Some Applications of Friction Physics to Dynamics of Dance

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Abstract:
This study is about applying the fundamentals of physics using the concepts of friction to understand and describe dance shoes and dance floor interaction. We have used sliding friction, skidding friction and rotational friction to describe our model independent study. Since we are performing a general and model independent study so we have taken the dancer to be of a uniform cylindrical shape. The various frictions used are time dependent.

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1 Introduction

We all have seen a dance of some style at some point of our lives. There are various types of dances which are favorites through various cultures since a long time. Some of these dances are ballet, salsa & bachata, hip-hop, contemporary, jazz and many more. Dance movements have long being considered to be extremely complex and the dancers put in years of practice to become experts [1, 2]. There are studies which have regularly shown that the dance movements are increasingly complex and principles of physics are some of the most robust tools in understanding and describing them[3]. There are a vaiety of dances and majority of which require muscle strength because of the factors involving the surroundings, some of which are also seen in fields such engineering [4–6].

The dance movements involve various parts of the body and it deeply involves the mechanics of the various body parts [3, 7–9]. In this study we are trying to apply the concepts of friction to a model independent dance style. However, this study can be applied to any dance style by incorporating all the intricate and coarse movements of that particular dance. The movements of the dancers incorporate conepts of physics such as Newton's laws, inertia, center of mass, rotational interia and many more. All these concepts are involved and used by the dancer knowingly and unknowingly alike. As the dancer moves across the floor, friction is acting on the dancers feet and the dance floor [10, 11]. There are other situations which affect the dance motions and dynamics, some of which are,

- friction being different at various locations on the dance floor
- the wear and tear of the dance floor
- wear and tear of dance shoes
- the material of wood the dance floor is made of
- the linear speed and rotational speed of the dancer as they move across the dance floor
- the airflow in the dance room
- the jumping and hopping movements of the dance and which part of the foot lands on the floor

Along with all of the above, there are coarse movements, fine movements and superfine movements in all types of dances. All these different types of movements require a whole range of motion including coarse motor skills and fine motor skills. These motor skills depend on the fundamentals of physics, specifically friction, movements, and biomechanics.

This study is the first step towards a understanding and incorporating physics with dance movements. In this article, we have described a model independent dance and treated the dancer like a uniform cylinder due to the cylinder being a symmetrical object.

2 Defining a Coordinate System

We start with the assumption that the dancer is a uniform cylinder. This allows us to understand the dynaimcs in a uniform way, and keeps us from from any deviations which happen unnaturally. Also, since our study is a model independent, so we would like to keep it in such a way that it gives us an average of all the movements a dancer makes and treat the physical aspects of the dancer through uniform movements. Here we have used the references [1, 5, 6, 12]. The terms which we are using are

- **Position Vector:** $\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j}$
- **Velocity Vector:** $\vec{v}(t) = \dot{x}\hat{i} + \dot{y}\hat{j}$, with magnitude $v = \sqrt{\dot{x}^2 + \dot{y}^2}$
- **Angular Velocity:** $\vec{\omega}(t) = \omega(t)\hat{k}$
- **Moment of Inertia of a uniform cylinder:** $I = \frac{1}{2}mR^2$

3 Time-Dependent Friction

This study assumes that the friction between the dancer's feet and the dance floor decreases over time. The reason behind this assumption is that when the dancer starts their routine, the shoe sole is not smoothened out due to the movements across the floor. These movements include walking, rotating, running, hopping, jumping, sliding amd skidding. With time the smothness of the shoes increase, therby decreasing friction. Here we have taken the friction forces in a way that they decrease exponentially over time t due to smoothing of the dance shoes and dance floor conditions. The coefficient of friction is hence taken to be time dependent and is given by,

$$\mu(t) = \mu_0 e^{-\lambda t} \tag{3.1}$$

where μ_0 is the initial coefficient of friction and λ is the decay rate of the coefficient of friction.

3.1 Sliding and Skidding Friction

Here we have included sliding and lateral skidding friction forces into a net kinetic friction force which is constantly opposing the velocity vector. So the velocity is inversely proportional to the friction [3, 13, 14].

$$\vec{F}_k = -\mu(t)mg\frac{\vec{v}}{v} = -\mu_0 e^{-\lambda t} mg\frac{\vec{v}}{v} \tag{3.2}$$

3.2 Rotational Friction

When a dancer makes their moves on the dance floor, there are multiple rotational motions which they perform. These rotation motions have a high degree of friction acting towards reducing rotation. Since rotation is connected with torgue, so we have included the torque which is generated by twisting friction across the contact patch of the shoe sole with the dance floor. This torque is written as,

$$\tau_{\mathrm{R}} = -\frac{2}{3}\mu(t)mgR\,\mathbf{Z}(\omega) = -\frac{2}{3}\mu_0 e^{-\lambda t} mgR\,\mathbf{Z}(\omega) \tag{3.3}$$

4 Newtons Laws and Equations of Motion

Applying Newton's Second Law $\vec{F} = m\vec{a}$ and the equation between torque, moment of interia and angular acceleration $\tau = I\alpha$, we can write the equations of motion as,

4.1 Linear Acceleration

$$m\frac{d\vec{v}}{dt} = -\mu_0 e^{-\lambda t} mg\frac{\vec{v}}{v} \tag{4.1}$$

re arranging, we can write,

$$\frac{d\vec{v}}{dt} = -\mu_0 g e^{-\lambda t}\frac{\vec{v}}{v} \tag{4.2}$$

4.2 Angular Acceleration

$$\frac{1}{2}mR^2\frac{d\omega}{dt} = -\frac{2}{3}\mu_0 e^{-\lambda t} mgR\,\mathbf{Z}(\omega) \tag{4.3}$$

which can be rearranged as,

$$\frac{d\omega}{dt} = -\frac{4\mu_0 g}{3R}e^{-\lambda t}\,\mathbf{Z}(\omega) \tag{4.4}$$

5 Energy Dissipation

The total mechanical energy because of motion the dancer makes, E, at any given instant can be written as the sum of linear kinetic energy and rotational kinetic energy, which is:

$$E(t) = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \tag{5.1}$$

The power dissipated can be calculated from the energy by,

$$\frac{dE}{dt} = \vec{F}_k \cdot \vec{v} + \tau_{\text{rot}}\omega = -\mu_0 e^{-\lambda t} mg\left(v + \frac{2}{3}R|\omega|\right) \tag{5.2}$$

6 Wear and Tear of Dancing Shoes

6.1 Derivation of Archard's Wear Law

Archard's wear model describes a quantitative aspect of adhesive and abrasive wear volume resulting from the sliding of two surfaces which have friction. In our study, this is the dance shoe and the dance floor. The assumtion here is that contact occurs at microscopic positions which are known as asperities. These asperities deform inelastically under a force.

6.1.1 Single Asperity Model

Let us consider a single contact point which can be thought of as a circular connection of radius a. The real contact area A_i of this individual asperity is given by:

$$A_i = \pi a^2 \tag{6.1}$$

Assuming perfect inelastic deformation, the normal force F_i supported by this single asperity is constrained by the hardness H of the softer material:

$$F_i = HA_i = H\pi a^2 \tag{6.2}$$

As the dance shoe slides against the dance floor, the asperity interaction gives us a sliding distance which is proportional to its contact diameter, $s_i = 2a$. Archard postulated that the resulting wear fragment forms a hemisphere of radius a, resulting in a wear volume V_i:

$$V_i = \frac{2}{3}\pi a^3 \tag{6.3}$$

The ratio of wear volume to sliding distance can be written as,

$$\frac{V_i}{s_i} = \frac{\frac{2}{3}\pi a^3}{2a} = \frac{1}{3}\pi a^2 = \frac{1}{3}A_i \tag{6.4}$$

Substituting the relationship for the local contact area ($A_i = \frac{F_i}{H}$) into Equation (4) yields:

$$\frac{V_i}{s_i} = \frac{1}{3}\frac{F_i}{H} \tag{6.5}$$

6.1.2 Wear Coefficient

Adding all the individual forces ($F_T = \sum F_i$) and total wear volumes ($V = \sum V_i$) across all contacts over a sliding distance s produces the total wear rate:

$$\frac{V}{s} = \frac{1}{3}\frac{F_T}{H} \tag{6.6}$$

Experimentally, the wear is not generated at every single asperity contact. To account for this probability, a dimensionless wear coefficient K is introduced. This coefficient gives us the formulation of Archard's wear law:

$$V = K\frac{F_T \cdot s}{H} \tag{6.7}$$

Using Archard's Wear Law, the cumulative material wear volume V_{w} increases based on the work done by friction:

$$\frac{dV_{\mathrm{w}}}{dt} = k \cdot F_{\mathrm{friction}} \cdot v_{\mathrm{relative}} = k\mu_0 e^{-\lambda t} mg \left(v + \frac{2}{3}R|\omega| \right) \tag{6.8}$$

Where k represents the material wear coefficient.

In **Figure 2**, we can see the relationship between various physical quantities and how they evolve with time.

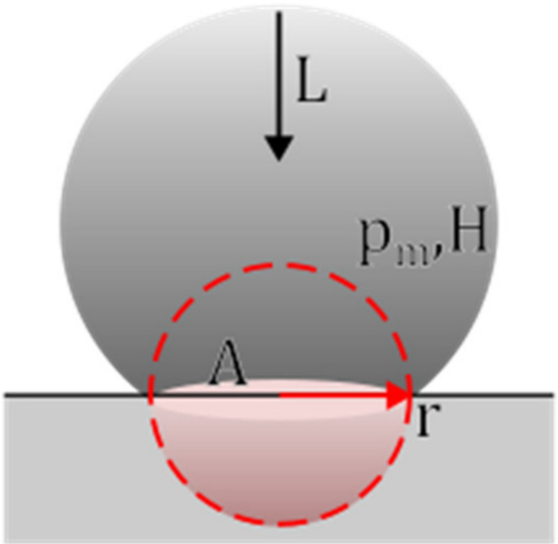

Figure 1. shows the area of contact under an applied load L and resulting in wear volume depicted by the red shaded region within the dashed contact radius. [15]

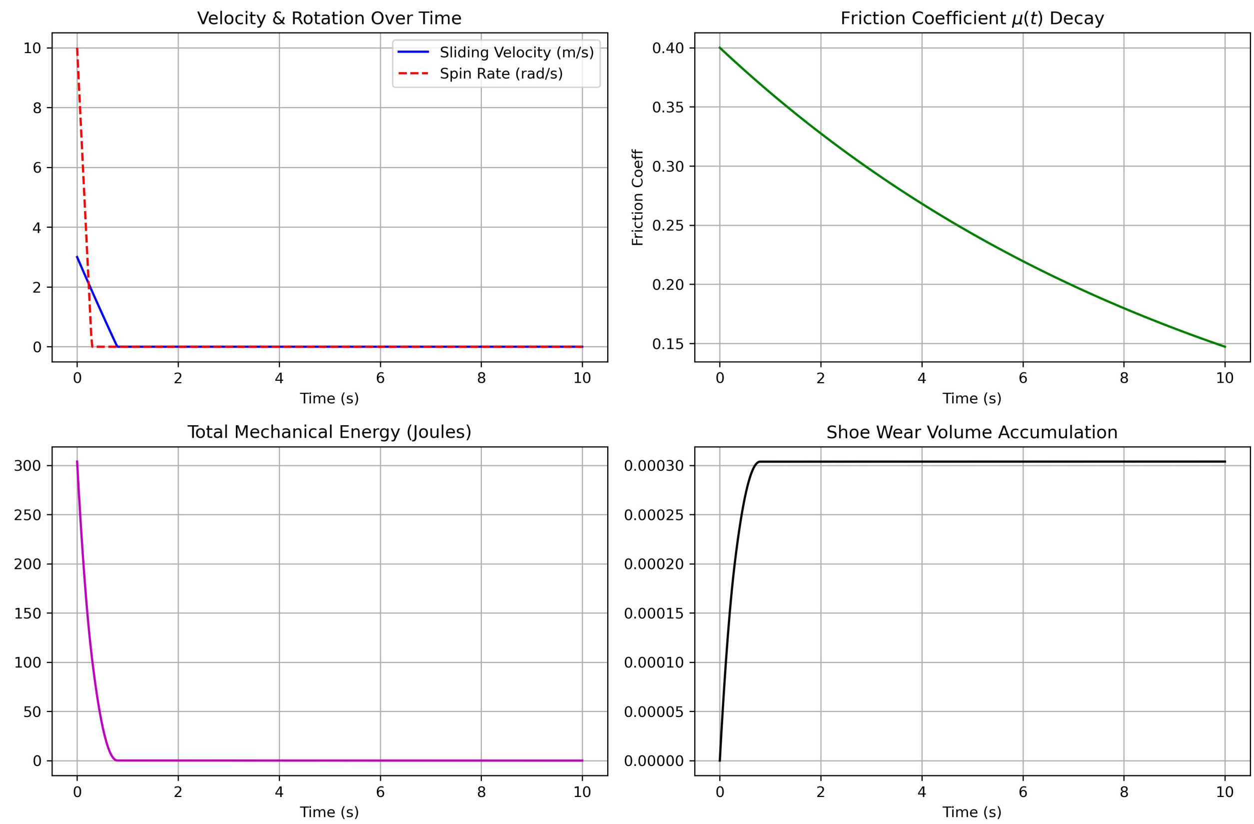

Figure 2. Various aspects of dance physics with time

7 Conclusion

This study is done on a model independent dance assuming that the dancer is a uniform cylinder. Then we have applied friction of various types and the studied the motion of the dancer including the energy dissipated. In the results we have shown the time evolution of "Velocity and Rotation Over Time", "Friction Coefficient Decay", "Total Mechanical Energy" and "Shoe Wear Volume Accumulation". This study is the first step towards a more detailed study of understanding various dances through the concepts of physics.

8 Conflict of Interest

The authors declare no conflict of interest.

9 Acknowledgements

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11 Glossary

In this section we have described the variables and physical quantities used in the studny.

Center of Mass (CoM): The unique spatial point where the entire mass of the dancer can be considered concentrated for linear physics calculations.

$\vec{r}(t)$ [units: meters, m]**:** The **Position Vector** tracking where the dancer's center of mass is located on the flat 2D floor relative to a fixed origin $(0, 0)$.

$\hat{i}, \hat{j}, \hat{k}$ [units: dimensionless]**:** **Unit Vectors** pointing along the x (forward), y (sideways), and z (vertical/upward) coordinate axes to establish mathematical direction.

$\vec{v}(t)$ [units: meters per second, m/s]**:** The **Velocity Vector** representing the instantaneous speed and direction of the dancer's straight-line sliding movement.

v [units: meters per second, m/s]**:** The **Linear Speed**, which is the scalar magnitude (absolute value) of the velocity vector: $v = \sqrt{\dot{x}^2 + \dot{y}^2}$.

$\dot{x}, \dot{y}$ [units: meters per second, m/s]**:** The **Component Velocities** representing how fast the dancer is moving strictly along the x-axis and y-axis respectively. The single dot notation $(\cdot)$ denotes the first derivative with respect to time (d/dt).

$\vec{\omega}(t)$ [units: radians per second, rad/s]**:** The **Angular Velocity Vector** representing how fast and in what directional orientation the dancer is spinning.

ω [runits: adians per second, rad/s]**:** The **Spin Rate**, representing the numerical scalar magnitude of the angular velocity.

θ [units: radians, rad]**:** The **Angular Position** (or rotation angle) tracking the cumulative angular displacement the dancer has spun.

m [units: kilograms, kg]**:** The total **Mass** of the dancer, measuring their physical matter and quantitative resistance to changes in straight-line motion.

R [units: meters, m]**:** The **Radius** of the circular contact patch of the dancer's shoe sole resting on the floor surface.

I [units: kilogram meters squared, $\mathrm{kg \cdot m^2}$]**:** The **Moment of Inertia**, measuring a body's structural resistance to changes in its rotational rate based on mass distribution. For a uniform cylinder, $I = \frac{1}{2}mR^2$.

g [units: meters per second squared, $\mathrm{m/s^2}$]**:** The **Acceleration due to Gravity**, which is the constant pull of Earth's gravity downward (approximated as $9.81\,\mathrm{m/s^2}$).

F_N [units: Newtons, N]**:** The perpendicular **Normal Force**, which is the upward structural push exerted by the stage floor supporting the dancer's total weight ($F_N = mg$).

$\vec{F}_k$ [units: Newtons, N]**:** The **Kinetic Friction Force** vector acting parallel to the surface of the floor to directly oppose translational sliding.

$\tau_{\mathbf{rot}}$ [units: Newton meters, $\mathrm{N \cdot m}$]**:** The **Rotational Friction Torque**, a twisting moment resulting from micro-contact interactions across the shoe area that opposes and slows down rotational spins.

$\mathbf{Z}(\omega)$ [units: dimensionless]**:** The mathematical **Z** functions is defined in such a way that it gives an output of

$+1$ if $\omega > 0$

-1 if $\omega < 0$

0 if $\omega = 0$

This function makes sure that the friction torque always dynamically opposes the exact direction of rotation.

$\mu(t)$ [units: dimensionless]**:** The **Time-Dependent Friction Coefficient**, representing the instantaneous lubricity or slippery state of the shoe-floor interface.

μ_0 [units: dimensionless]**:** The **Initial Friction Coefficient**, mapping the maximum static-to-kinetic grip threshold between the shoe and floor at the exact start of the motion ($t = 0$).

e [units: dimensionless]**:** **Euler's Number**, a fundamental mathematical constant ($e \approx 2.71828$) used to model natural exponential decay profiles.

λ [units: inverse seconds, 1/s]**:** The **Friction Decay Rate**, determining the transient speed at which the stage interface becomes slippery or the shoe smooths down over time.

t [units: seconds, s]**:** **Time**, the continuous independent variable tracking the ongoing duration of the dance performance.

$E(t)$ [units: Joules, J]**:** The system's total **Mechanical Energy**, which is equal to the sum of all linear and rotational kinetic terms.

$K_{\mathbf{T}}$ [units: Joules, J]**:** **Translational Kinetic Energy**, the mechanical kinetic energy stored in a moving body due to linear velocity ($K_{\mathrm{T}} = \frac{1}{2}mv^2$).

$K_{\mathbf{R}}$ [units: Joules, J]**: Rotational Kinetic Energy**, the mechanical kinetic energy stored due to mass, in this case the dancer, spinning around an axis ($K_{\mathrm{R}} = \frac{1}{2}I\omega^2$).

$\frac{dE}{dt}$ [units: Watts, W]**:** The **Power Dissipation Rate**, which shows the rate of conversion of mechanical energy into heat and sound. This also signifies that part of energy which cannot be recovered. It can be said that this much energy is lost.

$V_{\mathbf{w}}(t)$ [units: cubic millimeters, mm^3]**:** The **Cumulative Wear Volume**, that points to the amount or quantity of material completely sheared or worn off.

$\frac{dV_{\mathbf{w}}}{dt}$ [units: cubic millimeters per second, mm^3/s]**:** The **Wear Rate**, which shows how rapidly the sole of the dancing show degrades or wears out at any moment in time.

k [units: cubic millimeters per Joule, mm^3/J]**:** The **Archard Material Wear Coefficient**, a material constant which shows the vulnerability of the specific shoe compound against abrasive force.