Observational Predictions and Prospective Tests of the Cosmological Special Theory of Relativity

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Abstract

The Cosmological Special Theory of Relativity (CSTR) relates a local, unexpanded spatial coordinate to the physically expanded coordinate of a Robertson-Walker universe through a cosmological expansion function $\tau(t_0)$, and re-derives relativistic wave equations, electrodynamics, and general relativity within the resulting cosmological inertial frame (Yi, 2020, 2025). Companion reviews of this program have each found, for the specific quantities they consider – Yukawa interaction ranges and cross sections, Coulomb and hydrogen-like binding energies, Schwarzschild and Kerr-Newman horizon radii, parametrized post-Newtonian coefficients, and the fixed-epoch Lorentz transformation itself – that every physically measured quantity, once expressed in the physically expanded coordinate, recovers its ordinary, epoch-independent value, with the cosmological expansion function appearing only in the corresponding local-coordinate expression. This paper draws together that recurring finding into an explicit observational assessment: because CSTR predicts no departure from ordinary physics for any process confined to a single cosmological epoch, the theory is not constrained, but also not yet tested, by existing single-epoch observations, including present bounds on the time-variation of the fine-structure constant and the proton-to-electron mass ratio from quasar absorption spectroscopy. We argue that a genuine test of CSTR requires a process that spans a cosmologically significant interval of time within a single observation – such as the redshift-drift (Sandage-Loeb) test, multi-epoch photon propagation, or cosmological particle creation – precisely the regime in which a companion review has shown that CSTR's transformation law departs from a simple group structure. We summarize current observational bounds relevant to each sector of the theory and identify the redshift-drift test as the most concrete near-term observational program capable of constraining CSTR's genuinely multi-epoch predictions.

Keywords: cosmological special theory of relativity; fine-structure constant; proton-to-electron mass ratio; redshift drift; Sandage-Loeb test; observational cosmology

1. Introduction

Assessing a new theoretical framework's observational status is, alongside its internal mathematical consistency, one of the two basic requirements for taking it seriously as a candidate description of nature. For a framework proposed as a modification of an already extensively tested theory – as CSTR is, relative to ordinary special and general relativity – this assessment must proceed by identifying, as precisely as possible, the specific regime in which the new framework's predictions are expected to differ from those of the established theory, since it is only in that regime that existing or future data can, even in principle, discriminate between them. Outside that regime, however precise the available measurements, no discrimination is possible, and existing data can be neither cited in support of the new framework nor used against it. This paper carries out exactly this identification for CSTR, drawing on four companion reviews that have each, for a different physical sector of the theory, established that its predictions coincide exactly with those of ordinary relativity within a single cosmological epoch.

The Cosmological Special Theory of Relativity (CSTR) is built from a single operational rule: a local, unexpanded spatial coordinate x'^i is related to the physically expanded coordinate x^i of a spatially flat Robertson-Walker universe through a cosmological expansion function $\tau(t_0)$ of the cosmological time t_0, $x^i = \tau(t_0)x'^i$ (Yi, 2020), substituted directly into the coordinate transformations and differential operators of special relativity. Companion reviews have applied this rule to relativistic quantum mechanics and field quantization, to the Klein-Gordon-Maxwell theory of an interacting charged scalar field, to the Cosmological General Theory of Relativity (CGTR) and its Schwarzschild and Kerr-Newman solutions, and to the mathematical structure of the transformation itself (Yi, 2025). A pattern has recurred throughout these reviews: essentially every physically measured quantity considered – the Yukawa interaction range and scattering cross section, the Coulomb potential and hydrogen-like binding energy, the Schwarzschild and Kerr-Newman horizon radii, the parametrized post-Newtonian coefficients, and the fixed-epoch Lorentz transformation itself – recovers its ordinary, epoch-independent value once expressed in the physically expanded coordinate, with the expansion function $\tau(t_0)$ appearing explicitly only in the corresponding local-coordinate expression.

This recurring pattern is not a coincidence particular to any one calculation: a companion review's group-theoretic analysis shows that, at a single fixed cosmological time, the cosmological Lorentz transformation reduces exactly to the ordinary Lorentz group acting on the physical coordinate, so that any quantity computed entirely within a single epoch must, as a matter of mathematical necessity, be built from objects transforming under the ordinary Lorentz group alone (Yi, 2025). The single-epoch triviality of CSTR is therefore not an accident of the specific calculations performed in the companion reviews, but a direct consequence of the theory's underlying group-theoretic structure, and it is this structural fact that motivates the systematic observational assessment carried out in this paper: rather than checking each new physical calculation individually for epoch-independence, as the companion reviews have done case by case, this paper asks what that already-established structural fact implies for the observational program as a whole.

This paper asks directly what this recurring pattern implies for the observational status of CSTR. If every single-epoch physical prediction of the theory is identical to that of ordinary special and general relativity, then no single-epoch observation – however precise – can distinguish CSTR from the standard theory, and existing bounds from laboratory and astrophysical tests of fundamental physics neither constrain nor rule out the framework. A genuine observational test must instead probe the theory's specifically *multi-epoch* content: the groupoid, rather than group, structure of the transformation across different cosmological times identified in a companion review, and the open question of how quantized fields and propagators extend across epochs. Section 2 briefly recapitulates the cosmological substitution rule. Section 3 reviews existing observational bounds relevant to the quantum sector of CSTR. Section 4 reviews the gravitational sector. Section 5 identifies the redshift-drift test and related multi-epoch probes as the most direct route to a genuine test of the theory. Section 6 summarizes the resulting observational assessment and its relation to the open questions raised in companion reviews.

For reference, the companion reviews drawn upon throughout this paper are: a consolidated review of the full CSTR and CGTR program; a review concentrating on the quantum-theoretic sector (the Klein-Gordon and Dirac equations, the Yukawa potential, the cosmological uncertainty principle, the Klein-Gordon-Maxwell theory, and the canonical quantization of the scalar field); a review of the Cosmological General Theory of Relativity and its Schwarzschild and Kerr-Newman solutions; and a review of the mathematical structure of the cosmological Lorentz transformation itself, establishing the group-versus-groupoid distinction central to this paper's argument. Each is cited by its principal result as it becomes relevant below, and Table 5 in the appendix indexes the specific results drawn from each.

2. The Cosmological Substitution Rule

The Robertson-Walker line element underlying CSTR is

$$ds^2 = -c^2 dt_0^2 + \tau(t_0)^2 \left[dx'^2 + dy'^2 + dz'^2\right], \qquad (1)$$

with $\tau(t_0)$ normalized so that $\tau(t_0^{\rm obs}) = 1$ at the present epoch, and identified with the cosmological redshift through $1 + z = \tau(t_0^{\rm obs})/\tau(t_0^{\rm em})$ (Yi, 2020). Every wave equation and field theory considered in the companion reviews is obtained by substituting the coordinate relation $x^i = \tau(t_0)x'^i$ directly into the corresponding flat-space equation, replacing $\partial/\partial x^i \to \tau(t_0)^{-1}\partial/\partial x'^i$ throughout. A companion review of the mathematical structure of this substitution has shown that, at a single fixed cosmological time, the resulting transformations reduce exactly to the ordinary Lorentz group, while across different cosmological times the transformation law fails to close within the same class of transformations, and is instead correctly described as a groupoid (Yi, 2025). It is this distinction – exact triviality within a single epoch, genuine (but so far only partially characterized) structure across epochs – that organizes the observational assessment of this paper.

2.1. Methodology of This Assessment

For each observational sector considered below, this paper follows a uniform three-step procedure. First, the relevant single-epoch physical prediction of CSTR is identified from the appropriate companion review and expressed in the physical (as opposed to local) coordinate, where it invariably coincides with the ordinary special- or general-relativistic prediction. Second, the best current observational bound relevant to that prediction is summarized, drawing on the mainstream observational literature rather than on any CSTR-specific reanalysis of the underlying data, since no such reanalysis has been performed in the papers reviewed here. Third, the paper states explicitly whether the existing bound constrains CSTR, is neutral with respect to it, or – in the small number of cases where an open theoretical question identified in a companion review prevents a definite single-epoch prediction from being stated at all – remains genuinely undetermined pending further theoretical work. This three-step procedure is repeated for the quantum sector in Section 3 and the gravitational sector in Section 4, before Section 5 turns to the qualitatively different multi-epoch sector, for which no single-epoch prediction exists to be compared against data in the first place.

3. Observational Bounds: The Quantum Sector

The quantum sector of CSTR comprises the cosmologically modified Klein-Gordon, Dirac, and Klein-Gordon-Maxwell theories developed in a companion review, together with the associated Yukawa potential, Coulomb limit, hydrogen-like spectrum, and cosmological uncertainty relation. Following the methodology of Section 2, each subsection below identifies the relevant single-epoch prediction from that review, summarizes the best current observational bound on the corresponding physical quantity, and states explicitly whether the comparison constrains CSTR.

3.1. The Yukawa Range and the Fine-Structure Constant

A companion review shows that the cosmologically modified Yukawa potential, $V(r') = -(g^2/r')\exp[-(mc/\hbar)\tau(t_0)r']$, has a local-coordinate screening length that varies with $\tau(t_0)$, while the corresponding physical screening length, $\hbar/(mc)$ measured in the physical coordinate $r_c = \tau(t_0)r'$, is independent of the cosmological epoch (Yi, 2025). The closest existing observational analogue to a test of this prediction is the search for cosmological time-variation of the fine-structure constant α, which controls the overall strength of the electromagnetic interaction entering atomic and molecular transition energies. High-resolution quasar absorption spectroscopy with the Very Large Telescope's UVES spectrograph constrains the fractional variation of α over roughly the last ten billion years to $|\Delta\alpha/\alpha| \lesssim 0.6$ parts per million, the strongest astronomical constraint of this kind to date. More recently, James Webb Space Telescope observations of [O III] emission lines in galaxies at redshifts $2.5 \lesssim z \lesssim 9.5$ give the independent constraint $\Delta\alpha/\alpha = (0.2 \pm 0.7) \times 10^{-4}$, consistent with zero over most of cosmic history. Because CSTR predicts that the physical (as opposed to local-coordinate) interaction range and coupling strength are epoch-independent, these null results are fully consistent with, but do not specifically test, CSTR: an ordinary ΛCDM universe with strictly constant α predicts exactly the same null result.

A complementary, geologically based constraint comes from the natural fission reactor that operated roughly two billion years ago at Oklo, Gabon: the isotopic abundances left by this reactor constrain $|\Delta\alpha/\alpha|$ at that epoch (corresponding to a very low redshift, $z \lesssim 0.14$) to below approximately 10^{-7}, tighter than the quasar bounds but restricted to a much more recent cosmological epoch. Taken together, the quasar, emission-line, and Oklo constraints span roughly the full range of cosmological time from $z \sim 9.5$ down to $z \sim 0.14$, and all are consistent with $\Delta\alpha/\alpha = 0$ throughout. The methodology behind the quasar constraints, the so-called many-multiplet method, compares the relative wavelength shifts of multiple atomic transitions with different sensitivities to α within the same absorption system, a technique considerably more precise than comparing a single transition's wavelength alone; subsequent work has emphasized the importance of controlling systematic effects in the line-fitting procedure itself, since several early claims of a nonzero $\Delta\alpha/\alpha$ were later attributed, at least in part, to such systematics rather than to genuine physical variation. This methodological caution is itself relevant to any future attempt to search for a CSTR-specific signature in this class of data: distinguishing a genuine multi-epoch CSTR effect from an unmodeled systematic would require at least the same level of care already established in this literature for the ordinary, single-parameter variation of α.

It is worth being explicit about the limits of this analogy. The companion review's Yukawa potential describes a short-range interaction mediated by a massive field, whereas the electromagnetic interaction probed by α is mediated by the massless photon and is long-range; the two are not the same physical interaction, and the observational bound on $\Delta\alpha/\alpha$ does not directly constrain the specific Yukawa coupling g or mass m appearing in the companion review's formula. The relevance of the α-variation literature to CSTR is instead structural rather than direct: both cases concern whether an interaction's characteristic strength or range varies over cosmological time, and both receive the same qualitative answer from CSTR – no variation in the physically measured quantity, only in its local-coordinate expression – for the same underlying group-theoretic reason established in the companion mathematical-structure review. A direct, quantitative test of the companion review's specific Yukawa-potential prediction would instead require an observable directly sensitive to a short-range, massive-mediator interaction at different cosmological epochs, such as constraints on new short-range forces from precision tests of gravity at sub-millimeter scales combined with astrophysical constraints on new light mediators, rather than the electromagnetic α-variation literature discussed here for its structural, rather than direct, relevance.

3.2. The Proton-to-Electron Mass Ratio

A related, independent probe is the proton-to-electron mass ratio μ, constrained by molecular hydrogen absorption lines in quasar spectra to $\Delta\mu/\mu \sim (1\text{–}2) \times 10^{-6}$ at redshifts $z > 1$. The cosmological hydrogen-like binding-energy formula of a companion review, $E_n \propto m\,\tau(t_0)^2/n^2$ in the local coordinate, was shown there to raise an open question as to the exact power of $\tau(t_0)$ with which this expression reduces to the ordinary, epoch-independent Bohr formula once re-expressed in the physical coordinate (Yi, 2025). Existing bounds on $\Delta\mu/\mu$ are precise enough, in principle, to constrain any leftover physical epoch-dependence in this formula at the part-per- million level, so that resolving the open power-counting question identified in that companion review is a prerequisite for stating whether current molecular absorption-line data already constrain CSTR's hydrogen-like sector, are neutral with respect to it, or would require still more precise data to do so.

The molecular hydrogen constraint on μ is derived from the relative wavelengths of Lyman and Werner band vibro-rotational transitions of H_2, whose sensitivity to μ differs systematically from transition to transition in a manner analogous to the many-multiplet method used for α; the tightest such constraints to date come from a small number of exceptionally bright, high-column-density damped Lyman-α absorption systems suitable for this measurement, since detecting molecular hydrogen absorption at all requires a sufficiently dense and shielded gas cloud along the quasar sight line. This observational scarcity – in contrast to the much larger samples available for the atomic many-multiplet method used for α – means that current $\Delta\mu/\mu$ constraints, while individually precise, are based on a handful of sight lines rather than the fifty or more systems typically combined in the leading α studies, and expanding the sample of suitable absorption systems is itself an active observational priority independent of any specific theoretical motivation such as CSTR.

3.3. Atomic Clocks and Laboratory Bounds

Laboratory frequency comparisons between atomic clocks based on different transitions, conducted over periods of years within a single, essentially fixed cosmological epoch, constrain the present-day rate of change of α and μ to levels far more stringent than any astrophysical bound, but necessarily probe only $\dot{\tau}(t_0^{\rm obs})$ (the present-day Hubble parameter) and not the integrated history of $\tau(t_0)$ accessible to quasar absorption studies. Because a companion review's adiabaticity estimate shows that $\hbar H(t_0)/(mc^2)$ is smaller than order unity by some 46 orders of magnitude for the electron (Yi, 2025), CSTR predicts that laboratory atomic-clock tests should show no detectable drift from this source whatsoever, a prediction trivially consistent with the null results of all such experiments performed to date, but again not a distinguishing test of the theory.

Modern optical atomic clocks compare transition frequencies with fractional precision at the 10^{-18} level or better over integration times of hours to days, and repeated comparisons over multi-year baselines constrain any present-day drift in α to of order 10^{-18} per year. Even at this extraordinary precision, the predicted CSTR contribution from the ordinary cosmological expansion rate, of order $H(t_0) \sim 10^{-18}$ per second (equivalently, of order 10^{-10} per year), so vastly exceeds the corresponding contribution to any epoch-dependence of α or μ predicted by CSTR – which, per the adiabaticity estimate above, is suppressed by a further factor of order 10^{-46} relative to $H(t_0)$ for the electron – that no plausible near-term improvement in atomic-clock precision could detect it. This numerical comparison illustrates concretely why laboratory tests, however precise, are not the appropriate observational arena for testing CSTR's cosmological content: the theory's predicted single-epoch effects are simply too small, by a vast margin, to be laboratory-detectable, quite apart from the more fundamental point, established above, that single-epoch predictions coincide exactly with ordinary physics regardless of their size.

4. Observational Bounds: The Gravitational Sector

The gravitational sector of CSTR is governed by the companion CGTR review's perturbative embedding of a localized mass, or a rotating charged mass, within the Robertson-Walker background of Section 2. As in the quantum sector, the companion review's central finding is that every physically measured geometric or dynamical property of the resulting Schwarzschild and Kerr-Newman solutions – horizon radii, photon-sphere radius, tidal forces, frame dragging, and the parametrized post-Newtonian coefficients – is independent of the cosmological epoch once expressed in the physical radial coordinate, so that the same three-step methodology of Section 2 applies here as well: each of the following subsections identifies the relevant single-epoch prediction, summarizes the best current observational bound, and states explicitly that existing gravitational tests, however precise, are consistent with but do not specifically test CSTR.

4.1. Black Hole Shadow Imaging

A companion review of the Cosmological General Theory of Relativity (CGTR) shows that the photon-sphere radius of a cosmologically embedded Schwarzschild black hole, $r_{\rm ph} = 3GM/c^2$ in the physical radial coordinate, is independent of the cosmological epoch, identical to the ordinary general-relativistic value (Yi, 2025). Very-long-baseline interferometric imaging of the black-hole shadows at the centers of M87 and the Milky Way, both at negligible cosmological redshift, is therefore not expected, within CSTR, to show any departure from the ordinary general-relativistic prediction, and existing shadow measurements are consistent with this expectation, as they are with ordinary general relativity itself. A distinguishing test would require imaging a black-hole shadow at cosmologically significant redshift, which lies beyond the angular resolution of existing very-long-baseline interferometric arrays.

The angular size of a black-hole shadow scales as the physical photon-sphere radius divided by the angular-diameter distance to the source; because angular-diameter distance itself increases with redshift only up to a maximum and then decreases at very high redshift in standard cosmology, the shadow angular size does not simply shrink monotonically with distance, but the practical obstacle to imaging a cosmologically distant black hole is instead the source's intrinsically small physical photon-sphere radius relative to its host galaxy and the telescope resolution achievable at the correspondingly large physical distance, combined with the difficulty of identifying and resolving an individual supermassive black hole's immediate environment at such distances with current very-long-baseline interferometric baselines, which are set by the diameter of the Earth. Next-generation concepts involving space-based very-long-baseline interferometry could, in principle, extend the reachable distance and hence redshift range for shadow imaging, but no such capability is presently operational, and CSTR makes no specific quantitative prediction for how the shadow angular size should differ from the ordinary general-relativistic expectation even at such redshifts, beyond the qualitative point, established in the companion CGTR review, that the physical photon-sphere radius itself does not depend on the epoch of observation.

4.2. Solar-System and Pulsar Tests

The companion CGTR review further shows that the parametrized post-Newtonian coefficients of the cosmologically embedded Schwarzschild solution take their ordinary general-relativistic values, $\beta_{\rm PPN} = \gamma_{\rm PPN} = 1$, at every cosmological epoch (Yi, 2025). Solar-system tests of light bending, Shapiro delay, and perihelion precession, and binary-pulsar tests of orbital decay via gravitational-wave emission, are conducted at negligible cosmological redshift and are therefore predicted, within CSTR, to agree with ordinary general relativity to the same precision as they already do; no existing solar-system or pulsar-timing result either constrains or is in tension with CSTR specifically.

Cassini-era Shapiro-delay measurements constrain $\gamma_{\rm PPN}$ to agree with unity at the 10^{-5} level, and lunar laser ranging constrains related PPN combinations at a comparable or better precision; double neutron-star and neutron-star–white-dwarf binary pulsars provide complementary, strong-field tests of orbital decay through gravitational-wave emission, consistent with general relativity at the sub-percent level for the best-timed systems. All of these measurements are made at redshifts of order 10^{-4} or smaller (solar-system distances) or, for pulsars, at most a few kiloparsecs – entirely negligible on cosmological scales – so that, per the companion CGTR review's finding that $\beta_{\rm PPN} = \gamma_{\rm PPN} = 1$ identically at every epoch, CSTR predicts these tests should continue to agree with general relativity at whatever precision future observations achieve, with no epoch-dependent correction of any kind entering at this level.

4.3. Gravitational-Wave Standard Sirens

Gravitational-wave events from compact-binary mergers, used as standard sirens to measure cosmological distances, in principle sample a range of redshifts from near zero out to $z \sim 1$ or beyond for future detectors. Because the companion CGTR review's horizon and orbital results are all epoch-independent in the physical coordinate, CSTR predicts that the waveform of a merger occurring at any single epoch, and hence the inferred source-frame masses and luminosity distance, are identical to the ordinary general-relativistic prediction at that epoch; any departure would again require a genuinely multi-epoch effect (for instance, an imprint of the merger's own duration relative to the cosmological expansion timescale) rather than a single-epoch modification of the waveform itself, and no such effect has been computed in the papers reviewed here.

The duration of a typical compact-binary merger observable by current ground-based detectors, from entry into the sensitive frequency band through coalescence, ranges from a fraction of a second to at most a few minutes for the lowest-mass systems, utterly negligible compared with the Hubble time $1/H(t_0)$ of order ten billion years; the same is true of the light-travel time across the binary system itself. This vast separation of timescales is what justifies treating each individual merger as a single-epoch process for the purposes of the waveform calculation, even though the population of detected mergers as a whole spans a range of cosmological epochs via their different redshifts. Any CSTR-specific correction to the merger waveform itself would therefore have to arise from physics operating on timescales far shorter than the Hubble time, which is precisely the regime in which the fixed-epoch triviality established throughout this paper applies most straightforwardly, making a detectable single-merger CSTR signature especially unlikely on purely dimensional grounds, independent of the detailed calculation.

A related question concerns the cosmological redshifting of the gravitational-wave signal itself as it propagates from source to detector: the observed frequency is redshifted by the same factor $1 + z = \tau(t_0^{\rm obs})/\tau(t_0^{\rm em})$ that applies to electromagnetic radiation, and the companion mathematical-structure review's derivation of the redshift relation directly from the coordinate relation $X = \tau(t_0)x'$ applies equally to a gravitational as to an electromagnetic wave, since both are massless excitations obeying the same wave equation in the weak-field limit. This is the same, single-epoch-per-emission redshifting already accounted for in the ordinary general-relativistic treatment of standard sirens, and CSTR predicts no further modification of it, consistent with the pattern established throughout this section: the redshift itself is a well-understood, already-tested single-epoch-emission phenomenon, and it is only the multi-epoch propagation *history* of the signal – rather than its net redshift – that carries any potentially CSTR-specific content, as elaborated in Section 5.

4.4. The Speed of Gravitational Waves

The joint detection of the binary neutron-star merger GW170817 and its electromagnetic counterpart constrained the speed of gravitational-wave propagation to agree with the speed of light c to within a few parts in 10^{15}, one of the tightest tests of the equivalence of the two propagation speeds performed to date. A companion mathematical-structure review establishes that c is invariant under the fixed-epoch cosmological Lorentz transformation by construction, entering Equation (1) in exactly its ordinary, unmodified form, and that this invariance is not itself a consequence of, or threatened by, the multi-epoch groupoid structure identified there, since that structure concerns only the relation between spatial coordinates at different epochs. CSTR therefore predicts, with no room for ambiguity, that gravitational and electromagnetic waves emitted at the same event and epoch propagate at exactly the same speed c, consistent with the GW170817 result and offering no distinguishing signature relative to ordinary general relativity for this particular test.

5. Multi-Epoch Probes: Where a Genuine Test Might Lie

Having established in Sections 3 and 4 that no single-epoch observation can distinguish CSTR from ordinary relativity, this section turns to the complementary question of which existing or prospective observational programs are, at least in principle, sensitive to the theory's multi-epoch content – the groupoid structure of the cosmological Lorentz transformation identified in a companion review, and the related open questions concerning the two-time propagator and cosmological particle creation. Three candidate probes are considered in turn below; none currently constitutes a completed test of CSTR, since none has yet been matched to a specific CSTR prediction derived from the multi-epoch structure, but each identifies a concrete direction in which such a prediction could eventually be sought.

5.1. The Redshift-Drift (Sandage-Loeb) Test

The redshift-drift or Sandage-Loeb test directly measures the rate of change of a distant source's redshift over an observational baseline of years to decades, by repeated high-precision spectroscopy of the same Lyman-α forest absorption features toward a bright, stable quasar. Because this measurement directly probes $d\tau(t_0)/dt_0$ evaluated along the observer's own worldline over an extended interval, rather than a single instantaneous comparison, it is a genuinely multi-epoch observation in the sense of Section 2: it compares the cosmological expansion function's behavior at two different observation times, rather than probing a single-epoch physical process. A companion review's analysis of the multi-epoch composition of cosmological Lorentz transformations, and its identification of the resulting groupoid structure, is directly relevant to how such a measurement should be interpreted within CSTR, since the redshift-drift signal is precisely the observable counterpart of the rescaling factor $\tau(t_2)/\tau(t_1)$ that was shown there to obstruct closure of the transformation group. Existing redshift-drift measurements are not yet precise enough to detect the predicted signal in the standard ΛCDM cosmology, and next-generation extremely large telescopes equipped with high-precision, high-stability spectrographs are the most concrete near-term facility capable of performing this measurement at the required precision.

The test was first proposed by Sandage in 1962 and revived, in a form practical for realistic spectrographs, by Loeb in 1998, who noted that the tiny drift in redshift over a human observational baseline, $dz/dt_{\mathrm{obs}} = (1+z)H_0 - H(z)$ in the standard cosmological parametrization, corresponds to a spectroscopic velocity drift of only a few centimeters per second per decade for sources at $z \sim$ 2–4 – extraordinarily small, but in principle within reach of the extreme radial-velocity precision and long-term wavelength calibration stability being developed for extremely large telescope spectrographs, motivated originally by the search for terrestrial exoplanets. In ordinary ΛCDM cosmology, this quantity depends only on the standard cosmological parameters (H_0, the matter and dark-energy density parameters) and provides an independent, purely geometric probe of the expansion history, free of the standard-candle or standard-ruler calibration uncertainties that affect supernova and BAO distance measurements. Within CSTR, the same observable directly measures the quantity $\tau(t_2)/\tau(t_1)$ for two observation times t_1, t_2 separated by the human observational baseline, which is exactly the rescaling factor identified in a companion review as the obstruction to closing the cosmological Lorentz transformation into a single group across different epochs. Deriving whether CSTR predicts any correction to the standard ΛCDM redshift-drift formula – for instance, from the groupoid composition law itself, rather than merely from whatever background expansion history $\tau(t_0)$ is assumed – is precisely the calculation identified in Section 6 as the most valuable next theoretical step for this program.

5.2. Cosmic Chronometers and Differential Age Dating

A related, already well-established technique measures $H(z)$ not through redshift drift but through the differential ages of passively evolving galaxies at slightly different redshifts, using the relation $H(z) = -(1+z)^{-1} dz/dt_{\mathrm{age}}$: by dating pairs of galaxies with similar stellar populations but different redshifts, this "cosmic chronometer" method infers the age difference dt_{age} corresponding to the redshift difference dz between them, and hence $H(z)$, without invoking a specific expansion-history model. Unlike the redshift-drift test, cosmic chronometers compare two *different* sources at two different redshifts rather than the same source observed twice, and therefore probe the background expansion history $\tau(t_0)$ itself rather than the multi-epoch composition law of a companion review's group-theoretic analysis; in this sense, cosmic chronometers are best classified as a single-epoch-per-source probe of the background $\tau(t_0)$, analogous to supernova and BAO distance measurements, rather than as a genuinely multi-epoch probe of CSTR's transformation structure in the sense of Section 2. It is included here nonetheless because it illustrates, by contrast, exactly what makes the redshift-drift test distinctive: only a measurement performed on the *same* source across an observational baseline directly probes the quantity $\tau(t_2)/\tau(t_1)$ relevant to the groupoid structure identified in the companion mathematical-structure review, whereas a comparison across different sources at fixed observation time, however precise, remains a single-epoch measurement of the background expansion history alone.

5.3. Multi-Epoch Photon Propagation and the Two-Time Propagator

A companion quantum-theoretic review identifies the extension of the equal-time Feynman propagator to field insertions at two different cosmological times as an open, uncompleted calculation (Yi, 2025). Any observational process that depends on the propagation amplitude of a quantum field between two genuinely different cosmological epochs – for instance, the detailed spectral shape of a signal that has propagated over a cosmologically significant redshift interval, as opposed to its overall, single-epoch redshift alone – would be sensitive to this presently unresolved sector of the theory, and no observational program currently targets this specific signature, since the required theoretical prediction has not yet been derived.

A related, already-operating class of observation that is sensitive, at least qualitatively, to genuinely multi-epoch propagation is pulsar timing array detection of the nanohertz-frequency stochastic gravitational-wave background, in which the signal of interest is encoded in tiny, correlated timing deviations accumulated by radio pulses that have propagated for years to decades before detection. While this technique targets classical gravitational-wave strain rather than the specifically quantum two-time propagator identified above, it illustrates the general observational strategy relevant to CSTR's multi-epoch sector: precision timing of a signal over the longest practical observational baseline, so as to accumulate a detectable imprint of physics that operates too slowly to affect any single, short observation. Adapting this general strategy to a genuinely propagator-sensitive observable specific to CSTR would first require completing the two-time propagator calculation identified as open in the companion quantum-theoretic review.

5.4. Cosmological Particle Creation

A companion review's comparison of CSTR's canonical field quantization with the mainstream theory of quantum fields in curved spacetime identifies, as an open question, whether the theory's fixed-epoch annihilation and creation operators are related across different epochs by a nontrivial Bogoliubov transformation, analogous to the cosmological particle creation of the mainstream theory (Yi, 2025). Should CSTR predict a definite, calculable particle-creation rate once this question is resolved, cosmological probes of relic particle backgrounds – analogous to existing searches for signatures of primordial particle production in the cosmic microwave background or in a stochastic gravitational-wave background – would become a further, presently unavailable avenue for testing the theory; until the underlying calculation is completed, no quantitative prediction exists to compare against such data.

Existing cosmic microwave background measurements already constrain deviations of the background radiation spectrum from a pure blackbody form to remarkably small fractional levels, and any additional photon or particle production predicted by a resolved CSTR Bogoliubov calculation would need to be compared against these existing spectral-distortion bounds, as well as against big-bang nucleosynthesis constraints on any additional relativistic energy density at early times, before it could be considered observationally viable. Because the mainstream cosmological particle-creation effect for ordinary quantum fields in a Robertson-Walker background is itself generally tiny except for fields with mass comparable to or smaller than the expansion rate at the epoch in question, any analogous CSTR effect would likely be similarly suppressed for ordinary massive particles at the present epoch, and would be most relevant, if at all, for the very early universe rather than for present-day astrophysical sources – a regime in which the adiabatic, fixed-background approximation used throughout the companion reviews may itself break down and require a fully time-dependent treatment beyond their scope.

6. Discussion

6.1. Limitations of This Assessment

Before summarizing the resulting observational picture, three limitations of the assessment carried out in this paper are worth stating explicitly. First, the single-epoch triviality result on which this paper's central argument rests – that every physically measured quantity considered in the companion reviews is epoch-independent – has been established case by case in those reviews, together with a companion group-theoretic argument for why this should hold generally (Section 2); it has not been proven as a fully general theorem covering every conceivable observable constructible within CSTR, and a specific counterexample, if found, would require revising the observational assessment given here accordingly. Second, several of the bounds summarized in Sections 3 and 4 are drawn from a fast-moving observational literature, and the specific numerical values cited should be understood as illustrative of the current state of the art rather than as a definitive, permanently current compilation. Third, this paper has not attempted an independent statistical reanalysis of any existing dataset in light of CSTR specifically, since no CSTR-specific prediction differing from the standard single-epoch expectation has been derived for any of the observables considered; such a reanalysis would only become meaningful once the theoretical work identified in Section 5 has been completed.

Sector	**Best current bound**	**Constrains CSTR?**
Fine-structure constant (α)	$\lesssim$ 0.6 ppm (10 Gyr)	No (single-epoch)
Proton-electron mass ratio (μ)	~ 1–2×10^{-6}	Open (Sec. 4.2)
Atomic clocks (lab)	Far below $\dot{\tau}$ scale	No (single-epoch)
Black hole shadows	Consistent w/ GR	No (single-epoch)
Solar system / pulsars	Consistent w/ GR	No (single-epoch)
GW standard sirens	Consistent w/ GR	No (single-epoch)
Redshift drift	Not yet detected	Yes, in principle

Table 1: Observational status of the principal sectors of CSTR discussed in this paper.

6.2. Summary of the Observational Assessment

Table 1 collects the observational status of each sector of CSTR considered in this paper.

The picture that emerges is that CSTR, as currently formulated, is observationally indistinguishable from ordinary special and general relativity for every process confined to a single cosmological epoch, because every physically measured single-epoch quantity considered in the companion reviews recovers its ordinary value. This is not a weakness unique to CSTR – it is a direct and unavoidable consequence of the fixed-epoch triviality established in a companion review's group-theoretic analysis – but it does mean that the extensive existing body of precision tests of fundamental physics, however constraining for other proposed modifications of relativity, neither supports nor disfavors CSTR specifically.

This should be distinguished carefully from the situation faced by theories that make single-epoch predictions genuinely different from general relativity or the Standard Model (for instance, specific modified-gravity theories with a distinctive PPN signature, or specific varying-constant theories with a predicted functional form for $\Delta\alpha/\alpha(z)$): those theories are directly constrained, and in many cases ruled out or tightly bounded, by exactly the observational programs surveyed in Sections 3 and 4. CSTR's situation is different in kind, not merely in degree: its single-epoch predictions are not merely consistent with the data at some level of precision, but are mathematically identical to those of ordinary relativity, as a direct consequence of the fixed-epoch group-theoretic result established in a companion review. No improvement in single-epoch measurement precision, however dramatic, could therefore ever distinguish CSTR from ordinary relativity; only a measurement of the multi-epoch type identified in Section 5 could do so, in principle.

6.3. What Would Constitute a Genuine Test

A genuine observational test of CSTR, as distinct from ordinary relativity, requires a measurement that is sensitive to the theory's multi-epoch structure: the redshift-drift test, a two-time propagator signature, or a particle-creation rate, each identified in Section 5. Of these, the redshift-drift test is the only one for which both an observational program (extremely large telescope spectroscopy of the Lyman-α forest) and a well-established theoretical prediction (in ordinary ΛCDM cosmology) already exist; what is presently missing, and is identified here as a concrete task for further theoretical work, is the specific CSTR prediction for this observable, which would need to be derived from the groupoid-structured multi-epoch transformation of a companion review rather than assumed to coincide with the ordinary ΛCDM prediction.

Deriving that prediction would, on the analysis of this paper, proceed in three steps: first, expressing the redshift-drift observable explicitly in terms of the local coordinate and the groupoid morphism of Section 2 connecting the two observation times, rather than purely in terms of the physical coordinate as in the standard derivation; second, determining whether the resulting expression differs from the ordinary ΛCDM formula at any order in the ratio of the observational baseline to the Hubble time, given the adiabaticity estimates already established in companion reviews for other multi-epoch quantities; and third, comparing the resulting prediction, if it differs at a level within reach of current or next-generation instrumental sensitivity, against the redshift-drift measurement programs already being developed for other cosmological purposes. Because the observational baseline for any realistic redshift-drift measurement – years to decades – is itself an utterly negligible fraction of the Hubble time, the same adiabatic suppression that renders CSTR's single-epoch predictions indistinguishable from ordinary relativity might plausibly render any multi-epoch correction to the redshift-drift formula similarly small; establishing whether this is in fact the case, or whether the groupoid structure introduces a qualitatively different, non-adiabatically-suppressed correction, is exactly the calculation that remains to be carried out.

6.4. Relation to Falsifiability

A theory that is observationally indistinguishable from an established one across an entire, well-tested regime is not thereby unfalsifiable in principle, but it does place the burden of a genuine test specifically on the regime where the two theories are expected to differ. For CSTR, this paper has argued that this regime is the multi-epoch sector identified in Section 5, and that deriving a specific, quantitative prediction for at least one multi-epoch observable – most promisingly the redshift-drift signal – is the most direct route to making the theory falsifiable in practice rather than only in principle. This recommendation is consistent with, and adds an observational dimension to, the purely theoretical open questions already raised by companion reviews of CSTR's mathematical structure and by an independent critical review of the framework (A Critical Review, 2025).

A useful historical comparison is the early observational status of general relativity itself relative to Newtonian gravity: in the weak-field, low-velocity regime relevant to most solar-system phenomena, general relativity's predictions differ from Newtonian gravity only at a small, calculable post-Newtonian level, so that the two theories were, for most practical purposes, observationally similar prior to the precision tests (perihelion precession of Mercury, light bending, and later Shapiro delay) that specifically targeted the regime of difference. CSTR's situation is more extreme – its single-epoch predictions do not merely approximate those of ordinary relativity at some small parameter, but coincide with them exactly, as a consequence of the group-theoretic argument of Section 2 – but the underlying logic of identifying, and then specifically targeting, the regime of expected difference is the same. For general relativity, that regime was identified theoretically well before it was tested observationally; the analogous theoretical step for CSTR, identifying precisely how its multi-epoch predictions differ from the corresponding ΛCDM or standard-model expectation, is the outstanding task highlighted throughout Section 5 and is a necessary precursor to any observational test, exactly as the post-Newtonian formalism was a necessary theoretical precursor to the classical tests of general relativity.

Probe	**Theory status**	**Observational status**
Redshift drift	CSTR prediction not yet derived	Program exists (ELT-class facilities)
Two-time propagator	Open calculation	No dedicated program
Particle creation / Bogoliubov	Open calculation	CMB/BBN bounds apply once derived

Table 2: Readiness of the three multi-epoch probes identified in Section 5.

Table 2 summarizes the three multi-epoch probes identified in Section 5, together with their current theoretical and observational readiness.

Of the three, the redshift-drift probe is unique in already having a mature, funded observational program on the facility side, so that the rate-limiting step for a genuine test of CSTR via this channel is entirely theoretical: deriving the specific CSTR prediction, if any, for the redshift-drift signal beyond the ordinary ΛCDM expectation already targeted by that program.

7. Conclusion

This paper has assessed the observational status of the Cosmological Special Theory of Relativity by drawing together the recurring finding of companion reviews that every single-epoch physical prediction of the theory – in the quantum sector (Section 3) and the gravitational sector (Section 4) alike – coincides exactly with the corresponding prediction of ordinary special and general relativity once expressed in the physically expanded coordinate. Existing precision tests of fundamental physics, including quasar-spectroscopic bounds on the fine-structure constant and proton-to-electron mass ratio, laboratory atomic-clock comparisons, black-hole shadow imaging, solar-system and pulsar tests, and gravitational-wave standard sirens, are accordingly consistent with CSTR but do not test it specifically, since they probe only single-epoch physics. A genuine test requires a measurement sensitive to the theory's multi-epoch structure (Section 5), among which the redshift-drift (Sandage-Loeb) test, targetable by next-generation extremely large telescopes, is identified as the most concrete near-term observational program, contingent on deriving CSTR's specific prediction for this observable from the groupoid-structured multi-epoch transformation law characterized in a companion review. Completing that derivation is identified as the single most valuable next theoretical step for placing CSTR on a genuinely testable observational footing.

More broadly, this paper has illustrated a general methodological point that may be of use beyond CSTR itself: when a theoretical framework is built by modifying a well-tested theory through a substitution rule that is provably trivial within some restricted domain (here, a single cosmological epoch), a systematic observational assessment should explicitly identify that domain and set it aside, concentrating observational scrutiny – and, more importantly, theoretical development – on the complementary domain in which the modification has genuine content. For CSTR, that domain is the multi-epoch sector characterized, but not yet fully worked out, by companion reviews of the theory's quantum field theory and group-theoretic structure. The redshift-drift test, the two-time propagator, and cosmological particle creation identified in this paper are not merely three unrelated observational suggestions, but three different observational windows onto what is, at the theoretical level, a single open problem: how the cosmological inertial frame at one epoch is properly related to the cosmological inertial frame at another. Resolving that problem is the common prerequisite for turning any of the three into an actual, quantitative test of the theory.

Finally, it is worth emphasizing what this paper's assessment does *not* claim. It does not claim that CSTR is confirmed by existing data: a theory whose single-epoch predictions are identical to those of an already well-confirmed theory is, in that regime, exactly as well supported as the underlying theory, no more and no less, and the additional multi-epoch content of CSTR remains entirely untested. Nor does it claim that CSTR is disfavored: no existing observation contradicts any prediction actually derived in the papers reviewed here. The honest summary is one of a theoretically well-posed but observationally untested extension of relativity, whose testability depends entirely on completing a specific, identifiable body of theoretical work – chiefly, the multi-epoch composition law and its consequences for the propagator and for particle creation – rather than on any further refinement of the already extensive single-epoch observational programs surveyed in this paper.

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A. Glossary of Key Symbols

B. Observational Data Sources

Table 4 records the specific observational figures cited in this paper and the general class of measurement from which each is drawn, for readers wishing to consult the primary observational literature directly.

Symbol	Meaning
t_0	Cosmological time
$\tau(t_0)$	Cosmological expansion function (scale factor)
α	Fine-structure constant
μ	Proton-to-electron mass ratio
$H(t_0)$	Hubble parameter, $\dot{\tau}(t_0)/\tau(t_0)$
$\beta_{\rm PPN}$, $\gamma_{\rm PPN}$	Parametrized post-Newtonian parameters
$r_{\rm ph}$	Photon-sphere radius
CSTR	Cosmological Special Theory of Relativity
CGTR	Cosmological General Theory of Relativity

Table 3: Glossary of symbols used in this paper.

Quantity	Measurement class
$\Delta\alpha/\alpha \lesssim 0.6$ ppm	VLT/UVES quasar absorption spectroscopy
$\Delta\alpha/\alpha = (0.2 \pm 0.7) \times 10^{-4}$	JWST emission-line galaxy spectroscopy
$\lvert\Delta\alpha/\alpha\rvert \lesssim 10^{-7}$	Oklo natural reactor isotopic abundances
$\Delta\mu/\mu \sim (1\text{–}2) \times 10^{-6}$	Molecular hydrogen quasar absorption
Atomic clock drift $\sim 10^{-18}$/year	Optical atomic clock frequency comparison
$\gamma_{\rm PPN} - 1 \sim 10^{-5}$	Cassini Shapiro-delay tracking

Table 4: Observational data sources cited in this paper.

C. Index of Companion Reviews

Table 5 indexes the specific results drawn from each companion review of the CSTR program, all consolidated in Yi (2025) and originating from Yi (2020).

Companion review	Results used in this paper
Consolidated review	Overall framework, redshift identification
Quantum-theoretic review	Yukawa range/cross section, uncertainty relation, Coulomb/hydrogen sector, adiabaticity estimate, two-time propagator, Bogoliubov question
CGTR review	Schwarzschild/Kerr-Newman horizons, photon sphere, PPN parameters
Mathematical-structure review	Group-versus-groupoid distinction, redshift derivation

Table 5: Index of results drawn from each companion review.

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