Strings with exponential tensions and target space scale invariant Dynamics

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Abstract

The string tensions can be dynamical as one of us (EG) has studied in recent publications. For example, in the case when string theories are formulated in the modified measure formalism where string and brane tensions appear as an additional dynamical degree of freedom. It is found that the tension values assigned to these strings and branes may not fixed (universal), but rather that each string and brane acquires its own tension with a different value for each string and brane. We have displayed cases where the spontaneous breaking of target space scale invariance is produced in the process of string-tension-generation, except for some limits where the tensions dynamically approach infinity. The introduction of internal gauge fields governs the dynamical tensions of the strings, and specific sources for these gauge fields provide diverse tensions. In this work it is shown that in special cases where the tensions have the exponential form $\exp(\alpha\sigma_1)$ or $\exp(\alpha\sigma_0)$, in the conformal gauge, the target space scale invariance can be restored by considering shifts in either the (spatial) σ_1 or (temporal) σ_0 string world-sheet coordinates. In the first case, the (open) strings must be of infinite length and cannot have boundaries in order to respect target space scale invariance. While in the second case we can consider closed or open strings without breaking target space scale invariance. World sheet conformal invariance implies a Ricci flat condition for an effective metric and which is consistent with the expected equations of motion of the strings. Boundary conditions can be introduced by introducing charges that couple to the internal gauge fields. Point like sources break the scale invariance point-wise which can lead to very interesting consequences, like the construction of semi-open strings which are strings which connect to a charge only at one point and where along the other direction the tension decays. These serve as models of deconfined quarks. Generalizations of these effects to brane like solutions in Milne and Wesson spaces, etc. are also discussed.

1 Introduction

String and Brane Theories have been studied as candidates for the theory of all matter and interactions including gravity, in particular string theories [1]. But string theory has a dimensionful parameter, the tension of the string, in its standard formulation, the same is true for brane theories in their better known formulations. Another troublesome aspect of String theory is the fact that it has to be formulated in space times dimensions higher than four, although we should of course end up with a four dimensional space time for the low energy effective theory.

The appearance of a dimensionful string tension and brane theories from the start appears somewhat unnatural. Previously however, in the framework of a Modified Measure Theory, a formalism originally used for gravity theories, see for example [2, 3, 5, 4, 6, 7, 8, 9], the tension was derived as an additional degree of freedom [10, 11, 12, 13, 14, 15, 16]. See also the treatment by Townsend and collaborators [17, 18].

A floating cosmological constant is a generic feature of the modified measure theories of gravity [2, 3, 5, 4, 6, 7, 8, 9], including the covariant formulation of the unimodular theory [19], which is in fact a particular case of a modified measure theory, as reviewed in [20].

The tension of the string plays a very similar role to the cosmological constant in four dimensional gravity, but the analogous situation and the role of the cosmological constant is quite different to that of the string tension, because while several world sheets of strings can exist in the same universe and in this way many strings can probe at the same time the same region of space time,but the same is not the case for the cosmological constant, where every cosmological constant defines necessarily a different universe.

This paper is organized as follows. After this section, the introduction, in Section 2 we review the modified-measure approach in the string context. In Section 3 we review the modified-measure theory in the brane context. The modified-measure theories of strings and branes rely on two basic elements, the modified measure and the existence of internal gauge fields in the strings and branes, the equations of motion of these gauge fields lead to an equation of motion whose integration constant is the Tension of the extended object. In Section 4 we discuss the fact that this tension generation (the integration constant) could take place independently for each world sheet separately, which would mean that the string or brane tension is not a fundamental coupling in nature and it could be different for different strings or branes. In section 5 we discuss possible background fields that can be introduced into the theory for the bosonic case and find that a new field can be introduced that changes the value of the tension of the extended object along its world sheet, we call this the tension scalar . We present first the coupling of gauge fields in the extended objects to currents in the world sheet of the extended object that couple to the gauge fields, as a consequence this coupling induces variations of the tension along the world sheet of the extended object. Then we consider a bulk scalar and how this scalar naturally can induce this world sheet current that couples to the internal gauge fields, the equation of motion of the internal gauge field lead to the remarkably simple equation that the local value of the tension along the string is given by $T = g\phi + T_i$, where g is a coupling constant that defines the coupling of the bulk scalar to the world sheet gauge fields and T_i is an integration constant which can be different for each string in the universe. In section 6 we introduce the target space scale invariance or space time scale symmetry of the theory, which does not exist in the standard string theory , We do this following [35], but in that paper the focus was on the study of strings living in the same region of space time, which lead to interesting phenomena, like braneworlds, avoidance of swampland constraints, etc, but in this paper we focus on the case where all string tensions are equal, leading to strict, unbroken target space scale invariance. Then, in section 7, we study effective theories of gravity and strings as matter. The effective theory is required to inherit the target space scale invariance of the fundamental theory, which then implies that the dimensionality of the effective theory must be four space time dimensions.

In section 8 we study strings with an exponential tension which are associated to Rindler hyperbolic accelerated trajectories of a continuum of point-particles, and display the target-space scale invariance given in terms of the scaling of coordinates. In this section we are considering another approach to the study of dynamical strings. Instead of finding the tension field from the independent consistent quantization of two types of strings with different tensions, it is based in working with a single string type and in determining the functional form of the string tension by demanding global target space scale invariance. Since we select a particular functional form for the tension, this procedure is not well defined unless one states which is the world-sheet coordinate system one is working with. We will be working in the conformal gauge, where the world sheet metric is conformally flat.

Section 9 deals with the same issues as in section 8 and involving unbroken target space scale invariance of strings with exponential tensions, but now using the formalism of modified measures and world sheet currents.

In section 10 we consider tensions of the form $exp(-\alpha\sigma_1)$, and find that we cannot impose open or closed boundary conditions if we insist on strict target space scale invariance, since these would require either periodic boundary conditions for σ_1, or interval (segment) conditions for σ_1, indicating at which value of σ_1 the string starts and at which value of σ_1 the string ends. These boundary conditions will violate target scale invariance. Hence, target space scale invariance would allow only infinite size open strings in the case that the tension is of the form $exp(-\alpha\sigma_1)$. By considering charges in the world sheet (that break the target scale invariance) we find (oriented) strings that can start from a certain point and whose tension exponentially decrease as $\sigma_1 \to \infty$, resulting in a model of an unconfined quark. Other models for bound states of quark-anti quarks or glue balls are considered.

In section 11, we find that when a string tension is dependent on the temporal world-sheet coordinate σ_0, there are no obstacles in implementing target space scale invariance if one considers either open or closed strings. By analogy with section 10, the surfaces with $\sigma_0 = constant$ are considered. We focus on cosmological scenarios. To study them it is useful to consider the flat spacetime metric in the Milne representation and consider the transformations of Milne metrics under time inversion ($t \to C/t$). The latter are conformally related to the original Milne Universe where the conformal factor is associated to the variable string tension.

In this section, a very similar exercise is performed in the case of Wesson spaces. These are solutions to the vacuum Einstein field equations similar to the Milne metric, where instead of the temporal coordinate, a spatial coordinate l plays the key role. These are Ricci flat solutions in higher dimensions discovered by Wesson and collaborators. The slices with $l = constant$ represent de Sitter or Schwarzschild-de Sitter spaces. Once again, motivated by what is done in Milne spaces, one considers the inversions under ($l \to C/l$) leading to a space which is conformally related to the original Wesson

Universe. In doing so, we can again relate the expression of the conformal factor to a variable string tension. This is a new way to generate a brane-world scenario with a brane-world cosmological constant, although the higher dimensional gravitational field is flat!

In section 12 we find that for strings with tensions proportional to $exp(\alpha\sigma_1)$, a reflection in $\sigma_1 : \sigma_1 \to -\sigma_1$, leads to an inversion of the tension of the form $T \to T_0^2/T$, where T_0 is the value of the tension associated with $\sigma_1 = 0$. A similar transformation for the charges that generate these tensions is found and which resembles the inverse (EM duality) relation between electric and magnetic charges.

For completeness purposes, in section 13 we review how target space scale invariance in the actions for the Nambu-Goto String and Gravity can be achieved in $D = 4$. Finally, in section 14 we study Dp branes with modified measures and how their tension can be generated dynamically. Then finally in section 15, we present some concluding remarks are discussed pertaining to new avenues of research in the future.

2 The Modified Measure Theory String Theory

The standard world sheet string sigma-model action using a world sheet metric is [21], [22], [23]

$$S_{sigma-model} = -T \int d^2\sigma \frac{1}{2}\sqrt{-\gamma}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu g_{\mu\nu}. \tag{1}$$

Here γ^{ab} is the intrinsic Riemannian metric on the 2-dimensional string worldsheet and $\gamma = det(\gamma_{ab})$; $g_{\mu\nu}$ denotes the Riemannian metric on the embedding spacetime. T is a string tension, a dimension full scale introduced into the theory by hand.

From the variations of the action with respect to γ^{ab} and X^μ we get the following equations of motion:

$$T_{ab} = (\partial_a X^\mu \partial_b X^\nu - \frac{1}{2}\gamma_{ab}\gamma^{cd}\partial_c X^\mu \partial_d X^\nu) g_{\mu\nu} = 0, \tag{2}$$

$$\frac{1}{\sqrt{-\gamma}}\partial_a(\sqrt{-\gamma}\gamma^{ab}\partial_b X^\mu) + \gamma^{ab}\partial_a X^\nu \partial_b X^\lambda \Gamma^\mu_{\nu\lambda} = 0, \tag{3}$$

where $\Gamma^\mu_{\nu\lambda}$ is the affine connection for the external metric.

There are no limitations on employing any other measure of integration different than $\sqrt{-\gamma}$. The only restriction is that it must be a density under arbitrary diffeomorphisms (reparametrizations) on the underlying spacetime manifold. The modified-measure theory is an example of such a theory.

In the framework of this theory two additional worldsheet scalar fields $\varphi^i (i = 1, 2)$ are introduced. A new measure density is

$$\Phi(\varphi) = \frac{1}{2}\epsilon_{ij}\epsilon^{ab}\partial_a\varphi^i\partial_b\varphi^j. \tag{4}$$

Then the modified bosonic string action is (as formulated first in [10] and latter discussed and generalized also in [11])

$$S=-\int d^2\sigma\Phi(\varphi)(\frac{1}{2}\gamma^{ab}\partial_a X^\mu\partial_b X^\nu g_{\mu\nu}-\frac{\epsilon^{ab}}{2\sqrt{-\gamma}}F_{ab}(A)), \tag{5}$$

where F_{ab} is the field-strength of an auxiliary Abelian gauge field A_a: $F_{ab}=\partial_a A_b-\partial_b A_a$.

It is important to notice that the action (33) is invariant under conformal transformations of the intrinsic measure combined with a diffeomorphism of the measure fields,

$$\gamma_{ab}\rightarrow J\gamma_{ab}, \tag{6}$$

$$\varphi^i\rightarrow\varphi^{'i}=\varphi^{'i}(\varphi^i) \tag{7}$$

such that

$$\Phi\rightarrow\Phi^{'}=J\Phi \tag{8}$$

Here J is the jacobian of the diffeomorphim in the internal measure fields which can be an arbitrary function of the world sheet space time coordinates, so this can called indeed a local conformal symmetry.

To check that the new action is consistent with the sigma-model one, let us derive the equations of motion of the action (33).

The variation with respect to φ^i leads to the following equations of motion:

$$\epsilon^{ab}\partial_b\varphi^i\partial_a(\gamma^{cd}\partial_c X^\mu\partial_d X^\nu g_{\mu\nu}-\frac{\epsilon^{cd}}{\sqrt{-\gamma}}F_{cd})=0. \tag{9}$$

It implies

$$\gamma^{cd}\partial_c X^\mu\partial_d X^\nu g_{\mu\nu}-\frac{\epsilon^{cd}}{\sqrt{-\gamma}}F_{cd}=M=const. \tag{10}$$

The equations of motion with respect to γ^{ab} are

$$T_{ab}=\partial_a X^\mu\partial_b X^\nu g_{\mu\nu}-\frac{1}{2}\gamma_{ab}\frac{\epsilon^{cd}}{\sqrt{-\gamma}}F_{cd}=0. \tag{11}$$

We see that these equations are the same as in the sigma-model formulation (2), (3). Namely, taking the trace of (38) we get that $M=0$. By solving $\frac{\epsilon^{cd}}{\sqrt{-\gamma}}F_{cd}$ from (24) (with $M=0$) we obtain (2).

A most significant result is obtained by varying the action with respect to A_a:

$$\epsilon^{ab}\partial_b(\frac{\Phi(\varphi)}{\sqrt{-\gamma}})=0. \tag{12}$$

Then by integrating and comparing it with the standard action it is seen that

$$\frac{\Phi(\varphi)}{\sqrt{-\gamma}}=T. \tag{13}$$

That is how the string tension T is derived as a world sheet constant of integration opposite to the standard equation (1) where the tension is put ad hoc.The variation with respect to X^{μ} leads to the second sigma-model-type equation (3). The idea of modifying the measure of integration proved itself effective and profitable. This can be generalized to incorporate super symmetry, see for example [11], [13], [12] , [14]. For other mechanisms for dynamical string tension generation from added string world sheet fields, see for example [17] and [18]. However the fact that this string tension generation is a world sheet effect and not a universal uniform string tension generation effect for all strings has not been sufficiently emphasized before. Now we go and review the Modified Measure Brane Theory

3 The Modified Measure Brane Theory

The standard world sheet string sigma-model action using a world sheet metric is [24]:

$$S_{sigma-model} = -T \int d^d\sigma \frac{1}{2}\sqrt{-\gamma}(\gamma^{ab}\partial_a X^{\mu}\partial_b X^{\nu} g_{\mu\nu} + 2\Lambda). \tag{14}$$

Notice that now a cosmological term has been added as well, that was not needed in the usual formulation of the string theory or in the modified measure formulation of the string theory. In the modified measure formulation of the brane theory such term will not be required. As it is well known, and as we will review, in the standard formulation , this cosmological term needs to be fine tuned. Here again γ^{ab} is the intrinsic Riemannian metric on the d-dimensional brane worldsheet and $\gamma = det(\gamma_{ab})$; $g_{\mu\nu}$ denotes the Riemannian metric on the embedding spacetime. T is a brane tension, a dimension full scale introduced into the theory by hand.

From the variations of the action with respect to γ^{ab} and X^{μ} we get the following equations of motion:

$$T_{ab} = (\partial_a X^{\mu}\partial_b X^{\nu} - \frac{1}{2}\gamma_{ab}\gamma^{cd}\partial_c X^{\mu}\partial_d X^{\nu})g_{\mu\nu} - \gamma_{ab}\Lambda = 0, \tag{15}$$

$$\frac{1}{\sqrt{-\gamma}}\partial_a(\sqrt{-\gamma}\gamma^{ab}\partial_b X^{\mu}) + \gamma^{ab}\partial_a X^{\nu}\partial_b X^{\lambda}\Gamma^{\mu}_{\nu\lambda} = 0, \tag{16}$$

where $\Gamma^{\mu}_{\nu\lambda}$ is the affine connection for the external metric.

From (15) we see that without the cosmological constant term, or if this term is not fine tuned, the equations of motion are inconsistent upon considering the trace for example. This feature is absent in the modified measure brame theory, where an integration constant, that takes the role of the cosmological term is determined dynamically by the consistency of the equations of motion.

To show this, again , as in the string case, we consider other measure of integration different than $\sqrt{-\gamma}$. The only restriction is that it must be a density under arbitrary diffeomorphisms (reparametrizations) on the underlying spacetime manifold, as we have seen before while reviewing the modified measure string theory, a metric independent measure build along the lines of what we did in the previous section is an example for that

In the framework of a d dimensional brane d additional worldsheet scalar fields $\varphi^i (i = 1, 2, ..., d)$ are introduced. The new measure density is defined now as

$$\Phi(\varphi) = \epsilon_{ijk...m}\epsilon^{abc....d}\partial_a\varphi^i\partial_b\varphi^j......\partial_d\varphi^m \tag{17}$$

Then the modified bosonic brane action is

$$S = -\int d^d\sigma\Phi(\varphi)(\frac{1}{2}\gamma^{ab}\partial_a X^\mu\partial_b X^\nu g_{\mu\nu} - \frac{\epsilon^{abcd...}}{2\sqrt{-\gamma}}F_{abcd...}(A)), \tag{18}$$

where $F_{abcd...}$ is the rank d totally antisymmetric field-strength derived from an auxiliary abelian gauge field $A_{acd...}$ of rank $d-1$: $F_{ab....cd} = \partial_a A_{bcd..} + ...$, the totally antisymmetrized curl of a $d-1$ potential $A_{bcd......}$

In the brane modified theory there is a scale invariance, although it is only a global one, as opposed to the string case, where it is local,

$$\gamma_{ab} \to J\gamma_{ab}, \tag{19}$$

$$\varphi^i \to \varphi'^i = \lambda^{ij}(\varphi^j) \tag{20}$$

such that

$$\Phi \to \Phi' = J\Phi \tag{21}$$

That is if $det(\lambda^{ij}) = J$, where J, λ^{ij} are just a constant number and a constant matrix now, and finally the antisymmetric tensor field $A_{bcd......}$ must transform as

$$A_{bcd......} \to J^{\frac{d-2}{2}} A_{bcd......} \tag{22}$$

To check that the new action is consistent with the sigma-model one, let us derive the equations of motion of the action (14). To start, the variation with respect to φ^i leads to the following equations of motion:

$$K^a_b\partial_a(\gamma^{cd}\partial_c X^\mu\partial_d X^\nu g_{\mu\nu} - \frac{\epsilon^{cdef...}}{\sqrt{-\gamma}}F_{cdef...}) = 0. \tag{23}$$

Where the matrix K^a_b is just like the measure, except that one factor of a derivative of a measure field is missing, so that there are two free indices therefore the determinant of K^a_b is a power of the measure, so if the measure is non vanishing, this implies

$$\gamma^{cd}\partial_c X^\mu\partial_d X^\nu g_{\mu\nu} - \frac{\epsilon^{cdef...}}{\sqrt{-\gamma}}F_{cdef...} = M = const. \tag{24}$$

The equations of motion with respect to γ^{ab} are

$$T_{ab} = \partial_a X^\mu\partial_b X^\nu g_{\mu\nu} - \frac{1}{2}\gamma_{ab}\frac{\epsilon^{cdef...}}{\sqrt{-\gamma}}F_{cdef...} = 0. \tag{25}$$

We will see now that the final equations equations are the same as in the sigma-model formulation (2), (3). Namely, taking the trace of (25), using also (24), and then reinserting into (25), we get that $M \neq 0$ now. By solving $\frac{\epsilon^{cdef..}}{\sqrt{-\gamma}}F_{cdef...}$ from (24) we obtain a relation between the intrinsic metric and the induced metric if $p \neq 1$, (where p is the number of spacelike world sheet coordinates ($p = d-1$)) that if we do not deal with the string case ($p = 1$),

$$\gamma_{ab} = \frac{1-p}{M} \partial_a X^{\mu} \partial_b X^{\nu} g_{\mu\nu} \tag{26}$$

Using the global scale symmetry of the modified measure brane theory we can set $\frac{1-p}{M} = 1$ if M is negative if we wish, for $p \neq 1$. That is, the intrinsic metric in the world sheet becomes equal to the induced metric from the embedding metric, while in the string case there is only a conformal equivalence between the intrinsic metric in the world sheet and the induced metric. varying the action with respect to $A_{abc...}$:

$$\epsilon^{abc..d} \partial_d (\frac{\Phi(\varphi)}{\sqrt{-\gamma}}) = 0. \tag{27}$$

Then by integrating and comparing it with the standard action it is seen that

$$\frac{\Phi(\varphi)}{\sqrt{-\gamma}} = T. \tag{28}$$

That is how the brane tension T is derived as a world sheet constant of integration, opposite to the standard equation (1) where the tension is put in from the start. One can see indeed that the string energy excitations will be proportional to T in exactly the same way as it is the case in the standard case. The variation with respect to X^{μ} leads to the second sigma-model-type equation. The idea of modifying the measure of integration proved itself effective and profitable.

If we look at a single string, the dynamical string tension theories and the standard string theories appear indeed indistinguishable, there are however more than one string and/or one brane in the universe then, let us now observe indeed that it does not appear that the string tension or the brane tension derived in the sections above correspond to ¨the¨ string or brane tensions of the theory. The derivation of the string or brane tensions in the previous sections holds for a given string or brane, there is no obstacle that for another string or brane these could acquire a different string or brane tension. In other words, the string or brane tension is a world sheet constant, but it does not appear to be a universal constant same for all strings and for all branes. Similar situation takes place in the dynamical string generation proposed by Townsend for example [17], in that paper worldsheet fields include an electromagnetic gauge potential. Its equations of motion are those of the Green-Schwarz superstring but with the string tension given by the circulation of the worldsheet electric field around the string. So again ,in [17] also a string will determine a given tension, but another string may determine another tension. If the tension is a universal constant valid for all strings, that would require an explanation in the context of these dynamical tension string theories, for example some kind of interactions that tend to equalize string tensions, or that all strings in the universe originated from the splittings of one primordial string or some other mechanism.

In any case, if one believes for example in strings , on the light of the dynamical string tension mechanism being a process that takes place at each string independently, we must ask whether all strings have the same string tension.

4 Equations for the Background fields and a new background field

As discussed by Polchinski for example in [26] , gravity can be introduced in two different ways in string theory. One way is by recognizing the graviton as one of the fundamental excitations of the string, the other is by considering the effective action of the embedding metric, by integrating out the string degrees of freedom and then the embedding metric and other originally external fields acquire dynamics which is enforced by the requirement of a zero beta function. These equations fortunately appear to be string tension independent for the critical dimension $D = 26$ in the bosonic string for example, so they will not be changed by introducing different strings with different string tensions, if these tensions are constant along the the world sheet .

However, in addition to the traditional background fields usually considered in conventional string theory, one may consider as well an additional scalar field that induces currents in the string world sheet and since the current couples to the world sheet gauge fields, this produces a dynamical tension controlled by the external scalar field as shown at the classical level in [25]. In the next two subsections we will study how this comes about in two steps, first we introduce world sheet currents that couple to the internal gauge fields in Strings and Branes and second we define a coupling to an external scalar field by defining a world sheet currents that couple to the internal gauge fields in Strings and Branes that is induced by such external scalar field. This is very much in accordance to the philosophy of Schwinger [27] that proposed long time ago that a field theory must be understood by probing it with external sources.

As we will see however, there will be a fundamental difference between this background field and the more conventional ones (the metric, the dilaton field and the two index anti symmetric tensor field) which are identified with some string excitations as well. Instead, here we will see that a single string does not provide dynamics for this field, but rather when the condition for world sheet conformal invariance is implemented for two strings which sample the same region of space time, so it represents a collective effect instead.

4.1 Introducing world sheet currents that couple to the internal gauge fields in Strings

If to the action of the brane (18) we add a coupling to a world-sheet current $j^{a_2 \cdots a_{p+1}}$, $p = d - 1$ (the case $p = 1$ represents a string) i.e. a term

$$S_{\text{current}} = \int d^{p+1}\sigma A_{a_2 \ldots a_{p+1}} j^{a_2 \ldots a_{p+1}}, \tag{29}$$

see [], [28], [29] , [30], [31] for different applications of this. Then the variation of the total action with respect to $A_{a_2 \ldots a_{p+1}}$ gives

$$\epsilon^{a_1 \ldots a_{p+1}} \partial_{a_1} \left(\frac{\Phi}{\sqrt{-\gamma}} \right) = j^{a_2 \ldots a_{p+1}}. \tag{30}$$

We thus see indeed that, in this case, the dynamical character of the brane is crucial here.

4.2 Coupling to a bulk scalar field, the tension field and resolution of the internal gauge field dynamics equations determining the string tension

Suppose that we have an external scalar field $\phi(x^\mu)$ defined in the bulk. From this field we can define the induced conserved world-sheet current

$$j^{a_2 \ldots a_{p+1}} = e \partial_\mu \phi \frac{\partial X^\mu}{\partial \sigma^a} \epsilon^{a a_2 \ldots a_{p+1}} \equiv e \partial_a \phi \epsilon^{a a_2 \ldots a_{p+1}}, \tag{31}$$

Notice that for the string case, i.e, for $p = 1$ which is the most interesting, the current $j^{a_2 \ldots a_{p+1}}$ has only one index and the current is j^{a_2} where e is some coupling constant.

Then (30) can be integrated to obtain

$$T = \frac{\Phi}{\sqrt{-\gamma}} = e\phi + T_i, \tag{32}$$

The constant of integration T_i may vary from one string to the other.

Notice that dynamics of the string tension is determined by the internal gauge field, from the canonical point of view and the tension is not directly a dynamical field, but is rather the canonically conjugate variable of the A^1 field. Exactly analogous to an electric like field (in this case a world sheet gauge field) .

5 For the string case there is a lagrange multiplier measure formulation

Things are special for strings, for example for branes that are not strings ($p \neq 1$ the world sheet metric can be solved by eq. (26), but if $p = 1$, eq. (26) implies that the world sheet metric is proportional to 0/0, because for the string case we found that the Lagrangian $L = M = 0$. When the case of the string is consider by itself of course eq. (26) cannot be applied directly, instead, we find the the world sheet metric is determined up to an arbitrary conformal factor. This special value of $M = 0$ is also related to simpler formulation that gives equivalent results but without the need to consider additional degrees of freedom to define the measure Φ , which now can be simply considering Φ as an independent density, that is, a Lagrange multiplier in fact. Then the modified bosonic string action (before we introduce the tension field) is

$$S = -\int d^2\sigma \Phi (\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu g_{\mu\nu} - \frac{\epsilon^{ab}}{2\sqrt{-\gamma}} F_{ab}(A)), \tag{33}$$

where $F_{ab}(A)$ is the field-strength of an auxiliary Abelian gauge field A_a : $F_{ab} =_a A_{bb} A_a$, or in terms of the tension T,

$$S = -\int d^2\sigma \sqrt{-\gamma} T (\frac{1}{2}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu g_{\mu\nu} - \frac{\epsilon^{ab}}{2\sqrt{-\gamma}} F_{ab}(A)), \tag{34}$$

It is important to notice that the action (33) is invariant under conformal transformations of the intrinsic measure combined with a diffeomorphism of the measure fields,

$$\gamma_{ab} \rightarrow J\gamma_{ab}, \tag{35}$$

and

$$\Phi \rightarrow \Phi' = J\Phi \tag{36}$$

To check that the new action is consistent with the sigma-model one, let us derive the equations of motion of the action (33).

The variation with respect to Φ in (33) or T in (34) leads to the following equation :

$$\gamma^{cd}\partial_c X^\mu \partial_d X^\nu g_{\mu\nu} - \frac{\epsilon^{cd}}{\sqrt{-\gamma}} F_{cd} = 0. \tag{37}$$

The equations of motion with respect to γ^{ab} are the same, either using (33) or (34) supplemented with (37), if $T \neq 0$,

$$T_{ab} = \partial_a X^\mu \partial_b X^\nu g_{\mu\nu} - \frac{1}{2}\gamma_{ab}\frac{\epsilon^{cd}}{\sqrt{-\gamma}} F_{cd} = 0. \tag{38}$$

We see that these equations are the same as in the sigma-model formulation (2), (3). By solving $\frac{\epsilon^{cd}}{\sqrt{-\gamma}} F_{cd}$ from (37) we obtain (2).

A most significant result is obtained by varying the action with respect to A_a:

$$\epsilon^{ab}\partial_b(\frac{\Phi(\varphi)}{\sqrt{-\gamma}}) = 0. \tag{39}$$

Then by integrating and comparing it with the standard action it is seen that

$$\frac{\Phi(\varphi)}{\sqrt{-\gamma}} = T. \tag{40}$$

The measure as a Lagrange multiplier is equivalent to consider the Tension as a Lagrange multiplier that sets $L = 0$, so independent variations of the Tension are legitimate, but this is the case only for strings, not p-branes with $p > 1$. Only for the string case ($p = 1$) it is consistent to consider the measure as a Lagrange multiplier, and consequently one can consider choosing $M = 0$, because for p-branes ($p > 1$) the value of $M \neq 0$ is required for consistency of the formulation.

6 Target Space Scale Invariance and its SSB in the Dynamical String Tension Theory

Notice that the string theory, has world sheet conformal invariance at the classical level, and this world sheet conformal invariance is requires to be extended to the quantum level.

At the classical level, the ordinary string theory does not have target space scale invariance, which is very much related to the fact that there is a definite scale in the theory, the string tension.

Indeed, in the ordinary string theory, a scale transformation of the background metric

$$g_{\mu\nu} \to \omega g_{\mu\nu}$$

where ω is a constant, is not a symmetry of the Polyakov action, but in the dynamical tension string theory, this transformation is a symmetry provided the world sheet gauge fiends and the measure transforms as

$$A_a \to \omega A_a$$

$$\Phi(\varphi) \to \omega^{-1}\Phi(\varphi)$$

and the tension field transforms in a similar way,

$$\phi \to \omega^{-1}\phi$$

As we have seen, the integration of the equations of motions leads to the spontaneous generation of the string tension and at the same time, the spontaneous generation of the target space global scale invariance, since for this case, (32) is satisfied. or equivalently

$$\Phi = \sqrt{-\gamma}(e\phi + T_i), \tag{41}$$

Notice that the interaction is metric independent. Notice that in the absence of these constants of integration, i.e. , if $T_i = 0$ there is no breaking of the Target space scale invariance, since the measure and the tension field transform in the same way, but the introduction of non zero constants of integration introduces a spontaneous breaking of the Target space scale invarince. The role of the constants of integrations T_i is analogous to the role of the integrations M_i that we discussed in the context of the gravitational theories.

One may interpret (41) as the result of integrating out classically (through integration of equations of motion) or quantum mechanically (by functional integration of the internal gauge field, respecting the boundary condition that characterizes the constant of integration T_i for a given string). Then replacing $\Phi = \sqrt{-\gamma}(e\phi + T_i)$ back into the remaining terms in the action gives a correct effective action for each string. Each string is going to be quantized with each one having a different T_i. The consequences of an independent quantization of many strings with different T_i covering the same region of space time was studied studied in Ref. A similar exercise can be considered for the Target space scale invariance and its spontaneous breaking for the Modified Measure Dynamical Brane Tension Theory.

Our emphasis in this paper will be on unbroken Target space time scale symmetry which is achieved for the case where all the constants of integration Ts are zero, or if they are equal, since if they are all equal they can be also eliminated by a shift of the Tension field.

7 Determination of the Dynamical tensions in the string case, a review of previous work and new way forward for scale invariant realizations

Among the extended objects, the case of the strings is of course different, because the strings can be quantized and the consistency conditions imply the vanishing of the beta function. If we have a scalar field coupled to a string or a brane in the way described in the sub section above, i.e. through the current induced by the scalar field in the extended object, according to eq. (41), so we have two sources for the variability of the tension when going from one string to the other: one is the integration constant T_i which varies from string to string and the other the local value of the scalar field, which produces also variations of the tension even within the string or brane world sheet.

As we discussed in the previous section, we can incorporate the result of the tension as a function of scalar field ϕ, given as $e\phi + T_i$, for a string with the constant of integration T_i by defining the action that produces the correct equations of motion for such string, adding also other background fields, the anti symmetric two index field $A_{\mu\nu}$ that couples to $\epsilon^{ab}\partial_a X^{\mu}\partial_b X^{\nu}$ and the dilaton field φ that couples to the topological density $\sqrt{-\gamma}R$

$$S_i = -\int d^2\sigma(e\phi+T_i)\frac{1}{2}\sqrt{-\gamma}\gamma^{ab}\partial_a X^{\mu}\partial_b X^{\nu}g_{\mu\nu}+\int d^2\sigma A_{\mu\nu}\epsilon^{ab}\partial_a X^{\mu}\partial_b X^{\nu}+\int d^2\sigma\sqrt{-\gamma}\varphi R. \tag{42}$$

Notice that if we had just one string, or if all strings will have the same constant of integration $T_i = T_0$.

In any case, it is not our purpose here to do a full generic analysis of all possible background metrics, antisymmetric two index tensor field and dilaton fields, instead, we will take cases where the dilaton field is a constant or zero, and the antisymmetric two index tensor field is pure gauge or zero, then the demand of conformal invariance for $D = 26$ becomes the demand that In any case, it is not our purpose here to do a full generic analysis of all possible background metrics, antisymmetric two index tensor field and dilaton fields, instead, we will take cases where the dilaton field is a constant or zero, and the antisymmetric two index tensor field is pure gauge or zero, then the demand of conformal invariance for $D = 26$ becomes the demand that

will satisfy simultaneously the vacuum Einstein´s equations,

7.1 The case where all all string tensions are the same, i.e., $T_i = T_0$, and the appearance of a target space conformal invariance

If all $T_i = T_0$, we just redefine our background field so that $e\phi + T_0 \to e\phi$ and then in the effective action for all the strings the same combination $e\phi g_{\mu\nu}$, and only this combination will be determined by the requirement that the conformal invariance in the world sheet of all strings be preserved quantum mechanically, that is , that the beta function be zero. So in this case we will not be able to determine $e\phi$ and $g_{\mu\nu}$ separately, just the product $e\phi g_{\mu\nu}$, so the equation obtained from equating the beta function to zero will have the target space conformal invariance $e\phi \to F(x)e\phi$, $g_{\mu\nu} \to F(x)^{-1}g_{\mu\nu}$.

That is, there is no independent dynamics for the Tension Field in this case. So in conclusion, if we just look at one string, or if we look at a set of strings, all of them equal string tensions, then the tension field is not observable, can be gauged away by the target space conformal invariance explained above. Another way to see this is to realize that there is no string excitation of a single string that corresponds to the tension field, unlike the metric, the dilaton or the two index antisymmetric potential. On the other hand, if there are at least two types of string tensions, that symmetry will not exist and there is the possibility of determining separately $e\phi$ and $g_{\mu\nu}$ (setting the dilaton and the two index antisymmetric potential to be trivial).

So the tension field dynamics corresponds to a non trivial collective dynamics of strings that cannot be manifest for the equal string tensions case or the single string case. Of course collective effects are very familiar in physics, for example the statistics of fermions and bosons can be understood only when we consider many particles. Here we will see in the next subsection how the Tension field gets dynamics in the case of two strings with different tensions, [28], [29] , [30], [31].

7.2 The case where not all string tensions are the same, with special emphasis of two types of strings with $T_1 \neq T_2$,

The interesting case to consider is therefore many strings with different T_i, let us consider the simplest case of two strings, labeled 1 and 2 with $T_1 \neq T_2$, then we will have two Einstein´s equations, for $g^{(1)}_{\mu\nu} = (e\phi + T_1)g_{\mu\nu}$ and for $g^{(2)}_{\mu\nu} = (e\phi + T_2)g_{\mu\nu}$,

$$R_{\mu\nu}(g^{(1)}_{\alpha\beta}) = 0 \tag{43}$$

and , at the same time,

$$R_{\mu\nu}(g^{(2)}_{\alpha\beta}) = 0 \tag{44}$$

These two simultaneous conditions above impose a constraint on the tension field ϕ, because the metrics $g^{(1)}_{\alpha\beta}$ and $g(2)_{\alpha\beta}$ are conformally related, but Einstein´s equations are not conformally invariant, so the condition that Einstein´s equations hold for both $g^{(1)}_{\alpha\beta}$ and $g^{(2)}_{\alpha\beta}$ is highly non trivial. Suppose this can be achieved. Then for these situations, we have,

$$e\phi + T_1 = \Omega^2(e\phi + T_2) \tag{45}$$

which leads to a solution for $e\phi$

$$e\phi = \frac{\Omega^2 T_2 - T_1}{1 - \Omega^2} \tag{46}$$

which leads to the tensions of the different strings to be

$$e\phi + T_1 = \frac{\Omega^2(T_2 - T_1)}{1 - \Omega^2} \tag{47}$$

and

$$e\phi + T_2 = \frac{(T_2 - T_1)}{1 - \Omega^2} \tag{48}$$

and we thus determined the tension field. We just have to determined the two space times and the conformal transformation that connects them, like when

$$ds_1^2 = \eta_{\alpha\beta}dx^\alpha dx^\beta \tag{49}$$

where $\eta_{\alpha\beta}$ is the standard Minkowski metric, with $\eta_{00} = 1$, $\eta_{0i} = 0$ and $\eta_{ij} = -\delta_{ij}$. This is of course a solution of the vacuum Einstein´s equations.

We now consider the conformally transformed metric, [30], [31],

$$ds_2^2 = \Omega(x)^2\eta_{\alpha\beta}dx^\alpha dx^\beta \tag{50}$$

where conformal factor coincides with that obtained from the special conformal transformation. In this case, it is found that there are two surfaces where the string tensions can go to infinity, and the strings are confined between these two surfaces, giving rise to a braneworld scenario.

In particular, strings are pushed preferentially to one of these surfaces which has an induced metric which is de Sitter [36], thus providing an explicit construction of de Sitter space in dynamical tension string theory, something that the swampland conjectures do not allow for the standard string theories.

Strings with different tensions could provide a mechanism for explaining dark matter [37], since string interactions , like spliting or joining of strings are only defined for string with the same tension. Also [38] these Strings with a different tension are argued to be producing with a pattern of giving rise to dark copies of the Standard Model.

8 Exponential Tension leading to Rindler solutions and target space scale invariance in terms of scaling of coordinates

In this section we will consider another approach to dynamical strings, which instead of finding the tension field from the independent consistent quantization of two types of strings with different tensions, one works with a single string type and determines the functional form of the string tension by imposing global target space scale invariance.

Since we set a particular functional form for the tension, this procedure is not well defined unless one specifies which world sheet coordinate system one is working with. We shall specify that one is working in the conformal gauge where the world sheet metric γ_{ab} is conformally flat.

8.1 Rindler string

We shall consider now a new interpretation behind the idea of dynamical string tensions which can be described independently of the formalism of the previous sections. And afterwards we shall relate them in order to obtain further generalizations.

Let us begin by pointing out the connection [39] between open strings and the one-parameter family of Rindler hyperbolic trajectories in $D = 1 + 1$ given by

$$t(\eta,\sigma) \ = \ \frac{\exp(g_o\sigma)}{g_o} \ \sinh(g_o\eta), \quad x(\eta,\sigma) \ = \ \frac{\exp(g_o\sigma)}{g_o} \ \cosh(g_o\eta) \tag{51}$$

$$(d\tau)^2 \ = \ (dt)^2 \ - \ (dx)^2 \ = \ \exp(2g_o\sigma) \ \big((d\eta)^2 - (d\sigma)^2\big) \tag{52}$$

Each Rindler hyperbolic trajectory corresponds to setting $\sigma =$ constant in eq-(33) such that the proper time along each one of these trajectories becomes $(d\tau) = \exp(g_o\sigma)d\eta \Rightarrow \tau = \exp(g_o\sigma)\eta$, and one can write

$$g_o \ \eta \ = \ [\exp(-g_o\sigma) \ g_o] \ [\exp(g_o\sigma) \ \eta] \ = \ g(\sigma) \ \tau; \quad g(\sigma) = \exp(-g_o\sigma) \ g_o \tag{53}$$

such that the expressions in (51) can be rewritten as

$$t(\tau,\sigma) \;=\; \frac{1}{g(\sigma)}\;\sinh[g(\sigma)\tau], \quad x(\tau,\sigma) \;=\; \frac{1}{g(\sigma)}\;\cosh[g(\sigma)\tau] \tag{54}$$

The solutions (51) obey the string equations of motion associated with the Polyakov string action in the conformal gauge[1]

$$\frac{\partial^2 t}{\partial \eta^2} \;-\; \frac{\partial^2 t}{\partial \sigma^2} \;=\; 0, \quad \frac{\partial^2 x}{\partial \eta^2} \;-\; \frac{\partial^2 x}{\partial \sigma^2} \;=\; 0 \tag{55}$$

whereas the solutions in (54) do *not*

$$\frac{\partial^2 t}{\partial \tau^2} \;-\; \frac{\partial^2 t}{\partial \sigma^2} \;\neq\; 0, \quad \frac{\partial^2 x}{\partial \tau^2} \;-\; \frac{\partial^2 x}{\partial \sigma^2} \;\neq\; 0, \tag{56}$$

The variables η, σ appearing in (55) are the appropriate temporal and spatial variables associated with the two-dimensional world-sheet with a constant tension, whereas τ in (56) is the proper time affine parameter associated with the world-line of a point mass and whose spatial σ-dependent proper acceleration $g(\sigma)$ obeys the relation

$$\left(\frac{\partial^2 t}{\partial \tau^2}\right)^2 \;-\; \left(\frac{\partial^2 x}{\partial \sigma^2}\right)^2 \;=\; -\; g^2(\sigma), \quad g(\sigma) = g_o\; \exp(-g_o\sigma) \tag{57}$$

The physical interpretation of (57) is the following : Imagine an infinite open string stretching from x_o all the way to $x = \infty$ and comprised of a continuum of point masses m where each one of these point masses is subjected to a σ-dependent proper force given by $F(\sigma) = mg(\sigma) = mg_o \exp(-g_o\sigma)$ endowing each one of these point masses with a hyperbolic trajectory such that the σ-dependent proper acceleration $g(\sigma)$ is $g_o \exp(-g_o\sigma)$. One could rewrite the expression for $F(\sigma)$ as $m(\sigma)g_o = [m_o \exp(-g_o\sigma)]g_o$, and in turn, interpret the proper force at each value of σ as that experienced by a σ-dependent mass distribution of points given by $m(\sigma)$ where each one of these mass points is experiencing the *same* proper acceleration g_o.

Because the physical units of a proper force F are the same as a tension when $c = 1$: $[mc^2/L] = [Tc^2]$ (the tension T is just mass per unit length m/L) one could view this continuum arrangement of accelerated point masses as belonging collectively to an infinite open string with a variable tension $T(\sigma)$ which sweeps a two-dim world-sheet lying inside a region of the Rindler wedge [40]. The view of a string as a collection of point particles has been studied earlier on by [41].

Equating the tension T with the mass density $\frac{dm(\sigma)}{d\sigma}$ gives a variable but negative tension

$$T(\sigma) \;=\; \frac{dm(\sigma)}{d\sigma} \;=\; -m_o g_o \exp(-g_o\sigma) \tag{58}$$

Strings with a dynamical (variable) tension (both in space and time) $T(\eta,\sigma)$ have been extensively studied in the previous sections of this paper based on the modified measure formulation which may appear to differs from the mechanism described in this section, but as we will see the two approaches can be reconciled, and they turn out to

[1]The Dirichlet boundary conditions $t = x = 0$, and Neumann boundary conditions $\partial_\sigma t = \partial_\sigma x = 0$ are obeyed at $\sigma = -\infty$, but not at $\sigma = \infty$. It should be mentioned that a string with constant tension of infinite length is not physically feasible.

give the unbroken scale invariant realization of the dynamical string tension, necessary for the arguments concerning why $D = 4$ for example-

Some remarks are in order about the issue of a negative tension. The literature on negative-tension strings, branes is extensive. In particular, there are ways how to avoid instabilities. See, for example [45]. Their explicit construction of sensible negative-tension objects such as the orientifolds within string theory hints at how the aforementioned instability is avoidable. Rather, negative-tension branes are arranged to be localized at special points, such as orbifold fixed points or space-time boundaries, and hence do not carry fluctuating dynamical variables causing an instability as the tension T becomes negative-valued. One of us (EG) has provided string and brane actions with a variable tension that may acquire negative values in certain regions of the world sheet (world volume).

We have discussed above how to introduce a variable tension of the form $T(\sigma)$. The next question is how to introduce an additional temporal dependence. Because the string was comprised of a continuum of point masses that are being accelerated, if they emit gravitational radiation, there will be a loss of mass, and in this fashion, one could attain the sought after temporal dependence leading to a (negative) dynamical tension as well $T(\eta, \sigma)$.

We should emphasize that the interpretation of Rindler trajectories as strings with a variable tension is *heuristic* and not derived from a fundamental string action, like the phase-space string action which was introduced by [39]

8.2 Rindler Hyperboloids

We shall discuss now the case of Rindler hyperboloids associated with closed p-branes of spherical topology moving in flat target Minkowski backgrounds of dimensions $D = p + 2$, instead of the family of Rindler hyperbolae associated with a spacetime filling open *string* in $2D$ discussed in the previous subsection. For an early discussion about what follows see [40].

So far we have discussed the open string. One can envision an accelerated closed string as an expanding loop (circle) in the $x - y$ plane whose radius increases in size with a uniform *proper* acceleration. In this case each one of the continuum of point masses comprising the closed string has the same proper acceleration given by $g(\sigma) = g_o \exp(-g_o\sigma)$, where σ parametrizes the family of closed strings (loops). Each initial loop's radius (the initial radius of the closed string parametrized by σ) is $R(\sigma) = \frac{1}{g(\sigma)}$, and such that the world-sheets swept by these radially expanding loops in a $3D$ target Minkowski spacetime background have the shape of a family of hyperboloids whose throat sizes are $R(\sigma) = \exp(g_o\sigma)/g_o$.

The expanding accelerated radial motions of the loops across the target $3D$ spacetime background can be visualized as a family of hyperboloids (of varying throat sizes) which can be described in terms of the Rindler temporal and spatial parameters η, σ, and the additional angular coordinate θ of the circles of radii $R = R(\eta, \sigma) = \frac{1}{g(\sigma)} \cosh(g_o\eta) = \frac{\exp(g_o\sigma)}{g_o} \cosh(g_o\eta)$, as follows

$$x = \frac{\exp(g_o\sigma)}{g_o} \cosh(g_o\eta) \cos(\theta), \quad y = \frac{\exp(g_o\sigma)}{g_o} \cosh(g_o\eta) \sin(\theta) \tag{59}$$

$$t = \frac{\exp(g_o\sigma)}{g_o} \sinh(g_o\eta) \tag{60}$$

such that the algebraic equation

$$t^2 - x^2 - y^2 \;=\; -\frac{1}{g^2(\sigma)} \;=\; -\;\frac{\exp(2g_o\sigma)}{g_o^2} \tag{61}$$

is consistent with the algebraic equation of a one-parameter family of hyperboloids in three dimensions parametrized by σ. The radius of each loop increases in size according to $R = R(\eta,\sigma) = \frac{1}{g(\sigma)}\cosh(g_o\eta)$. The initial size of the radii (throat sizes of the hyperboloids) are obtained by setting $\eta = 0$ giving $R = R(\eta = 0,\sigma) = \frac{1}{g(\sigma)} = \frac{1}{g_o}\exp(g_o\sigma)$.

From (61) one can infer that the global scalings of the target spacetime coordinates $(t,x,y) \to (\lambda t, \lambda x, \lambda y)$ with λ constant, are tantamount to a shift of the σ variable of the form $\sigma \to \sigma + \frac{1}{g_o}\ln\lambda$ such that

$$\lambda\exp(g_o\sigma) \;=\; \exp(g_o\sigma + \ln\lambda) = \exp[g_o(\sigma + \frac{1}{g_o}\ln\lambda)] \tag{62}$$

Namely, given a point P with coordinates (t,x,y) belonging to the hyperboloid parametrized by σ, under global scalings it is mapped to the point $P' = \lambda(t,x,y)$ belonging to the hyperboloid parametrized by $\sigma' = \sigma + \frac{1}{g_o}\ln\lambda$. If one wishes to implement global scale invariance then the variable σ must be unbounded. There is a correspondence of the target space scale invariance for these strings formulated at the level of the target space coordinates with the target space scale invariance formulated by involving the embedding metric instead.

Given (59), (60), the flat target spacetime metric in $3D$ can be rewritten in terms of the world volume coordinates (η,σ,θ) of a spacetime *filling* membrane (the bulk region of the hyperboloid) as

$$(d\tau)^2 \;=\; (dt)^2 - (dx)^2 - (dy)^2 \;=\; e^{2g_o\sigma}\;[\cosh^2(g_o\eta) - \sinh^2(g_o\eta)\cos^2(\theta) -$$

$$\cosh^2(g_o\eta)\sin^2(\theta)](d\eta)^2 \;+\; e^{2g_o\sigma}[\sinh^2(g_o\eta) - \cosh^2(g_o\eta)\cos^2(\theta) -$$

$$\sinh^2(g_o\eta)\sin^2(\theta)](d\sigma)^2 \;-\; \frac{e^{2g_o\sigma}}{g_o^2}\;[\cosh^2(g_o\eta)\sin^2(\theta) + \sinh^2(g_o\eta)\cos^2(\theta)]\;(d\theta)^2 \;+$$

$$\frac{1}{g_o}\;e^{2g_o\sigma}\;\sin(2\theta)\;d\sigma d\theta$$

The above metric is a generalization of the Rindler metric in $2D$. One may note that all the terms of the metric involve the exponential factor $\exp(2g_o\sigma)$ as it should to ensure that the global scaling of the t,x,y coordinates can be re-absorbed as *shifts* in σ as shown above.

To sum up, (61) described in terms of the (x,y,t) target space coordinates, represent a one-parameter family of hyperboloids (of varying throat sizes parametrized by σ) and associated with the world-sheets swept by a one-parameter family of closed strings (parametrized by σ). θ,η are the world-sheet spatial and temporal coordinates of the *closed* strings, respectively, and σ is the parameter which labels each member of the family of closed strings. The throat size of each hyperboloid coincides with the initial radius size of each closed string. As their radius begin to grow the world-sheets swept by each closed string begin to wrap up their corresponding hyperboloids.

Before we had an open string of infinite size and comprised of a continuum of mass points that are being uniformly accelerated with a σ-dependent acceleration $g(\sigma) = g_o \exp(-g_o\sigma)$. Here we have now a continuum of closed strings (loops) whose different radii are increasing in size according to the law $R(\eta, \sigma) = \frac{1}{g(\sigma)} \cosh(g_o\eta) = \frac{1}{g_o} \exp(g_o\sigma)\cosh(g_o\eta)$. This continuum of closed strings foliate/slice the unbounded disc (membrane) into a continuous infinite family of rings. And their temporal evolution foliate/slice the Rindler-like *wedge* region of the $3D$ Minkowski target spacetime background.

The evolution of the origin of coordinates (0, 0, 0) represents a loop of zero size (point) that begins to radially expand with an $infinite$ proper acceleration sweeping the two-dimensional light-cone (horizon) and leading to a string with an $infinite$ tension. The (almost) spacetime filling membrane is an unbounded *disc* (of infinite size) whose evolution spans two three-dim Rindler-like *wedge* regions (left and right wedges) of the $3D$ flat target spacetime background.

One can generalize this construction to closed p-branes whose world volume are $p + 1$-dimensional and that can realized in terms of expanding p-spheres S^p (bubbles) whose radii are accelerating with a uniform proper acceleration according to the law $R = R(\eta, \sigma) = \frac{\exp(g_o\sigma)}{g_o} \cosh(g_o\eta)$. For instance, the accelerating expanding two-sphere S^2 is described by the following embedding functions in a flat $3 + 1$ target Minkowski spacetime background

$$x = \frac{\exp(g_o\sigma)}{g_o} \ \cosh(g_o\eta) \ \sin(\theta) \ \cos(\phi), \quad y = \frac{\exp(g_o\sigma)}{g_o} \ \cosh(g_o\eta) \ \sin(\theta) \ \sin(\phi) \quad (63)$$

$$z = \frac{\exp(g_o\sigma)}{g_o} \ \cosh(g_o\eta) \ \cos(\theta), \quad t = \frac{\exp(g_o\sigma)}{g_o} \ \sinh(g_o\eta) \quad (64)$$

As they expand, the spheres S^2 sweep the three-dimensional family of hyperboloids parametrized by σ and defined by the algebraic equation

$$t^2 - x^2 - y^2 \ - z^2 \ = \ -\frac{1}{g^2(\sigma)} \ = \ - \ \frac{\exp(2g_o\sigma)}{g_o^2} \quad (65)$$

Replacing S^2 for p-spheres S^p (higher dimensional bubbles) lead to a p+1-dimensional family of hyperboloids parametrized by σ and embedded in a flat $D = p$+2-dimensional background defined by the algebraic equation

$$t^2 - x_1^2 - x_2^2 \ - \ldots \ - x_{p+1}^2 \ = \ -\frac{1}{g^2(\sigma)} \ = \ - \ \frac{\exp(2g_o\sigma)}{g_o^2} \quad (66)$$

The family of four-dim de Sitter spaces dS_4 of varying throat sizes $\frac{\exp(g_o\sigma)}{g_o}$ parametrized by σ is associated to the case when $p = 3$ and corresponds to 3-spheres S^3 sweeping 4-dimensional hyperboloids embedded in a flat 5-dimensional Minkowskii background. We will see in the next section how these surfaces reappear in the Unbroken Target Space Scale Invariance with Exponential Tensions in the formalism of modified measures,

9 Unbroken Target Space Scale Invariance with Exponential Tensions in the formalism of modified measures and world sheet currents. Case of one species of scale invariant strings only

9.1 Tension field with exponential dependence on world sheet coordinates from target space scale invariance

In our first examples, in the context of the modified measures, we have explored the situation where two species of strings generate their own string tensions and the tension field is determined through the demand that both strings satisfy the condition that two corresponding string bulk metric satisfy the Ricci Flat condition in 26 space time dimensions.

This raises the question : can we just consider one string species where the Tension field does not need to be determined ?, or the opposite where the tension is assumed to have a form that automatically satisfies the target space scale invariance?. How does this unbroken phase for dynamical string tension works in the formalism of modified measures and world sheet currents?. These questions will allow us to make contact with the construction in the previous section.

In particular, we will find similar hyperboloids to those discussed in the previous section after setting $T_i = 0$ in Eq-(32)

$$T = \frac{\Phi}{\sqrt{-\gamma}} = e\phi = K\exp(g_o\sigma_1) \tag{67}$$

where ϕ is taken as defined by eq. (67), i.e; not a dynamical field, but rather a given function of σ_1 and for this choice, T transforms in a linear form, required for target space scale invariance after a shift of σ, $\sigma_1 \to \sigma_1 + \frac{1}{g_o}\ln\lambda$ such that

$$\lambda\exp(g_o\sigma_1) \;=\; \exp(g_o\sigma_1 + \ln\lambda) = \exp[g_o(\sigma_1 + \frac{1}{g_o}\ln\lambda)]$$

, when the target space metric is also scaled, by $\lambda = 1/\omega$, ω being the scale parameter of the embedding metric. More generically, ϕ can be taken as an exponential function of σ, τ. A similar procedure can be used by considering exponentials of τ.

$$T = \frac{\Phi}{\sqrt{-\gamma}} = e\phi = K\exp(g_o\sigma_0) \tag{68}$$

and the relevant transformation would be $\sigma_0 \to \sigma_0 + \frac{1}{g_0}\ln\lambda$ such that

$$\lambda\exp(g_o\sigma_0) \;=\; \exp(g_o\sigma_0 + \ln\lambda) = \exp[g_o(\sigma_0 + \frac{1}{g_o}\ln\lambda)]$$

, when the target space metric is also scaled, by $\lambda = 1/\omega$, ω being the scale parameter of the embedding metric. To be completely unambigous, we assume these exponential forms in the conformal gauge of the string metric γ

The consideration of exponentials of σ_1 leads to infinite open strings if we want to respect scale invariance, but not closed strings, while the exponential in τ allows closed and open strings and scale invariance since the shift in τ does not affect boundary conditions defined in terms of σ_1

Now the measure of integration can be resolved once the tension T is given, for example if we use (67)

$$\Phi = T\sqrt{-\gamma} = K\exp(g_o\sigma_1)\sqrt{-\gamma}$$

, then introducing the measure into the action , we get,

$$S = -K\int d^2\sigma \exp(g_o\sigma_1)\frac{1}{2}\sqrt{-\gamma}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu g_{\mu\nu} + \int d^2\sigma A_{\mu\nu}\epsilon^{ab}\partial_a X^\mu \partial_b X^\nu + \int d^2\sigma \sqrt{-\gamma}\varphi R. \tag{69}$$

In any case, it is not our purpose in this work to provide a full generic analysis of all possible background metrics, antisymmetric two index tensor field and dilaton fields. Instead, we will consider the cases when the dilaton field is a constant or zero, and the antisymmetric two index tensor field is pure gauge or zero, then the requirement to have conformal invariance on the worldsheet is tantamount to imposing that the effective target spacetime metric $g_{\mu\nu} = K\exp(g_o\sigma_1)\eta_{\mu\nu}$ must satisfy the vacuum Einstein field equations (Ricci flat solutions $R_{\mu\nu} = 0$) as the flat metric $\eta_{\mu\nu}$ does as well. These two Ricci flat spaces are related by a conformal transformation $K\exp(g_o\sigma_1)$.

Notice that the string tension is first introduced as $K\exp(g_o\sigma_1)$ defined in the world sheet of the string, however, beyond this, we consider this string tension field to be extended into the bulk space, since the string can be at any point in target space$X$, and by considering σ_1 to be then a function in target space, then the Ricci flat condition for $g_{\mu\nu} = K\exp(g_o\sigma_1)\eta_{\mu\nu}$. We will see that through the requirement of of Ricci flat condition for a conformally transformed metric we determine the dependence of the tension, which is in fact the conformal between the two metrics . This tension was first determined in the world sheet was determined in the previous section by other arguments independent of the Ricci flat condition for a conformally transformed metric. The flat spacetime in Minkowski coordinates is,

$$ds_1^2 = \eta_{\alpha\beta}dx^\alpha dx^\beta \tag{70}$$

where $\eta_{\alpha\beta}$ is the standard Minkowski metric, with $\eta_{00} = 1$, $\eta_{0i} = 0$ and $\eta_{ij} = -\delta_{ij}$. This is of course a solution of the vacuum Einstein´s equations.

We now consider the conformally transformed metric, [30], [31],

$$ds_2^2 = \Omega(x)^2\eta_{\alpha\beta}dx^\alpha dx^\beta \tag{71}$$

The most general solution for the conformal factor $\Omega^2(x)$ leading to a Ricci flat metric is [42]

$$\Omega^2(x) = \frac{1}{(1 + 2a_\nu x^\nu + a^2x^2)^2} \tag{72}$$

where the conformal factor $\Omega^2(x)$ coincides with the square of the factor $\Omega(x)$ appearing in the denominator of the expression obtained from the special conformal transformations of the spacetime coordinates

$$x\prime^\mu = \frac{(x^\mu + a^\mu x^2)}{(1 + 2a_\nu x^\nu + a^2x^2)} \tag{73}$$

for a certain D vector a_ν. To sum up, we have two solutions for the Einstein´s equations, As noticed by Zumino [43], finite special conformal transformations mix up in a complicated way the topology of space time. For instance we can see that all the points with $x^2 \to +\infty$ or $x^2 \to -\infty$, that is both space like infinity and time like infinity all converge to the single point, $x^\mu \to a^\mu/a^2$. This is similar to a more elementary transformation, that is used in geology for example, the stereographic projection [44] where the north pole is projected to infinity.

The conformal transformation is between the flat space in Minkowski coordinates $g^{(1)}_{\alpha\beta} = \eta_{\alpha\beta}$ and

$$g^{(2)}_{\alpha\beta} = \Omega^2 \eta_{\alpha\beta} = \frac{1}{(1 + 2a_\mu x^\mu + a^2 x^2)^2} \eta_{\alpha\beta} \tag{74}$$

Let us find the algebraic locus of points (surfaces, hypersurfaces) in the target spacetime determined by equating the tension $K \exp(g_o \sigma_1)$ with the conformal factor Ω^2

$$\Omega^2 = \frac{1}{(1 + 2a_\mu x^\mu + a^2 x^2)^2} = K \exp(g_o \sigma_1) \tag{75}$$

Notice that the tension field is defined in the bulk, even in the regions where there are no strings, such that this framework determines a given value for the string tension when the string world-sheet has support on a certain region in the bulk. This allows us to remarkably connect with the results of the previous section. Let us explore several cases. In particular, one can obtain exactly the same hyperboloid whose throat size is proportional to the inverse of the string tension by equating the tension $K \exp(g_o \sigma_1)$ with the conformal factor Ω^2 and demanding that this conformal transformation corresponds to the special conformal transformation as described in eq. (75).

Case **1**

Let us choose the timelike a_μ (acceleration-like) vector in $D = 25 + 1$ spacetime dimensions given by $a_\mu = (A, 0, 0, \ldots, 0)$. Equating Ω^2 with $\frac{T(\sigma_1)}{T_o} = \exp(-4g_o\sigma_1)$ leads to

$$\pm(\frac{T(\sigma_1)}{T_o})^{1/2} \;=\; \pm \exp(-2g_o\sigma_1) \;=\; \Omega(x) \;=\; \frac{1}{1 + 2At + A^2(t^2 - x_1^2 - x_2^2 - \ldots - x_{25}^2)} \tag{76}$$

and yielding

$$\pm\frac{\exp(2g_o\sigma_1)}{A^2} \;=\; (t + \frac{1}{A})^2 - x_1^2 - x_2^2 - \ldots - x_{25}^2 \tag{77}$$

Choosing the $+$ sign in front of the square root, one arrives at case **1a** given by a one parameter family (parametrized by σ_1) of two-sheeted 25-dim hyperboloids embedded in a flat $25 + 1$-dim spacetime background (north/south branches) and whose throat sizes are $\frac{\exp(g_o\sigma_1)}{A}$

$$\frac{\exp(2g_o\sigma_1)}{A^2} \;=\; (t + \frac{1}{A})^2 - x_1^2 - x_2^2 - \ldots - x_{25}^2 \tag{78}$$

Choosing the $-$ sign leads to the case **1b** given by a family of one-sheeted hyperboloids and centered at the point $P = (-\frac{1}{A}, 0, 0, \ldots, 0)$

$$-\frac{\exp(2g_o\sigma_1)}{A^2} \;=\; (t + \frac{1}{A})^2 - x_1^2 - x_2^2 - \ldots - x_{25}^2 \tag{79}$$

As we mentioned earlier, the tension field is assigned to be a bulk field, it is defined in regions where the string lives and in regions where there are no strings.

We can make contact with the findings of the previous section related to the Rindler hyperbolae by taking a 2-dim slice of the one-sheeted hyperboloids by setting $x_2 = x_3 = \ldots = x_{25} = 0$. In doing so, one finds that the trajectory associated with the value σ_1 is,

$$-\frac{1}{A^2}\exp(2g_o\sigma_1) \;=\; (t+\frac{1}{A})^2 - x_1^2 \tag{80}$$

which is the algebraic equation for a family of hyperbolas centered at the point $P = (-\frac{1}{A}, 0)$.

After comparing the above algebraic equation for the family of hyperbolas in 2-dim with the family of Rindler hyperbolas of the previous section

$$-\frac{1}{g_o^2}\;\exp(2g_o\sigma_1) \;=\; (t-t_0)^2 - x_1^2 \tag{81}$$

whose throat sizes are $\frac{\exp(g_o\sigma_1)}{g_o}$, and centered at $(t_0, 0)$, one finds that the value of A required for the throat sizes to coincide must obey the condition

$$\frac{1}{A^2}\exp(2g_o\sigma_1) \;=\; \frac{1}{g_o^2}\;\exp(2g_o\sigma_1)\;\Rightarrow A = g_o \tag{82}$$

Therefore, one finds an exact agreement with the results of the previous section when $A = g_o$, and $t_0 = -\frac{1}{A} = -\frac{1}{g_0}$.

Case **2**

Let us choose the spacelike a_μ vector in $D = 25+1$ spacetime dimensions given by $a_\mu = (0, A, 0, 0, \ldots, 0)$. Repeating the previous procedure, after completing now the square in the variable x_1, and taking the $+$ sign in front of the square root it leads to case **2a**

$$\frac{1}{A^2}\;[\;\exp(2g_o\sigma_1)\;] \;=\; -t^2 + (x_1 - \frac{1}{A})^2 + x_2^2 + \ldots + x_{25}^2 \tag{83}$$

and one arrives now at a one parameter family of one-sheeted 25-dim hyperboloids embedded in a flat $25+1$-dim spacetime background, parametrized by σ_1, centered at the point $P = (0, \frac{1}{A}, 0, 0, \ldots, 0)$, and whose throat sizes are $\frac{\exp(g_o\sigma_1)}{A}$

Taking the $-$ sign in front of the square root gives case **2b**

$$-\frac{1}{A^2}\;[\;\exp(2g_o\sigma_1)\;] \;=\; -t^2 + (x_1 - \frac{1}{A})^2 + x_2^2 + \ldots + x_{25}^2 \tag{84}$$

and one arrives at a one parameter family of two-sheeted 25-dim hyperboloids embedded in a flat $25+1$-dim spacetime background and centered at the point $P = (0, \frac{1}{A}, 0, 0, \ldots, 0)$.

If one wishes now to make contact with the findings of the previous section related to the Rindler hyperbolae

$$-\frac{1}{A^2}\;\exp(2g_o\sigma_1) \;=\; t^2 - (x_1 - \frac{1}{A})^2 \tag{85}$$

centered at $P = (0, \frac{1}{A})$ and with throat size $A^{-1}\exp(g_o\sigma_1)$, after taking a $1+1$-dim slice of the one-sheeted hyperboloids by setting $x_2 = x_3 = \ldots = x_{25} = 0$, and equating the throat sizes,

$$\frac{1}{A}\ \exp(g_o\sigma_1) \ = \ \frac{1}{g_o}\ \exp(g_o\sigma_1) \tag{86}$$

one ends up by having $A \ = \ g_o$

To sum up, out of all these four cases **1a**, **1b**, **2a**, **2b**, the two cases **1b**, **2a** associated with the one-sheeted hyperboloids and leading to $A = g_o$ (constant) are physical and compatible with the results of the previous section

Case **3**

A null-like vector $a_\mu = (A, A, 0, 0, \ldots, 0)$; $a_\mu a^\mu = 0$ leads to a family of null *hyperplanes* in $D = 25+1$-dim, parametrized by σ and whose algebraic equation is

$$t \ = \ x_1 \ - \ \frac{1 \pm \exp(2g_o\sigma_1)}{2A} \tag{87}$$

The two-dim *slice* of these null-hyperplanes yields a family of light-like lines in $D = 2$.

Case **4** (Infinite Tension regions)

In the previous section we found that the vertex of the Rindler coordinate grid is located at $\sigma_1 = -\infty$, and such that the tension blows up, consequently the denominator appearing in the expression for $\Omega(x)$ must be zero. When the a_μ vector is timelike, the zero denominator expression gives

$$(t + \frac{1}{A})^2 - x_1^2 - x_2^2 - \ldots x_{25}^2 \ = \ 0 \tag{88}$$

and leading to a light-like cone in $D = 25+1$ and centered at $P = (-\frac{1}{A}, 0, 0, \ldots, 0)$. Taking the two-dim slice of the light-cone by setting $x_2 = x_3 = \ldots = x_{25} = 0$ gives two light-like lines of slope ± 1 and whose algebraic equations are respectively

$$t = x_1 - \frac{1}{A}, \quad t = -x_1 - \frac{1}{A} \tag{89}$$

Hence, the tension becomes infinite on those light-like lines in the world-sheet.

When the a_μ vector is spacelike, the zero denominator expression yields the algebraic expression

$$t^2 - (x_1 - \frac{1}{A})^2 - x_2^2 - \ldots - x_{25}^2 \ = \ 0 \tag{90}$$

and leading to the light-cone in $D = 25+1$ flat spacetime centered at $P = (0, \frac{1}{A}, 0, 0, \ldots, 0)$. Taking the two-dim slice of this hyperboloid by setting $x_2 = x_3 = \ldots = x_{25} = 0$ gives the algebraic equation of the two light-like directions

$$t = x_1 - \frac{1}{A}, \quad t = -x_1 + \frac{1}{A} \tag{91}$$

such that the tension becomes infinite along the points belonging to the above two null directions along the world-sheet.

And when the a_μ vector is null-like, the zero denominator expression, after taking the two-dim slice, leads to an infinite tension region in the world-sheet along the following light-like trajectory

$$t = x_1 - \frac{1}{2A} \tag{92}$$

9.2 One dimensional string motion solutions from the tension field obtained from the special conformal transformation and the tension field as the analog to the Brout-Englert-Higgs field for strings

Of course there should be other solutions that involve more than one dimension. Here each segment associated with a given value of σ_1 is associated with a certain hyperbola. This hyperbola can be obtained in two different ways : firstly, using the method of the previous section, or secondly, by just assuming a one-dimensional motion along x_1

$$(t, x_1, 0, 0, ...0)$$

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using the expression of the tension field as a function of the world sheet coordinate σ_1, as obtained from the formula for the special conformal transformation, which was identified as the tension in the full bulk space and assuming a one dimensional trajectory $(t, x_1, 0, 0, ...0)$, we obtain equating this to the expression for the tension field as an exponential of $\alpha\sigma_1$, we obtain then the hyperbolic trajectories for $\alpha\sigma_1 = constant$ world sheet points in the target spacetime obtained also from a more intuitive in section 8. This shows self consistency between the Ricci flat condition that determines the conformal transformation and the motion of the different sections of the string with different tensions determined by an independent analysis.

There could be also many string solutions, each evolving in different directions, multi string condensation between two surfaces where the solutions for the tension field determines that the string tensions become infinite at these surfaces, resulting in a new type of braneworld scenario, as discussed in the spontaneously broken target scale invariant case discussed in ref. [35] and references there.

It appears that the tension field, responsible for generating the tension of the strings is the analog for strings to the the role of the Brout-Englert-Higgs field for particles in the standard model of particle physics, which is responsible for generating the masses of particles. As the Brout-Englert-Higgs field expectation value is defined in all of the space, the Tension field is defined everywhere in the bulk, not just in the exact locations where the strings are.

10 Class of σ_1 dependent String Solutions

10.1 Target Space Scale invariant Strings

In the case of a tension of the form $exp(-\alpha\sigma_1)$, we cannot impose open or closed boundary conditions if we insist on strict target space scale invariance, since these would require either periodic boundary conditions for σ_1, or interval (segment) conditions for σ_1, indicating at which value of σ_1 the string starts and at which value of σ_1 the string ends. These boundary conditions will violate target scale invariance. Hence, target space scale invariance would allow only infinite size strings in the case that the tension is of the form $exp(-\alpha\sigma_1)$.

Whereas in the case of a string tension that is dependent on the temporal world-sheet coordinate σ_0, there would not be a target space scale invariance obstacle in considering either open or closed strings.

10.2 Convergent semi infinite strings from inclusion of charges and convergent single quark type solutions

Notice that although the derivative of the tension field is discontinuous, the tension itself is continuous, By placing point charges in the world sheet, the tension can be made discontinuous, the discontinuity being equal to the value of the point charge. We can see in more detail by considering sources in the world sheet action. If to the action of the string we add a coupling to a world-sheet current j^{a_2}, i.e. a term

$$S_{\text{current}} = \int d^2\sigma A_{a_2} j^{a_2}, \tag{93}$$

see [28], [29] , [30], [31] for different applications of this. Then the variation of the total action with respect to A_{a_2} gives

$$\epsilon^{a_1 a_2}\partial_{a_1}(\frac{\Phi}{\sqrt{-\gamma}}) = j^{a_2}. \tag{94}$$

Naturally, j^{a_2} is comprised of a part that is induced by bulk tension field, but now we shall focus on a singular-delta function contribution to it of the form

$$j^0 = ... + Q\delta(\sigma_1 - \sigma_1^*) \tag{95}$$

and where $j^1 = ...$ is regular. The dots represent non singular contributions and the delta function has support at $\sigma_1 = \sigma_1^*$. Considering only the delta function contribution and integrating the 0 component of eq(94) from $\sigma_1 = \sigma_1^* - \epsilon$, to $\sigma_1 = \sigma_1^* + \epsilon$ we obtain that the discontinuity of the tensions is equal to the value of the point charge

$$T_2 - T_1 = Q \tag{96}$$

In particular, on one side of the charge the tension can be zero, let us say $T_1 = 0$, such that the tension flows to the other side with a strength $T_2 = Q$. Furthermore, we can choose the decaying leg of the string tension to be along this direction.

This leaves us with a situation where the tension emerges from the particle, but since it is decaying rapidly as we reach large values of $\sigma_1 > 0$, we may obtain a model where an isolated ¨quark¨ does not have infinite energy and does not need to be confined which is compatible with a realization of the ideas by [47].

10.3 Introducing special points of Target Scale Symmetry Violation

The origin of the divergence of the tension stems form our insistence of choosing the exponential dependence of the tension in the variable σ_1 . We can cure this divergence in either one of the limits $\sigma_1 = \pm\infty$ by to replacing σ_1 by $|\sigma_1|$. This procedure leads to a string tension of the form

$$T = e\phi = K\exp(g_o|\sigma_1|) \tag{97}$$

we consider $g_o < 0$, so the tension vanishes as $\sigma_1 = \pm\infty$. The resulting theory still has a guaranteed shift symmetry

$$\sigma_1 \to \sigma_1 + \Delta$$

provided

$$|\Delta| < |\sigma_1|$$

so that the transformation does not bring us across the point $\sigma_1 = 0$. Also, of course, if $\sigma_1 > 0$ and $\Delta > 0$, there is no further restriction on Δ , likewise, if $\sigma_1 < 0$ and $\Delta < 0$, there is no further restriction on Δ .

At this point $\sigma_1 = 0$,there is a change of sign in the derivative of the tension field because of the change of sign of the derivative of the $|\sigma_1|$ function, which causes a discontinuity of the world sheet charge defined in eq. (31), for $p = 1$, at the point $\sigma_1 = 0$.

It is interesting to see if there is a source that can generate only a discontinuity in the derivative of the tension, but not a discontinuity in the tension itself. An net electric charge contribution to the world sheet current does produce a jump in the tension, so for the purpose of the configuration studied in this section we must exclude this contribution. Since we are given the tension, we can calculate the charge distribution by using equation (94), which implies that the charge density is $Kg_o \exp(g_o\sigma_1)$ for $\sigma_1 > 0$ and $-Kg_o \exp(-g_o\sigma_1)$ for $\sigma_1 < 0$. Thus, the net charge of the system zero.

It can be considered as a model for a glue-ball or a quark-antiquark system, where the wave functions do not overlap so they do not automatically annihilate.

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11 Temporal world-sheet coordinate dependence of the Tension

So far we have discussed string tensions depending on the spatial world-sheet coordinate. Next we shall focus on tensions depending on the temporal world-sheet variable, and later on the most general case with $T = T(\sigma_0, \sigma_1)$.

Specifically, we turn our attention to dynamical tensions of the form $T = T_0 \exp(\alpha\sigma_0)$, such that the scalings $T \to \lambda T$ can be reabsorbed as shifts in $\sigma_0 : \sigma_0 \to \sigma_0 + \frac{\ln\lambda}{\alpha}$.

11.1 Working by analogy with the case of the previous section

We shall follow the same framework as before, but instead of choosing a σ_1-dependent conformal factor $\Omega^2 = f(\sigma_1) = T(\sigma_1)$, now we shall choose a σ_0-dependent one $\Omega^2 = f(\sigma_0) = T(\sigma_0)$. For each given fixed (constant) value of σ_0, it leads to the following conditions on the conformal factors of the form $(1 + 2a_\mu x^\mu + a^2x^2)^2$ = constant. Choosing for example $a_\mu = (A, 0, 0, 0, 0...))$, we obtain

$$(1 + 2At + A^2(t^2 - x_1^2 - x_2^2 - ....)) = constant$$

, where the constant can be positive or negative. This condition can be expressed as

$$(1+At)^2 - A^2(x_1^2 + x_2^2 + ...) = constant$$

After performing a time translation, it becomes simply

$$A^2t^2 - A^2(x_1^2 + x_2^2 + ...) = constant$$

and leading to the algebraic equation of a hyperboloid whose throat size is proportional to A^{-1}, and which coincides exactly with the results obtained in the previous section .

In the case where $a_\mu = (0, A, 0, ...)$, where the D vector a_μ is space like, after an appropriate shift in x, the equation is exactly the same as for the timelike case, of the form $A^2t^2 - A^2(x_1^2 + x_2^2 + ...) = constant$. However, one may notice that the constant is obtained from the square root of the tension, and one is allowed to have a positive square root and a negative square root. In one case, we obtain the timelike motion, and in the other case, a tachyonic or space like motion.

If one chooses the light like vector, where $a_\mu = (A, A, 0, ...)$, we find that the locus of points in the target spacetime corresponding to $\sigma =$ constant is given by $x - t =$ constant, that is the locus of points with constant tension are associated with a family of light-like trajectories rather than hyperbolic motion.

11.2 Cosmological framework. String world sheet time as a function of cosmic time

In this section we shall focus on cosmological scenarios. To study them, it is useful to consider the flat spacetime metric in the Milne representation in $D = 4$

$$ds^2 = -dt^2 + t^2(d\chi^2 + sinh^2\chi d\Omega_2^2) \tag{98}$$

where $d\Omega_2$ is the infinitesimal solid angle associated with the two-dim sphere. In $D > 4$ dimensions the infinitesimal solid angle is $d\Omega_{D-2}$ and corresponds to the $(D-2)$-dim sphere leading to the following D-dim background flat spacetime metric $g_{\mu\nu}^{(B)}$

$$ds_B^2 = -dt^2 + t^2(d\chi^2 + sinh^2\chi d\Omega_{D-2}^2) \tag{99}$$

For the string metric $g_{\mu\nu}^{(S)}$ we will choose the metric that one would obtain from the coordinate transformation $t \to 1/t \Rightarrow dt \to -(dt/t^2)$ so that $(dt)^2 \to (dt)^2/t^4$(using Minkowski coordinates x^μ, this corresponds to the inversion transformation. $x^\mu \to x^\mu/(x^\nu x_\nu)$). And followed by an overall multiplication by a constant C. In doing so, the chosen string metric $g_{\mu\nu}^{(S)}$ is given by

$$ds_S^2 = \frac{C}{t^4}\left(-dt^2 + t^2(d\chi^2 + sinh^2\chi d\Omega_{D-2}^2\right) \tag{100}$$

Therefore, one can read-off the conformal factor relating the background and string metric to be $\Omega^2 = \frac{C}{t^4}$. By writing the conformal factor between the metrics as Ω^2, we are explicitly avoiding negative-valued conformal factors and, consequently, negative string tensions.

Therefore, given $\Omega^2 = \frac{C}{t^4}$, with $C > 0$, and following our procedure, we shall identify this conformal factor as the string tension. Upon doing so, one obtains the relation between the cosmic time and the world sheet temporal string coordinate σ_0 to be of the form

$$\frac{C}{t^4} = T_0 \exp(\alpha\sigma_0)$$

It is not necessary, however if one wishes, one may choose both t, σ_0 to be monotonically increasing when $\alpha < 0$.

11.3 Exponential string tensions in Wesson warped space times

One may wonder if there are similar solutions to the vacuum Einstein´s equations similar to the Milne space but where instead of time some spacial coordinate would play a similar way. The answer to this question is yes, and these are the solutions in higher dimensional vacuum General Relativity discovered by Wesson and collaborators, see [46] and references there. In five dimensions for example the following warped solution is found,

$$ds^2 = l^2 dt^2 - l^2 cosh^2 t \ (\frac{dr^2}{1 - r^2} + r^2 d\Omega_2^2) - dl^2 \tag{101}$$

where l is the fourth dimension and the signature is chosen to be $(+, -, -, -, -)$.

As pointed out by [46], in these metrics the coordinates t and r are chosen to be dimensionless, with the physical dimension of ds carried by the coordinate l. If we want to keep the dimensionality of the coordinates t and r , one must introduce explicitly a length parameter L (which could be set to unity), into the metric [46]. These two formulations are equivalent of course, but we choose the coordinates t and r to be dimensionless, with the physical dimension of ds carried by the coordinate l, so that string tension generation will be directly associated to a choice of l.

One may notice that with respect to the coordinate l (fourth dimension) such metric is homogeneous of degree two, just as the Milne space time was homogeneous of degree two with respect to the time. Notice that maximally symmetric $4D$ de Sitter space metrics are obtained from the $5D$ Wesson metric simply by setting $l =$ constant, instead of euclidean spheres that appear in the Milne Universe for $t =$ constant.

The list of space times of this type is quite large, for example, one can find $4D$ Schwarzschild-de Sitter metrics by setting for $l =$ constant in the $5D$ Wesson metric

$$ds^2 = \frac{l^2}{3} \left(dt^2 (1 - \frac{2M}{r} - \frac{\Lambda r^2}{3}) - \frac{dr^2}{1 - \frac{2M}{r} - \frac{\Lambda r^2}{3}} - r^2 d\Omega_2^2 \right) - dl^2 \tag{102}$$

This $5D$ metric can be extended to D dimensions, where we choose $D-1$ dimensions to have a l^2 warp factor as follows

$$ds^2_{(2)} = l^2 \bar{g}_{\mu\nu}(x) dx^\mu dx^\nu - dl^2 \tag{103}$$

where $\bar{g}_{\mu\nu}(x)$ is a $(D-1)$-dim Schwarzschild-de Sitter metric for example [46]. This we will be our metric $g^{(2)}_{\mu\nu}$ labeled by the suffix 2.

In any case, by working with a generic metric of the form (103), but now in D dimensions, we can perform the inversion transformation $l \to \frac{1}{l}$, and after multiplying by a factor C one obtains the conformally transformed metric which also satisfies the vacuum Einstein´s field equations. Thus, the conformally related intervals are given by

$$ds^2_{(1)} = Cl^{-2}\bar{g}_{\mu\nu}(x)dx^\mu dx^\nu - C\frac{dl^2}{l^4} = Cl^{-4}ds^2_{(2)} \tag{104}$$

From this point on the equations leading to the solutions for the tensions of the strings are analogous to those for the cosmological Milne case, just that now one will have $\Omega^2 = Cl^{-4}$, instead of Ct^{-4}. And in doing so, by equating

$$T = \frac{\Phi}{\sqrt{-\gamma}} = e\phi = T_0 \exp(g_o\sigma_1) = \Omega^2 = Cl^{-4} \tag{105}$$

one is able to correlate two spacelike coordinates, one in target space given by the extra dimension l, and the other in world sheet given by σ_1.

From Eq-(105) one finds that when $l =$ constant it implies $\sigma_1 =$ constant such that a specific value for a cosmological constant in the submanifold is generated.

11.4 A new type of Brane where strings have one point of contact only

Along the lines of Wesson's approach discussed above we could view each $l =$ constant slice as a universe of its own, but in the context of string theory we could contemplate the scenario involving a region of space close to an $l = l^*$ constant slice that could be viewed as a thick brane.

The construction looks like this : let us consider open strings aligned along the l direction ending at $l = \infty$, where the tension is zero, and decreasing exponentially from the selected value $l = l^*$ constant > 0, where the string begins. We could call such string a semi-infinite open string. Since the values of the tension decay exponentially away from the constant l slices, the energy density generated from a collection of semi-infinite open strings is mainly localized in those regions of l near the chosen $l = l^*$ constant slice, and in this fashion, we may engineer a thick-brane configuration.

These kind of strings are similar to the semi infinite strings discussed before in the free quark construction. This picture differs from the D-brane formulation where open strings have their end-points residing on the D-branes. The two end-points could reside in the same D-brane or in different D-branes.

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Because for each cut $l = constant$, in the Wesson spaces [104] as shown by Wesson et. al, is described as a spacetime with a cosmological constant $2/l^2$ (although the full space is Ricci flat), we will obtain that for a Dp brane consisting of an $l = constant$ cut of a Wesson spaces, the relevant induced metric in this slice will be a de Sitter space with a cosmological constant $2/l^2$- ——————

12 A duality for Strings with tensions proportional to $exp(\alpha\sigma_1)$

It is interesting to see that for strings with tensions proportional to $exp(\alpha\sigma_1)$, under a reflection in σ_1,

$$\sigma_1 \to -\sigma_1$$

it leads to an inversion of the tension of the form

$$T \to T_0^2/T$$

where T_0 is the value of the tension associated with $\sigma_1 = 0$.

If we consider open (oriented) strings and postulate that the mechanism by which the tension emanates from a point is due to a charge located at this point, then if we take in eq. (96) that the tension can be zero on one side of the charge, say T_1, leaving the tension $T_2 = T_0^2/T = q$ on the other side of the charge. By comparing this scenario with the situation before the reflection, where $T = Q$, we conclude that

$$q = T_0^2/Q$$

leading to

$$qQ = T_0^2$$

This resembles an electromagnetic duality equation if Q represents an electric charge and q represents the magnetic charge obtained after an electric/magnetic duality transformation. Indeed, as shown by Dirac, see [49], [50], in the presence of an electric charge e and a magnetic charge g , the relation, in natural units where the speed of light $c = 1$ and $h/2\pi = T_0^2 = 1$, is,

$$eg = N/2$$

which for $N = 2$ is analogous to our relation between q and Q.

13 Target space scale invariance with Nambu-Goto Strings and Gravity in D = 4

We would like to consider an effective gravitational theory based on a modified measure theory, because then the gravitational part of the action, linear in the curvature scalar, can be scale invariant (the Riemannian measure of integration does not allow this possibility). The appropriate integration measure in the space of D scalar fields $\varphi_a, (a = 1, 2, ...D)$, is

$$dV = d\varphi_1 \wedge d\varphi_2 \wedge \ldots \wedge d\varphi_D \equiv \frac{\Phi}{D!} d^D x \tag{106}$$

where

$$\Phi \equiv \varepsilon_{a_1 a_2 \ldots a_D} \varepsilon^{A_1 A_2 \ldots A_D} (\partial_{A_1} \varphi_{a_1})(\partial_{A_2} \varphi_{a_2}) \ldots (\partial_{A_D} \varphi_{a_D}). \tag{107}$$

We label the coordinates relevant to the effective theory by the indices $A, B, C, \ldots$ to differentiate them from the Greek spacetime indices of the fundamental theory. We also use G^{AB} for the metric of the effective theory.

Accordingly, the total action in the D-dimensional space-time should be written in the form

$$S = \int \Phi L d^D x \tag{108}$$

Our choice for the total Lagrangian density is

$$L = -\frac{1}{\kappa} R(\Gamma, G) + L_{strings} \tag{109}$$

and as mentioned before, the R contribution is multiplied by the measure and this can be target space scale invariant provided $\Phi \to \omega^{-1}\Phi$, as $G^{AB} \to \omega G^{AB}$ and this is true in either first or second order formalisms, since in either case $R \to \omega R$. We will see if this linear transformation in ω is also possible for the string matter part. The Lagrangian density for the string matter contribution is

$$L_{string} = -T \int d\sigma d\tau \frac{\delta^D(x - X(\sigma,\tau))}{\sqrt{-G}} \sqrt{Det(G_{AB} X^A_{,a} X^B_{,b})} \tag{110}$$

where $\int L_{string}\sqrt{-G}d^Dx$ would be the action of a string embedded in a D-dimensional space-time in the standard theory; a, b label the coordinates of the string world sheet and T is the string tension. Notice that under a transformation $G^{AB} \to \omega G^{AB}$, $L_{string} \to \omega^{(D-2)/2} L_{string}$, therefore L_{string} is a homogeneous function of G^{AB} of degree one, and the target space scale invariance of the effective theory is satisfied only if $D = 4$.

It would appear that we have introduced a dimensionful string tension T and a dimensionful Planck scale through $\frac{1}{\kappa}$, but this is not so, since under the transformation $G^{AB} \to \omega G^{AB}$, $T \to \omega T$ and $\frac{1}{\kappa} \to \omega \frac{1}{\kappa}$, so only the ratio of the string tension to the Planck scale is meaningful. This ratio will be dependent on details of the compactification to four dimensions, etc. Full details of the results of this section which were briefly reviewed here to complete our discussion of strings with target space scale invariance are given in a longer publication [48].

14 Standard Dp-Branes and Dp-Branes with modified measures and target space scale invariance

14.1 Standard Dp-Branes

We reviewed models of Gravity coupled to strings in the Nambu-Goto formalism after using modified measures which acquire target space scale invariance in $D = 4$.

We have examined strings/branes with dynamical tensions. It would be desirable if one could extend this work to the study of Dp-branes whose tension are *constant* and given by

$$T_{D_p} \;=\; \frac{1}{g_s (2\pi)^p (\alpha')^{(p+1)/2}} \tag{111}$$

where g_s is the closed string coupling that is related to the open-string coupling by $g_s = g^2_{open}$. α' is the Regge slope whose units are $(length)^2$. The type IIA string coupling constant is used if p is even, and the type IIB coupling constant is used if p is odd [51].

Constant Dp-brane tensions are required to implement T-duality that maps two theories of the same type, one whose compactification on a circle of radius R is physically equivalent to a compactification on a circle of radius $\tilde{R} = \alpha'/R$. Namely, momentum excitations in one description correspond to winding modes excitations in the dual description and vice versa [51]. Therefore, it would be very challenging to accommodate dynamical tensions if one wishes to implement T-duality.

It has been known for some time that the low-energy effective D-brane actions is given by the Dirac-Born-Infeld (DBI)-action [51]

$$S \; = \; -T_{D_p} \int d^{p+1}\sigma \; \sqrt{-det(G_{ab} + 2\pi\alpha' F_{ab})}$$

where $G_{ab} = \partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu}$ is the induced world-volume metric as a result of embedding the Dp-brane in a flat background, and F_{ab} is the Maxwell field strength.

14.2 Dp-Branes with modified measures and target space scale invariance

Similarly to the string case we now introduce a modified world-volume integration measure density in terms of $p+1$ auxiliary scalar fields φ^i $(i = 1, \dots, p+1)$:

$$\Phi(\varphi) \equiv \frac{1}{(p+1)!} \epsilon_{i_1 \dots i_{p+1}} \epsilon^{a_1 \dots a_{p+1}} \partial_{a_1} \varphi^{i_1} \dots \partial_{a_{p+1}} \varphi^{i_{p+1}} \; , \tag{112}$$

and use it to construct the following new p-brane-type action (the coupling to the Ramond-Ramond background field is omitted for simplicity). Some time ago a Polyakov-type action classically equivalent to the original DBI-action of the Dp-brane was proposed in [52] :

$$S = -T \int d^{p+1}\sigma \, \sqrt{-\zeta} \Big(\; e^{-\beta U} \zeta^{ab} \; (G_{ba} - F_{ba}) - (p-1)) \Big)$$

In this last sub-section we discuss the modified measure formalism for the Dp-brane first discussed in [53] and show now that it has target space scale invariance :

$$S = - \int d^{p+1}\sigma \, \Phi(\varphi) \left(e^{-\beta U} \zeta^{ab} \; (G_{ba} - F_{ba}) + \frac{\varepsilon^{a_1 \dots a_{p+1}}}{(p+1)\sqrt{-\zeta}} F_{a_1 \dots a_{p+1}} \right) \tag{113}$$

where

$$F_{a_1 \dots a_{p+1}} = \nabla_{[} a_1 A_{a_2 \dots a_{p+1}]}$$

Splitting the auxiliary tensor variable $\zeta^{ab} = \zeta^{(ab)} + \zeta^{[ab]}$ into symmetric and anti-symmetric parts and setting $\zeta^{[ab]} = 0$, the action (113) reduces to the action of the modified-measure model of ordinary p-branes, eq. (18) The advantage of the present more general action (113) is that not only it yields variable dynamical brane tension and naturally introduces additional higher-rank world-volume gauge fields, but also it does not need a “cosmological” term as it was the case of the p brane eq. (18). This action has target space scale invariance

$$A_{a_2 \dots a_{p+1}} \to \omega A_{a_2 \dots a_{p+1}}$$

$$\Phi(\varphi) \to \omega^{-1} \Phi(\varphi)$$

,

$$G_{ba} \to \omega G_{ba}$$

,

$$F_{ba} \to \omega F_{ba}$$

The integration of the equations of motion breaks the target space scale invariance. For example, the integration of the equation of motion of the degrees of freedom that define the measure leads to a constant Lagrangian density of the form

$$e^{-\beta U}\zeta^{ab}\ (G_{ba}-F_{ba})+\frac{\varepsilon^{a_1\ldots a_{p+1}}}{(p+1)\sqrt{-\zeta}}F_{a_1\ldots a_{p+1}}=M$$

Furthermore, the integration of the equation obtained from the variation of the world sheet gauge fields ensures that

$$\frac{\Phi}{\sqrt{-\zeta}}=C=constant$$

By considering the variation with respect to ζ^{ab}, we can in fact solve for ζ^{ab} and its inverse, ζ_{ab}, and upon inserting back these solutions into the action we get the Nambu Goto type of action of the form

$$S^{DBI}=-T\int d^{p+1}\sigma\, e^{-\beta' U}\sqrt{-\det||G_{ab}-F_{ab}||}\ , \tag{114}$$

$$T\equiv\ C(2M)^{-\frac{p-1}{2}}(p-1)^{\frac{p+1}{2}}\quad ,\quad \beta'\equiv\frac{p+1}{2}\beta\ , \tag{115}$$

If we wish, we could also include some contribution to the tension of the brane from the expectation value of the dilaton field U. It is interesting to establish a correspondence between the tension obtained from the standard formulation of the Dp-brane and the result above based on the modified measure of Dp-branes.

15 Review of Results in this paper.

The consideration of dynamical tension generation lead us to theories where each string generates dynamically its own string tension. This has profound consequences. In particular, a new background scalar field, the ¨tension¨ field, is discussed that can change dynamically the value of the tension along the world-sheet (world-volume) of the extended objects. This new field introduced in the theory of extended objects appears to have very important consequences in various cosmological and other scenarios. In previous papers, this tension field was considered as a dynamical field which was consistently determined by considering the consistency of two sets of strings which acquire different tensions through a process of spontaneous symmetry breaking of the target scale invariance.

Here, by contrast, we consider the dynamical string tension as an exponential function of σ or τ. This dependence can be obtained in two different ways, by considering for example that each value of σ in the string is undergoing a hyperbolic motion. The resulting string trajectories display target space scale invariance such that a scaling of the target space coordinates can be compensated by a shift in σ.

The second approach, a bit more formal but leading to the same result, is to realize that when one is solving for the measure in the string action, the scaled metric $Tg_{\mu\nu}$, must satisfy the vacuum Einstein Equations. In other words, it must be Ricci flat. Because we also considered the embedding space metric $g_{\mu\nu}$ itself to be associated to a flat space-time background one ends up with two Ricci flat spaces whose metrics are related by a conformal factor given by the tension T.

As Culetu showed this conformal factor provided now by the tension can only be the one obtained from a special conformal transformation parametrized by a D vector a_μ. For example, in the timelike case where $a_\mu = (A, 0, 0, ...)$, this yields a very special expression for the tension field. After interpreting the latter as a bulk field, we find that the family of curves of constant tension corresponding to a given value of σ_1, and associated to a tension of the form $exp(\alpha\sigma_1)$, are provided by a family of hyperbolas in space-time. After considering, for example, the motion in the region $(t, x, 0, 0, 0, ...)$ we obtain again the results explored in section 9 in a more intuitive way.

Other results can be obtained in the light like case, where $a_\mu = (A, A, 0, ...)$, and one finds that the locus of points with $\sigma_1 =$ constant corresponds in the target space to the locus of points with $x - t =$ constant, and representing the motion associated with null-like trajectories rather than hyperbolic ones.

Further generalizations when the tension fields have the form $exp(\alpha\sigma_0)$ were examined. A cosmological Milne space was considered as well as the higher dimensional Wesson-type space-times. These are indeed Ricci flat space-times displaying de Sitter space slices. An inversion transformation of t in the Milne space, or of l in the Wesson spaces, leads to conformally related Ricci flat space-times whose conformal factor is provided by the tension field.

We then considered ways to introduce boundary conditions for strings whose tension have an an exponential form. If the exponential is $exp(\alpha\sigma_0)$, it allows closed and open strings involving periodic or interval-like boundary conditions in σ_1, respectively, without affecting the target space scale symmetry.

For strings whose tension have a dependence of the form $exp(-\alpha\sigma_1), \alpha > 0$, leave us with no possibilities of considering closed strings, so we are left with open strings if we wish to respect strictly target space scale invariance. We may allow the breaking of target space scale invariance at special points, with the aim of eliminating the regions where the tension goes to infinity. We can cure this divergence by replacing σ_1 by $|\sigma_1|$. So now the tension vanishes both for very large positive values of σ_1 and for very large negative values of σ_1

Another possibility is to add a charge that couples to the tension, so that we can eliminate the region were the tension diverges. This would be a model of a deconfined quark, like it was proposed by [47] as discussed above.

Likewise, we could have these semi infinite strings could end up in only in one brane, with the rest without being reabsorbed into the same or other brane, since if in this direction we have a decaying string tension, this would prevent an infinite energy.

We introduced string interactions of strings with different tensions via the inclusion of the tension field. However, these interactions are clearly beyond the interactions described in the standard model. According to our ideas of dark matter, strings with different tension [37], [38] are leading candidates for the interactions of standard model matter and dark matter. It would be interesting to see the role that these interactions play in some brane-world scenarios where the tension field has an essential role.

Finally, we discussed Dp-branes with a dynamical tension endowed with target space scale invariance, and which is spontaneously broken by the constants of integration M and C. The tension of the modified measure formalism for the Dp-brane depends on these two constants of integration, while the tension of the Dp-brane in the standard formulation depends on the string tension of the fundamental strings, etc. It should be possible to understand this difference,

In the context of Dp-branes with a modified measure, implying also Dp-branes with dynamical tension, we can introduce a tension field as an external field with an exponential dependence on one of the world sheet variables, such that the target space scale invariance is satisfied provided a shift in the appropriate world sheet coordinate is performed, just as it was done in the fundamental strings case. All this will be studied in future publications.

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