A Vacuum Field-Particle Interpretation of Hawking-Unruh Equivalency

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Abstract

The *Rindler Horizons* are known to *causally disconnect* regions of the flat spacetime from an accelerating observer. In this article it is shown that the past and future horizons of such observer indeed meet at the event horizon of the Unit Black Hole (UBH) of the c-SRQM theory. Furthermore, the Unruh temperature registered by an accelerating observer is shown to be the *red shifted* Hawking temperature of the UBH at such a hovering distance where the local UBH gravity is equal to the acceleration of the observer. Thermodynamics of the gravity and acceleration are unified.

Keywords — Hawking-Unruh equivalency, Rindler horizon, Dark matter halo

1 Background

According to the c-SRQM theory [1] a particle moving in *absolute vacuum* [1] under a definite momentum p has the following inherent uncertainties in its spacetime coordinates:

$$\begin{aligned} \delta x' &= \frac{Av'}{\sqrt{c^2 - v'^2}} \\ c\,\delta t' &= \frac{Ac}{\sqrt{c^2 - v'^2}} \end{aligned} \tag{1}$$

where x' and t' are the position and time coordinates of any *inertial system* wherein the particle momentum p is found true, v' is the particle velocity, A is a physical constant named the *resolution interval* and c is the speed of light. The significance of the resolution interval A, according to the theory, is in that if a quantum particle has the Compton Wavelength $\lambda = h/mc > A$ then its rest mass m would be so feeble that it would physically be *unresolvable.* Such particles are best described as *physically* massless while their rest mass, *numerically*, might not be precisely zero. From Eqn 1, it is clear that the spacetime coordinate uncertainties of physical particles trace a north-opening hyperbola as follows:

$$(c\,\delta t')^2 - (\delta x')^2 = A^2 \tag{2}$$

[1] Devoid from any form of physical matter, gravity or radiation

Subsequently the inherent spacetime coordinate uncertainties $(\delta x', c\delta t')$ of a particle with rest mass m was shown to quantize as follows:

$$\begin{aligned} \delta x' &= A\sqrt{\frac{\bar{m}}{m}}\, n \\ c\,\delta t' &= A\sqrt{1+\frac{\bar{m}}{m}\, n^2} \end{aligned} \tag{3}$$

where $n = 1, 2, ...$ is the *quantum state* of the particle and wherein $n = 1$ corresponds to the nearest state that the particle could get to an absolute rest condition. In Eqn 3, the entity with rest mass $\bar{m}$ is interpreted to be the *constituent of the absolute vacuum* whose Compton wavelength *fixes* the value of the resolution interval through $A = h/\bar{m}c$. The existence of such resolution interval in nature, then necessitates the existence of *acceleration upper limit* $a_u = c^2/A$. According to the theory [2],

$$\lim_{a\to a_u} \frac{d}{dt'}(\delta x') = c \tag{4}$$

This indicates that any acceleration higher than the limit a_u would result in a superluminal expansion of the spatial uncertainty, therefore, result in a direct violation of the General Relativity. Using the upper limit acceleration a_u the theory then arrives at the concept of the Unit Black Hole (UBH) with horizon gravity $a_u = c^4/4GM_1$ and therefore the mass $M_1 = Ac^2/4G$ and horizon diameter $D_1 = A$. The UBH is the smallest possible black holes in the universe. It was then shown in [3] that the mass M_b of all black holes and the diameters of their event horizon D_b and core C_b (or singularity) can be quantized using those of the UBH as follows:

$$\begin{aligned} M_b &= b^3 M_1 \\ D_b &= b^3 A \\ C_b &= bA \end{aligned} \tag{5}$$

where b is the *quantum index of the black holes*. It is clear that for the UBH, with index $b = 1$, we then have $D_1 = C_1 = A$, ie. for a stand alone UBH the core and the event horizon coincide on diameter A, as expected. Comparing the core diameter C_b in Eqn 5 with the spatial uncertainty $\delta x'$ in Eqn 3, it is evident that for the vacuum constituent $\bar{m}$ under the condition $n \cong b$ we then have $\bar{\delta}_b = C_b = bA$, i.e. the vacuum constituent's spatial uncertainty $\bar{\delta}$ in quantum state $n = b$ matches the diameter of the shell b in the singularity of the black holes. It is, therefore, hypothesized that the black holes and their singularities are the *condensed states of the constituents of the vacuum* and therefore should not be viewed as the physical objects. Therefore, according to the c-SRQM theory outlined in [3], condensates of the vacuum constituent at quantum states $n = 1, 2, ..., b$ generate internal spherical shells $j = 1, 2, ..., b$ of the singularity of a black hole of index b.

To establish a continuity in ideas presented earlier in our cosmological model [5], the vacuum constituent $\bar{m}$ is named Primordial Stem Particle (PSP) to indicate that : 1) *its existence predates the Big Bang*; and 2) *the physical matter has stemmed from it at the Big Bang, and does return to that form upon falling into a black hole.*

As shown in Fig 1, the number of the particles $\bar{N}_j$ on each shell j was shown to be given by:

$$\bar{N}_j = (3j^2 - 3j + 1)\bar{N}_1 \qquad\qquad j = 1, 2, ..., b \tag{6}$$

where $\bar{N}_1$ is the number of PSP constituents on the innermost shell $j = 1$, i.e. the UBH, given by:

$$\bar{N}_1 = \frac{M_1}{\bar{m}} = \frac{1}{4}(\frac{A}{L_p})^2 \tag{7}$$

where L_p is the Planck length. From Eqn 7 we noted that since *perfect squares of odd numbers are never divisible by* 4, the necessity of $\bar{N}_1 \in \mathbb{N}$ then demands that *the Planck length L_p must be an* *even divisor of the resolution interval A.* Finally, taking the $\bar{N}_1$ as the *quanta* of particle count in black hole singularity, the number of total particles $\bar{N}_b$ in a black hole with index b is given by:

$$\bar{N}_b = \sum_{j=1}^{b}(3j^2 - 3j + 1)\bar{N}_1 = b^3\,\bar{N}_1 \tag{8}$$

We also noted that the PSP count on the core shells makes the overall density ρ_s of all black hole

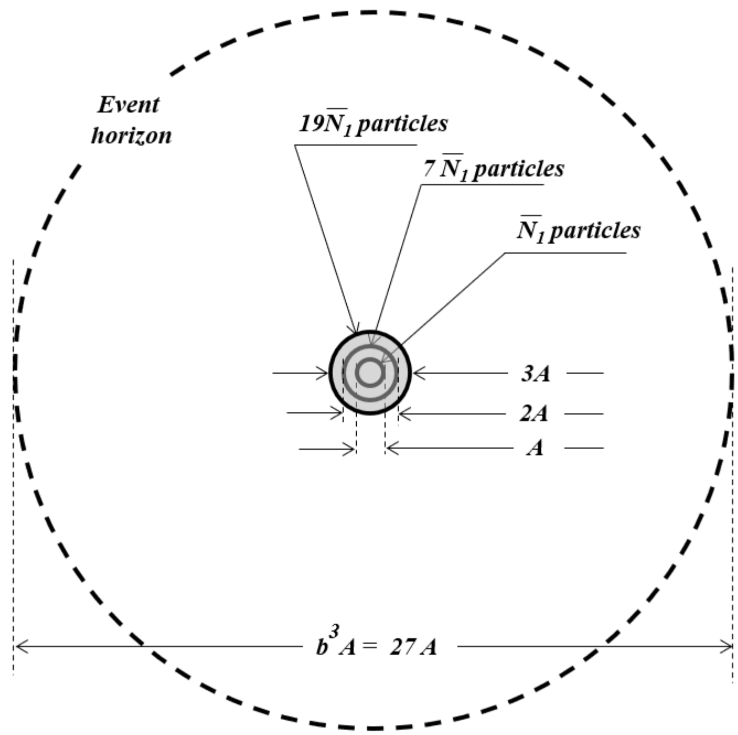

Figure 1: Event horizon and core of a b=3 black hole with the number of PSPs on each shell

singularities to be the constant:

$$\rho_s = \frac{3c^2}{2\pi G A^2} \tag{9}$$

2 Worldline of an accelerating particle

The analysis of the rectilinear motion of a particle under a constant acceleration a can be found in many numbers of books and articles, such as [6] and [7]. As shown in Appendix A, however, the *worldline* of such a particle can also be written in terms of the inherent uncertainties $\delta x'$ and $c\delta t'$ *prior* to the acceleration, the limit acceleration a_u and the UBH diameter or the resolution interval A as follows:

$$[\frac{a}{a_u}(x' - x'_1) + c\delta t'_1]^2 - [\frac{a}{a_u}(ct' - ct'_1) + \delta x'_1]^2 = A^2 \tag{10}$$

Eqn 10 is a west-opening branch of a hyperbola and corresponds to the relativistic worldline of a particle under a constant acceleration a in absolute vacuum, under initial conditions $v'(t') = v'_1$ and $x(t') = x'_1$ at time $t' = t'_1$ measured in an arbitrary *inertial frame of reference* (x', ct'). The initial uncertainties in the spacetime coordinates (x'_1, ct'_1) are given by $\delta x'_1$ and $c\delta t'_1$, respectively. First thing to note, is that the appearance of the spacetime uncertainties in the worldline of the accelerated motion is such that by taking $a = 0$, Eqn 10 reduces to $(c\,\delta t')^2 - (\delta x')^2 = A^2$, as obtained in Eqn 2 for the motion under a constant momentum. From Eqn 10, it is also clear that any acceleration $+a$ results in an increase of the inherent uncertainties in the particle's coordinate in spacetime by traversing the uncertainty locus *up towards the asymptote*. A decelerated $-a$ motion, results in a decrease of the inherent uncertainties from their initial values $(\delta x'_1, c\delta t'_1)$ by traversing the uncertainty locus *down towards the ordinate*, ie. $c\delta t'$ axis.

Variation of the phase angle θ can be calculated for an accelerating particle by taking the derivative of Eqn 10 as follows:

$$\tan\theta = \frac{\frac{a}{a_u}x' + c\delta t'_1}{\sqrt{(\frac{a}{a_u}x' + c\delta t'_1)^2 - A^2}} \tag{11}$$

Also, note that again by taking $a = 0$, Eqn 11 reduces to the following:

$$\tan\theta_1 = \frac{c\delta t'_1}{\delta x'_1} \tag{12}$$

which is the slope of worldline of a particle under a constant momentum where $a = 0$. Now, note that since we also have $\tan\theta_1 = c/v'_1$, comparing the latter with Eqn 12 we arrive to the following realization that:

$$v'_1 = \frac{\delta x'_1}{\delta t'_1} \tag{13}$$

This indicates that according to the c-SRQM theory, the *velocity of a particle in absolute vacuum can also be viewed as the ratio of its inherent spatial to temporal uncertainties.* Since $c/v' >= 1$, the instantaneous slope of the worldline of an accelerated particle will always vary between the limits $\pi/4 < \theta < \pi/2$, the upper limit $\pi/2$ corresponds to that of a stationary particle and the lower limit $\pi/4$ to that of massless particles.

3 Rindler horizon and the UBH event horizon

The $\pm$45-degree asymptotes of the hyperbolic worldline of a particle accelerating under local acceleration a in flat spacetime is then given by:

$$ct' = \pm x' + \Delta \tag{14}$$

where the distance Δ of the non-inertial particle from its Rindler horizon is given from Eqn 10 as:

$$\Delta = A\,\frac{a_u}{a} \tag{15}$$

Therefore, as shown in Fig 2, the Rindler past and future horizons always meet at the *far side* of the event horizon of the UBH. The worldline shown in the figure is based on the a_u/a ratio 5000, $a_u = 4.09 \times 10^{19}(\mathrm{m/s^2})$ and the resolution interval $A = 0.0022(\mathrm{m})$ from the work presented in [3]. From Eqn 15, it is then evident that the lower the magnitude of the local acceleration in absolute vacuum, ie. $a \to 0$, then $\Delta \to \infty$ and therefore the condition of the particle resembles to hovering very far away from the UBH horizon. As the acceleration of the particle approaches to the upper limit $a \to a_u$, the distance $\Delta \to A$, reducing the particle distance to the UBH event horizon. When the local acceleration of a particle is $a = a_u$, then $\Delta = A$, ie. as if the particle is hovering at the *near side* of the UBH event horizon.

In the cosmological model presented in [5], the initial condition at the Big Bang corresponds to the condition $a = a_u$, or $\Delta = A$, as shown in Fig 4, which will be briefly discussed later.

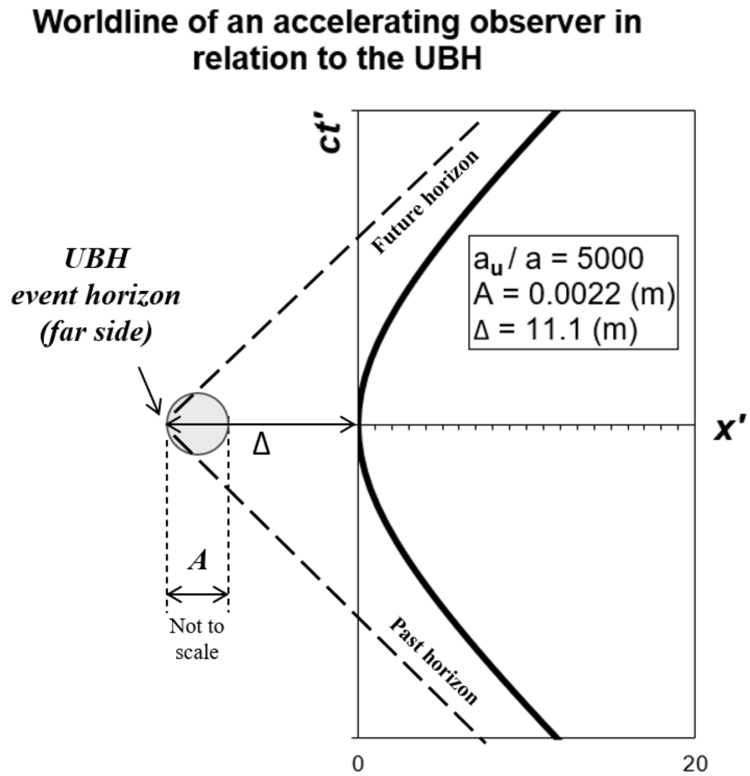

Figure 2: Rindler horizon intersects in relation to the UBH event horizon

4 Hawking-Unruh equivalency

The Hawking temperature of a black hole with horizon gravity g_s is given by:

$$T = \frac{\hbar\, g_s}{2\pi c \kappa} \tag{16}$$

where κ is the Boltzmann constant. For the UBH with mass $M_1 = Ac^2/4G$ and horizon gravity $g_1 = c^4/4GM_1$ the Hawking temperature T_1 of the UBH will be given by:

$$T_1 = \frac{\hbar c}{2\pi A \kappa} \tag{17}$$

Now consider acceleration a of a particle in absolute vacuum such that $a \cong g(R)$ where $g(R)$ corresponds to the local gravitational strength of the UBH at the hovering distance R from it. The Unruh temperature of the vacuum experienced by such an accelerating particle will be given by $U = (\hbar a)/2\pi c\kappa$. Substituting for a by its equivalent $g(R) = GM_1/R^2$, the Unruh temperature of the vacuum will be registered as:

$$U(R) = \frac{\hbar c A}{8\pi \kappa R^2} \tag{18}$$

Comparing Eqn's 17 and 18 we arrive at:

$$U(R) = \frac{A^2}{4R^2}\, T_1 \tag{19}$$

indicating the Unruh temperature $U(R)$ is equal to the UBH Hawking temperature T_1 red shifted by a factor $(A/2R)^2$. Evidently, the distance Δ in the flat spacetime and R in the curved spacetime of the UBH are related through $R = 0.5\sqrt{A\Delta}$. As discussed earlier, at low accelerations, the hovering distance $R \to \infty$ and with that the corresponding vacuum temperature $U \to 0$, ie. T_1 suffers an infinite red shift. At the limiting condition, when the acceleration of the particle reaches to the upper limit $a_u = c^2/A$, *the thermodynamic properties of the vacuum local to the particle is physically equivalent to that of a particle hovering at the UBH event horizon.* For that condition, ie. at the distance $R = \frac{A}{2}$ from the center of the UBH, from Eqn 19 we then have $U(\frac{A}{2}) = T_1$, exactly. Now recall that the number of configurational microstates Ω_1 of the PSP condensate in the UBH singularity is given by $\Omega_1 = e^{2\pi^2 \bar{N}_1}$ and subsequently the resulting horizon entropy by $S_1 = 2\pi^2 \kappa \bar{N}_1$ [4]. This leads us to a profound deduction as follows:

The Hawking-Unruh equivalency suggests that the thermodynamic properties of a black hole horizon and of the absolute vacuum *local to accelerating matter*, must arise from the underlying degrees of freedom that exists in the configurational-microstates of their PSP constituents. Under this interpretation, the black hole singularity, absolute vacuum and the dark matter represent a vastly different number of PSPs that are in the state of *quantum coherence*. The black hole singularity represents the maximum number of PSPs that can ever be in the state of quantum coherence forming a region of space. The vacuum corresponds to the maximally disordered state of free PSPs with no quantum coherence among them. Bounded by those limiting extremes, the dark matter represents a range of intermediate number of PSPs that are in a coherent state, either induced locally by the accelerated motion of ordinary matter, or globally by the gravity.

5 Initial condition at the Big Bang

Since the ordinary matter, upon falling into black holes, return to the PSP form on the singularity, it is not unreasonable to think that *a reverse process* could have happened at the Big Bang where PSPs are freed to form both the vacuum and the physical matter. On that basis, in the cosmological model introduced in [5], the universe was initially assumed to be in the form of a PSP condensate in the core of a super-massive Primordial Mother Black Hole (PMBH). During the Big Bang, the PSP spherical shells of the core undergo a sudden velocity change from the initial condition zero to the speed of light c within one cosmological time step A/c. This represents the upper limit acceleration c^2/A.

As illustrated in Fig 3, consider the spherical shell j of mass dm on a sphere of radius R and mass m, concentric with the PMBH core. For the mass m of the sphere R we can now write:

$$m = \frac{4}{3}\pi R^3 \rho_s \tag{20}$$

where ρ_s, from Eqn 9, is the *mass density distribution* within the PMBH core. The equation of motion of the shell R under the sole influence of gravity is then given by:

$$\ddot{r} + \frac{4}{3}\pi \frac{GR^3}{r^2}\rho_s = 0 \tag{21}$$

where $r(t, R)$ is the radial coordinate of shell R versus cosmic time t any time after the Big Bang. A detailed review of solution to such matter dominated model can be found in [8]. Substituting the *dimensionless scale factor*:

$$a = r/R \tag{22}$$

and its second derivative $\ddot{a} = \ddot{r}/R$ in Eqn 21 we arrive at:

$$\ddot{a} + \frac{4}{3}\pi \frac{G}{a^2}\rho_s = 0 \tag{23}$$

Evidently, the *steady state condition* of the PMBH core prior to the Big Bang is as follows:

$$a(0) = \frac{R}{R} = 1 \tag{24}$$

$$\dot{a}(0) = 0$$

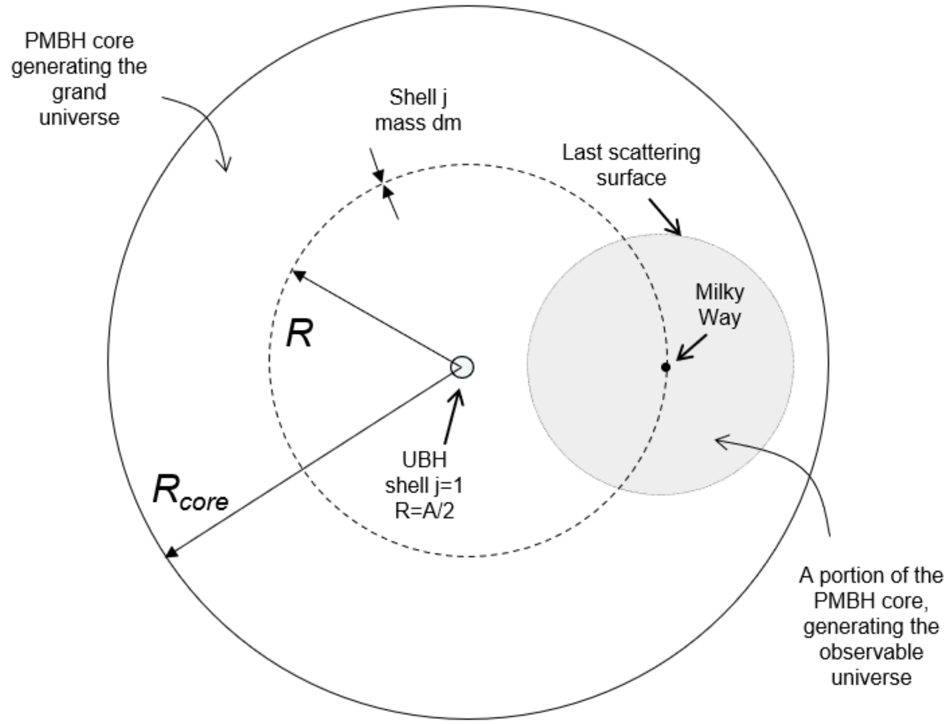

Figure 3: Galaxies originating from a PMBH shell have common comoving coordinates

According to the theory, since the acceleration $a_u = c^2/A$ is *both* an upper limit and the gravitational strength of the UBH, it would be *the only possible initial condition* that the Big Bang can occur in this model, i.e the expansion rate in one hand cannot be higher than a_u, and in the another hand, it cannot be lower than a_u either. Therefore, it can be shown that the initial conditions at the Big Bang is given by:

$$a(0^+) = \frac{R}{R} = 1 \qquad (25)$$
$$\dot{a}(0^+) = \frac{2}{A}c$$

The initial radial coordinate r and velocity $\dot{r}$ of any singularity shell j of radius R can then be obtained from Eqn 25 as follows:

$$r(0^+, R) = R \qquad (26)$$
$$\dot{r}(0^+, R) = \frac{2R}{A}c$$

From the first equation above, it is evident that at time $0^+ = A/c$, i.e at the *end* of the Big Bang, the coordinate R of the singularity shells are still unchanged. From the second equation, however, it is evident that the initial velocity corresponding to the boundary of the UBH, i.e shell $j = 1$ where $R = A/2$, changes from zero to c during the time A/c of the Big Bang. This corresponds to the expansion the innermost shell at the upper limit c^2/A, as discussed earlier. Since the PMBH singularity shells are incompressible, the **rest frame** of the higher singularity shells $j > 1$ gain the initial velocity c due to the expansion of the shell $j = 1$ alone. At the Big Bang, however, not only the UBH but **all** of the PMBH shells expand *relative to their rest frames*; and as a result, a linear compound of the initial velocities of the rest frames occurs from the boundary of the UBH all the way to the outermost shell at radius R_{core}. Hence, from the total initial velocity $\frac{2R}{A}c = jc$, the contribution of shell j relative to its own pre-Big-Bang rest frame is c and the remaining, i.e $(j-1)c$, is due to the expansion of the shells $j-1, j-2, ..., 1$ underneath.

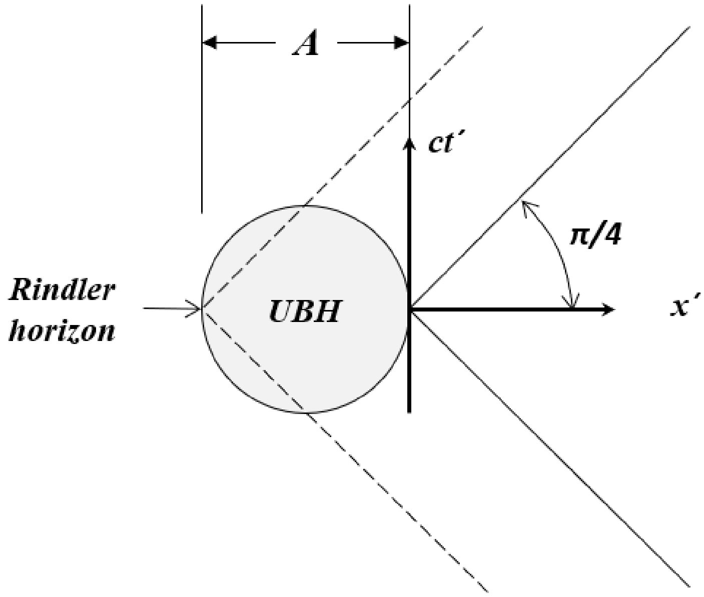

Figure 4: Initial condition at big bang corresponds to the upper limit accel c^2/A [5]

6 Conclusion

It is shown that the past and the future Rindler horizons of any accelerating particle do meet at the far side of the event horizon of the UBH. The Unruh temperature of the vacuum experienced by such an accelerating particle is shown to be identical to the Hawking temperature of the UBH, red-shifted to a hovering distance from it where the local gravity and the particle acceleration are equal. It was previously shown that the entropy of the UBH horizon originates from the number of configurational microstates of its PSP condensate. From the Hawking-Unruh equivalency, therefore, it is concluded that acceleration of ordinary matter must induce a similar quantum coherent state in a number of dark matter PSPs local to the particle. Macroscopic effects of such *dark matter halo*, local to and co-moving with an accelerating particle, are experienced as the Unruh temperature, the force of inertia and the relativistic mass.

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A Appendix A - accelerated motion

Unlike the proper-velocity and the proper-acceleration, the coordinate-velocity $v' = dx'/dt'$ and the coordinate-acceleration $a' = dv'/dt'$ do NOT obey Lorentz transformation between the inertial frames of reference. According to the Special Relativity, the equation of motion of an accelerating particle, as viewed by an arbitrarily chosen inertial observer I', is given by the derivative of its momentum p with respect to the coordinate time t' of the observer as follows:

$$\frac{dp}{dt'} = ma \tag{A.1}$$

where a is the local-acceleration of the particle as discussed earlier. Restricting ourself to a rectilinear motion only (where the acceleration and velocity vectors are co-aligned with the x' axis of the arbitrary inertial frame) we therefore have:

$$\frac{d}{dt'}\left(\frac{mv'}{\sqrt{1-\frac{v'^2}{c^2}}}\right) = ma \tag{A.2}$$

Eliminating rest mass m and taking derivative of v' with respect to t' we arrive at a relation between the local acceleration a and coordinate acceleration a' as follows:

$$a' = \frac{dv'}{dt'} = (1 - \frac{v'^2}{c^2})^{3/2}\, a \tag{A.3}$$

Re-arranging and integrating Eqn A.3 we have:

$$= \int_{v'_1}^{v'} \frac{dv'}{(1-\frac{v'^2}{c^2})^{3/2}} = \int_{t'_1}^{t'} a\, dt' \tag{A.4}$$

Assuming a constant acceleration a and integrating Eqn A.4 with the initial condition $v'(t'_1) = v'_1$:

$$a(t' - t'_1) + z_1 = \frac{v'(t')}{\sqrt{1 - \frac{v'(t')^2}{c^2}}} \tag{A.5}$$

where

$$z_1 = \frac{v'_1}{\sqrt{1 - \frac{{v'_1}^2}{c^2}}} \tag{A.6}$$

Considering the equation of spatial uncertainty $\delta x'$ from Eqn 1 we therefore note that the integration constant z_1 is a function of the particle spatial uncertainty *prior* to acceleration as follows:

$$z_1 = \frac{c}{A}\delta x'_1 \tag{A.7}$$

Solving for $v'(t')$ in Eqn A.5 we arrive at:

$$v'(t') = \frac{a'(t' - t'_1) + z_1}{\sqrt{1 + \frac{1}{c^2}[a(t' - t'_1) + z_1]^2}} \tag{A.8}$$

Further simplifying gives a relation between the velocity and acceleration ratios v'/c and a/a_u:

$$\frac{v'(t')}{c} = \frac{\frac{a}{a_u}(ct' - ct'_1) + \delta x'_1}{\sqrt{A^2 + [\frac{a}{a_u}(ct' - ct'_1) + \delta x'_1]^2}} \tag{A.9}$$

Next, by integrating Eqn A.8 with respect to t' we arrive at the equation of motion of the accelerated particle as viewed in I'. First, we substitute $a(t'-t'_1)+z_1 = w$ and $dt' = dw/a$ in Eqn A.8 to arrive at:

$$dx' = \frac{c}{a}\frac{wdw}{\sqrt{c^2+w^2}} \tag{A.10}$$

Integrating EqnA.10 we arrive at:

$$x' = \frac{c}{a}\sqrt{c^2+[a(t'-t'_1)+z_1]^2} + z_2 \tag{A.11}$$

Note that in a non-relativistic accelerated motion where speed $at' << c$, Eqn A.11 would reduce to the elementary *parabolic* equation $x'(t') = \frac{1}{2}at'^2$, as expected. With the initial condition of $x'(t'_1) = x'_1$ we have the integration constant z_2 as follows:

$$z_2 = x'_1 - \frac{c^2}{a\sqrt{1-\frac{v_1'^2}{c^2}}} \tag{A.12}$$

Again, considering the equation of temporal uncertainty $c\delta t'$ from Eqn 1, we therefore note that integration constant z_2 is a function of the particle temporal uncertainty *prior* to acceleration as follows.

$$z_2 = x'_1 - \frac{a_u}{a}c\delta t'_1 \tag{A.13}$$

Substituting for z_1 and z_2 from Eqns A.7 and A.13 in Eqn A.11 we arrive at:

$$x'(t') = \frac{a_u}{a}\sqrt{A^2+[\frac{a}{a_u}(ct'-ct'_1)+\delta x'_1]^2} + x'_1 - \frac{a_u}{a}c\delta t'_1 \tag{A.14}$$

Multiplying both sides by a/a_u and simplifying, the last equation can be finally re-cast in a hyperbola form as follows:

$$[\frac{a}{a_u}(x'-x'_1)+c\delta t'_1]^2 - [\frac{a}{a_u}(ct'-ct'_1)+\delta x'_1]^2 = A^2 \tag{A.15}$$