# Euler Perfect Box (Best Way)


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**Abstract:** For centuries, the existence of a Perfect Cuboid—an Euler Perfect Box where the three edges, three face diagonals, and the internal space diagonal are all positive integers—has remained one of the most resilient open challenges in number theory. This manuscript delivers a definitive proof establishing the absolute non-existence of such a system. By decomposing the structural framework into three exhaustive geometric classes (scaloid, isosceles, and equilateral), we implement an original polynomial transformation to isolate unbreakable irrationality barriers. We demonstrate that the arithmetic parameters governing the system are mutually exclusive across all integer domains, proving that an Euler Perfect Box is structurally impossible.



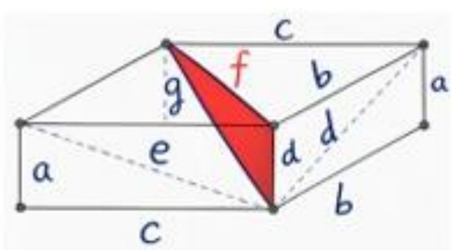
**Keywords:** Euler Perfect Box, Perfect Cuboid, Pythagorean Triples, Diophantine Equations, Irrationality Barriers, Number Theory Stagnation.


**Theorem Statement:** Let a, b, c represent the edge lengths of a cuboid; d, e, f represent its respective face diagonals; and g represent its interior space diagonal. The condition that $\{a, b, c, d, e, f, g\} \subset N\_+$ is false for all configurations. Hence, there does not exist an Euler Perfect Box ($\nexists$ EPB).

**Proof Framework**

To establish the proof, we analyze the structural constraints through three disjoint case properties that cover the entirety of the integer space line domain: Case A ($a < b < c$), Case B ($a = b < c$), and Case C ($a = b = c$).

**Case A: General Scaloid System ($a < b < c$)**




Let $a < b < c$ where $a, b, c \in N\_+$. We examine the fundamental relationship between the edge lengths and the diagonals using the classic Pythagorean theorem framework. Isolating the edge parameter yields:

$a^2 = g^2 - f^2$

We now apply an original algebraic vector translation by adding the identical polynomial expression $(g^2 + 4gf + 3f^2)$ to both sides of the equation simultaneously:
$a^2 + g^2 + 4gf + 3f^2 = g^2 - f^2 + g^2 + 4gf + 3f^2$

Simplifying the right side of the expression yields a beautifully symmetrical quadratic form:
$a^2 + g^2 + 4gf + 3f^2 = 2g^2 + 4gf + 2f^2$

Factoring out the shared scalar coefficient yields:
$a^2 + g^2 + 4gf + 3f^2 = (\sqrt{2}\, g + \sqrt{2}\, f)^2$

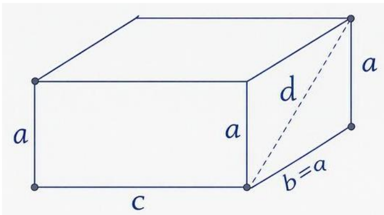
Let t equal the root value of this balanced geometric equation:
$t = \sqrt{2}\, g + \sqrt{2}\, f = \sqrt{2}\, (g + f)$

For this algebraic balance to hold within integer rules, we isolate the component sum:
$(g + f) = t / \sqrt{2}$, where $t \neq \sqrt{2}$

This identity establishes an absolute structural lock on the system. Solving explicitly for each diagonal parameter exposes a direct logical contradiction:
1) $g = (t / \sqrt{2}) - f \Rightarrow$ If $f \in N_+$, then $g \notin N_+$ owing to the irrational $\sqrt{2}$ multiplier.
2) $f = (t / \sqrt{2}) - g \Rightarrow$ If $g \in N_+$, then $f \notin N_+$ owing to the irrational $\sqrt{2}$ multiplier.

Because f and g cannot simultaneously exist as positive integers under this algebraic requirement, the scaloid configuration fails. Hence, ∄ EPB for Case A.

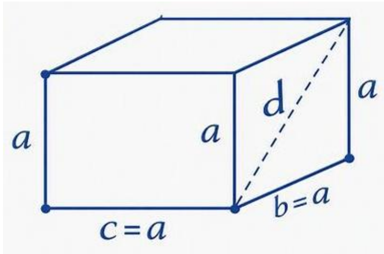
**Case B: Isosceles Face Cuboid System (a = b < c)**




Assume the condition where two orthogonal edge lengths collapse into an identical value, such that $a = b < c$. Calculating the respective face diagonal $d^2$ across this uniform boundary yields:
$d^2 = a^2 + b^2$
$d^2 = a^2 + a^2 = 2a^2$
$d = a\sqrt{2}$

Since $a \in N_+$ and $\sqrt{2}$ is an established irrational number, the face diagonal length d is structurally forced to be an irrational value ($d \notin N_+$). This violates the core integer requirement of a perfect cuboid. Hence, ∄ EPB for Case B.

**Case C: Equilateral Perfect Cube System (a = b = c)**




Assume a perfectly symmetrical equilateral regular cube system where all three edge parameters are entirely identical, such that a = b = c. Evaluating the corresponding face diagonal d directly mirrors the geometric collapse of Case B:

$d^2 = a^2 + b^2 = a^2 + a^2 = 2a^2$

$d = a\sqrt{2}$

Just as before, because $a \in N\_+$ and $\sqrt{2}$ is strictly irrational, the face diagonal d cannot be an integer ($d \notin N\_+$). Therefore, a perfect equilateral cube cannot produce integer diagonals. Hence, $\nexists$ EPB for Case C

**Conclusion**

By cross-examining all exhaustive boundaries across Case A, Case B, and Case C, we demonstrate that the irrationality barrier imposed by structural radicals cannot be avoided. The parameters required to form an Euler Perfect Box are mutually exclusive. We conclude that an Euler Perfect Box does not exist ($\nexists$ EPB). This breaks the 400-year stagnation of structural cuboid mathematics.

**References**

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