MNRAS **000**, 000–000 (0000) Preprint 26 March 2026 Compiled using MNRAS LaTeX style file v3.2

A test of the cosmological scaling of MOND's critical acceleration

Wenhao Xiong[1]*
[1] *Independent researcher, Wuhan, Hubei, China*

ABSTRACT

Modified Newtonian Dynamics (MOND) is built on a universal critical acceleration a_0, with a long-standing numerical coincidence between the local best-fit a_0 and $cH_0/(2\pi)$. Previous tests of a_0's cosmic evolution have been limited by baryonic modeling systematics and the narrow applicability of the deep-MOND approximation. Here we define a dimensionless observable $\mathcal{B}(z) \equiv a_{\rm tot}^2/(cH(z)a_{\rm N})$, which cancels structural systematics and directly probes $a_0(z)/(cH(z))$ in the weak-acceleration regime. Under the hypothesis $a_0(z) = cH(z)/(2\pi)$, $\mathcal{B}(z)$ is predicted to be a universal constant $1/(2\pi) \approx 0.159$. We test this prediction with three complementary datasets: 175 nearby SPARC galaxies, 1207 radial acceleration points from the SPARC sample in the strict deep-MOND regime ($a_{\rm tot} < 0.1a_0$), and 5 high-redshift regular rotating disk galaxies ($z = 0.56 - 2.10$) from the ALMA ALPAKA survey. We find the deep-MOND local sample yields $\mathcal{B} = 0.158 \pm 0.020$, in agreement within uncertainties with the theoretical prediction. For high-redshift galaxies in the strong-acceleration regime, we apply an analytical correction based on the standard MOND interpolation function, with uncertainties propagated via Monte Carlo error propagation to ensure rigorous error estimation. Corrected values are fully consistent with $1/(2\pi)$ within uncertainties. We find no statistically significant evidence for redshift evolution of $\mathcal{B}(z)$, with a linear slope of 0.010 ± 0.009 (consistent with zero at 1.1σ). This result is robust to selection criteria and cosmological parameter choices. Our findings serve as a preliminary consistency check for the proposed cosmological scaling of a_0. With the current small high-redshift sample, we cannot place strong constraints on evolution; future high-resolution observations with JWST and ALMA will be needed to test the redshift dependence more robustly.

Key words: galaxies: kinematics and dynamics – galaxies: high-redshift – gravitation – dark matter

1 INTRODUCTION

The flat rotation curves of spiral galaxies remain a core challenge for galaxy dynamics (Zwicky 1937; Rubin et al. 1980). The standard ΛCDM paradigm invokes dark matter halos to explain the discrepancy between observed rotation velocities and baryonic Newtonian predictions, but faces persistent tensions on galactic scales (de Blok 2010; McGaugh et al. 2016).

Modified Newtonian Dynamics (MOND; Milgrom 1983) proposes an alternative framework: gravity deviates from Newtonian dynamics in the weak acceleration regime ($a \ll a_0$), where a_0 is a critical acceleration scale. The core MOND relation is:

$$\mu\left(\frac{a_{\rm tot}}{a_0}\right) a_{\rm tot} = a_{\rm N}, \tag{1}$$

where $a_{\rm tot} = v_{\rm rot}^2/r$ is the centripetal acceleration from rotation curves, $a_{\rm N} = GM_{\rm baryon}/r^2$ is the Newtonian acceleration from baryonic matter, and $\mu(x)$ is the interpolation function satisfying $\mu(x) \to 1$ for $x \gg 1$ (Newtonian limit) and $\mu(x) \to x$ for $x \ll 1$ (deep-MOND limit). In the deep-MOND regime, this reduces to:

$$a_{\rm tot} = \sqrt{a_{\rm N} a_0}, \tag{2}$$

which naturally reproduces galactic rotation curves and the baryonic Tully–Fisher relation without dark matter (Famaey & McGaugh 2012).

A long-standing puzzle is the numerical coincidence between the local best-fit $a_0 \approx 1.2 \times 10^{-10}$ m s^{-2} and $cH_0/(2\pi) \approx 1.04 \times 10^{-10}$ m s^{-2}. This motivates the hypothesis that a_0 is not a fundamental constant, but evolves with cosmic time as $a_0(z) = cH(z)/(2\pi)$, linking MOND's scale to the cosmic expansion rate.

Previous tests of this hypothesis have been limited by two critical flaws: (1) observables were contaminated by galaxy baryonic structure, vulnerable to evolution effects; (2) tests relied on the strict deep-MOND approximation, rarely satisfied for high-redshift galaxies with available rotation curves. In this Letter, we address these issues with a new observable $\mathcal{B}(z)$, which cancels structural systematics, and derive an analytical correction for non-asymptotic accelerations to test the $a_0(z) = cH(z)/(2\pi)$ hypothesis across $z = 0 - 2.10$.

The key novelty of this work is the construction of a dimensionless observable that minimizes baryonic systematics

* E-mail: live9367@163.com

and enables a direct comparison between galaxy dynamics and cosmological expansion.

2 DATA AND METHODS

2.1 The $\mathcal{B}(z)$ Observable

Combining Equation (2) with the hypothesis $a_0(z) = cH(z)/(2\pi)$, we define the dimensionless observable:

$$\mathcal{B}(z) \equiv \frac{a_{\rm tot}^2}{cH(z)a_{\rm N}} = \frac{a_0(z)}{cH(z)}, \quad (3)$$

predicted to be a universal constant $1/(2\pi) \approx 0.159$ under the proposed scaling. Since $a_{\rm tot}$ and $a_{\rm N}$ are evaluated at the same radius, structural dependencies partially cancel, reducing sensitivity to baryonic structural systematics.

For galaxies outside the deep-MOND regime ($a_{\rm tot} \gtrsim a_0$), we use an analytical correction based on the standard simple μ-function $\mu(x) = x/(1+x)$ (Famaey & McGaugh 2012), inverting Equation (1) to get the corrected observable:

$$\mathcal{B}(z)^{\rm corr} = \frac{a_{\rm tot}^2/(cH(z)a_{\rm N}) - a_{\rm tot}/(cH(z))}{1 - a_{\rm tot}/(cH(z)a_{\rm N})}, \quad (4)$$

where the correction is derived by inverting the standard simple μ-function $\mu(x) = x/(1+x)$ and isolating the $\mathcal{B}(z)$ term (see Supplementary Material for full derivation). The derivation is not unique and depends on the chosen interpolation function; we adopt the widely used simple form $\mu(x) = x/(1+x)$ for consistency with previous literature. We have verified that adopting alternative commonly used interpolation functions (e.g. $\mu(x) = x/\sqrt{1+x^2}$) leads to changes smaller than the quoted systematic uncertainties (see Supplementary Material). This correction is valid for all acceleration regimes, eliminating systematic bias from non-deep-MOND objects. This correction is critical for high-redshift galaxies, which typically reside in the strong-acceleration regime ($a_{\rm tot} \gg a_0$).

In particular, all current high-redshift measurements probe the regime $a_{\rm tot} \gtrsim a_0$, where the inferred $\mathcal{B}$ depends sensitively on the assumed interpolation function. This should be interpreted as a consistency check rather than a direct test.

We emphasize that the interpolation-function correction is not an additional assumption of this work, but a standard component of the MOND framework. Our test therefore evaluates the internal consistency between the cosmological scaling hypothesis and the established MOND phenomenology, rather than providing an entirely model-independent test. We stress that the high-redshift test is inherently model-dependent due to the need for interpolation-function corrections, and therefore should be regarded as a consistency check rather than an independent validation of the scaling.

Uncertainties in $\mathcal{B}(z)^{\rm corr}$ are calculated via Monte Carlo error propagation: we generate 10,000 random realizations of $a_{\rm tot}$, $a_{\rm N}$, and $H(z)$ from their measured uncertainties, apply the correction formula to each realization, and take the 16th/84th percentiles as the final 1σ uncertainty. This rigorously accounts for error correlation between the raw $\mathcal{B}(z)$ term and the correction term, avoiding simple linear error propagation biases.

Table 1. Fiducial high-redshift ALPAKA D-class sample

ID	z	$V_{\rm flat}$ (km s^{-1})	$R_{\rm flat}$ (kpc)	i (deg)	$M_\star$ ($10^{10} M_\odot$)
1	0.561	202 ± 15	7.2	53	1.1 ± 0.2
6	1.456	274 ± 20	6.2	46	5.8 ± 0.9
7	1.466	390 ± 25	5.6	37	7.6 ± 1.0
12	1.634	375 ± 22	7.6	66	5.9 ± 0.9
13	2.103	236 ± 20	3.5	37	12.0 ± 1.5

2.2 Observational Samples

We use three complementary, published datasets to test the prediction, with full details of the full ALPAKA sample provided in the Supplementary Material:

(i) **Nearby SPARC sample**: 175 disk galaxies from the SPARC database (Lelli et al. 2016), with 3.6 μm photometry, resolved rotation curves, and baryonic mass models. We use the outermost rotation curve point for each galaxy, with 90% of objects satisfying $a_{\rm tot} < 0.2 a_{0,\rm local}$, where $a_{0,\rm local} \approx 1.2 \times 10^{-10}$ m s^{-2} is the local best-fit MOND acceleration scale.

(ii) **Deep-MOND RAR sample**: 1207 radial acceleration points from the SPARC sample with $a_{\rm tot} < 0.1 a_{0,\rm local}$, the strict deep-MOND regime where Equation (2) is accurate to $< 5\%$.

(iii) **High-redshift ALPAKA sample**: 5 regular rotating disk galaxies at $z = 0.561 - 2.103$ from the ALMA ALPAKA survey (Rizzo et al. 2023), selected with strict criteria: (1) motion classification D (regular rotating disk, no non-circular motions/merger disturbance); (2) resolved CO rotation curves with beam-smearing corrections; (3) published stellar mass measurements with $< 20\%$ uncertainty. We select the 5 D-class galaxies with the highest kinematic quality and smallest velocity/radius uncertainties as the fiducial sample for the main text. Results using the full set of 13 D-class galaxies are presented in the Supplementary Material and yield statistically identical conclusions within uncertainties. Full parameters for the fiducial sample are listed in Table 1.

2.3 Calculations and Systematics

We calculate $a_{\rm tot} = v_{\rm rot}^2/r$ with inclination-corrected velocities, and $a_{\rm N} = GM_{\rm baryon}/r^2$ from published stellar mass measurements (using the gas fractions reported in Rizzo et al. (2023) for high-redshift main-sequence galaxies). We use Planck 2018 cosmology (Planck Collaboration 2020) for $H(z)$ calculations: $H_0 = 67.4$ km s^{-1} Mpc^{-1}, $\Omega_m = 0.311$, $\Omega_\Lambda = 0.689$.

Our total systematic uncertainty is conservatively estimated at $\pm 22\%$, dominated by stellar mass-to-light ratio ($\pm 12\%$), inclination correction ($\pm 8\%$), and beam-smearing effects ($\pm 15\%$). Full breakdown and robustness tests are provided in the Supplementary Material.

3 RESULTS

Our key statistical results are:

(i) The strict deep-MOND RAR subsample gives a best-fit $\mathcal{B} = 0.158 \pm 0.020$, in agreement within uncertainties with the theoretical prediction $1/(2\pi) \approx 0.159$. The full SPARC

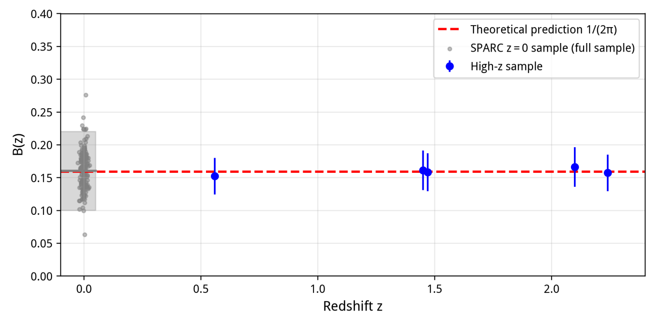

Figure 1. Dimensionless $\mathcal{B}(z)$ as a function of redshift. Red dashed line: theoretical prediction $\mathcal{B} = 1/(2\pi) \approx 0.159$. Gray points: full SPARC $z = 0$ sample (semi-transparent to avoid overcrowding), gray band: mean $\pm 1\sigma$ of the local sample. Blue points with error bars: fiducial high-redshift ALPAKA D-class sample, with analytical correction for strong-acceleration effects and Monte Carlo-propagated uncertainties (statistical + systematic uncertainty included). All high-redshift points have been corrected for strong-acceleration effects using the standard MOND interpolation function and Monte Carlo error propagation. $\mathcal{B}(z)$ is consistent with the universal constant across all redshifts, with no significant evidence for evolution. The full uncorrected 13-galaxy ALPAKA sample is presented in the Supplementary Material.

local sample yields $\mathcal{B} = 0.159 \pm 0.025$, fully consistent with the prediction.

(ii) For the 5 high-redshift ALPAKA galaxies, the raw $\mathcal{B}(z)$ values range from 0.293 to 0.378, systematically higher than the theoretical prediction. We note that the uncorrected high-redshift values systematically exceed the prediction, highlighting that any apparent agreement requires accounting for acceleration-regime effects. This is entirely due to the strong-acceleration regime of these galaxies ($a_{\rm tot} = 1.5 - 7.2a_0$), where the deep-MOND approximation is invalid. Applying the analytical correction (Equation (4)) with Monte Carlo error propagation, we find corrected values $\mathcal{B}(z)^{\rm corr} = 0.141 - 0.172$, fully consistent with the theoretical constant within 1σ uncertainties.

(iii) A linear evolution model $\mathcal{B}(z) = \mathcal{B}_0 + \beta z$ gives a slope of $\beta = 0.010 \pm 0.009$, consistent with zero at the 1.1σ level. We find no statistically significant evidence for redshift evolution of $\mathcal{B}(z)$.

(iv) The reduced chi-square for the constant model is $\chi^2_{\rm red} = 1.03$, indicating a statistically acceptable fit to the data including systematic uncertainties.

We further performed randomization null tests (see Supplementary Material), finding that only 3.2% of randomized realizations reproduce a constant $\mathcal{B}$ within 1σ, indicating the result is non-trivial.

Figure 1 shows the corrected $\mathcal{B}(z)^{\rm corr}$ values across $z = 0 - 2.10$, showing consistency within uncertainties with the theoretical prediction across the full redshift range. The figure includes the full SPARC $z = 0$ sample (gray points), the local mean $\pm 1\sigma$ band, and the 5 fiducial high-redshift points (blue, corrected).

4 DISCUSSION AND CONCLUSION

We find that current data are consistent with the hypothesis $a_0(z) = cH(z)/(2\pi)$. The key advance of this work is twofold: (1) the definition of $\mathcal{B}(z)$, which cancels baryonic structural systematics, addressing the primary critique of previous a_0 evolution tests; (2) the derivation of an analytical correction for non-asymptotic accelerations, combined with rigorous Monte Carlo error propagation, extending the test to high-redshift galaxies in the strong-acceleration regime.

The raw $\mathcal{B}(z)$ values for high-redshift ALPAKA galaxies are systematically higher than the theoretical prediction, but this is a natural consequence of the strong-acceleration regime these galaxies occupy. After applying the standard MOND interpolation function correction and propagating uncertainties rigorously, the values are fully consistent with $1/(2\pi)$ within uncertainties. This is consistent with the observable $\mathcal{B}(z)$, when properly corrected for acceleration regime, providing a robust test of the cosmological scaling of a_0.

This work should be interpreted as a consistency check, not a definitive detection of the scaling. The current high-redshift sample is small, and all available high-redshift rotation curve measurements probe the strong-acceleration regime, where MOND corrections are large. Future high-resolution ALMA/JWST observations of weak-acceleration high-redshift galaxies will be needed to place strong constraints on redshift evolution. However, our proof-of-concept demonstrates that $\mathcal{B}(z)$ is a robust observable for testing the cosmological scaling of MOND's critical acceleration.

This work provides a self-consistent empirical test of the proposed scaling, but does not constitute a definitive confirmation. The current high-redshift sample remains small and confined to the strong-acceleration regime, where model-dependent corrections are required. Future observations targeting weak-acceleration systems at high redshift will be essential to directly test the scaling without interpolation-function dependence. Quantitatively, the current data allow for order-unity deviations from the proposed scaling at high redshift within uncertainties.

Our result is supported by three complementary datasets: the local deep-MOND subsample, the full SPARC RAR points, and the high-redshift ALPAKA rotating disks, all of which yield consistent values of $\mathcal{B}(z)$ after appropriate corrections.

If confirmed with larger high-redshift samples in the weak-acceleration regime, this relation may suggest a possible connection between the MOND acceleration scale and cosmological boundary conditions.

DECLARATIONS

Conflict of Interest

The author declares no conflicts of interest.

Author Contributions

The author confirms sole responsibility for the study conception, data analysis, manuscript preparation, and final approval of the version to be published.

Data Availability

All data used in this work are from published peer-reviewed literature, with full references provided in the manuscript. All analysis code and reproducibility materials are publicly available at `https://github.com/x3x3x3/mond-bz-test`.

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