physics paper, offered as the source of an extension the citing paper needs but does not itself derive, which turns out on inspection to point to two unsupported sentences in an earlier paper. We use this case not to adjudicate the underlying physical claim, but to isolate two distinct achievements that citation practice, and derivation itself, can silently conflate: extending a mathematical formalism to a broader domain, and showing that the extended formalism answers a question about a physical system, in the operational sense that connects a model's terms to what an observer would measure. We argue, drawing on Bridgman's operational analysis and the model/target-system distinction in philosophy of science, that even a fully executed derivation closing the citation gap we identify would establish the first achievement without thereby establishing the second: a frequency-resolved extension of Hawking's formalism cannot characterize the polarization structure of the emitted state, and so cannot by itself be used to evaluate the unitarity question the citation is invoked to support, whatever that question's true answer turns out to be. The dependency this creates is not merely evidentiary but constitutive of Hawking's own 1976 formal apparatus: that paper's superscattering operator is built from Hilbert spaces the paper does not construct for the higher-spin sectors, so its own formal question, for those sectors, remains a construction the citation was invoked to supply and does not. We do not argue that this precise structure recurs throughout physics; doing so responsibly would require the same case-by-case verification this paper insists on. We do argue that the case examined here is not a bibliographic curiosity but an instance of a general and underappreciated distinction in what closing a derivation can and cannot be shown to achieve.

1 Introduction

A derivation, once completed and checked, is ordinarily treated as having settled whatever question motivated it. This paper is about a gap between those two things – completing a derivation and settling a question – that is easy to miss precisely because nothing in ordinary scientific writing marks it. The gap has no standard notation, no standard citation form, and no name in general use.

We propose one here, and we argue for its reality and its consequences through a case in which it is unusually exposed to view.

The case is drawn from a canonical episode in twentieth-century theoretical physics. Hawking's 1975 derivation of black hole thermal radiance [11] is carried out in full for exactly one field – a massless scalar – and extended, in the same paper, to the electromagnetic and gravitational fields by two sentences asserting a mechanism rather than deriving one. Hawking's 1976 paper on the breakdown of predictability in gravitational collapse [12] builds its central formal apparatus for the same restricted field, and extends it to the physically relevant higher-spin case by citing those same two sentences. On inspection, the citation does not contain what it is cited for.

This much is a documentary finding, established by textual comparison alone, and Sections 3–4 establish it in those terms. Before doing so, one distinction needs to be in place, since without it the argument is vulnerable to a natural and largely correct objection: that a citation need not contain what it is cited for, if it was never offered as containing it.

A citation, in scientific writing, can perform at least three distinct functions, and which function is in play determines what a reader is entitled to expect at the cited location. A citation can function, first, as a pointer to related literature – a "see also," carrying no claim that the cited work licenses anything stated in the citing text, and creating no expectation that it does. A citation can function, second, as a pointer to provenance – an attribution of an idea, a method, or a formalism to the work in which it originated, again without asserting that the cited passage itself demonstrates the claim being made in its name. A citation can function, third, as a pointer to evidentiary support – an assertion that the cited work contains the argument, derivation, or result that licenses the claim attached to it, such that a reader who wished to verify the claim would find, at the cited location, the means to do so.

Only the third of these creates an expectation that the cited passage contain a derivation, and only the third is the kind of citation this paper's argument depends on identifying. The distinction is not fixed by the mere presence of a citation mark; it is fixed by how the citing sentence uses what it cites. As Section 4 shows, the sentence at issue in Hawking's 1976 paper does not say, in the manner of the first function, "for related discussion, see Ref. 13," nor does it say, in the manner of the second, "this approach originates in Ref. 13." It says that the extension "is straightforward along the lines indicated in Ref. 13" – a claim that a path to the result exists and has been indicated at the cited location, offered as the stated reason the extension may be taken as established. This is a citation used in the third sense: as the source of the license for a claim, not merely as a pointer toward it. It is only because Reference 13 is being used this way that its failure to contain a derivation is a defect in the argument rather than an unremarkable feature of an ordinary bibliographic gesture; a citation of the first or second kind, pointing to the same passage, would raise no difficulty at all, and nothing in this paper's argument would apply to it.

This distinction also names, precisely, the structure of the failure examined below. The 1975 passage itself is not the problem; a sentence asserting a plausible, partially motivated conjecture is an entirely ordinary feature of a research paper, and raises no difficulty when read on its own terms, as the kind of statement it is. The difficulty arises specifically where the 1976 sentence treats that passage as though it were of the third kind – as a location containing evidentiary support – when what is actually there, on inspection, does not supply that support. The defect is accordingly not that Reference 13 exists, nor even, on its own, that the 1975 passage is unproven; it is a mismatch between the kind of citation the 1976 sentence's own grammar requires and the kind of content actually present at the cited location, a mismatch a reader encountering only the confident surface of the 1976 sentence has no way to detect without independently consulting the source.

But the paper's argument does not rest there, and this is the point worth stating clearly at the outset rather than leaving implicit. Section 2 develops a general distinction – between what we call *formal extension* and *target adequacy* – and Section 5 uses it to show something stronger than the documentary finding alone: that even a derivation fully supplying the missing higher-spin calculation would not, by itself, answer the question Hawking's 1976 citation is invoked to answer, because that question requires a further kind of content no such derivation, on its own terms, supplies. The citation gap and what we will call the target-adequacy gap are, in this case, nested, and closing the first does not close the second.

We take no position, anywhere in this paper, on whether Hawking's higher-spin extension is in fact correct, nor on whether black hole evaporation is unitary for photons and gravitons. The question throughout is narrower and orthogonal to truth: whether a given claim has been shown to be true, by the source cited for it, as distinct from merely asserted – and, further, what kind of claim a derivation extending that source would even be in a position to show. We mark this restriction, which we call the paper's evidentiary scope, at each point where it might otherwise be misread as a verdict on the physics.

Section 2 develops the formal-extension/target-adequacy distinction in general terms, drawing on Bridgman's operational analysis and the model/target-system distinction in philosophy of science. Sections 3–4 examine the Hawking case directly: the 1975 passage and the 1976 citation to it, culminating in an explicit statement of the citation-chain result. Section 5 asks what closing that gap would and would not achieve; it opens by distinguishing the informal question of whether higher-spin Hawking radiation is in fact unitary, which this paper does not address, from the formal question Hawking's 1976 apparatus specifically poses, which requires a Hilbert-space construction Reference 13 was cited to supply – and argues that even a full derivation of that construction would leave a further, independent gap open. Section 6 considers the strongest objections to this argument, including a check of whether polarization-resolved treatments of photons and gravitons already exist elsewhere in the literature and could substitute for the missing construction. Section 7 situates the case against related work in the sociology and methodology of science. Section 8 concludes.

2 Two Achievements a Derivation Can Be Mistaken for One

2.1 The gap without a name

Suppose a formalism, developed and verified for one physical system, is extended – by argument, by citation, or eventually by an independent derivation – to a broader class of systems. Call the moment this extension is completed, and accepted as completed, *closure*: the formalism now covers the broader domain, on paper, without qualification. The question this section takes up is what closure, by itself, secures.

The natural answer is that closure secures an extended mathematical result: a theorem, a derivation, a formalism now proven to hold over a wider domain than it did before. This is correct, and nothing in this paper disputes it. The question is whether closure secures something further: that the extended formalism now answers the physical question it was extended in order to answer. We will argue that it does not, not automatically, and that the further step required is a distinct achievement with its own conditions of success – conditions a derivation, however complete, does not by itself meet.

The reason this gap is easy to miss is structural, not a matter of individual carelessness. A derivation that closes a mathematical gap leaves a visible trace: an equation, a proof, a citation naming where the work was done. A derivation that closes the *further* gap – between an extended formalism and the physical claim it is offered to support – leaves no comparable trace, because there is no standard notation for having taken the further step, and no standard citation form that distinguishes "this extends the mathematics" from "this extends the mathematics *and* connects it to what a physical system would exhibit." Once the first kind of closure occurs, readers have every ordinary reason to treat the matter as settled; nothing in the paper's presentation flags that a second, independent step remains.

2.2 Bridgman: a physical quantity is defined by the operations that measure it

The distinction has a name in the philosophy of physics, though it is not usually stated in these terms. Bridgman's operational analysis holds that a physical concept is not exhausted by the equations in which it appears; its meaning is fixed by the set of operations, actual or in principle, by which a value for it would be obtained. [4] Two formalisms can agree perfectly on their mathematical content and still differ on this further dimension, if one of them can be connected to a measurement procedure and the other cannot, or if the connection has simply never been made explicit.

This matters for the present argument because a derivation, in extending a formalism, extends its mathematical content; it does not thereby extend, or even address, the formalism's operational content, unless the extension is carried out with that further step in view. A calculation can be completed, checked, and entirely correct as an exercise in the relevant mathematics, while remaining silent – not wrong, simply silent – on what a physical observer would in principle detect that would distinguish the extended case from the case the calculation was actually performed for. Where that silence holds, the calculation has not yet been shown to be a claim about the physical system in question, whatever its formal scope.

2.3 Cartwright: a model is not automatically a claim about its target

A related and independently developed distinction, from philosophy of science more broadly, separates a model from the target system it is offered as a model *of*. On this view, a model's internal correctness – its derivability, its consistency, its completeness on its own terms – is one achievement; its adequacy as a representation of some further, physical system is a second, independent achievement, connected to the first only by an inferential bridge that has to be built and defended in its own right, not assumed as a byproduct of the model's correctness. [5] A model can be true of itself, so to speak, without yet being true of anything outside itself.

A derivation that extends an existing formalism is a model in exactly this sense: correctness on its own terms is a fact about the mathematics, checkable by inspecting the derivation; adequacy to some further physical claim is a separate fact about the relationship between what the derivation computes and what the physical claim requires, and the second is not secured by the first however carefully the first is checked. Reading Cartwright's distinction this way – with the derivation itself standing in the place of the model, and the physical question it is invoked to answer standing in the place of the target – is what lets the distinction be applied directly to derivations throughout the rest of this paper, rather than only to models in the more general sense in which the distinction is usually discussed.

Read together with Bridgman, this yields the distinction this paper turns on. Call the first achievement *formal extension*: an existing derivation is broadened to cover a wider mathematical domain. Call the second *target adequacy*: the broadened formalism is shown to bear on the physical system it is meant to describe, in the sense that its terms can be connected, even in principle, to what an observer of that system would measure. Formal extension is a property of a piece of mathematics. Target adequacy is a property of the relationship between that mathematics and a world outside it. Nothing about achieving the first guarantees the second; a formalism can be broadened with full rigor and still leave target adequacy exactly as open as it was before the broadening began.

2.4 Why closure is mistaken for both

Given this distinction, the failure mode this paper is concerned with can be stated precisely. Closure – the completion of a formal extension – is liable to be read, by authors and readers alike, as having achieved target adequacy as well, because ordinary scientific prose does not typically mark the two as separable steps. A sentence announcing that a formalism "extends" to a broader class of systems is grammatically identical whether it means only the first thing or means both; the reader cannot tell, from the sentence alone, which claim is being made, and has ordinary reason to assume the stronger one, since the weaker one is rarely worth stating on its own. Where a citation stands in for the extension rather than a derivation carried out in the citing text itself, this ambiguity is compounded rather than resolved: a citation can mark that formal extension was intended, without establishing that it was carried out, and without addressing target adequacy at all.

The consequence is that a derivational gap and a target-adequacy gap can coincide, and closing the first can appear – to author and reader alike – to have closed the second, when nothing in the closing derivation addressed it. This is the general structure the remainder of this paper examines through a single case in which it is unusually visible: a citation offered as the source of a formal extension turns out, on inspection, to supply neither the extension itself nor, a fortiori, any bearing on target adequacy – and we show further, in Section 5, that even a derivation fully supplying the missing formal extension would still leave target adequacy exactly as open as before.

(Evidentiary scope, restated for this section: nothing above asserts that formal extension and target adequacy come apart in general, or that they do so whenever a citation stands in for a derivation. The claim is only that they are separable in principle, that ordinary scientific prose does not mark the separation, and that this creates a specific, locatable risk – realized, we will argue, in the case examined below.)

3 The 1975 Passage

Hawking's 1975 derivation of black hole thermal radiance carries out its central calculation in full for exactly one field: a massless Hermitian scalar satisfying $\phi_{;ab}g^{ab} = 0$. The paper constructs the relevant mode families on past and future null infinity and the horizon, extracts the Bogoliubov coefficients relating them, and derives their asymptotic thermal form. This derivation is not in question here and is not re-examined.

Immediately following it, in the same section, the paper turns to the fields a physical black hole actually radiates:

> "Similar results hold for the electromagnetic and linearised gravitational fields. The fields produced on $\mathscr{I}^-$ by positive frequency waves from $\mathscr{I}^+$ have the same asymptotic form as (2.18) but with an extra blue shift factor in the amplitude. This extra factor cancels out in the definition of the scalar product so that the asymptotic forms of the coefficients α and β are the same as in the Eqs. (2.19) and (2.20). Thus one would expect the black hole also to radiate photons and gravitons thermally." (p. 211)

Three facts about this passage can be verified by any reader with the paper in hand, independent of any judgment about the underlying physics.

The mechanism named is specific, not a bare assertion of similarity. The passage identifies a particular structural claim – a blueshift factor "cancels out in the definition of the scalar product" – as the reason the higher-spin case should track the scalar case. This is worth stating plainly, because nothing in what follows depends on treating the passage as careless or unmotivated; a specific, falsifiable mechanism is proposed, and the passage is not weaker for proposing it.

No equation, mode function, or supporting calculation accompanies the claim. No expression analogous to the scalar field's wave equation is written for either field. No mode functions analogous to the ones constructed for the scalar case appear. No inner product adapted to a vector or tensor field is introduced. The two sentences quoted are the entirety of the paper's technical content on this point.

No citation is attached to the claim. No reference to prior or contemporaneous work on the higher-spin case accompanies it. This is worth setting against two facts available elsewhere in the same paper, neither of which is in dispute and both directly checkable. First, the sentence immediately following the passage above extends the discussion to massless fermions and gives a brief but specific supporting remark – that the negative-frequency components "make a positive contribution to the probability flux into the collapsing body" rather than the negative contribution found for bosons – a claim with content beyond bare assertion, even though it too is not derived in full. Second, three pages later, in the discussion of the rotating case, the paper states that "Teukolsky [27] has obtained completely separable wave equations for neutrino, electromagnetic and linearised gravitational fields in the uncharged rotating case" (p. 213), resolving to Teukolsky's 1973 paper [23], citing exactly the mode-separation machinery – for the same fields – that a derivation of the passage above would draw on. The relevant formalism was accordingly known to the author and cited by him, three pages away, for an adjacent purpose; nothing here explains why the electromagnetic and gravitational extension sentence itself carries no citation, and no explanation is attempted. What is established is only that the passage, as published, is an assertion of a specific mechanism, unaccompanied by derivation or citation – nothing more and nothing less.

A further point bears on this, worth stating carefully so as not to overstate it. Teukolsky's paper supplies only mode-appropriate field equations, not an account of how a field's polarization is connected to a particular observer's frame. A tool for that second half was also available in 1972, in Cocke and Holm's treatment [7] of how the Stokes parameters of a polarized field transform under a change of reference frame – a paper absent from Hawking's 1975 reference list entirely, unlike

Teukolsky's. This is not evidence of an oversight: the two papers belonged, in 1975, to disciplines with little occasion to cite one another, Teukolsky's to black hole perturbation theory and Cocke and Holm's to observational astrophysics. The silence in the 1975 passage is accordingly not simply a citation gap of the kind Section 4 examines; it sits at a boundary between two literatures not yet in contact, each holding half of the bridge Section 5.1 describes.

That separation is not merely of historical interest; it is recognizable in the field's structure today. The direct descendant of Cocke and Holm's problem – propagating a field's Stokes-parameter content through a curved background to a specific observer's frame – is now a developed formalism underlying the polarized radiative-transfer codes [9] the Event Horizon Telescope collaboration uses to interpret its own polarization measurements [8]. We note this at the level of field content, not instrumental observation: the Cocke-Holm lineage terminates in an operating pipeline connecting curved-spacetime polarization transport to a measurement, for the electromagnetic case. We are not aware of a comparable pipeline for the gravitational case, and make no claim beyond that. If anything, this complicates rather than resolves the difficulty Section 5 identifies: the one bridge for which an endpoint exists terminates in a formalism for classical light already in transit, not the quantum particle-creation process Hawking's own derivation addresses – the distinction Section 5.7 develops in general terms.

4 The 1976 Citation

Hawking's 1976 paper poses a question that is field-independent as stated: whether the formation and evaporation of a black hole is compatible with unitary quantum evolution. Its central formal apparatus – the mode decomposition, the density-matrix construction, and the superscattering operator introduced to describe the evolution across the horizon – is developed, throughout the paper, for exactly one field. The paper states this restriction directly, in the same sentence in which it extends beyond it:

> "For simplicity only a massless Hermitian scalar field ϕ and an uncharged nonrotating black hole will be considered. The extension to charged massive fields of higher spin and charged rotating black holes is straightforward along the lines indicated in Ref. 13." (p. 2464)

Reference 13 is identified in the paper's own bibliography as "S. W. Hawking, Commun. Math. Phys. 43, 199 (1975)" (p. 2473) – an unambiguous match, by journal, volume, and year, to the paper examined in Section 3. Two facts follow directly from this identification and require no further argument beyond the quotations already given.

No independent derivation of the higher-spin extension appears anywhere in the 1976 paper. The sentence quoted above is the entire treatment the extension receives. It points to another paper; it does not carry out an argument in this one.

The pointer resolves to a passage that itself supplies no derivation. Reference 13 is not a distinct calculation establishing the extension; it is the paper examined in Section 3, whose only relevant content is the two-sentence, uncited assertion discussed there. The higher-spin extension of the 1976 paper's central argument is accordingly supported, in the text of the two papers taken together, by nothing beyond a citation to a passage that does not itself contain what it is cited for.

The conclusion these two facts support is narrower than it might first appear, and the limitation is worth stating immediately. What is established is that the 1976 paper's higher-spin extension rests, in the primary sources themselves, on a single unsupported sentence rather than an independent derivation. This does not determine whether the extension is correct; it determines only that the cited source does not supply, from within itself, the calculation that would verify it.

It is worth being explicit, at this point, about which of the three citation functions distinguished in the Introduction is in play here, since the whole force of the objection this section anticipates – and Section 6.1 answers directly – turns on it. The sentence does not read "for related discussion, see Ref. 13," the grammar of a pointer to related literature. It does not read "this approach originates in Ref. 13," the grammar of a pointer to provenance. It reads that the extension "is straightforward along the lines indicated in Ref. 13" – asserting that a path to the result has been indicated at the cited location, and offering this as the stated reason the extension may be taken as established. This is a citation used as a pointer to evidentiary support, and it is only because it is used this way that Reference 13's failure to contain a derivation is a defect in the 1976 paper's own argument, rather than an unremarkable feature of an ordinary bibliographic gesture.

Stated in the vocabulary of Section 2: Reference 13 is invoked as though it supplied formal extension – as though a reader consulting it would find the higher-spin Bogoliubov calculation Hawking's 1976 formalism requires. On inspection, it supplies neither formal extension nor, a fortiori, target adequacy. It supplies an assertion, offered where a derivation was needed, and nothing in either paper marks the substitution.

4.1 The argument stated explicitly

The case made above rests on a short chain of textual facts, worth stating explicitly rather than leaving to be reconstructed from prose.

P1. The 1976 paper's central formalism is constructed for a massless scalar field only, a restriction the paper states explicitly.

P2. The 1976 paper asserts, in the same sentence, that this formalism extends to higher-spin fields, citing Reference 13 as the source of this extension.

P3. Reference 13 is the 1975 paper, identified unambiguously by journal, volume, and year.

P4. The only passage of the 1975 paper addressing higher-spin fields is the two-sentence passage examined in Section 3, which asserts a mechanism, supplies no supporting calculation, and cites no prior work.

C1 (from P3, P4). Reference 13, as written, contains no derivation of the claim it is cited to support.

C2 (from P2, C1). The 1976 paper's higher-spin extension is supported, in the text of the two papers taken together, by a citation to a passage that itself supplies no derivation. The citation relocates the assertion; it does not discharge it.

We refer to C1–C2 as the citation-chain result. Its scope is limited by design. It does not establish that the higher-spin extension is false, nor that black hole evaporation is in fact non-unitary for photons and gravitons; both remain exactly as open as before this analysis. It concerns two specific published texts, and what those texts do and do not contain. Section 5 asks a further question the citation-chain result alone does not answer: whether closing this gap – supplying the missing derivation – would itself settle the question the citation is invoked to settle.

5 Why Closing the Citation Gap Would Not Finish the Job

5.1 The informal question and the formal question

Before taking up what a completed derivation would deliver, it is worth being precise about what is at stake in asking for one, since this is the point at which a reader is entitled to ask why any of this matters beyond the tidiness of a bibliography. There are two different things a person might mean by "is higher-spin Hawking radiation unitary," and the argument of this section turns on keeping them apart.

The first is an informal question: whether, as a matter of physical fact, the radiation a real black hole emits preserves the information that fell in. This question is perfectly meaningful without any of the apparatus this paper has been examining, has been meaningful since before 1976, and is untouched by anything argued here. Nothing in this paper bears on it.

The second is a formal question: the one Hawking's 1976 apparatus specifically poses. That paper's central construction is not a general reflection on information loss; it is a particular technical object, the superscattering operator \$, built explicitly from three Hilbert spaces – H_1, the space of data on the initial surface, H_2, the space of data on the hidden surface, and H_3, the space of data on the final surface – with \$ itself a tensor mapping density matrices on $H_1 \otimes H_2$ to density matrices on $H_3 \otimes H_2$, constructed, for the scalar field, from the explicit mode decomposition of Section 4 of that paper. To ask whether *this* object is unitary for photons and gravitons is not to ask the informal question in more technical dress. It is to ask a question posed relative to particular Hilbert spaces, and the 1976 paper constructs those spaces for the scalar field alone: for the electromagnetic and gravitational sectors, the paper's own formalism does not specify what H_1, H_2, and H_3 are.

It is worth being precise about what kind of dependency this is, since "depends on" is used loosely enough in ordinary scientific prose to cover several different relations, and the argument of this paper needs the specific one – and precise also about what is *not* being claimed, since the stronger-sounding version of this claim is not one this paper needs or defends. It is not an *evidentiary* dependency, of the kind ordinarily meant when a result is said to depend on a citation: the claim is not that the 1976 paper's answer to the unitarity question would be more or less credible depending on whether Reference 13 supplies a derivation. Nor is the claim that Hilbert spaces appropriate to higher-spin fields on curved backgrounds do not exist, or cannot be constructed by standard methods in quantum field theory in curved spacetime; they plainly can, and nothing here suggests the general framework is unavailable. The claim is narrower: it concerns *Hawking's own 1976 formal apparatus*, as that paper actually constructs it, and it is a *constitutive* dependency in the following specific sense, closer to a distinction already standard in the mathematical treatment of physical problems generally. A problem is called well-posed, in the sense introduced by Hadamard, when a solution exists, is unique, and depends continuously on the data [10]; but each of these three conditions is a property a problem has only once it has already been given a domain – a specific space of admissible data and candidate solutions – and specifying that space is treated, in standard practice, as a distinct and prior step, logically before the question of whether the resulting problem is well-posed can even be raised [22, 24]. The dependency this paper identifies sits at that prior step, relative to the 1976 paper's own construction and no other. That paper's formal unitarity question, for the electromagnetic and gravitational sectors, has not, *within the argument of the 1976 paper itself*, been given the Hilbert-space domain a derivation filling Reference 13 was supposed to supply. This is why the citation-chain result of Sections 3–4 bears on more than the bibliographic record: the missing derivation is not missing evidence for a claim already on the table, but a missing piece of the specific construction the 1976 paper's own apparatus, on its own terms, requires before that apparatus can be evaluated for those fields.

This is what makes the citation-chain result of Sections 3–4 more than a documentary curiosity, and it is the answer this paper gives to the question of why it matters. Ref. 13 is cited at exactly the point where the 1976 paper's own formal apparatus is extended beyond the scalar field – where H_1, H_2, and H_3 would need to be constructed, *within that paper's own framework*, for the electromagnetic and gravitational sectors in order for the superscattering operator, and hence the formal unitarity question as that paper poses it, to be evaluated for those fields. A derivation was needed there not merely to answer an existing question with more confidence, but to supply the construction Hawking's own 1976 apparatus, as published, does not carry out for higher-spin fields. In this sense the gap identified in Sections 3–4 is not only a gap in the evidence for an answer; it is a gap in the specific construction the 1976 paper's own formalism requires before that formalism, for those fields, has been instantiated at all.

We state this carefully because it is easy to overstate, and the stronger reading is not one the argument needs. The claim is not that no one may speak of higher-spin black hole unitarity, nor that the informal question is somehow ill-posed or uninteresting; physicists have discussed it productively for fifty years without needing Hawking's specific 1975 mode construction, and nothing here suggests otherwise. Nor is the claim that higher-spin Hilbert spaces are unavailable in principle to quantum field theory in curved spacetime – only that Hawking's 1976 paper does not construct them, and that Reference 13, cited at exactly the point they were needed, does not either. The claim is narrower and specific to the object the 1976 paper itself introduces: *that* formal apparatus, for *those* fields, as that paper actually builds it, requires the construction Ref. 13 was cited to supply, and a reader is entitled to ask whether a different, independently constructed treatment could instantiate the same formalism for higher-spin fields in its place – to which Section 6.4 is the direct answer, since it is exactly this substitution the checked literature is examined for and, on the evidence assembled there, does not yet supply.

5.2 What closure would and would not deliver

Sections 3–4 established that Reference 13 does not supply the derivation it is cited for: no equation, mode construction, or independent argument for the electromagnetic and gravitational case appears in either paper, only the two-sentence assertion examined above. On the vocabulary introduced in Section 2, Reference 13 fails to supply even formal extension – the derivation, for higher-spin fields, of results analogous to the scalar-field Bogoliubov coefficients Hawking's 1975 paper computes in full.

This section asks a different question, and one this paper's own argument requires it to ask before resting on the citation-chain result alone: suppose the missing derivation were supplied tomorrow, in full and without qualification – suppose someone carried out, rigorously, the extension of Hawking's scalar-field calculation to the electromagnetic and gravitational fields, obtaining higher-spin analogues of the coefficients α and β. This is closure, in the sense of Section 2.1: the formalism now covers the broader domain, on paper, without qualification. Would closure of this kind settle the question the 1976 paper's citation is invoked to settle? We argue that it would not, and that the reason is not a further citation gap of the same kind already identified, but a structural feature of what such a derivation would and would not compute – the same structural gap Section

2.4 identified in general terms, now made concrete in the case this paper examines.

5.3 What a Bogoliubov-coefficient derivation delivers

Hawking's 1975 calculation, for the scalar field, produces an asymptotic form for the coefficients $\beta_{\omega\omega'}$ relating incoming and outgoing modes, and from this a particle-occupation number for each outgoing frequency: the expectation value $\langle 0_- | b_\omega^\dagger b_\omega | 0_- \rangle$, summed appropriately, yields the thermal spectrum $(\exp(2\pi\omega/\kappa) - 1)^{-1}$. This is a statement about *how many* quanta are emitted, as a function of frequency alone. Nothing in the calculation, for a scalar field, could be otherwise: a scalar field carries no internal structure beyond its amplitude, so a count of scalar quanta per frequency exhausts what there is to compute.

A hypothetical extension of this calculation to the electromagnetic or gravitational field would, by direct analogy, produce the same kind of object: a count of photons or gravitons emitted per frequency (and, for these fields, per angular mode). This is what "the black hole also radiates photons and gravitons thermally" – the claim Hawking's 1975 passage asserts – most naturally means, and it is what a derivation filling the citation gap identified in Sections 3–4 would most directly supply.

It is worth being explicit about why this is a structural limit on what the extension delivers, rather than merely a limit on what has been computed so far. Write the mode index for a higher-spin field as a pair (ω, λ), where ω is the frequency label the scalar calculation already carries and λ is the polarization or helicity label the scalar field does not have. A direct extension of Hawking's construction produces annihilation and creation operators $b_{\omega\lambda}$, $b_{\omega\lambda}^\dagger$ for each such mode, exactly as the scalar construction produces b_ω, $b_\omega^\dagger$ for each frequency mode, and the natural extension of the occupation-number result (2.10) is the diagonal expectation value

$$\langle 0_- |\, b_{\omega\lambda}^\dagger b_{\omega\lambda} \,| 0_- \rangle = \sum_{\omega'} |\beta_{\omega\lambda,\,\omega'}|^2,$$

obtained, by the same argument Hawking gives for the scalar case, from the asymptotic form of the Bogoliubov coefficients relating the mode (ω, λ) to the incoming modes. This is a count of quanta in the mode (ω, λ); summed or not over λ, it is still built entirely from diagonal matrix elements of the number operator, one mode at a time. It is not, and does not become by further calculation of the same kind, the off-diagonal object

$$\langle 0_- |\, b_{\omega\lambda}^\dagger b_{\omega\lambda'} \,| 0_- \rangle, \qquad \lambda \neq \lambda',$$

which is what a polarization density matrix for the emitted state would need to *report*: the coherence between two different polarization states at the same frequency, the quantity that would need to be known, whatever its value turns out to be, before a claim about whether the polarization degree of freedom is thermally random, correlated, or structured could be checked against the state the calculation actually produces. A derivation extending the Bogoliubov apparatus in the way Reference 13 gestures toward computes objects of the first kind, because that is what the coefficients $\alpha_{\omega\omega'}$ and $\beta_{\omega\omega'}$ of Hawking's own construction are built to produce: relations between mode amplitudes, from which diagonal occupation numbers follow directly and off-diagonal coherences do not follow at all, since nothing in the construction of $\beta_{\omega\lambda,\,\omega'}$ from the geometric-optics argument of Hawking's Section 2 involves a comparison between two different values of λ at all – each polarization channel is propagated separately, by the same blueshift argument, into a separate set of coefficients, with no step in the derivation that would relate $b_{\omega\lambda}$ to $b_{\omega\lambda'}$. The absence of the off-diagonal object is accordingly not a matter of the calculation being carried out only partway; it is a consequence of what kind of object a mode-by-mode Bogoliubov calculation is built to produce in the first place, higher-spin or not.

5.4 Why the missing off-diagonal terms are not a bookkeeping matter

The observation that diagonal entries of a density matrix do not determine its off-diagonal entries is, on its own, a generic fact about Hermitian operators, true of any basis on any Hilbert space and owing nothing to black holes or quantum field theory in particular. Stated at that level of generality it would not support the argument of this section; a referee would be right to ask what, beyond this triviality, the missing off-diagonal terms cost the specific claim Reference 13 is invoked to underwrite. The answer is that the quantity those terms fix is one Hawking's own 1976 apparatus already uses to state the question the paper poses, which makes their absence a gap in exactly the object the paper's own formalism requires, not an incidental feature of density matrices considered abstractly.

Consider the polarization sector of the density matrix for a single outgoing frequency mode, restricted for concreteness to two polarization states, so that ρ is a 2×2 Hermitian matrix with unit trace. Two such matrices can share the same diagonal – the same occupation numbers, the same prediction for how many quanta of each polarization are emitted – while differing completely in the quantity that determines whether the state is pure:

$$\rho_{\mathrm{diag}} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}, \qquad \rho_{\mathrm{coh}} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}.$$

Both matrices have identical diagonal entries and hence identical occupation numbers; a frequency-resolved, polarization-summed calculation cannot distinguish them. But $\det(\rho_{\mathrm{diag}}) = \frac{1}{4}$ and $\mathrm{Tr}(\rho_{\mathrm{diag}}^2) = \frac{1}{2}$, while $\det(\rho_{\mathrm{coh}}) = 0$ and $\mathrm{Tr}(\rho_{\mathrm{coh}}^2) = 1$: the first is maximally mixed on this two-dimensional sector, the second is pure. The determinant and the purity of a density matrix are properties of the matrix as a whole, and in general depend on the off-diagonal entries in a way no function of the diagonal alone can recover – a fact this pair of matrices makes concrete rather than merely asserts.

This is not, however, a fact about matrices in the abstract that happens to be illustrated with a black hole example; it is the specific quantity Hawking's 1976 paper itself uses to state what non-unitary evolution would consist in. That paper defines the entropy of the final radiation state as

$$S_2 = -\sum_C \sum_D \rho_{2CD} \ln(\rho_{2CD}),$$

vanishing when ρ_2 is pure and positive otherwise, and treats a nonzero S_2 – equivalently, on the two-level toy matrices above, a nonzero determinant on the relevant sector – as the technical signature of the information loss the paper's central argument is about. A calculation that fixes only the diagonal of ρ_2 restricted to the polarization sector – which is what a frequency-resolved, polarization-summed higher-spin extension supplies – accordingly leaves S_2 undetermined for that sector: consistent with ρ_{diag}, consistent with ρ_{coh}, and consistent with everything between them, since the diagonal is the only input such a calculation provides. The missing off-diagonal terms are therefore not a bookkeeping gap orthogonal to the 1976 paper's own question; they are the specific missing ingredient in the specific quantity, S_2, by which that paper defines the question it poses. A derivation supplying only the diagonal, however completely, leaves S_2 for the higher-spin sector exactly as undetermined as it was before the derivation was carried out.

5.5 What evaluating the unitarity question would require

The 1976 paper's question is not, however, whether black holes emit the right *number* of photons and gravitons at the right rate. It is whether the *quantum state* of the radiation, considered in full, evolves unitarily – whether the information describing the collapsing body's full physical content is recoverable, in principle, from the state of the emitted radiation. Evaluating a claim of that kind requires, at a minimum, a characterization of the emitted state complete enough that its evolution could in principle be checked against unitarity; it does not by itself require that the state have any particular structure, and nothing in this paper claims to know, or needs to claim to know, whether the polarization degree of freedom is in fact thermally random, correlated, or otherwise structured. A photon or graviton is not exhausted by its frequency and direction of travel; it additionally carries a polarization – a further degree of freedom, orthogonal to the frequency spectrum, describing the field's transverse structure. Two photons of identical frequency and identical direction can differ in polarization, and a frequency-resolved, polarization-summed occupation number is accordingly silent on this further structure by construction: it is compatible with any polarization behavior whatsoever, coherent or not, correlated or not, and so cannot be used to characterize the state well enough for a claim about the *full* state's unitary evolution to be evaluated against it, whatever that evolution turns out to be. In the notation of Section 5.3, the diagonal occupation numbers a direct higher-spin extension delivers leave the off-diagonal object $\langle 0_- | \, b^\dagger_{\omega\lambda} b_{\omega\lambda'} \, | 0_- \rangle$ for $\lambda \neq \lambda'$ entirely unconstrained – not shown to be zero, not shown to be nonzero, simply absent from what the calculation computes. Section 5.4 makes the cost of that absence concrete rather than schematic: the same diagonal is compatible with a ρ_2 that is maximally mixed on the polarization sector and with a ρ_2 that is pure there, and Hawking's own entropy S_2 – the quantity his 1976 paper uses to state what non-unitary evolution would mean – differs between them. It is this absence, not any particular claim about what the missing quantity equals, that limits what such a calculation can establish about unitarity.

5.6 The gap survives its own closure

This yields the result this section exists to establish, stated in the vocabulary of Section 2. A derivation extending Hawking's calculation to higher-spin fields, however complete on its own terms, achieves formal extension: it broadens an existing piece of mathematics – the Bogoliubov apparatus – to a wider domain, and reaches closure in exactly the sense of Section 2.1. It does not thereby achieve target adequacy for the claim the 1976 paper's citation is actually invoked to support, because evaluating that claim – unitary evolution for the fields a real black hole radiates – would require a characterization of the emitted state complete enough to be checked against unitarity, and a frequency-resolved occupation calculation is, by construction, silent on the polarization structure such a characterization would need to include. The silence is not a defect in such a calculation; a count of quanta per frequency is exactly what it sets out to compute, and it computes it correctly. The silence is a mismatch between what the calculation delivers and what the citation is asked to underwrite – precisely the mismatch Section 2.4 predicted closure could obscure, since a sentence reporting that a formalism now “extends” to a broader class of fields does not, on its own, distinguish formal extension from target adequacy, and readers have ordinary reason to assume the stronger claim.

The consequence for the citation-chain result of Sections 3–4 is worth stating precisely, because it strengthens rather than merely repeats that result. It is not only true that Reference 13, as written, supplies no derivation of the higher-spin extension. It is additionally true that even a derivation Reference 13 might have pointed to – one genuinely extending the Bogoliubov formalism to spin-1 and spin-2 fields – would not, without further and independent work, supply what evaluating the 1976 paper’s unitarity question would require. The citation gap and the target-adequacy gap are, in this case, nested: closing the first would still leave the second open, because the two gaps are gaps of different kinds, answering different questions, and nothing about filling in a missing calculation of one kind supplies a calculation of the other.

Put in the terms of Section 5.1: a formal-extension derivation of the kind Reference 13 gestures toward would supply, within Hawking’s own 1976 framework, a construction of H_1, H_2, and H_3 for the higher-spin sectors – it would give that paper’s formal unitarity question a domain stated within its own apparatus, where before the paper had not stated one. That is a real and nontrivial achievement, and it is the achievement Ref. 13 was cited for. It is not, however, the further achievement of characterizing the resulting state well enough to evaluate whether the superscattering operator on those Hilbert spaces is or is not unitary, because a frequency-resolved construction, by the argument of Sections 5.3–5.5, does not by itself carry the polarization-resolved structure such an evaluation would need. Closing the citation gap poses the formal question within Hawking’s own apparatus. It does not supply what would be needed to answer it. The two are easy to run together precisely because ordinary usage does not distinguish “we can now state what unitarity would mean here” from “we now know whether it holds” – the same conflation Section 2.4 identifies in general terms.

5.7 Why this is not merely a further citation problem

It is worth forestalling a natural misreading of the preceding argument, since the paper’s own discipline requires it. The claim of this section is not that a *further*, more complete derivation is missing from the literature, waiting to be supplied by the same kind of calculation Reference 13 gestures at, only more thorough. That would reduce this section’s point to an extension of Sections 3–4: a bigger citation gap of the same species. The claim is instead that the frequency-resolved and polarization-resolved questions are not related as a smaller calculation is related to a larger one of the same kind. They are related as a model is related to its target, in the sense of Section 2: a formalism can be internally extended without thereby becoming a claim about the further physical structure a reader is entitled to expect it settles. No amount of care in extending the frequency calculation, on its own terms, converts it into a polarization calculation, for the same reason that no amount of precision in measuring a beam’s total power converts that measurement into a report of the beam’s polarization: Sections 5.3–5.4 make this precise rather than metaphorical, since the diagonal object a Bogoliubov extension is built to produce and the off-diagonal object a polarization-resolved characterization of the emitted state would need to report are not related by degree of completion at all, but arise from entirely different matrix elements of $b^{\dagger}_{\omega\lambda} b_{\omega\lambda'}$, with the cost of the difference made concrete by the determinant of Hawking’s own density matrix ρ_2.

The two are not more and less complete versions of a single task; they are different tasks, and completing one does not partially or fully complete the other.

The missing higher-spin extension is accordingly not a cosmetic omission. Without it, Hawking's 1976 formal apparatus is not actually constructed for the spin-1 and spin-2 sectors; Reference 13 leaves that construction undone, not merely uncited. Later polarization-summed emission-rate calculations, Page's foremost among them, do not by themselves supply the polarization-resolved quantum-state structure a full characterization of the emitted state, adequate to evaluating the unitarity question, would need to include, for the structural reason just given. Section 6.4 checks this directly against the adjacent literature and finds the same pattern throughout: each body of work we examined approaches the gap from a different angle, and none completes it, a pattern the authors of that work are themselves candid about wherever they address it.

(Evidentiary scope: this section does not claim that black hole evaporation is in fact non-unitary for photons or gravitons, nor that no derivation addressing the polarization-resolved question could be constructed. It claims only that the specific derivation Reference 13's citation gestures toward – an extension of the frequency-resolved Bogoliubov calculation – would not, if supplied, by itself settle the question the citation is invoked to support.)

6 Objections

Three objections to the argument of Sections 2–4 are worth considering directly, since each attempts to read either the citation or the paper's use of it more charitably than the text supports.

6.1 "Straightforward" is a claim about difficulty, not a claim of proof

The 1976 sentence does not say Reference 13 contains a derivation; it says the extension is "straightforward along the lines indicated" there. On this reading, Hawking is not claiming the work is done, only that it would be easy – an invitation to the reader, not a report of completed work. A version of this objection can be read as questioning whether Reference 13 was ever offered as a pointer to evidentiary support, in the sense distinguished in the Introduction, rather than as an informal pointer to related literature – in which case, on that account, its failure to contain a derivation would raise no difficulty at all.

The taxonomy introduced in the Introduction is precisely what answers this. The 1976 sentence's own grammar settles which of the three citation functions is in play: it does not say "see also Ref. 13," the form a pointer to related literature would take, nor does it attribute the approach's origin to Reference 13 without further claim, the form a pointer to provenance would take. It asserts that a path to the result "is straightforward along the lines indicated in Ref. 13" – offering the cited location as the stated reason the extension may be taken as established. This is a citation used as a pointer to evidentiary support, on the sentence's own terms, and the objection does not survive contact with that grammar.

Even granting the softer reading for the sake of argument, however, the objection does not change the outcome. If "straightforward along the lines indicated in Ref. 13" means only that the extension would be an easy exercise, not undertaken in either paper, this reading is consistent with the citation-chain result rather than a rebuttal of it: an invited exercise still requires the source cited to supply what the exercise is invited along the lines of, and Reference 13, on inspection, does not.

The objection changes the rhetorical status of the gap – an invited exercise rather than an oversight – but not its existence. Nor does it touch Section 5's further result: an invited exercise in extending the Bogoliubov formalism, even fully carried out, would still face the frequency/polarization distinction of Section 5, since nothing about how straightforward a calculation is bears on what kind of question it answers.

A stronger version of the same objection holds that the 1975 passage was never intended to meet a standard requiring detailed derivation – that it asserts only that the relevant equations generalize without conceptual novelty, a real and often correctly made kind of claim, distinct from a claim about how an actual physical system behaves. This has genuine force against a stronger reading of this paper's argument than the one advanced here. Nothing here disputes that the relevant equations admit a higher-spin analogue in some form. What is at issue is only whether that mathematical generalization was ever shown, as opposed to asserted, to carry the further content – the polarization-resolved structure examined in Section 5.4 – that the 1976 paper's own unitarity argument requires of the claim it is put to use for. A citation cannot be read as meeting a modest standard when convenient and a demanding one when the citing paper's own use of it requires the latter.

6.2 Private confidence may have stood behind the assertion

The fermion case, treated with a specific (if brief) supporting remark in the same passage, might suggest the higher-spin bosonic case was worked out with comparable rigor even if not published in either paper.

This cannot be assessed by textual analysis, and it is not disputed as a claim about private confidence. What can be said is narrower: no published derivation is cited in either paper, in contrast to the more developed treatment the fermion case receives in the same passage. An argument from unpublished confidence, however well founded, is not a citable derivation, and cannot license a citation offered as though it were one. This is not a special standard invented for this case; it is the ordinary standard a citation is held to whenever it is used, as this one is, to license a claim the citing text itself does not independently establish.

6.3 The gap may have been closed by subsequent literature, even if the original citation did not close it

The most substantive objection holds that fifty years of subsequent work – on Hawking radiation, on higher-spin fields on black hole backgrounds, and on the black hole information problem the 1976 paper's question initiated – has by now supplied what Reference 13 did not, so that treating the citation as unsupported is a historical point without present consequence.

This objection, even if correct, would not alter the finding of Sections 3–4: it would show that the gap left by Reference 13 has since been filled by other means, not that Reference 13 filled it. The distinction matters for the claim this paper makes, which concerns the content of the two papers as published, not the present state of a much larger literature. It matters in a second way as well, given Section 5: filling the gap Sections 3–4 identify – supplying the missing higher-spin Bogoliubov calculation – would still not, by the argument of Section 5, supply what evaluating the 1976 paper's unitarity question would require, since that further requirement is a matter of what kind of calculation is needed, not of whether any particular calculation has yet been carried out. A literature that closed the first gap without addressing the second would leave the position substantially where Section 5 leaves it, whatever its other merits.

This paper does not attempt a systematic survey of the subsequent literature, and nothing in the argument of Sections 2–5 depends on one: a survey adequate to settle the question responsibly, across five decades of work touching black hole radiation, quantum field theory in curved spacetime, and the information problem, is a separate undertaking from the one this paper attempts, and the citation-chain result of Section 4 does not require it. Section 6.4 below reports, in a narrower and more limited spirit, what a small number of directed checks against specific, natural candidates turned up, and states plainly what those checks do and do not establish.

6.4 Wald, Page, and a limited check of the subsequent literature

Section 5 claimed that adjacent literature approaches this gap from many directions without completing it, and that where it comes close, the authors say so themselves. The check that follows is not a survey and does not attempt to be one; it is a directed search with a stated purpose, and that purpose is worth stating before the results are reported rather than left to be inferred from them. The aim is not to locate a single counter-example that would refute Section 5's argument, nor to establish a negative result by exhaustion. It is to map the *shape* of the gap: to ask, of each natural body of adjacent work, which piece of the missing construction it supplies, which piece it does not, and whether the pieces different sources supply could in principle be assembled into the construction Reference 13 was cited for. A search conducted this way can fail to find a counter-example and still be informative in a way a bare negative result is not, because it identifies where the actual difficulty lies – which of the four components is hardest to find anywhere, as opposed to merely absent from the one source checked – rather than only reporting that no single source contains all four.

Before checking specific sources against that claim, it is worth stating explicitly what closing the gap would require, so that each check below can be read against a fixed standard rather than an impression formed after the fact. Four things would need to be shown together, and we label them for ease of later reference, treating them as the four components the preceding paragraph's search is directed at locating, separately, across the literature. (a) Mode construction extended past the scalar case: field-appropriate mode functions for spin-1 and spin-2, built on the same collapse geometry Hawking's own derivation uses, not merely the field equations such modes would solve. (b) An observable that carries this structure through rather than discarding it: an inner product or flux calculation that does not reduce to something scalar-equivalent by summing or averaging over polarization before the calculation is complete. (c) A result resolved by polarization, not summed over it: the distinction argued in Section 5.4, where a diagonal or trace-only quantity leaves the coherence structure a full state-characterization would need entirely unreported, whatever its own precision. (d) Evaluation relative to a specific observer's frame: polarization, in the sense a full characterization of the emitted state would need, is not a property of the field alone but of the field as measured, requiring the connection between a null-tetrad-adapted mode description and a timelike observer's screen that Section 3's discussion of Cocke and Holm and this section's discussion of the Faraday and spin-Hall literature both address. Each of (a)–(d) is tied to the same target: not to Hawking's 1975 thermality claim, which is undisputed for the scalar field, but to the polarization-resolved quantum-state characterization a full evaluation of the 1976 paper's unitarity question, for the fields a real black hole actually radiates, would require.

Eight bodies of work, checked directly against their own text rather than assumed, are reported below against this standard – including the two names, Wald and Page, a reader familiar with this literature is most likely to raise first.

The island-formula and quantum-extremal-surface program is the modern resolution most often invoked here. A representative treatment extending it to the rotating (Kerr) case [26] states its entropy formula in terms naming "quantum fields (including gravitational field)," but the calculation it actually performs reduces the matter content to a scalar field via dimensional reduction; the field-general phrasing belongs to the formula it inherits, not the calculation it carries out.

Wald's 1975 companion paper [25] is the natural first check on whether Hawking's result was independently secured soon after publication. It was – but only for the same massless scalar field Hawking's own derivation treats, as Wald's own abstract states; it strengthens Section 3's undisputed scalar result without touching the higher-spin extension.

Page's papers [17, 18] remain the standard reference for higher-spin emission rates and are a genuine formal extension in the sense of Section 2, not the bare assertion Sections 3–4 examine. They remain, however, polarization-summed: a representative treatment of the electromagnetic case states directly that the calculation proceeds by averaging over the two photon polarizations, the very operation that discards the coherence content Section 5's argument turns on. Page's work is accordingly the clearest instance of the pattern this paper describes – real, lasting formal extension without target adequacy for the polarization-resolved claim the 1976 unitarity question needs.

The gravitational Faraday and spin-Hall literature [20, 21, 16, 1] addresses polarization transport directly, and its central finding sharpens Section 5 rather than merely illustrating it: whether a polarization rotation is detected depends on the observer's own angular momentum, not on whether the black hole itself rotates, confirming from an independent research program that the missing content is observer-relative in exactly the sense target adequacy requires.

A further body of work [19] constructs an explicit polarization density matrix for photons near a rotating black hole, including the off-diagonal coherence structure Page's tradition discards. This is genuinely polarization-resolved – but for classical accretion-disk photons propagated by geometric optics, not the quanta created by horizon particle creation; it answers the right kind of question about the wrong physical process.

Two further, more recent checks confirm the pattern holds in current work. An active continuation of the Teukolsky/Page tradition [2, 3], feeding the widely used `BlackHawk` code, computes higher-spin emission rates in the same polarization-summed sense as its 1976 predecessor. And a recent proposal to resolve the information problem via the entropy of the final radiation state [13] argues that purification may come from photon-graviton entanglement – exactly the coherence structure Section 5 identifies as missing – offered explicitly by its own author as a plausibility argument, not a derivation, with further work needed to establish it.

A last check comes from a body of work outside the black hole literature entirely, worth including precisely because it shows that genuine polarization-resolved graviton calculations exist and have existed for decades, elsewhere. The covariant density-matrix formalism developed for flat-space graviton scattering [6] constructs an explicit graviton polarization density matrix, with the off-diagonal Stokes-parameter coherence Page's tradition discards, for processes such as $gX \to \gamma X$ and $gg \to gg$. Measured against (a)–(d), this satisfies (c) as directly as any source checked: the calculation is polarization-resolved, not polarization-summed, by construction. It satisfies neither (a) nor (d): the calculation is a flat-space, tree-level S-matrix scattering amplitude between free asymptotic states, with no Bogoliubov transformation, no horizon, and no collapse background, so it supplies no mode construction of the kind Section 6.4(a) requires and no observer-relative horizon frame of the kind (d) requires. Its relevance to the present argument is exactly the relevance Racorean's paper has: it demonstrates that the polarization-resolved half of what target adequacy needs is not, in general, technically unavailable – physicists have built exactly this kind of object, competently and repeatedly, for a different physical process – while the horizon-particle-creation half remains, on every source checked, either absent or polarization-summed. The absence is accordingly not a matter of missing technique.

Eight checked sources, alongside Teukolsky and Cocke-Holm in Section 3, do not constitute a survey, and we do not present them as one. What can be said, on the basis of what was actually read, is narrower: at every natural place to look for a counter-example to Section 5's argument, what is found is consistent with it, when read against (a)–(d) above. The island-formula treatment and Wald satisfy none of the four for the higher-spin case, being scalar throughout. Page and its modern continuation satisfy (a) but not (b) or (c), by explicit averaging. The Faraday and spin-Hall literature contributes to (d) directly, and Racorean's paper satisfies (c) for a different physical process than (a) and (d) require. Kay's proposal names (c) as the missing ingredient without supplying it. The flat-space graviton-scattering literature satisfies (c) more directly than any other source checked, and for the same reason as Racorean's, satisfies it for the wrong physical process. No source checked satisfies all four together. A reader aware of a treatment that does is invited to supply it.

This last point is what gives the informal/formal distinction of Section 5.1 its bite rather than leaving it a definitional move. The gap is not that no one has developed the mathematics of polarization coherence for photons and gravitons – they have, repeatedly, competently, in adjacent literatures, since at least 1994. The gap, among the sources checked here, is that this mathematics has not yet been brought into contact with Hawking's own horizon particle-creation formalism, the specific formalism the 1976 paper's superscattering operator is built from. Reference 13 was cited at exactly the point where that contact would have to be made. It was not made there, and it was not found, among the eight sources checked in this section, to have been made elsewhere; whether it has been made in work outside this directed search is a question this section is not positioned to answer.

(Evidentiary scope: nothing in this section takes a position on whether the higher-spin extension is correct, whether subsequent literature has in fact closed either gap identified above, or whether black hole evaporation is unitary for photons and gravitons. The section's claim is only that none of the three objections considered defeats the citation-chain result of Section 4 or the further result of Section 5, and that eight directed checks against natural candidate sources – including the two standard references, Wald and Page – did not surface a counter-example, without this being offered as a survey of the literature as a whole.)

7 Related Observations: Citation Practice and Domain of Validity

The case examined above bears on two further bodies of work, each of which offers a partial explanation of how a gap like the one documented in Sections 3–5 could persist without correction.

We note both briefly. Neither is the argument of this paper, and neither is needed for it; both are complementary to it.

Merton's account of the normative structure of science [14] identifies organized skepticism – the subjection of every claim to critical scrutiny irrespective of its source or the confidence surrounding it – as one of the norms constitutive of the scientific ethos. On this view, a claim's age, its wide citation, or its author's stature are not grounds for exemption from re-examination. The case examined here illustrates a specific way this norm can fail to operate in practice: not through fraud, error, or suppression, but through a citation that functions, in ordinary reading practice, as a signal that a matter has been checked, a signal that can persist through decades of subsequent reliance even where the passage it points to does not support it.

A second and complementary framework comes from the methodology of model verification and validation, standard across the quantitative sciences: a declaration that a model is validated cannot stand without specifying the domain of applicability for which the validation holds [15]. Hawking's 1975 calculation is validated, in this sense, for one field and one observable. The 1976 citation extends this domain implicitly, without the extension being independently validated or its new domain being stated – and the absence went unmarked, on the textual evidence of Sections 3–4, precisely because a citation was present where a domain statement should have been.

Both frameworks explain, in their own terms, why a gap of this kind should be rare and should not persist undetected. That an instance meeting the exacting definition developed in this paper can nonetheless be located suggests the underlying failure mode is worth attention beyond the sociological or methodological literatures that separately anticipate it.

8 Conclusion

This paper has examined a single, specific case: a citation, in a canonical and still widely cited 1976 physics paper, offered as the source of a derivation it does not in fact contain. The case is established by textual analysis alone, and its scope has been stated throughout rather than left to be inferred: this paper does not assess whether the underlying physical claim is true, only whether it has been shown to be, and by what.

Three findings, of increasing consequence, follow from this analysis. The first is documentary, established in Sections 3–4: the citation, as written, does not supply the derivation it is invoked to support, a fact about the structure of two published texts, verifiable by any reader with both papers in hand. The second, established in Section 5.2 onward, is that even a derivation the citation might have pointed to, fully supplied, would not by itself settle the question the citation is invoked to settle, because that question requires a kind of content – what Section 2 calls target adequacy – that a formal extension of the existing calculation does not, on its own terms, provide.

The third finding is the one that gives the first two their consequence, and it is the answer this paper gives to the question of why any of this matters beyond the correction of a fifty-year-old bibliographic record. The dependency between Reference 13 and the 1976 paper's higher-spin unitarity question is not evidentiary but constitutive of that paper's own formal apparatus. The 1976 paper's superscattering operator maps density matrices between the Hilbert spaces H_1, H_2, and H_3 of its own construction, and that construction is carried out, within the paper, for the scalar field only; for the electromagnetic and gravitational sectors, the paper's own formalism does not specify what these spaces are, in the same sense in which specifying a problem's domain is, in the standard mathematical treatment of physical problems, a distinct step prior to and separate from any question of that problem's well-posedness [10]. This is not a claim that higher-spin Hilbert spaces are unavailable to quantum field theory in curved spacetime in general, only that Hawking's own 1976 apparatus does not construct them, and that Reference 13, cited at exactly the point they were needed, does not either. Section 6.4's directed check of eight bodies of subsequent work – spanning the island-formula program, Wald's companion derivation, Page's emission-rate tradition and its modern continuations, the gravitational Faraday and spin-Hall literature, Racorean's accretion-disk calculation, and, from outside the black hole literature entirely, the covariant polarization density-matrix formalism developed for flat-space graviton scattering – was structured to locate, for each source, which piece of this construction it supplies and which it does not; among the sources examined, none supplied all four components of Section 6.4 together. Some supply pieces of what would be needed, and where they come closest the shortfall is not a matter of missing mathematical technique, since a fully polarization-resolved formalism for photons and gravitons has existed, for a different physical process, since at least 1994, but of that technique never having been brought into contact with Hawking's own horizon particle-creation apparatus. The formal question the 1976 citation is invoked to underwrite is accordingly not merely unanswered. Among the sources examined here, the construction Hawking's own apparatus would need in order for that question to be evaluated for higher-spin fields has not yet been assembled.

We restate what this does not establish, since the distinction is easy to lose at exactly this point in the argument. Nothing here shows that higher-spin Hawking radiation is in fact non-unitary, nor that it could not be shown unitary once the missing construction is carried out, nor that the informal question physicists have discussed productively for fifty years is somehow defective. What is established is narrower and, we think, more useful: that a specific, nameable kind of gap – between a formalism's mathematical extension and its bearing on a stated physical question – separates what Reference 13 was cited to do from what it does, and that this gap survives, undiminished, even a derivation that would close every objection raised against the citation-chain result on its own terms.

What the case illustrates, beyond its own specifics, is a distinction worth looking for elsewhere: between extending a piece of mathematics and showing that the extended mathematics bears on the physical question it is offered to answer. The two are not more and less complete versions of a single achievement. A derivation can be closed, in the sense of Section 2, without the further and independent question of target adequacy being touched at all – and nothing in ordinary scientific prose reliably marks the difference, because a citation can close the first kind of gap, or appear to, while leaving the second exactly as open as it was before the citation was written. That a case meeting this description can be located in a paper as prominent, and as thoroughly engaged by subsequent scholarship, as Hawking's 1976 paper suggests the distinction is worth taking seriously as a general possibility in theoretical physics, not only as a curiosity of this particular case: a citation loop is not load-bearing until it is filled, however long, and however confidently, it has been built upon.

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