Solving the Hubble tension problem with the Dark Matter by Quantum Gravitation theory using Cosmic Chronometers and the PANTHEON+ project

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Abstract

This work presents the first application of the Dark Matter by Quantum Gravitation (DMbQG) theory to the study of dark matter (DM) at cosmological scales.

In the section 2.1, the first Friedmann equation is introduced within the DMbQG framework. A key feature of this approach is the explicit separation between baryonic matter density and dark matter density. It is shown that, in this theory, the dark matter density evolves with the cosmological scale factor following a power-law exponent of −2.5, in contrast to the ΛCDM model, where total matter density is not differentiated and scales as a^{-3}.

In Section 2.2, the second Friedmann equation and the cosmological conservation equation are presented. From the latter, the equation-of-state parameter w is derived for baryonic matter, dark matter, and dark energy. A novel result of this work is that, for dark matter, $w = -1/6$, which implies that its effective density is one-half of its physical density.

In the section 3, it is calculated the current deceleration parameter in the ΛCDM model and the DMbQG theory being tested with some works published. Also it is studied the cosmological deceleration function depending on the z red-shift according to the two theoretical frameworks, yielding two quite different functions. This result may be tested by the scientific community using the most recent and sophisticated experimental project such as DESI.

In the Section 4, the age of the Universe is calculated first within the standard ΛCDM model and subsequently using the DMbQG framework. A central result in this section is that the age of the Universe calculated with DMbQG theory is 14.3 Gyr using the Hubble constant $H_0 = 73.4$ km s^{-1} Mpc^{-1},i.e. the value obtained by the PANTHEON+ project, based on Type Ia Supernovae, placed into the Local Universe.

In contrast, applying this same value H_O within ΛCDM leads to an age of 12.8 Gyr, in tension with the oldest known astrophysical objects. Consequently, ΛCDM relies on the Planck Collaboration value, $H_0 = 67.4$ km s^{-1} Mpc^{-1}, which gives an age of 13.85 Gyr.

Given that the SHOES or PANTHEON+ projects provide a more direct measurement of the present-day expansion rate, while the Planck value is inferred from early-Universe data (cosmic microwave background), the former may better represent the current Hubble parameter.

The DMbQG theory offers the possibility to reconcile a higher Ho with a consistent estimate of the Universe's age.

The section five makes an important contribution to solve the "Hubble tension" problem because it is calculated the Ho value, using the two theoretical models, and the values obtained matches with the $H_{O\text{- PLANK}}$ in the framework ΛCDM model whereas matches with $H_{O\text{-PANTHEON+}}$ for the DMbQG theory.

The dataset used for this calculus is 34 points [z,H(z)] measured by the Cosmic Chronometers method. The mathematical tool to do the calculations is an elemental statistical method, however in spite its simplicity produces the two results explained in the previous paragraph.

This work offers to the scientific community an important advance about the solution of “Hubble tension” problem. Now it is the time for other researcher specialists to test the DMbQG theory using the same or different dataset and more sophisticated mathematical tools in order to solve this cosmologic problem that remains open for a decade.

1. Introduction

According to the Dark Matter by Quantum Gravitation theory (hereafter DMbQG), dark matter (DM) grows without spatial limit, and the total mass associated with a galaxy or a cluster is calculated using the so-called *direct mass* formula. The domain of this function is the halo, i.e., the region where the baryonic density is negligible compared to the DM density. This formula is derived in Section 8.8 of [1] Abarca (2024),

$M_{DIRECT}(<r) = \frac{a^2 \cdot \sqrt{r}}{G}$ where a^2 is the characteristic parameter of the galaxy or cluster.

Since baryonic matter is confined within galaxies, it is reasonable to assume that the halo region contains only DM. As a consequence of the direct mass formulation, the DM density function can be written as

$D_{DM} = L \cdot r^{-2.5}$ where $L = \frac{a^2}{8\pi G}$. Similarly to the *direct mass*, its domain is the halo region.

This radial dependence with exponent −2.5 is translated into the Hubble function because the DM density exhibits the same dependence on the cosmic scale factor (a), the variable associated with the Hubble expansion. The exponent −2.5 for the DM density constitutes the main novelty of this work and explains why the estimated age of the Universe is increased relative to the value obtained in the standard ΛCDM model.

The DMbQG theory was initially developed through the study of the rotation curves of the Milky Way (MW) and M31. In [1] Abarca (2024), the theory was formulated and successfully tested using several published datasets for the rotation curves of these galaxies. In [2] Abarca (2024), the theory was extended to galaxy clusters and tested using observational studies of the turnaround radius and its associated mass in the Virgo cluster.

In [3] Abarca (2025), it was shown that the Navarro–Frenk–White (NFW) profile for the virial mass is equivalent to the virial mass calculated within the DMbQG framework, using rotation curves of the MW and M31.

In [4] Abarca (2026), it was demonstrated that the NFW profile for the total mass is compatible with the direct mass formulation, the characteristic expression of total mass in the DMbQG theory. Consequently, it is also consistent with the velocity decay law in the halo region of rotation curves, characterized by the exponent −0.25. In that work, in addition to the MW and M31, six galaxies outside the Local Group were analyzed to test the proposed framework.

In [5] Abarca (2026), within the DMbQG framework, the mass associated with the turnaround radius of galaxy clusters was calculated, showing results approximately 5% lower than those obtained using the corresponding formula in the standard model.

This brief summary of the previous works highlights that the DMbQG theory was initially developed to describe dark matter in galaxies and was subsequently extended successfully to galaxy clusters.

The present work represents the first test of the theory at cosmological scale. In the section 2 are introduced the Friedmann equations using the DMbQG theory and the standard ΛCDM model. It is remarkable how in the DMbQG framework is demonstrated using the second Friedmann equation that the effective DM density is a half of the physical DM one. This theoretical finding has several successful consequences studied in the sections 3 and 4. Also in this section, it is calculated the transition between the decelerating-accelerating *eras*, by the two frameworks and the theoretical calculus are tested with other published results.

In the section 3, it is studied the current cosmologic deceleration parameter and his function associated depending on z red-shift according the two frameworks. The theoretical results are tested with other published results.

In the section 4, the age of the Universe is calculated within the DMbQG framework, yielding a value of 14.3 Gyr. This result is associated with the Hubble constant determined by the PANTHEON+ program, based on Type Ia supernovae ranging in red-shift from z = 0.001 to 2.26 , (H_0 = 73.5 km/s/Mpc). This method provides one of the most direct measurements of this cosmological parameter for the present-day Universe. See the paper [8] D.Brout. 2022. Additionally, in the frameworks of a flat ΛCDM model and PANTHEON projects the authors find Ω_m= 0.33 and this new value regarding the Ω_m= 0.31 (the Plank project) has improved some results in the new version of this work, namely the Universe age.

However, within the standard ΛCDM model, the value H_0 = 73 km/s/Mpc cannot be adopted because it would lead to an age of the Universe in tension with the oldest astrophysical objects. Consequently, the value obtained by the Planck collaboration is typically used instead, based on the most precise measurements of the cosmic microwave background (CMB), yielding (H_0 = 67.4 km/s/Mpc). See the paper [7] Plank collaboration. 2018.

Despite the fact that the Plank collaboration project has measured the CMB with an impressive precision, it is not the best method to calculate the current Hubble constant because it is needed a Universe evolution model, whereas the SHOES or PANTHEON+ projects are the most direct methods because the both ones use Type Ia supernovae. Precisely, the DMbQG theory calculates its best Universe age using the SHOES/PANTHEON+ Hubble constant.

The gap between the Hubble constant measured by the SHOES or PANTHEON + projects and the value calculated based on the Plank data of CMB generates an open problem in the current cosmology known as "Hubble tension" , which is one of the most significant over the past decade. See [9-10-11-12-13-14-15-16-17-18-19].

The section five makes an important contribution to solve the "Hubble tension" problem because it is calculated the Ho value, using the two theoretical models, the ΛCDM model and the DMbQG theory, and the values obtained matches with the $H_{O\text{- PLANK}}$ in the framework the ΛCDM model whereas matches with $H_{O\text{-PANTHEON+}}$ for the DMbQG theory.

The dataset used for this calculation consists of 34 [z, H(z)] data points measured using the Cosmic Chronometers method and compiled from ten different papers published by well-established international research teams. The mathematical procedure used for the calculation is based on a simple statistical method. Nevertheless, despite its simplicity, it yields the two results described in the previous paragraph.

2. Friedmann equations

2.1 First Friedmann equation

In this work it will supposed flat Universe because at the present knowledge it is a reasonable hypothesis and it will be neglected the radiation contribution as this work is focused on the *eras of matter and DE*.

With the previous hypothesis the first Friedman equation in the current ΛCDM paradigm is:

$$H^2(a) = \frac{8\pi G}{3}(\rho_m + \rho_\Lambda) \quad (2.1)$$

Where H(a) is the Hubble function, that by definition is: $H(a) = \frac{\dot{a}}{a}$, where a is the cosmic scale factor, normalised to unity today, ρ_m is the density of the matter (baryonic plus DM) and ρ_Λ is the density of the Dark Energy which is defined through the cosmological constant:

$$\rho_\Lambda = \frac{\Lambda \cdot c^2}{8\pi G} \quad (2.2)$$

In the framework of ΛCDM it is not necessary to distinguish baryonic versus DM, but in DMbQG it is. So the Friedman equation in this novel theory becomes:

$$H^2(a) = \frac{8\pi G}{3}(\rho_{BA} + \rho_{DM} + \rho_\Lambda) \quad (2.3)$$

The baryonic density decreases with the exponent -3, and ρ_Λ is constant, like in the ΛCDM paradigm, but the DM density decreases with the exponent -2.5 by the reason explained in the introduction, so the densities may be written like these:

$$\rho_{BA} = \rho_{BA}^0 a^{-3} \quad (2.4)$$

where the superscript 0 means the density at the present Universe.

$$\rho_{DM} = \rho_{DM}^0 a^{-2.5} \quad (2.5)$$

As ρ_Λ is constant may be considered that the exponent for a is zero, $\rho_\Lambda = \rho_\Lambda \cdot a^0$.

So the Friedman equation becomes:

$$H^2(a) = \frac{8\pi G}{3}\left(\rho_{BA}^0 a^{-3} + \rho_{DM}^0 a^{-2.5} + \rho_\Lambda\right) \quad (2.6)$$

The omega cosmological parameters are defined by the formulas:

$$\Omega_{BA} = \frac{\rho_{BA}^0}{\rho_C} \quad (2.7)$$

where $\rho_c = \frac{3H_0^2}{8\pi G}$ is the critic density of the Universe, H_0 is the Hubble constant at the present and ρ_{BA}^0 is the baryonic density at the present Universe.

$$\Omega_{DM} = \frac{\rho_{DM}^0}{\rho_C} \quad (2.8)$$

where ρ_{DM}^0 is the DM density at the present Universe, and

$$\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_C} \quad (2.9)$$

So using these parameters the first Friedman equation may be rewritten like this:

$$H^2(a) = H_0^2 \cdot (\Omega_{BA} a^{-3} + \Omega_{DM} a^{-2.5} + \Omega_\Lambda) \quad (2.10)$$

Similarly to the equation (2.10), the first Friedmann equation into the ΛCDM paradigm will be

$$H^2(a) = H_0^2 \cdot (\Omega_M a^{-3} + \Omega_\Lambda) \quad (2.11)$$

According to the paper [7] Plank Collaboration. 2018. The omega parameters are: $\Omega_{BA} = 0.05$, $\Omega_{DM} = 0.26$, $\Omega_\Lambda = 0.69$

According to the paper [8] PANTHEON+ project. 2022. In the framework of a flat-ΛCDM $\Omega_m = 0.334 \pm 0.018$ Then $\Omega_\Lambda = 0.67$ and supposing $\Omega_{BA} = 0.05$ it is obtained $\Omega_{DM} = 0.28$

The current Hubble constant value is an important cosmologic challenge during the last decade.

Namely the value H_0= 67.4±0.5 Km/s/Mpc was obtained by [7] Plank collaboration.2018 and the value H_0= 73.04±1.04 Km/s/Mpc was calculated by [6] A. Riess, SHOES team, 2022. The first one was obtained through the CBM data provided by the satellite Plank and the second one was got studying the supernovas into the Local Universe, z < 0.011 that approximately is equivalent to 47 Mpc. The PANTHEON+ project [8] uses a more wide set of supernovae, with z ranging from 0.001 to 2,26. The Hubble constant obtained is H_0= 73.5±1.1 Km/s/Mpc

2.2 Second Friedmann equation

An expression quite compact for the equation is:

$$\frac{\ddot{a}}{a} = \frac{-4\pi G}{3} \cdot \sum_i \rho_{EFFECTIVE} \quad (2.12)$$

where $\ddot{a}$ is the second time derivative of a and the effective density is referred to the baryonic matter, to DM and to DE.

The effective density, ρ_{EF} hereafter, is defined like this:

$$\rho_{EF} = \rho + \frac{3P}{c^2} \quad (2.13)$$

Being ρ the physical density, P is the pressure and c is the speed of light.

2.2.1 The cosmologic conservation equation

In general relativity (GR) the pressure verifies the equation:

$$P = w \cdot \rho \cdot c^2 \quad (2.14)$$

Where w is a factor that may be calculated by the cosmologic conservation equation on a specific condition.

The equation cosmologic conservation equation is:

$$\dot{\rho} + 3H\left(\rho + \frac{P}{c^2}\right) = 0 \quad (2.15)$$

where $\dot{\rho}$ is the time derivative, P is the pressure and H is the Hubble function H(a).

This equation has important consequences for each type of matter or DE because each one has a different effective density as it will be shown in the following theorem.

Theorem

Supposing that the formula for the density is $\rho = \rho_0 \cdot a^{-n}$, and adopting the formula for the pressure stated in the framework of G.R. The pressure is $P = w \cdot \rho \cdot c^2$, then by the cosmologic conservation equation may be demonstrated the following dependence:

$$3(1+w) = n \text{ or equivalently } w = n/3 - 1 \quad (2.16)$$

Proof: The temporal derivative of density is written by a change of variable and using the Hubble function ($H(a) = \dot{a}/a$) like this:

$$\dot{\rho} = \frac{d\rho}{dt} = \frac{d\rho}{da} \cdot \frac{da}{dt} = \frac{-n}{a} \cdot \rho \cdot \dot{a} = -n \cdot \rho \cdot H(a) \quad (2.17)$$

Substituting the previous result for the derivative $\dot{\rho}$ into the equation (2.15) it is obtained:

$$-n \cdot \rho \cdot H + 3H \cdot (\rho + w\rho) = 0 \ (2.18)$$

That is equivalent to: $H \cdot \rho \cdot \left(-n + 3 \cdot (1 + w)\right) = 0$ and from that equation it is obtained the thesis: n=3·(1+w)

In the table 1 is shown the factor w for the three types of substances:

Table 1	Factor w depending on exponent n	
Baryonic	n=3	W=0
DM	n= 2.5	W= -1/6
DE	n=0	W= -1

The factors w for the baryonic matter and for the DE are common in the ΛCDM paradigm but the factor w = -1/6 for the DM is exclusive in the DMbQG theory.

In the table 2 is shown the effective density for the three types of substances:

Table 2 Effective density for the baryonic mass, the DM and DE		
	Pressure	Effective density
Baryonic matter	$P = 0 \cdot \rho \cdot c^2$	$\rho_{BA}^{EF} = \rho_{BA}$
DM	$P = -1/6 \cdot \rho \cdot c^2$	$\rho_{DM}^{EF} = \rho_{DM}/2$
DE	$P = -1 \cdot \rho \cdot c^2$	$\rho_{DE}^{EF} = -2\rho_{\Lambda}$

The effective density for the DM is the novelty arisen in the framework of DMbQG theory whereas the other two ones are known in the ΛCDM model.

2.2.2 Second Friedmann equation and the transition decelerating-accelerating eras in the ΛCDM paradigm

By substitution of the effective densities calculated in the previous section is obtained:

$$\frac{\ddot{a}}{a} = \frac{-4\pi G}{3}(\rho_m - 2\rho_\Lambda) \quad (2.19)$$

This equation is useful to calculate the ratio a, when the acceleration was zero, i.e. the moment into the history of the Universe when the *era of the matter*, with acceleration negative changed to the *era of D.E.* with acceleration positive. So the acceleration zero verifies:

$$\rho_m - 2\rho_\Lambda = 0 \quad (2.20)$$

As in the ΛCDM model the matter decreases with the ratio a to the -3 power, like the baryonic matter, then $\boldsymbol{\rho_M = \rho_M^0 a^{-3}}$, the DE density is constant so the equation (2.20) becomes

$$\boldsymbol{\rho_M^0 a^{-3} - 2\rho_\Lambda} = 0 \quad (2.21)$$

that is equivalent to

$$\Omega_M a^{-3} - 2\Omega_\Lambda = 0 \quad (2.22)$$

Whose solution for a is: $a = \left(\frac{\Omega_M}{2\Omega_\Lambda}\right)^{1/3}$ (2.23)

By the formula z = 1/a -1 is calculated the red-shift associated to the factor scale *a*. In the table 3 are shown the values calculated for *a* and *z*. The subscript *Tda* means: Transition decelerating-accelerating.

Table 3 Transition decelerating-accelerating depending on Omega parameters - ΛCDM						
	Ω_{BA}	Ω_{DM}	Ω_M	Ω_Λ	a_{Tda}	z_{Tda}
Plank project [7]	0.05	0.26	0.31	0.69	0.608	0.645
PANTHEON+ [8]	0.05	0.28	0.33	0.67	0.627	0.595

If the parameter a is rounded to $a_{Tda} \approx 0.6$. that means that the Universe began to have a positive acceleration when he was a 60% of its current size.

2.2.3 Second Friedmann equation and the transition decelerating-accelerating eras in the DMbQG theory

By substitution of the effective densities, see the table 2, into the second Friedman equation is obtained

$$\frac{\ddot{a}}{a} = \frac{-4\pi G}{3}\left(\rho_{BA} + \frac{\rho_{DM}}{2} - 2\rho_\Lambda\right) \quad (2.24)$$

In this new framework, the acceleration zero has to verify the equation:

$$\rho_{BA} + \frac{\rho_{DM}}{2} - 2\rho_\Lambda = 0 \quad (2.25)$$

Introducing the dependence of the densities with the ratio a, like in the previous section is obtained:

$$\boldsymbol{\rho_{BA}^0 a^{-3}} + \frac{\rho_{DM}^0}{2}\cdot \boldsymbol{a^{-2.5}} - 2\boldsymbol{\rho_\Lambda} = 0 \quad (2.26)$$

which is equivalent to

$$\Omega_{BA} a^{-3} + \frac{\Omega_{DM}}{2} a^{-2.5} - 2\Omega_\Lambda = 0 \quad (2.27)$$

In the table 4 are shown the values calculated for a and z, where z =1/a -1.The subscript Tda means: Transition decelerating-accelerating.

Table 4 Transition decelerating-accelerating depending on Omega parameters - DMbQG						
	Ω_{BA}	Ω_{DM}	Ω_M	Ω_Λ	a_{Tda}	z_{Tda}
Plank project [7]	0.05	0.26	0.31	0.69	0.4649	1.15
PANTHEON+ [8]	0.05	0.28	0.33	0.67	0.4785	1.09

Discussion

The gap associated to z_{Tda} between ΛCDM vs DMbQG is virtually 0.5. This remarkable difference is originated especially by the fact that in the DMbQG theory the effective DM density is a half of physical density, although the exponent -2.5 instead -3 also has its importance.

2.2.4 Theoretical value of Z_{Tda} parameter in DMbQG theory versus published results

To study the transition decelerating-accelerating red-shift value is an impressive challenge that it has been possible thanks to the most sophisticated cosmologic experiments and the remarkable development of the theoretical cosmology in the last decades.

There are many others works, and the majority give a value for the parameter Z_{Tda} close to the one predicted by the ΛCDM model. However there are some others that gives a value closer to the one calculated by the DMbQG theory. See the three papers below [20],[21],[22]. Notice how the paper [21] also gives a value q_0 (the deceleration parameter that will be studied in the following chapter) compatible with the one calculated by the DMbQG theory.

[20]Verdugo.2026.

The authors present a generalized phenomenological parameterization of the deceleration parameter q(z). They constrained the free parameters of his model using late-time observational data from cosmic chronometers (CC), Pantheon+ Type Ia supernovae (SNIa),H II galaxies (HIIG), and intermediate-luminosity quasars (QSO). For the full data combination (CC+SNIa+HIIG+QSO), they obtain $q_0 = -0.25 \pm 0.04$ and a transition red shift $z_{Tda} \simeq 0.80$. Furthermore, they obtain a Hubble constant $H_0 = 72.9 \pm 0.6$ km/s/Mpc

[21] M. Koussour,.2026.

The authors, using 31 cosmic chronometer data points, 1701 Pantheon+ Type Ia supernovae, and 12 BAO measurements from DESI, they obtain the best-fit values $H_0 = 73.11 \pm 0.4$ km/s/Mpc, $q_0 = -0.524 \pm 0.06$ and $z_{Tda} \simeq 0.9$

[22] B. Santos.2011

Using the 557 type Ia supernovae from the Union2 Compilation set along with the current estimates involving the product of the CMB acoustic scale ℓ_A and the BAO peak at two different red shifts, and using a well-behaved parameterization for the deceleration parameter, the authors have estimated $z_{Tda} \approx 1$

3. The deceleration parameter

This parameter by definition is:

$$q = -\frac{a \cdot \ddot{a}}{\dot{a}^2} \quad \text{or equivalently } q = -\frac{\ddot{a}}{a \cdot H^2} \quad (3.1)$$

Notice that a negative deceleration parameter means an accelerating factor scale $\boldsymbol{a}$. i.e. if the Universe is accelerating the deceleration parameter is negative, as it happens at the present Universe.

3.1 Formula for the deceleration parameter

Before to obtain the formula it will be shown three identities used in the process:

1º As $H = \frac{\dot{a}}{a}$ then by the time derivative it is right to get $\frac{\ddot{a}}{a} = H^2 + \dot{H}$ (3.2)

2º The time derivative of the function Hubble may be changed like this:

$\dot{H} = \frac{dH}{da} \cdot \dot{a} = a \cdot H \cdot \frac{dH}{da}$ (3.3)

3º The Hubble function may be expressed by a dimensionless function like this:

In the framework of DMbQG theory, from the expression (2.10) $H^2(a) = H_0^2 \cdot (\Omega_{BA} a^{-3} + \Omega_{DM} a^{-2.5} + \Omega_\Lambda)$ it is defined the dimensionless function

$$E^2 = (\Omega_{BA} a^{-3} + \Omega_{DM} a^{-2.5} + \Omega_\Lambda) \quad (3.4)$$

In the framework of ΛCDM model, from the expression (2.11) $H^2(a) = H_0^2 \cdot (\Omega_M a^{-3} + \Omega_\Lambda)$

it is defined the dimensionless function

$$E^2 = (\Omega_m a^{-3} + \Omega_\Lambda) \quad (3.5)$$

So the Hubble function may be expressed like this:

$H^2(a) = H_0^2 \cdot E^2(a)$ or $H(a) = H_0 \cdot \sqrt{E^2(a)}$ (3.6)

And it is right to get the following identity

$\frac{1}{H} \cdot \frac{dH}{da} = \frac{1}{2E^2} \cdot \frac{dE^2}{da}$ (3.7)

Process to obtain the deceleration parameter formula

From the definition $q = -\frac{\ddot{a}}{a \cdot H^2}$ and by the identity (3.2) it is right to get

$q = -1 - \frac{\dot{H}}{H^2}$ (3.8)

And by substitution of identity (3.3) into (3.8) it is right to get:

$q = -1 - \frac{a}{H}\frac{dH}{da}$ (3.9)

And by substitution of identity (3.7) into (3.9) it is right to get:

$q = -1 - \frac{a}{2 \cdot E^2}\frac{dE^2}{da}$ (3.10)

Deceleration parameter formulas and the current value for the both theories

Finally it is possible to calculate the deceleration parameter using the function (3.4), for DMbQG theory or the function (3.5), for the ΛCDM model.

Below are written the formulas for the deceleration parameter as a compact expression for the two models:

$$q_{DMbQG} = \frac{0.5\cdot\Omega_{BA}a^{-3}+0.25\cdot\Omega_{DM}a^{-2.5}-\Omega_{\Lambda}}{\Omega_{BA}a^{-3}+\Omega_{DM}a^{-2.5}+\Omega_{\Lambda}} \quad (3.11)$$

Similarly for the ΛCDM model.

$$q_{\Lambda CDM} = \frac{0.5\cdot\Omega_{M}a^{-3}-\Omega_{\Lambda}}{\Omega_{M}a^{-3}+\Omega_{\Lambda}} \quad (3.12)$$

In the table 5 are represented the current deceleration parameter (a=1) according the two models and depending on the set of Omega parameters selects.

Table 5 Current decelerating parameter ΛCDM versus DMbQG

	Ω_{BA}	Ω_{DM}	Ω_{M}	Ω_{Λ}	$q_{0\text{ - }\Lambda CDM}$	$q_{0\text{- }DMbQG}$
Plank project [7]	0.05	0.26	0.31	0.69	-0.535	-0.6
PANTHEON+ [8]	0.05	0.28	0.33	0.67	-0.505	-0.575

3.2 Theoretical results versus published results

In the paper [23], year 2022, the authors, assuming a spatially flat Universe and having tested 8 kinematic parameterized models with H(z) data from Cosmic Chronometers and SNe Ia from Pantheon compilation, they have obtained the value H_0 =68.8 ±3.7 km/s/Mpc and q_0 = -0.58 ±0.13

In the paper [24],year 2023, named: Constrained evolution of effective equation of state parameter in non-linear f (R, Lm) dark energy model: Insights from Bayesian analysis of cosmic chronometers and Pantheon samples, the authors have obtained the value q_0= -0.61 ± 0.01

In the paper [6], year 2022, named: *A comprehensive Measurement of the Local Value of the Hubble Constant with 1 km/s/ Mpc uncertainty from the Hubble Space Telescope and the SH0ES Team*. The authors have obtained the value H_0 =73.30 ±1.04 km/s/Mpc and q_0= -0.51 ±0.024

Discussion

Although there are many other works, these three publications show that it is possible to find published results compatibles with the ΛCDM model (q_0 = - 0.505) and others compatibles with DMbQG theory (q_0 = - 0.575). It is remarkable the fact that the papers [23] and [24] both using the PANTHEON samples give a results that virtually match with the value $q_{0\text{- }DMbQG}$.

This results show that according to DMbQG theory, currently the Universe has a rate acceleration bigger than in the ΛCDM model because the contribution of DM density is a half of the baryonic density as it is shown clearly in the formulas (3.11) and (3.12) .

The deceleration parameter value obtained by DMbQG theory is consistent with the fact that current Hubble constant, measured by the project SHOES or PANTHEON+, is bigger than the one calculated by CMB in the framework of ΛCDM model, because according to the former model the Universe has a bigger rate of acceleration than in the last model.

The following natural step in the theoretical development will be to calculate the current Hubble value using the CMB Plank data in the framework of DMbQG theory. If the value obtained were close to the current H_0 measured by the projects SHOES and PANTHEON+ then the tension Hubble problem would be solved and as the reader knows, the tension Hubble is one of the most important open problems for the current cosmology.

3.3 Deceleration parameter as a function depending on z

The function depending on a, is given by the expression (3.9) $q(a) = -1 - \frac{a}{H}\frac{dH}{da}$ (3.9). The factor scale a depending on z is $a = {}^{1}\!/_{(1+z)}$ and conversely $z = \frac{1}{a} - 1$.

Using the change of variable it is right to get the deceleration parameter depending on z:

$q(z) = -1 + (1+z)\frac{1}{H}\frac{dH}{dz}$ (3.13)

Where H(z) =H_0·E(z) being:

Case DMBQG theory $E^2(z) = \Omega_{BA}(1+z)^3 + \Omega_{DM}(1+z)^{2.5} + \Omega_\Lambda$ (3.14)

Case ΛCDM model $E^2(z) = \Omega_m(1+z)^3 + \Omega_\Lambda$ (3.15)

By the expressions (3.13), (3.14) and (3.15) it is quite right to get the cosmologic deceleration function for the both theories:

$q_{DMbQG}(z) = \frac{\Omega_{BA}(1+z)^3 + 0.5\Omega_{DM}(1+z)^{2.5} - 2\Omega_\Lambda}{2\cdot E^2(z)}$ (3.16)

Where $E^2(z)$ is the equation (3.14)

Notice how the numerator of expression (3.16) is modulated by the coefficients (1, 0.5, -2) associated to the effectives densities in the framework of DMbQG theory.

Below is represented the cosmologic deceleration function for the ΛCDM model.

$q_{\Lambda\mathrm{CDM}}(z) = \frac{\Omega_m(1+z)^3 - 2\Omega_\Lambda}{2\cdot E^2(z)}$ (3.17)

Where $E^2(z)$ is the equation (3.15)

Notice how the numerator of expression (3.17) is modulated by the coefficients (1, -2) associated to the effectives densities in the framework ΛCDM model.

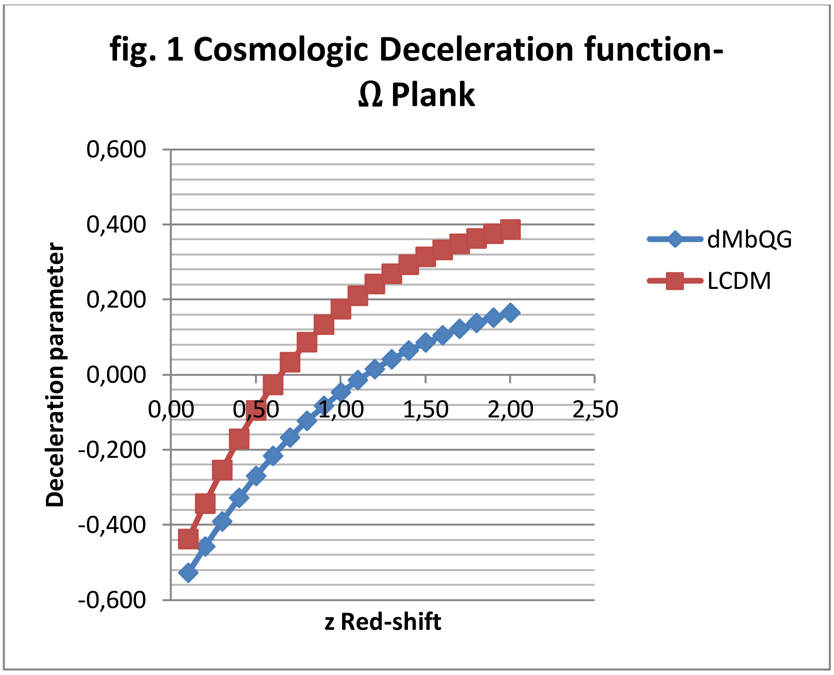
3.3.1 Deceleration function using data Plank for the Omega cosmologic parameters

According to the paper [7] Plank Collaboration. 2018. The omega parameters are: $\Omega_{BA} = 0.05$, $\Omega_{DM} = 0.26$, $\Omega_\Lambda = 0.69$

In this section are tabulated and plotted the functions (3.16) and (3.17) using the above omega parameters. The first column represents the factor scale a, given by a =1/(1+z). The value z = 0 corresponds to the present time.

Table 6 Cosmologic deceleration function – Ω Plank			
a	z	q(z) DMbQG	q(z) ΛCDM
1.000	0	-0.600	-0.535
0.909	0.10	-0.529	-0.439
0.833	0.20	-0.459	-0.344
0.769	0.30	-0.392	-0.255
0.714	0.40	-0.329	-0.172
0.667	0.50	-0.271	-0.096
0.625	0.60	-0.217	-0.028
0.588	0.70	-0.168	0.032
0.556	0.80	-0.124	0.086
0.526	0.90	-0.084	0.132
0.500	1.00	-0.048	0.174
0.476	1.10	-0.015	0.209
0.455	1.20	0.014	0.241
0.435	1.30	0.040	0.268
0.417	1.40	0.064	0.292
0.400	1.50	0.085	0.313
0.385	1.60	0.104	0.331
0.370	1.70	0.121	0.348
0.357	1.80	0.137	0.362
0.345	1.90	0.151	0.375
0.333	2.00	0.164	0.386

In the figure 1 is plotted the table 6 and it is seen how the gap between the both theories goes decreasing toward the current time.



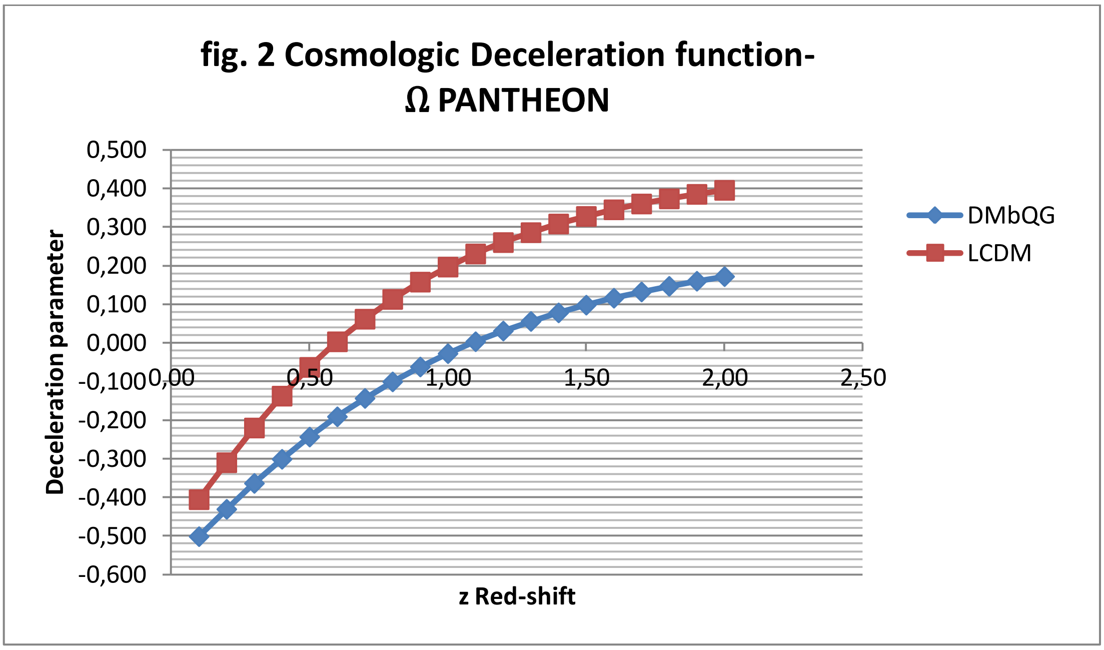
3.3.2 Deceleration function using the data PANTHEON for the Omega cosmologic parameters

According to the paper [8] PANTHEON+ project. 2022.In the framework of a flat-ΛCDM model $\Omega_m = 0.334 \pm 0.018$ Then $\Omega_\Lambda = 0.67$ and supposing $\Omega_{BA} = 0.05$ it is obtained $\Omega_{DM} = 0.28$

In this section are tabulated and plotted the functions (3.16) and (3.17) using the above omega parameters.

Table 7 Cosmologic deceleration function - Ω PANTHEON		
	DMbQG	ΛCDM
z	q(z)	q(z)
0	-0.575	-0.505
0.10	-0.502	-0.406
0.20	-0.431	-0.310
0.30	-0.364	-0.220
0.40	-0.301	-0.138
0.50	-0.244	-0.063
0.60	-0.191	0.003
0.70	-0.144	0.061
0.80	-0.101	0.113
0.90	-0.062	0.157
1.00	-0.028	0.196
1.10	0.003	0.230
1.20	0.031	0.260
1.30	0.056	0.285
1.40	0.078	0.308
1.50	0.098	0.328
1.60	0.116	0.345
1.70	0.132	0.360
1.80	0.147	0.373
1.90	0.160	0.385
2.00	0.172	0.395

In the figure 2 is plotted the table 7. It is clear how towards the ancient time Universe. the gap between the both theories is enormous and goes decreasing toward the current time.



These theoretical functions may be tested with recent results published of the DESI project. As the gap between the both functions is remarkable, this test may be crucial in order to validate or reject the DMbQG theory at the cosmological scales.

4. The age of the Universe

One of the most significant results obtained in this work is that the integral determining the age of the Universe is about 12.5% larger when computed within the DMbQG framework than in the standard ΛCDM model. See the section 4.4 and 4.5. This difference makes it possible to adopt the Hubble constant measured by the SHOES or PANTHEON+ programs i.e. Ho= 73.5 km/s/Mpc . Consequently, to calculate the age of the Universe, the DMbQG theory allows taking the value $H_{O\text{-}PANTHEON}$ whereas the ΛCDM model has banned this value because its Universe age would be incompatible with the age of the most ancient astrophysics objects, as it will be shown in the section 4.4.

4.1 The lower bound for the age Universe from the oldest astrophysical objects

As the age of the Universe has to be bigger than the oldest astrophysical object it is important to determine the age of this objects in order to compare with the age obtained by the cosmological models.

Although there are several types of the very ancient astrophysical objects, the globular clusters of stars placed in the galactic halos are one of the most reliable. By this method the best age range for the oldest stellar clusters is 13.0 Gyr to 13.5 Gyr. See [25-26-27-28-29].

4.2 The age of the Universe by the Hubble function

By definition $H(a) = \frac{\dot{a}}{a}$ (4.1)

where a(t) is the scale factor depending on the time and $\dot{a}$ is its time derivative. So considering $\dot{a} = \frac{da}{dt}$ it is right to write

$$dt = \frac{da}{a \cdot H(a)} \quad (4.2)$$

and by integration it is possible to obtain the age of the Universe

$$t_0 = \int_0^1 \frac{da}{a \cdot H(a)} \quad (4.3)$$

4.3 The Hubble time

The inverse of Hubble constant is called Hubble time. $t_H = \frac{1}{H_0}$

There are several techniques to measure H_0 and the intrinsic errors are lower than the gap between the greater and the lower values of H_0. See in the table 8 the different Hubble time associated to each Hubble constant.

Table 8 The Hubble time with different methods		
Method	H_0 Km/s/Mpc	Hubble time $1/H_0$ Gyr
Plank (CMB) See [7]	67.4±0.5	14.5
Supernova (SHOES) See [6]	73±1	13.4
PANTHEON+/SHOES See [8].[30]	73.5±1	13.3

4.4 The age of the Universe in the ΛCDM paradigm

As it was pointed in the section 2.1, the Hubble function in this cosmological model is:

$$H(a) = H_0 . \sqrt{\Omega_M a^{-3} + \Omega_\Lambda} \quad (4.4)$$

So the integral becomes

$$t_0 = \frac{1}{H_0} \int_0^1 \frac{da}{a \cdot \sqrt{\Omega_M a^{-3} + \Omega_\Lambda}} \quad (4.5)$$

In the integral has been neglected the *radiation era*, because it has no significance as its duration time is estimated to be lower than a million years.

So considering the Plank project parameters [7] $\Omega_M = 0.05 + 0.26 = 0.31$ and $\Omega_\Lambda = 0.69$ the integral value is:

$$I_{Plank} \approx 0.9553 \quad (4.6)$$

According to the paper [8] PANTHEON+ project. 2022. In the framework of a flat-ΛCDM model $\Omega_m = 0.334 \pm 0.018$ and $\Omega_\Lambda = 0.67$ so the integral value is:

$$I_{PANTHEON} \approx 0.9386 \quad (4.7)$$

From the formula of the age of the Universe it is clear that it depends on two factors: the Hubble time $1/H_0$ and the integral.

In the table 9 are shown the different result for the age of the Universe depending on the different values of H_0 and depending on the Ω parameters selected (Plank or PANTHEON).

Table 9 Universe age in the ΛCDM model

H_0 Km/s/Mpc	Hubble time Gyr.	Universe age Gyr. Ω- Plank par. $I_{Plank} \approx 0.9553$	Universe age Gyr. Ω-PANTHEON par. $I_{PANTHEON} \approx 0.9386$
Plank- 67.4±0.5	Plank 14.5	13.85	13.61
SHOES 73±1	SHOES 13.4	12.80	12.58
PANTHEON 73.5±1	PANTHEON 13.3	12.70	12.48

Currently the scientific community accept as the preferably age 13.85 Gyr by the H_0 (Plank) because the other values of H_0 give an Universe age a bit low if it is considered the age of the oldest astrophysical objects.

4.5 The age of the Universe in the DMbQG theory

In this new theory the Hubble function is:

$$H(a) = H_0 \cdot \sqrt{\Omega_{BA} a^{-3} + \Omega_{DM} a^{-2.5} + \Omega_\Lambda} \qquad (4.8)$$

So the integral becomes

$$t_0 = \frac{1}{H_0} \int_0^1 \frac{da}{a \cdot \sqrt{\Omega_{BA} a^{-3} + \Omega_{DM} a^{-2.5} + \Omega_\Lambda}} \qquad (4.9)$$

As in the previous section the *radiation era* has been neglected as well.

So considering the Plank project parameters [7] $\Omega_{BA} = 0.05$ $\Omega_{DM} = 0.26$ and $\Omega_\Lambda = 0.69$ the integral value is

$$I_{Plank} \approx 1.0948 \qquad (4.10)$$

According to the paper [8] PANTHEON+ project. 2022.In the framework of a flat-ΛCDM model $\Omega_m = 0.334 \pm 0.018$ the $\Omega_\Lambda = 0.67$ and supposing $\Omega_{BA} = 0.05$ it is obtained $\Omega_{DM} = 0.28$ so the integral value is:

$$I_{PANTHEON} \approx 1.0783 \qquad (4.11)$$

In the table 10 are shown the different result for the age of the Universe depending on the different values of H_0 and depending on the Ω parameters selected (Plank or PANTHEON).

Table 10 Universe age in the DMbQG theory			
H_0 Km/s/Mpc	Hubble time Gyr	Age Universe Gyr. Ω-Plank par. $I_{Plank} \approx 1.0948$	Age Universe Gyr. Ω-PANTHEON par. $I_{PANTHEON} \approx 1.0783$
Plank 67.4±0.5	Plank 14.5	15.87	15.63
SHOES 73±1	SHOES 13.4	14.67	14.45
PANTHEON 73.5±1	PANTHEON 13.3	14.56	14.34

The DMbQG theory gives values bigger than the oldest astrophysical objects for any one of the H_0 values, although the preferable would be 14.34 Gyr because is the nearest upper bound to the estimations of the oldest astrophysical objects.

Additionally its Hubble value associated, the $H_{O\text{-}PANTHEON}$, is got by the Supernovae type Ia placed into the Local Universe, so it is right to consider this technique as the most direct method to measure the current H_0.

By the contrary into the ΛCDM paradigm it is not possible to accept this Hubble value because its age of the Universe would be lower than the oldest astrophysical objects.

The SHOES/PANTHEON+ programs provide two of the most direct methods for measuring H_0, whereas the Planck program relies on observations of the cosmic microwave background (CMB), which reflects the state of the Universe at the onset of the matter-dominated era. Therefore, if the calculation of the age of the Universe requires the present-day value of the Hubble constant, it is more consistent to use the value obtained from direct measurements.

It is possible to state as conclusion that this chapter has solved partially the "tension Hubble" problem because using the DMbQG theory it is possible to use the value Ho obtained by the PANTHEON project in order to calculate the age of the Universe.

5. Determination of the Hubble constant using the ΛCDM model and the DMbQG theory with data from Cosmic Chronometers

In this chapter it will be estimated the Ho using the both theories: ΛCDM model and DMbQG theory. The data come from a dataset composed by 34 pairs, where the z ranges from 0.07 up to z =2. The method to obtain Ho is an elemental statistical process because the purpose is to show the different results obtained for Ho when it is used the ΛCDM model and the DMbQG theory in the same statistical framework. Despite the simplicity of this method, it will be demonstrated that the value for Ho obtained is quite close to the [7] Plank project result if it is used the ΛCDM model and the theoretical value for Ho matches with the one published in the framework of the PANTHEON project, [8] D.Brout.2022 if it is used the DMbQG theory.

Consequently, in this chapter it will be shown how the DMbQG theory could be a modification of ΛCDM model that worth to be tested extensively by the scientific community regarding the "tension Hubble" problem.

5.1 Data for the Hubble function up to red shift z=2

The data used in this chapter is the table 2 of the paper [31]. This dataset consists of 34 [z, H(z)] measurements obtained using the Cosmic Chronometers method. These data are independent because they come from 10 different papers, published since 2010 up to 2023 by well known international team of researchers. In the table 2 of the paper [31] J.A.Lozano are cited the original paper for each one of the data. The set of papers are the following ones: [32-33-34-35-36-37-38-39-40-41]

As it is usual, the units for the values of the Hubble function are Km/s/Mpc, z is dimensionless.

Table 11					
Data or.	z	H(z)	Data or.	z	H(z)
1	0.07	69	18	0.48	97
2	0.09	69	19	0.5929	104
3	0.12	68.6	20	0.6797	92
4	0.17	83	21	0.75	98.8
5	0.179	75	22	0.7812	105
6	0.199	75	23	0.8	113.1
7	0.2	72.9	24	0.8754	125
8	0.27	77	25	0.88	90
9	0.28	88.8	26	0.9	117
10	0.352	83	27	1.0370	154
11	0.38	83	28	1.26	135
12	0.4	95	29	1.3	168
13	0.4004	77	30	1.3630	160
14	0.4247	87.1	31	1.43	177
15	0.4497	92.8	32	1.53	140
16	0.47	89	33	1.75	202
17	0.4783	80.9	34	1.9650	186.5

5.2 Estimation of the Hubble constant by the ΛCDM model

As the scale factor a depend on z by the formula a =1/(z+1), the Hubble function depend on z in the ΛCDM model like this:

$$H(z) = H_0 \cdot \sqrt{\Omega_m \cdot (1+z)^3 + \Omega_\Lambda} \quad (5.1)$$

Similarly, the H(z) in the DMbQG theory is:

$$H(z) = H_0 \cdot \sqrt{\Omega_{BA} \cdot (1+z)^3 + \Omega_{DM} \cdot (1+z)^{2.5} + \Omega_\Lambda} \quad (5.2)$$

The radiation component has been neglected because the influence over the Hubble function is about 0.05%

In the table 12, the first and second columns contain the dataset given by the table 11.

In the column 3 has been calculated the expression (5.2) using

$\Omega_{ba} = 0.05$; $\Omega_{dm} = 0.265$; $\Omega_\Lambda = 0.685$, the values obtained in the paper [7] Plank.2018.

In the column 5 is calculated the expression (5.1) with $\Omega_m = 0.05 + 0.265 = 0.315; \Omega_\Lambda = 0.685$

The column 4 calculates the relative difference between the theoretical values, column 3, and the data shown in the column 2. Namely the formula used is Col. 4= (col.3-col.2)/col.3 and some similar it is done for the column 6, to calculate the relative difference between the theoretical values, column 5, and the data in the column 2 i.e. Col.6 = (col.5-col.2)/col.5

The procedure to obtain the optimal Ho according the ΛCDM model is the following one: Firstly it is selected an initial value Ho, and progressively it will be changed to achieve that the average value of the column 6 is virtually zero. See at the end of the table 12, that the average value of column 6 is $1.08\cdot10^{-8}$ whereas the average for the column 4 is only $-7.45\cdot10^{-2}$. This way is obtained the optimal Ho= 69.137197 using the Hubble function (5.1). i.e. ΛCDM model, because using such value Ho the average value for the column 6 is virtually zero in the column 6, whereas the average value in the column 4 is almost seven million times bigger than the column 6.

In the abstract of the paper [7] Plank results.2018, the value obtained Ho = 67.4 ±0.5 Km/s/Mpc. So the theoretical value differs only a 2.5%. This result shows that with the elementary statistical method is possible to obtain an acceptable theoretical value Ho. In the following section it will be calculated the theoretical Ho in the framework of DMbQG theory, which will match fully with the value obtained by [8] PANTHEON project.

Table 12 Optimal Ho by LCDM model and Omega mass 0.315 (Plank project)					
1	2	3	4	5	6
Data	Data	Exp.(5.2)	Relative diff.	Exp.(5.1)	Relative diff.
z	H(z)	Hz - DMbQG	Col.3 vs col.2	Hz- ΛCDM	Col.5 vs col.2
0.07	69	71.18412	0.03068	71.54576	0.03558
0.09	69	71.79827	0.03897	72.27844	0.04536
0.12	68.6	72.74345	0.05696	73.41421	0.06558
0.17	83	74.38160	-0.11587	75.40419	-0.10073
0.179	75	74.68468	-0.00422	75.77514	0.01023
0.199	75	75.36702	0.00487	76.61326	0.02106
0.2	72.9	75.40145	0.03318	76.65566	0.04899
0.27	77	77.88591	0.01137	79.74012	0.03436
0.28	88.8	78.25255	-0.13479	80.19923	-0.10724
0.352	83	80.97612	-0.02499	83.63794	0.00763
0.38	83	82.07409	-0.01128	85.03714	0.02396
0.4	95	82.87132	-0.14636	86.05734	-0.10392
0.4004	77	82.88737	0.07103	86.07792	0.10546
0.4247	87.1	83.87055	-0.03851	87.34087	0.00276
0.4497	92.8	84.89817	-0.09307	88.66616	-0.04662
0.47	89	85.74444	-0.03797	89.76140	0.00848
0.4783	80.9	86.09346	0.06032	90.21410	0.10324
0.48	97	86.16516	-0.12574	90.30717	-0.07411
0.5929	104	91.08582	-0.14178	96.74613	-0.07498
0.6797	92	95.07201	0.03231	102.02866	0.09829
0.75	98.8	98.42197	-0.00384	106.50726	0.07236
0.7812	105	99.94207	-0.05061	108.55018	0.03271
0.8	113.1	100.86766	-0.12127	109.79718	-0.03008

0.8754	125	104.65058	-0.19445	114.91645	-0.08775
0.88	90	104.88495	0.14192	115.23476	0.21899
0.9	117	105.90864	-0.10473	116.62658	-0.00320
1.0370	154	113.11860	-0.36140	126.49393	-0.21745
1.26	135	125.53565	-0.07539	143.71727	0.06066
1.3	168	127.84514	-0.31409	146.94875	-0.14326
1.3630	160	131.53019	-0.21645	152.12148	-0.05179
1.43	177	135.51111	-0.30617	157.73142	-0.12216
1.53	140	141.56669	0.01107	166.30618	0.15818
1.75	202	155.33892	-0.30038	185.97798	-0.08615
1.9650	186.5	169.34813	-0.10128	206.20723	0.09557
		Average →	-7.45E-02	Average →	1.08E-08
		Devest σ →	0.12207	Devest σ →	0.09126

In the table 13 is shown the value for Ho using the same procedure explained before, changing the mass ratio Ω_m with different values. Notice how for the mass ratio $\Omega_m = 0.334$ the same statistical procedure obtain a value for Ho quite close to the result obtained by [7] Plank project.2018. Ho = 67.4 ±0.5 Km/s/Mpc.

Table 13 Optimal Ho according ΛCDM for different mass ratios						
Ω_m	0.30	0.31	0.315 (Plank)	0.32	0.33	0.334
Ho	69.931991	69.3983	69.137197	68.87979	68.37567	68.177883

Conversely in the following section, it will be minimized the column 4, in order to look for the optimal Ho using the Hubble function (5.2) according the DMbQG theory. See table 14

5.3 Estimation of the Hubble constant by the DMbQG theory

In the table 14, the first and second columns contain the set of data given by the table 11.

In the column 5 is calculated the expression (5.1), the Hubble function for the ΛCDM model, with $\Omega_m = 0.334 \ and \ \Omega_\Lambda = 0.666$, the values published in the abstract of the paper [8] Brout.2022. PANTHEON+ Analysis.

In the column 3 has been calculated the expression (5.2), the one for DMbQG, using

$\Omega_{ba} = 0.05; \ \Omega_{dm} = 0.284 ; \ \Omega_\Lambda = 0.666$ the values obtained according $\Omega_m = 0.334$.[8] Brout.

The column 4 calculates the relative difference between the theoretical values by DMbQG, column 3, and the data shown in the column 2. Namely the formula used is:
Column 4 = (col.3-col.2)/col.3

Something similar it is done for the column 6, to calculate the relative difference between the theoretical values by ΛCDM model, column 5, and the data given in the column 2. i.e.
Column 6 = (col.5-col.2)/col.5

The procedure to obtain the optimal Ho according the DMbQG theory is the following one:
Firstly it is selected an initial value Ho, and progressively it will be changed to achieve that the average value of the column 4 is virtually zero. See at the end of the table 14, that the average

value of the column 4 is $-3.28\cdot10^{-9}$ whereas the average for the column 6 is only $7.13\cdot10^{-2}$. This way is obtained the optimal Ho=73.409086 using the Hubble function (5.2). i.e. DMbQG theory, because using such value Ho the average value for the column 4 is virtually zero, whereas the average value in the column 6 is almost 22 million times bigger than the column 4.

Notice how the procedure followed in this section is dual regarding the followed in the previous one, i.e. in the section 5.2 it is looked for a value Ho to achieve virtually zero the average for the column 6, whereas in this section it is done some similar for the column 4.

In the abstract of the paper [8] D.Brout. 2022, the value obtained for the Ho = 73.5 ±1.1 Km/s/Mpc . So there is almost a mathematical matching with the theoretical value Ho=73.409 This excellent matching shows how the effective is the elementary statistical procedure followed and the high quality of the dataset, table 11.

Table 14 Ho optimal by DMbQG theory and Omega mass 0.334					
1	2	3	4	5	6
Data	Data	Exp.(5.2)	Relative diff.	Exp.(5.1)	Relative diff.
z	H(z)	Hz-DMbQG	Col.3 vs col.2	Hz- ΛCDM	Col.5 vs col.2
0.07	69	75.70721	0.08859	76.11798	0.09351
0.09	69	76.39586	0.09681	76.94098	0.10321
0.12	68.6	77.45495	0.11432	78.21586	0.12294
0.17	83	79.28851	-0.04681	80.44707	-0.03173
0.179	75	79.62747	0.05811	80.86266	0.07250
0.199	75	80.39031	0.06705	81.80125	0.08314
0.2	72.9	80.42880	0.09361	81.84873	0.10933
0.27	77	83.20299	0.07455	85.29870	0.09729
0.28	88.8	83.61198	-0.06205	85.81170	-0.03482
0.352	83	86.64702	0.04209	89.64999	0.07418
0.38	83	87.86911	0.05541	91.20990	0.09001
0.4	95	88.75597	-0.07035	92.34665	-0.02873
0.4004	77	88.77382	0.13263	92.36957	0.16639
0.4247	87.1	89.86698	0.03079	93.77607	0.07119
0.4497	92.8	91.00893	-0.01968	95.25119	0.02573
0.47	89	91.94887	0.03207	96.46967	0.07743
0.4783	80.9	92.33642	0.12386	96.97314	0.16575
0.48	97	92.41603	-0.04960	97.07664	0.00079
0.5929	104	97.87274	-0.06260	104.22881	0.00220
0.6797	92	102.28479	0.10055	110.08565	0.16429
0.75	98.8	105.98757	0.06782	115.04474	0.14120
0.7812	105	107.66638	0.02477	117.30509	0.10490
0.8	113.1	108.68820	-0.04059	118.68431	0.04705
0.8754	125	112.86147	-0.10755	124.34272	-0.00529
0.88	90	113.11988	0.20438	124.69437	0.27824
0.9	117	114.24836	-0.02408	126.23174	0.07313
1.0370	154	122.18802	-0.26035	137.12095	-0.12310
1.26	135	135.83312	0.00613	156.09438	0.13514

1.3	168	138.36769	-0.21416	159.65047	-0.05230
1.3630	160	142.40999	-0.12352	165.34075	0.03230
1.43	177	146.77442	-0.20593	171.50937	-0.03201
1.53	140	153.40901	0.08741	180.93350	0.22624
1.75	202	168.48120	-0.19895	202.53667	0.00265
1.9650	186.5	183.79302	-0.01473	224.73299	0.17013
		Average→	-3.28E-09	Average→	7.13E-02
		Devstd σ→	0.11005	Devstd σ->	0.08406

In the table 15 are shown the value for Ho using the same procedure explained before, changing the mass ratio Ω_m with different values. Notice how for the mass ratio $\Omega_m = 0.334$ the value Ho matches fully with the result obtained by [8] D.Brout.2022. Ho = 73.5 ±1.1 Km/s/Mpc.

Table 15 Optimal Ho according DMbQG for the different mass ratios						
Ω_m	0.30	0.31	0.315	0.32	0.33	0.334
Ω_{DM}	0.25	0.26	0.265	0.27	0.28	0.284
Ho	75.00675	74.52319	74.285802	74.05125	73.59042	73.409086

5.4 Discussion

In the table 16 are compiled the tables 13 and 15, and there are calculated the absolute and the relative difference between the two models.

The absolute difference is similar for all of the ratio masses, about 5 km/s/Mpc and the relative difference as well, about a 7%.

The most spectacular result are obtained using the mass ratio 0.334, see the last column, because in the ΛCDM model the Ho almost match with the $H_{O\text{- PLANK}}$ whereas in the DMbQG theory the value obtained almost match with $H_{O\text{-PANTHEON+}}$, Ho= 73.5 ±1.1 Km/s/Mpc. See [8] D. Brout.2022

It is possible to state that in this chapter has been made an important contribution to solve the “Hubble tension “ problem because it has been demonstrated by an elementary statistical method how the DMbQG theory is able to obtain the value Ho obtained by the PANTHEON + project, [8] D. Brout, whereas in the framework of ΛCDM model, using the same data set, table 11, and the same statistical procedure, it is obtained a value Ho quite close the one obtained in the framework of the Plank project. See [7] Plank project.2018.

Table 16 Comparison Ho- ΛCDM versus Ho-DMbQG						
Ω_m	0,3	0,31	0,315	0,32	0,33	0,334
Ho- ΛCDM	69,931991	69,3983	69,137197	68,87979	68,37567	68,177883
Ho-DMbQG	75,00675	74,52319	74,285802	74,05125	73,59042	73,409086
Absol. Diff.	-5,07476	-5,12489	-5,14861	-5,17146	-5,21475	-5,23120
Relat. Diff.	-0,06766	-0,06877	-0,06931	-0,06984	-0,07086	-0,07126

It remains to the scientific community to test if the DMbQG theory, using the same or different dataset coming from the ancient Universe and improving the mathematical tools, is able to calculate the value of Ho obtained by the PANTHEON + project. Probably, thanks the DMbQG theory we are close to solve definitively the "Hubble tension" problem.

6. Concluding remarks

In the section 2, it is remarkable that, within the DMbQG framework, and using the cosmological conservation equation, the dark matter density is characterized by an equation-of-state parameter (w = -1/6). This implies that the effective density associated with dark matter is equal to one half of its physical density.

A direct consequence of these findings is that the transition from the matter-dominated era to the dark energy-dominated era shifts from (z = 0.6), as predicted by the ΛCDM model, to (z = 1.1) within the DMbQG framework.

In the section 3 is studied the cosmologic deceleration parameter and its associated function according to the two theoretical frameworks, yielding two quite different functions that could be tested by the recent results of the project DESI.

In the section 4 is estimated the age of the Universe by the two models: the standard ΛCDM and the DMbQG theory. However, within the ΛCDM model, it is not possible to adopt the SHOES/PANTHEON values of H_0, since this would lead to an age of the Universe in tension with the oldest astrophysical objects. Consequently, the ΛCDM framework must rely on the value of H_0 inferred from Planck data, despite the fact that it is needed a model for the evolution of the universe to calculate the current Ho.

By contrast, the DMbQG theory allows the age of the Universe to be computed consistently using the Hubble constant derived from local supernova observations. Namely, the SHOES or the PANTHEON+ projects.

The section five makes an important contribution to solve the "Hubble tension" problem because it is calculated the Ho value, using the two theoretical models, and the values obtained matches with the $H_{O\text{- PLANK}}$ in the framework the ΛCDM model whereas matches with $H_{O\text{-PANTHEON+}}$ for the DMbQG theory.

The dataset used for this calculus is 34 points [z,H(z)] measured by the Cosmic Chronometers method, compiled from ten different papers, which have been published by well known international team of researchers. The mathematical tool to do the calculation is an elemental statistical method, however in spite its simplicity produces the two results explained in the previous paragraph.

The scientific community must know the remarkable results obtained in this work, to replicate the calculus using the same or different dataset and more sophisticated mathematical tools in order to solve the "Hubble tension" problem thanks to the DMbQG theory.

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