A Vibrational Interpretation of the Schrödinger Equation for Hydrogen-Like Atoms

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Abstract

Physicists have been searching for a vibrational interpretation of the Schrödinger equation for a century; so far with limited success. This work presents a speculative vibrational interpretation in the special case of the Schrödinger equation for a hydrogen-like atom. Solutions of this equation are interpreted as high-frequency stationary vibrations of the atom's electromagnetic field driven by de Broglie's internal clock of the atom's electron, which is assumed to orbit around the atom's nucleus. The proposed vibrational interpretation challenges existing interpretations of the Schrödinger equation in various ways, which pose interesting research questions for future work.

1 Introduction

Since 1926, Schrödinger had compared wave functions to states of "a vibrating string or drumhead or metal plate, or of a bell that is tolling" [Sch52, page 113]. In his words, "[t]he achievement of wave mechanics was, that it found a general model picture in which the 'stationary' states of Bohr's theory take the rôle of proper vibrations, and their discrete 'energy levels' the rôle of the proper frequencies of these proper vibrations; and all this follows from the new theory, once it is accepted, as simply and neatly as in the theory of elastic bodies, which we mentioned as a simile" [Sch52, page 114]. However, he never found a satisfactory vibrational interpretation of the Schrödinger equation [Wes01]. In order to make progress, this work is limited to the special case of the Schrödinger equation for a single-electron wave function without spin in hydrogen-like atoms and relies on multiple speculative assumptions.

Specifically, the proposed vibrational interpretation is based on the assumption that de Broglie's hypothetical internal clock of the electron [dB25] manifests itself as a periodic oscillation of the electron's electromagnetic near-field, which is described in Section 2. This assumption is shared by several electron models including a modified Born-Infeld model of electrons [Kra23]. Additionally, this periodic oscillation of the electron is assumed to drive a stationary vibration of the atom's electromagnetic field while the electron orbits the atom's nucleus. As shown in Section 3, this kind of stationary vibration is described by the Schrödinger equation. While the significance of the presented vibrational interpretation in the special case of hydrogen-like atoms is limited, it leads to several interesting questions for future research as outlined in Section 4.

2 Electromagnetic Interpretation of de Broglie's Internal Clock

This section describes an electromagnetic oscillation in specific models of electrons, which can be interpreted as de Broglie's internal clock since it features the same frequency. First, the case of a resting electron is discussed, followed by a discussion of the more general case of a moving electron.

2.1 Case of Resting Electron

In his doctoral thesis [dB25], de Broglie associated each electron with a hypothetical periodic process, i.e., de Broglie's internal clock. He assumed that the angular frequency of this periodic process in a resting electron was the Compton angular frequency $\omega_C = mc^2/\hbar$ with the mass m of the electron, the speed of light c and the reduced Planck constant $\hbar = h/(2\pi)$. Historically, de Broglie's internal clock

was an extremely fruitful concept, as it provided a way to compute Bohr's energy levels of hydrogen and the observed wavelength of elementary particles in interference experiments, as well as leading to the Schrödinger equation and Bohm's pilot wave theory. On the other hand, de Broglie's relativistic derivation of the de Broglie wavelength $\lambda_B = h/p$ for a particle of momentum p has never been popular in the physics community, his original pilot wave theory has been ignored by most physicists [BV09, page 31], and he failed to develop his concept of phase waves into a field theory explaining the internal structure of elementary particles [Bro60]. The skepticism of many physicists towards de Broglie's hypothetical internal clock might have been caused in part by the fact that the theoretical frequency of the Zitterbewegung (German for "trembling motion") of Dirac electrons differs from the frequency of de Broglie's internal clock by a factor of 2.

However, there are no generally accepted experimental measurements neither of the Zitterbewegung frequency nor of the frequency of de Broglie's internal clock. Moreover, the standard calculation of the Zitterbewegung frequency might push the Dirac theory of electrons too far towards the Schwinger limit such that the calculated value might be incorrect [Kra24]. Therefore, this work is based on de Broglie's hypothetical internal clock ticking at the Compton frequency although this hypothesis has not been experimentally confirmed yet.

Some electron models are designed to include de Broglie's internal clock; for example, a modified Born-Infeld field-theoretical model of electrons [Kra23] and the non-relativistic bi-level electron model [Kra25b], which approximates the former model by the motion of two mass points: a center of mass, which represents the position of the whole electron, and a center of charge, which (in the rest frame of the electron) moves with the speed of light and the Compton angular frequency ω_C on a circular orbit around the center of mass at a radius given by the reduced Compton wavelength $\lambda\!\!\!^{-}_C = \hbar/(mc)$. This motion of the center of charge is called "spin motion". Since the details of the mentioned modified Born-Infeld model are less relevant in this work, the discussion here is limited to the bi-level electron model.

For any point outside the orbit of the center of charge (but sufficiently close to be affected by the field theoretical model [Kra23], i.e., within a few reduced Compton wavelengths), the spin motion causes a periodic change (at the rate of the Compton frequency) of the distance between the point and the center of charge. Consequently, the electric potential at the point due to the center of charge is periodically changing. Analogously, the magnetic vector potential at the point is also changing periodically. As a first approximation, a position-independent sinusoidal periodicity is assumed. The amplitude of this oscillation depends on the relative position of the point in a more complicated way. Fortunately, the details of this dependency are not relevant here, but it is assumed (already in the field theoretical model) that the amplitude is quickly falling off with increasing distance from the center of mass, which is necessary to avoid emission of electromagnetic radiation from resting electrons.

In order to describe the phase of this electromagnetic oscillation of angular frequency ω_C, a complex number $\exp(\mathrm{i}\omega_C t)$ is employed. It might be worth repeating that this high-frequency oscillation of the electromagnetic potential is limited to the near-field around the center of mass, i.e., within a few reduced Compton wavelengths.

2.2 Case of Moving Electron

If an electron is moving with speed v, the observation of the internal clock by a resting observer changes according to the theory of special relativity as discussed in detail by de Broglie [dB25]. Here, de Broglie's results imply that the angular frequency of the mentioned electromagnetic oscillation is increased to $\gamma\omega_C$ with the Lorentz factor $\gamma = 1/\sqrt{1 - v^2/c^2}$, i.e., it is increased in the same way as the so-called "relativistic mass". (In case of a periodically varying speed v, a time average of $\gamma\omega_C$ over one period appears to be suitable [Kra25a].) Furthermore, a dependency of the oscillation's phase on position is introduced due to the motion of the electron. This turns the position-independent oscillation into a wave of de Broglie wavelength $\lambda_B = h/p$ with $p = \gamma mv$. Thus, this wave features the same frequency and wavelength as de Broglie's phase wave of free electrons.

Therefore, the electromagnetic interpretation of de Broglie's internal clock from Section 2.1 naturally leads to an electromagnetic interpretation of de Broglie's phase wave. Note that in this interpretation, the phase wave of a single moving electron is characterized by an extremely high frequency $\gamma\omega_C$ and a wavelength λ_B, which is, for slowly moving electrons, much larger than the field's spatial extent of a few reduced Compton wavelengths. Also note that the electromagnetic field of a moving electron leads to another change of the electromagnetic field due to the changing distance from the electron's

charge and magnetic moment. This (relatively low-frequency) change and the high-frequency phase wave are assumed to superpose each other without affecting each other.

3 High-Frequency Vibration of a Hydrogen-Like Atom's Electromagnetic Field Driven by an Electron's Internal Clock

According to Bloch [Blo76], de Broglie's concept of phase waves inspired the Schrödinger equation. Thus, it is not surprising that the proposed electromagnetic interpretation of de Broglie's phase wave (see Section 2.2) leads to a new electromagnetic interpretation of the Schrödinger equation. The time-independent Schrödinger equation for a hydrogen-like atom, in particular, can be interpreted as describing stationary, high-frequency vibrations of the atom's electromagnetic field, which are driven by the internal clock of the atom's electron. To introduce the main features of this vibrational interpretation, this section begins by comparing it with vibrations of metal plates.

3.1 Analogy with Chladni Figures

Figure 1 illustrates how the German physicist Ernst Chladni (1756 – 1827) created vibrations of metal plates by drawing a violin bow over an edge of a metal plate, which he covered in sand to make the vibration visible. The resulting nodal patterns of stationary modes of vibration are still known as "Chladni figures". Ideally, the violin bow drives the vibration of the metal plate at a specific frequency, and all parts of the plate oscillate with the same frequency, while amplitudes and phases differ from point to point on the plate.

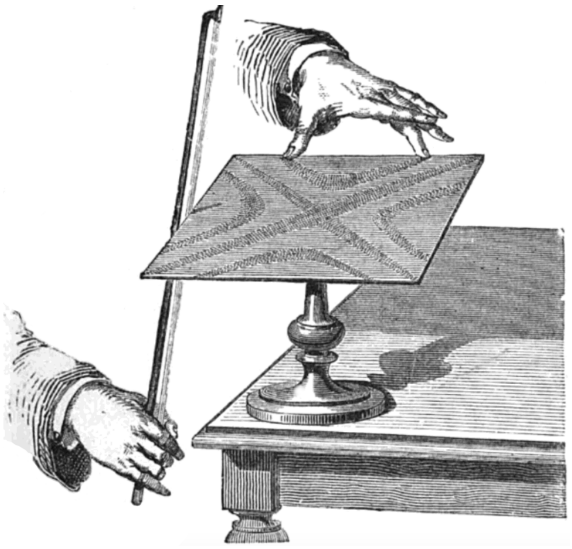

Figure 1: Ernst Chladni used a violin bow to drive vibrations in metal plates as depicted in this illustration reproduced from the textbook "Elementary Lessons on Sound" by William Henry Stone [Sto79, page 25].

In the proposed vibrational interpretation of the Schrödinger equation for a hydrogen-like atom, the atom's electromagnetic field is vibrating (in analogy with a vibrating metal plate), and this vibration is driven by the internal clock of the atom's electron (in analogy with a violin bow driving the vibration). For stationary solutions of the Schrödinger equation, every point of the vibrating electromagnetic field is oscillating with the same frequency (in analogy with stationary modes of vibration visualized by Chladni figures).

However, the analogy with Chladni's vibrating metal plates is far from being perfect. First, the electron is not in a fixed position; instead, it is assumed to move around the nucleus on a Bohr-Sommerfeld orbit, which would correspond to moving the violin bow along a curved edge of a metal plate in addition to drawing it over this edge. Secondly, in case of elliptic orbits, the speed of the electron is periodically changing; thus, the frequency of its internal clock (in the frame of the resting atom) is also periodically changing. This would correspond to a periodically changing frequency of the violin bow. Moreover, electronic orbits are precessing in various ways (as discussed in Section 3.3 below). In the analogy with a vibrating metal plate, this would correspond to a periodically changing shape of the plate's edge.

Thus, while the analogy with Chladni figures helps to understand Schrödinger's motivation to search for a vibrational interpretation of the Schrödinger equation, it also clarifies that an electronic internal clock moving on a precessing Bohr-Sommerfeld orbit is a much more complex driver of vibrations than a violin bow drawn with constant velocity at a fixed position.

3.2 Special Case of Non-Precessing Circular Orbits

Before considering the general case of an electromagnetic vibration driven by a moving electronic internal clock on a precessing Bohr-Sommerfeld orbit, it might be worthwhile to consider the special case of a circular Bohr orbit without any precession; in particular, because the speed of the electron and, therefore, the frequency of its internal clock (in the rest frame of the atom) is constant on a circular orbit.

In this case, it is sufficient (for our purposes) to consider a driven electromagnetic vibration on the circular orbit without considering the vibration of the electromagnetic field anywhere else. It turns out that the vibration on the circular orbit may be identified with de Broglie's phase wave of a free electron on the orbit [dB25, chapter 3][Kra25a] but for the fact that de Broglie assumed that there is no phase difference between the internal clock and its associated phase wave [dB25, page 9], which is not plausible for the phase difference between the driving oscillation of the internal clock and the driven vibration of the electromagnetic field at the internal clock's position. However, this issue is inconsequential as long as the phase difference is constant, which is a reasonable assumption for a stationary driven vibration.

As de Broglie showed, a "stable" phase wave of wavelength $\lambda_{\mathrm{B}} = h/p$ associated with a single electron of momentum p on a circular orbit of radius r has to satisfy the condition $n\lambda_{\mathrm{B}} = 2\pi r$ for a positive integer n, which is equivalent to Bohr's quantization condition $pr = n\hbar$. This result also applies to stationary vibrations driven by a single electron on a circular orbit. A similar result holds for non-precessing elliptic Bohr-Sommerfeld orbits [Kra25a]. It should be noted that these "stable" phase waves and stationary vibrations are *not* standing waves (with static nodal patterns); instead, they are superluminal waves racing with a phase velocity of about $c^2/v > c$ on the circular orbit in the same direction as the electron. A standing wave may be created by superposing two such waves moving in opposite directions; however, this would require two electrons.

3.3 General Case of Precessing Elliptic Orbits

Elliptic electronic orbits show various forms of precession; in particular, apsidal precession due to relativistic effects and Larmor precession due to the magnetic moment of the orbital angular momentum in external magnetic fields (including the magnetic field of the nucleus). In general, the precession of electronic orbits cannot be avoided. This has a direct consequence for the computation of stationary vibrations driven by electrons on elliptic orbits: a vibration is stationary for a specific elliptic orbit only if it is also stationary for all elliptic orbits that are generated from this orbit by precession. In the general case, it is therefore not sufficient to compute a stationary vibration for a specific closed orbit (as was the case in the previous section). Instead, a stationary vibration has to be stationary simultaneously for a whole family of orbits that are generated by precession. Otherwise, i.e., if a vibration is stationary only for some of those orbits, the inevitable precession of the electron's orbit would sooner or later result in an orbit with a non-stationary vibration, which means that the vibration is not stationary after all.

Thus, computing stationary vibrations for electrons on precessing elliptic orbits requires a very different approach from the one presented in the previous section. Schrödinger's wave mechanics addresses this issue by solving a wave equation on a region of space that includes all points that are part of the potentially relevant families of precessing elliptic orbits. This wave equation is discussed next.

3.4 Schrödinger's Wave Equation

One of the formal starting points for the single-electron Schrödinger equation, which Schrödinger provided in the second half of his second publication about wave mechanics [Sch26, page 509], is a

wave equation for a time-dependent wave function $\Psi(\mathbf{x}, t)$ in three-dimensional space with position-dependent phase velocity $u(\mathbf{x})$ [Sch26, Eq. (18)][Sch28, page 27]:

$$\nabla^2 \Psi(\mathbf{x}, t) - \frac{1}{u^2(\mathbf{x})} \frac{\partial^2}{\partial t^2} \Psi(\mathbf{x}, t) = 0. \tag{1}$$

Here, this wave equation is interpreted as describing a high-frequency electromagnetic vibration $\Psi(\mathbf{x}, t)$ driven by an electron on a family of precessing elliptic orbits. The phase velocity $u(\mathbf{x})$ is identified with the phase velocity of the phase wave associated with the electron because the phase difference between a stationary vibration and the driving electronic internal clock is assumed to be constant (see Section 3.2). This phase velocity $u(\mathbf{x})$ and the electron's speed $v(\mathbf{x})$ may be considered time-independent functions of $\mathbf{x}$ for an electron of constant mass m with time-independent potential energy $V(\mathbf{x})$ and constant, non-relativistic energy $E = mv^2(\mathbf{x})/2 + V(\mathbf{x})$.

A stationary vibration that solves this wave equation is characterized by a uniform angular frequency ω for all points $\mathbf{x}$, i.e., the time-dependent factor is $\exp(\mathrm{i}\omega t)$. The remaining (time-independent) part is named $\psi(\mathbf{x})$:

$$\Psi(\mathbf{x}, t) = \mathrm{e}^{\mathrm{i}\omega t} \psi(\mathbf{x}). \tag{2}$$

Inserting this equation into Eq. (1) and performing the time derivatives leads to a time-independent partial differential equation:

$$\nabla^2 \psi(\mathbf{x}) + \frac{\omega^2}{u^2(\mathbf{x})} \psi(\mathbf{x}) = 0. \tag{3}$$

For a uniform angular frequency ω, phase velocity $u(\mathbf{x})$ is directly related to wavelength $\lambda(\mathbf{x})$ [Sch26, Eq. (12)][Sch28, page 19]:

$$\lambda(\mathbf{x}) = \frac{2\pi u(\mathbf{x})}{\omega}. \tag{4}$$

Therefore, $\omega^2/u^2(\mathbf{x})$ in Eq. (3) can be replaced by $4\pi^2/\lambda^2(\mathbf{x})$:

$$\nabla^2 \psi(\mathbf{x}) + \frac{4\pi^2}{\lambda^2(\mathbf{x})} \psi(\mathbf{x}) = 0. \tag{5}$$

Wavelength $\lambda(\mathbf{x})$ is equal to the wavelength of the electron's phase wave, which is the de Broglie wavelength $\lambda_\mathrm{B}(\mathbf{x}) = h/p(\mathbf{x})$ with the electron's momentum $p(\mathbf{x})$. In non-relativistic approximation, $p(\mathbf{x})$ is given by $mv(\mathbf{x})$, where the speed $v(\mathbf{x})$ may be computed from the kinetic energy $E - V(\mathbf{x}) = mv^2(\mathbf{x})/2$:

$$\lambda(\mathbf{x}) = \frac{2\pi\hbar}{p(\mathbf{x})} = \frac{2\pi\hbar}{mv(\mathbf{x})} = \frac{2\pi\hbar}{\sqrt{2m(E - V(\mathbf{x}))}}. \tag{6}$$

Inserting this expression into Eq. (5) results in the following equation [Sch26, Eq. (18″)][Sch28, page 27]:

$$\nabla^2 \psi(\mathbf{x}) + \frac{2m(E - V(\mathbf{x}))}{\hbar^2} \psi(\mathbf{x}) = 0, \tag{7}$$

which leads to a well-known form of the time-independent Schrödinger equation:

$$-\frac{\hbar^2}{2m} \nabla^2 \psi(\mathbf{x}) + V(\mathbf{x}) \psi(\mathbf{x}) = E \psi(\mathbf{x}). \tag{8}$$

Thus, at least for stationary wave functions $\Psi(\mathbf{x}, t)$ of a single, non-relativistic electron in a time-independent potential $V(\mathbf{x})$, Eq. (8) is just an alternative form of Eq. (1). Therefore, the proposed vibrational interpretation of Eq. (1) equally applies to the time-independent Schrödinger equation Eq. (8).

3.5 Time-Dependent Schrödinger Equation

In the proposed vibrational interpretation, the wave equation Eq. (1) is more closely related to the physical scenario of a driven vibration than the time-independent Schrödinger equation Eq. (8). Many textbooks on quantum physics, however, do not mention Eq. (1) and derive the time-independent

Schrödinger equation Eq. (8) from the time-dependent Schrödinger equation (here with a time-independent potential energy $V(\mathbf{x})$):

$$\mathrm{i}\hbar\frac{\partial}{\partial t}\Psi(\mathbf{x},t) = -\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{x},t) + V(\mathbf{x})\Psi(\mathbf{x},t). \tag{9}$$

For a stationary wave function $\Psi(\mathbf{x},t) = \mathrm{e}^{\mathrm{i}\omega t}\psi(\mathbf{x})$ (see Eq. (2)), the time-dependent Schrödinger equation Eq. (9) can be derived from the wave equation Eq. (1) as follows:

$$\begin{aligned}
0 &= \nabla^2\Psi(\mathbf{x},t) - \frac{1}{u^2(\mathbf{x})}\frac{\partial^2}{\partial t^2}\Psi(\mathbf{x},t) && (10)\\
&= \nabla^2\Psi(\mathbf{x},t) - \frac{\mathrm{i}\omega}{u^2(\mathbf{x})}\frac{\partial}{\partial t}\Psi(\mathbf{x},t) && (11)\\
&= \nabla^2\Psi(\mathbf{x},t) - \frac{\mathrm{i}}{\omega}\underbrace{\frac{\omega^2}{u^2(\mathbf{x})}}_{4\pi^2/\lambda^2(\mathbf{x})}\frac{\partial}{\partial t}\Psi(\mathbf{x},t) && (12)\\
&= \nabla^2\Psi(\mathbf{x},t) - \frac{\mathrm{i}}{\omega}\frac{2m(E-V(\mathbf{x}))}{\hbar^2}\frac{\partial}{\partial t}\Psi(\mathbf{x},t) && (13)\\
&= \nabla^2\Psi(\mathbf{x},t) - \frac{2m}{\hbar^2}\frac{\mathrm{i}E}{\omega}\frac{\partial}{\partial t}\Psi(\mathbf{x},t) + \frac{2m}{\hbar^2}\frac{\mathrm{i}}{\omega}V(\mathbf{x})\underbrace{\frac{\partial}{\partial t}\Psi(\mathbf{x},t)}_{\mathrm{i}\omega\Psi(\mathbf{x},t)} && (14)\\
\mathrm{i}\hbar\underbrace{\frac{-E}{\hbar\omega}}_{?}\frac{\partial}{\partial t}\Psi(\mathbf{x},t) &= -\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{x},t) + V(\mathbf{x})\Psi(\mathbf{x},t) && (15)
\end{aligned}$$

Except for the factor $-E/(\hbar\omega)$, Eq. (15) matches the time-dependent Schrödinger equation Eq. (9). Thus, for stationary wave functions $\Psi(\mathbf{x},t)$, the time-dependent Schrödinger equation Eq. (9) is equivalent to the wave equation Eq. (1) if and only if

$$-E = \hbar\omega. \tag{16}$$

The minus sign is due to the plus sign in the factor $\exp(+\mathrm{i}\omega t)$, which is just a convention; thus, it is not of particular interest here. (Note that this work follows Schrödinger's convention, which differs from the convention of many modern textbooks.) Without the minus sign, Eq. (16) looks like the Planck relation between *photon energy* and *photon angular frequency*. Here, however, E is an electron's energy and ω is the angular frequency of a single-electron wave function. Thus, the physical content of Eq. (16) is not (directly) related to the Planck relation.

Eq. (16) represents a remarkable result for a couple of reasons. First, the value of $\omega = -E/\hbar$ does not (in general) agree with the time-average of $\gamma\omega_\mathrm{C}$ suggested by the proposed interpretation based on de Broglie's internal clock (see Section 2.2). Second, and more importantly, since $E = mv^2(\mathbf{x})/2 + V(\mathbf{x})$ and assuming that the zero point of the potential energy $V(\mathbf{x})$ is arbitrary, setting $\omega = -E/\hbar$ implies that the zero point of ω is just as arbitrary, and, therefore, it is difficult to assign physical meaning to the angular frequency $\omega = -E/\hbar$. On the other hand, the correctness of the standard form of the time-dependent Schrödinger equation Eq. (9) appears to hinge on Eq. (16).

One interpretation of this situation is that $\omega = -E/\hbar$ is a legitimate choice (which is required if Eq. (9) is to be used) because ω is as unobservable as the phase ωt. Therefore, in Eq. (4), i.e.,

$$\lambda(\mathbf{x}) = \frac{2\pi u(\mathbf{x})}{\omega},$$

the only observable quantity is the (de Broglie) wavelength $\lambda(\mathbf{x})$. Thus, for any value of the unobservable angular frequency $\omega \neq 0$, a suitable value of the unobservable phase velocity $u(\mathbf{x})$ can be chosen such that Eq. (4) produces the correct observable value of $\lambda(\mathbf{x})$. For these reasons, while the value of $\omega = -E/\hbar$ is not necessarily physically meaningful (i.e., not necessarily related to the angular frequency of de Broglie's internal clock), it is a legitimate choice, and (for this choice) the standard form of the time-dependent Schrödinger equation Eq. (9) is consistent with the wave equation Eq. (1).

Of course, not everyone is happy with this state of affairs, and the issue is still being debated among physicists [Gri95, Mun17a, Mun17b, Kra26].

Schrödinger himself was aware of the issue and commented that the relation $\nu = E/h$ makes sense only "if E is absolute and not, as in classical mechanics, indefinite to the extent of an additive constant" [Sch28, page 19]. On the other hand, in a later paper, he was apparently not concerned about this additive constant since he wrote that "a general additive constant, C, let us say, which should be added to all the ν_n's [...], (and corresponds to the 'rest-energy' of the particle) does not alter the essentials" [Sch28, page 44]. Similarly, in his report for the 1927 Solvay conference, he mentioned "that, considered from de Broglie's point of view, the multi-dimensional theory [of wave mechanics] is committed to a so to speak truncated view of the frequency, in that it subtracts once and for all from all frequencies the rest frequency ν_0" [BV09, page 457].

Is it true that "the essentials" are not altered by this "truncated view of the frequency"? The extraordinary success of the Schrödinger equation for the description of systems like the hydrogen atom and the harmonic oscillator appears to support Schrödinger's claim. In order to check it, let us consider the time average $\langle \gamma\omega_\mathrm{C} \rangle$, which is the value of ω suggested by the proposed vibrational interpretation (see Section 2.2):

$$\langle \gamma\omega_\mathrm{C} \rangle = \frac{\langle \gamma mc^2 \rangle}{\hbar} \approx \frac{mc^2}{\hbar} + \frac{\langle mv^2(\mathbf{x}) \rangle}{2\hbar}. \tag{17}$$

For the hydrogen problem, the time average of the kinetic energy $\langle mv^2(\mathbf{x})/2 \rangle$ equals $-\langle V(\mathbf{x})/2 \rangle$ because of the virial theorem. Thus:

$$\frac{\langle mv^2(\mathbf{x}) \rangle}{2} = -\frac{\langle V(\mathbf{x}) \rangle}{2} = -\frac{\langle mv^2(\mathbf{x}) \rangle}{2} - \langle V(\mathbf{x}) \rangle = -E. \tag{18}$$

With this relation, the time average $\langle \gamma\omega_\mathrm{C} \rangle$ becomes:

$$\langle \gamma\omega_\mathrm{C} \rangle \approx \frac{mc^2}{\hbar} - \frac{E}{\hbar}. \tag{19}$$

Thus, $\omega = -E/\hbar$ matches $\langle \gamma\omega_\mathrm{C} \rangle$ (in non-relativistic approximation) except for the additive constant $mc^2/\hbar$.

For the potential energy $V_\mathrm{SHO}(\mathbf{x})$ of a simple harmonic oscillator, the time-averaged kinetic energy $\langle mv^2(\mathbf{x})/2 \rangle$ equals $\langle V_\mathrm{SHO}(\mathbf{x}) \rangle$ because of the virial theorem. Thus:

$$\frac{\langle mv^2(\mathbf{x}) \rangle}{2} = \langle V_\mathrm{SHO}(\mathbf{x}) \rangle = \frac{E_\mathrm{SHO}}{2}. \tag{20}$$

Therefore, $\langle \gamma\omega_\mathrm{C} \rangle \approx mc^2/\hbar + E_\mathrm{SHO}/(2\hbar)$.

On first sight, these results might appear to support Schrödinger's claim that "the essentials" (in particular, differences between frequencies of stationary states) are not altered by the "truncated view of the frequency". Remember, however, that the virial theorem was necessary to derive these results. For other potential energies, comparable relations might not exist, and the "truncated view of the frequency" might be a much worse approximation than it is for the hydrogen problem and the harmonic oscillator. This might be part of the reason why the "truncated" time-dependent Schrödinger equation Eq. (9) is particularly useful for the analysis of a free particle, the hydrogen problem, and the harmonic oscillator.

3.6 Vibrational Interpretation of Single-Electron Wave Functions

Here, a wave function $\Psi(\mathbf{x}, t)$ is understood as a stationary solution of Eq. (1) of the form $\mathrm{e}^{\mathrm{i}\omega t}\psi(\mathbf{x})$ where $\psi(\mathbf{x})$ solves the time-independent Schrödinger equation Eq. (8) for a non-relativistic electron of constant energy E orbiting the nucleus of a hydrogen-like atom. Based on the vibrational interpretation of Eqs. (1) and (8) presented in Section 3.4, a wave function is interpreted as either a stationary electromagnetic vibration driven by the oscillation of the electron's internal clock or a superposition of such wave functions. Superpositions are included in this interpretation, because Eqs. (1) and (8) are homogeneous linear partial differential equations; thus, their solutions form linear vector spaces and, therefore, superpositions of solutions are also solutions.

However, the inclusion of superpositions means that not all wave functions can be interpreted as stationary vibrations driven by a single electron. The standing waves on circular orbits mentioned in Section 3.2 are examples of such superpositions of wave functions. Schrödinger commented on a related issue after de Broglie presented his report at the 1927 Solvay Conference [BV09, page 401]:

> Mr Schrödinger. — Mr de Broglie says that in the case of the hydrogen atom his hypothesis leads to *circular* orbits. That is true for the particular solutions of the wave equation that one obtains when one separates the problem in polar coordinates in space [...]. But in the case of a degeneracy (as he considers it here) it is, in reality, not at all the particular solutions which have a significance, but only an *arbitrary* linear combination, with constant coefficients, of all the particular solutions belonging to the same eigenvalue, because there is no means of distinguishing between them, all linear combinations being equally justified in principle. In these conditions, much more complicated types of orbit will certainly appear. But I do not believe that in the atomic domain one may still speak of 'orbits'.

While Schrödinger apparently interpreted this issue as evidence that the concept of electronic orbits is questionable in the atomic domain, one could also conclude that the Schrödinger equation (on its own) is insufficient to determine electronic orbits unambiguously. In any case, identifying those wave functions that can be interpreted as stationary vibrations driven by a single electron is a topic of future work, which is discussed in the next section.

In this section, one more property of the proposed vibrational interpretation of wave functions is discussed, namely the scaling of wave functions. In quantum mechanics, scaling a wave function by a complex number usually does not change the corresponding physical state. This is often motivated by the probability interpretation of normed wave functions according to the Born rule and the assumption that an overall phase of a wave function is not observable. For the proposed vibrational interpretation of wave functions, it should be noted that the amplitude of the oscillation of an electronic internal clock is fixed (in the rest frame of the electron) and, therefore, the amplitude of a stationary vibration driven by a single electron is also fixed. Thus, scaling the amplitude of a stationary vibration by a real number is physically impossible and all wave functions that differ only by a real scaling may be considered the same physical vibration. If we additionally assume that shifting the phase of a stationary vibration does not change the represented physical state, then all wave functions that differ only by scaling with a complex number may be considered the same physical vibration. In this sense, stationary vibrations may be scaled like wave functions in quantum mechanics.

4 Discussion and Future Work

The vibrational interpretation presented in this work is based on a modified Born-Infeld model of electrons [Kra23] and the concept of vibrations driven by periodic oscillations. It is motivated by Schrödinger's intuition that a vibrational interpretation of quantum mechanics should be feasible. In its current form, the presented interpretation is too speculative to serve as a classical model of Schrödinger's equation for hydrogen-like atoms. Thus, more research on the classical model implied by the presented interpretation is necessary to decide whether it is actually described by the Schrödinger equation.

Another point that warrants future research is mentioned in Section 3.5: not all solutions of the time-independent Schrödinger equation can be interpreted as stationary vibrations driven by a single electron. Identifying those solutions that allow for this interpretation would help to clarify their relation to the textbook solutions of the non-relatistic hydrogen problem. Some research related to this issue has been published by none other than Planck [Pla40] and more recently by Robert [Rob17].

Section 3.4 mentions the time-averaged angular frequency $\langle \gamma \omega_C \rangle$ of the proposed electromagnetic vibration driven by an electron's internal clock and its relation to the common choice $\omega = \pm E/\hbar$. Schrödinger's quote in Section 1 suggests that he considered the relation $\omega = \pm E/\hbar$ an important achievement of wave mechanics. Thus, a different value of the angular frequency in the proposed interpretation certainly justifies more research.

Related to the issue of the angular frequency of the proposed vibration is the interpretation of the time-dependent Schrödinger equation and, in particular, of the imaginary unit that is part of it. From the point of view of the presented interpretation and the discussion in Sections 2 and 3, complex

numbers are just a convenient way of quantifying phases of oscillations—comparable to the use of complex numbers in the analysis of electrical AC circuits. Whether complex numbers play a more fundamental role in quantum mechanics is another open question.

Further topics for future work include vibrational interpretations of the Schrödinger-Pauli equation (including the Pauli exclusion principle), the Dirac equation, unbounded solutions of these equations, the Born rule, etc.

5 Conclusion

This work presents a speculative interpretation of the time-independent and time-dependent Schrödinger equations for the electron of a hydrogen-like atom as a description of high-frequency stationary vibrations of the atom's electromagnetic field driven by the internal clock of the electron orbiting the atom's nucleus. While this interpretation is incomplete and too speculative to provide a classical model of the quantum mechanical energy levels of the electron, it sketches several features of such a model.

Some of these features challenge existing interpretations of the Schrödinger equation, and, therefore, warrant further research. One of these features, namely the angular frequency of the proposed vibration, suggests that Schrödinger's idea to "truncate" this frequency by consistently subtracting the electron's Compton frequency from it might not only be one of his most influential original contributions to quantum mechanics, but at the same time it might have prevented him from appreciating the importance of the electron's Compton frequency for a feasible vibrational interpretation of the Schrödinger equation, which he had been trying to develop for decades. *Isn't it ironic?*

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A Revisions

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