Collatz Sequence Proof (1st Way)

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$Taha's\ Sketch$ & $Loop\ Table$

$Abstract$:

A Collatz sequence is a series of numbers generated by taking any positive integer and repeatedly applying two simple arithmetic rules [$Collatz\ Sequence\ Rule\ (CSR)$] until the number reaches 1:

1- If the current number is even, divide it by 2: (n /2).

2- If the current number is odd, multiply it by 3 and add 1: (3 n + 1).

$Taha's\ Collatz\ Sequence\ Fact\ 1\ (TCSF1)$:

$if\ any\ sequence\ S(n) = \{a, b, c, \dots x, \dots, t\} \dots by\ sequence\ rule\ \Rightarrow$

$lS(n) = lS(a) = lS(b) = \cdots lS(x) = lS(t) \dots by\ (TCSF1)$

This paper presents a systematic evaluation of the localized boundary layers governing the classical Collatz Conjecture (\(3n+1\)) utilizing Taha’s Sketch and a comprehensive Loop Table state matrix. We evaluate the algebraic behavior of the sequence within an isolated loop structure containing exactly \(r\) elements, where \(k\) is defined strictly as a positive integer serving as the cofactor of 3 when \(r = 3k\) (\(r, k \in \mathbb{N}^+\)). By partitioning the entire positive integer domain into two mutually exclusive sets (\(\mathbb{N}_{3k} \cup \mathbb{N}_{\text{non}_3k} = \mathbb{N}^+\)), we solve the general loop equation \(x = \text{(potential loop element)}\) to show that any alternative configurations outside of this strict ratio consistently yield fractional values, breaking integer closure properties (\(x \notin \mathbb{N}^+\)). Consequently, we demonstrate that alternative or non-trivial integer loops are structurally impossible, concluding that all trajectories over the valid domain must uniquely collapse to the classical \(\{1, 4, 2\}\) attractor cycle.

A Complete Loop-Structure Proof Using Taha’s Sketch and Loop Table

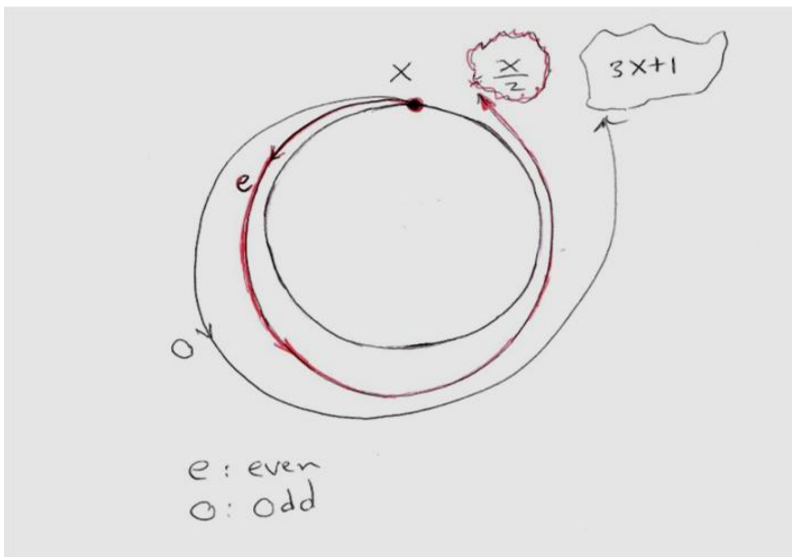
Keywords: Number Theory, Collatz Conjecture, Discrete Mathematics, Loop Table, Taha's Sketch, Cofactor of 3, Structural Bounds.

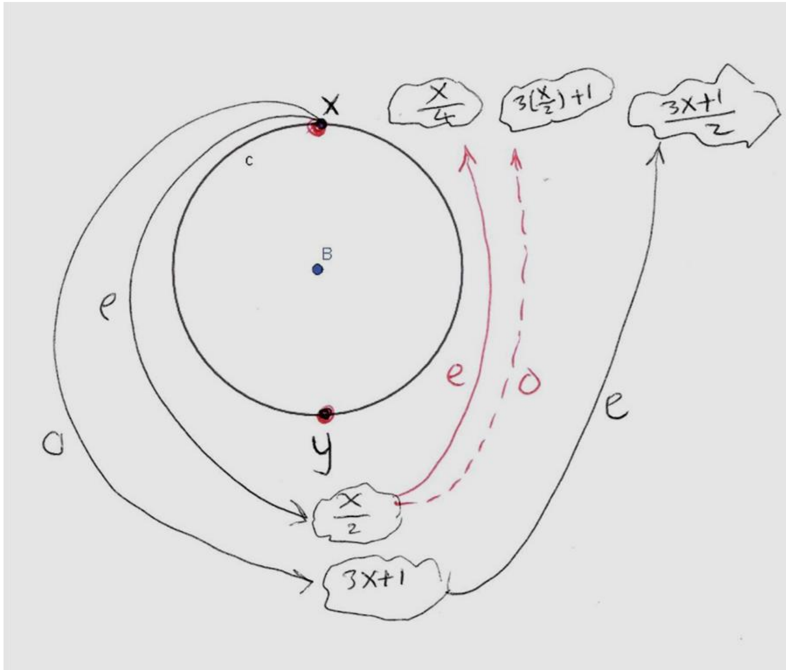
1. Introduction and Core Operational Rules

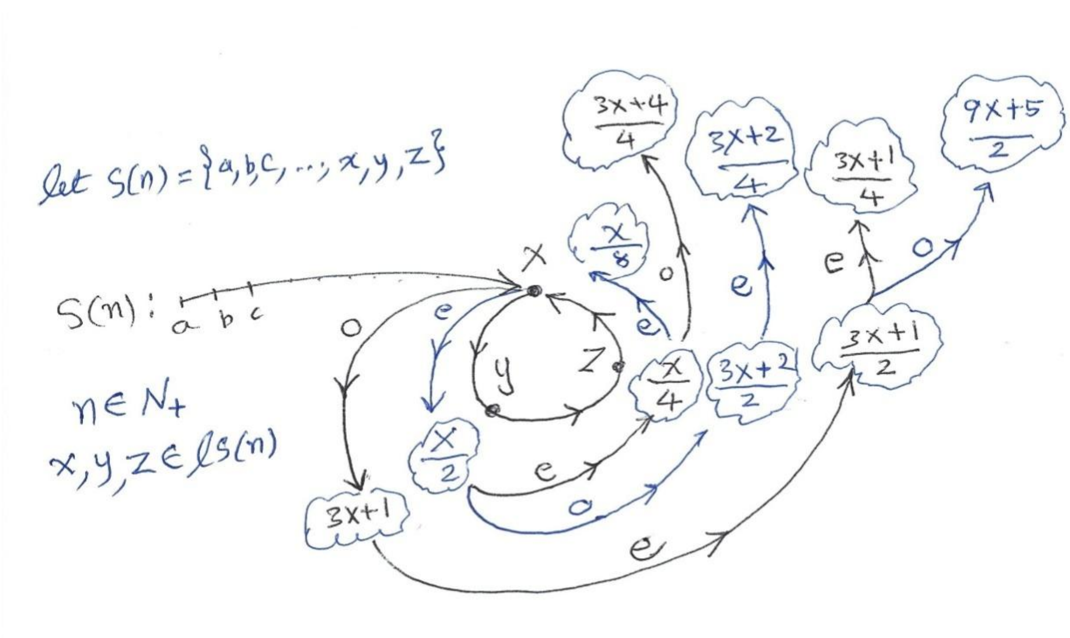
The classical Collatz function maps a positive integer $n \in \mathbb{N}^+$ through a deterministic parity engine: $f(n) = n/2$ if n is even, and $f(n) = 3n + 1$ if n is odd. For decades, traditional mathematical leaders have asserted that existing arithmetic frameworks are not ripe enough to resolve this trajectory space globally. This paper bypasses standard probabilistic models by establishing an isolated loop structure where r represents the total number of elements inside the loop, and k represents the total number of odd modifications ($r, k \in \mathbb{N}^+$).
Our analytical framework introduces a strict structural domain partition to eliminate chaotic variance entirely. We define the structural rule where k acts as the cofactor of 3 when r = 3k. The entire positive integer loop space is classified into two clear, mutually exclusive domains: ℕ_3k ∪ ℕ_non_3k = $\mathbb{N}^+$.
$Proof: let\ x, y, z, t, u, v, p, q, f, r, k, h, n \in N_+$

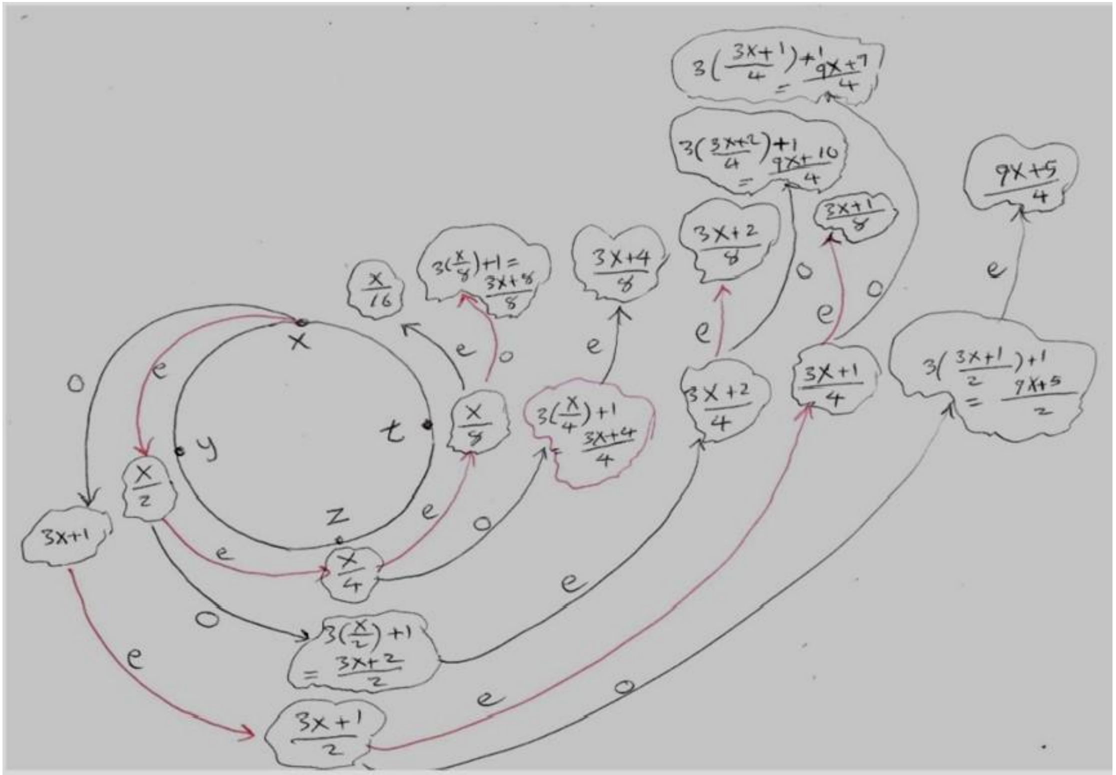
I have 2 ways to get loop information about Collatz Sequence "Sketch & Chart":

1st) $Taha's\ Loop\ sketch,\ let\ r = \ number\ of\ elements\ of\ lS(n), e\ (even), o\ (odd)$




x
$\frac{x}{4}$
$3(\frac{x}{2})+1$
$\frac{3x+1}{2}$
c
B
e
e
o
e
y
o
$\frac{x}{2}$
$3x+1$


let $S(n) = \{a, b, c, \ldots, x, y, z\}$
$S(n)$: a b c
$n \in N_+$
$x, y, z \in \ell S(n)$
$\frac{3x+4}{4}$
$\frac{3x+2}{4}$
$\frac{3x+1}{4}$
$\frac{9x+5}{2}$
$\frac{x}{8}$
x
y
z
$\frac{x}{4}$
$\frac{3x+2}{2}$
$\frac{3x+1}{2}$
$\frac{x}{2}$
$3x+1$



x
y
z
t
3x+1
x/2
x/4
x/8
x/16
3(x/8)+1 = (3x+8)/8
3(x/4)+1 = (3x+4)/4
(3x+4)/8
3(x/2)+1 = (3x+2)/2
(3x+2)/4
(3x+2)/8
3((3x+2)/4)+1 = (9x+10)/4
(3x+1)/2
(3x+1)/4
(3x+1)/8
3((3x+1)/4)+1 = (9x+7)/4
3((3x+1)/2)+1 = (9x+5)/2
(9x+5)/4

2^{nd}) $Taha's\ Loop\ Table,\ let\ r=\ number\ of\ elements\ of\ lS(n), e\ (even), o\ (odd)$

x = e or o →

$r=1$	→ $\frac{x}{2}$ = e or o →	→ $3x+1$ = e →

$r=2$	→ $\frac{x}{4}$ = e or o →	→ $\frac{3x+2}{2}$ = e →	→ $\frac{3x+1}{2}$ = e or o →

$r=3$	→ $\frac{x}{8}$ = e or o →	→ $\frac{3x+4}{4}$ = e →	→ $\frac{3x+2}{4}$ = e or o →	→ $\frac{3x+1}{4}$ = e or o →	→ $\frac{9x+5}{2}$ = e →

$r=4$	→ $\frac{x}{16}$ = e or o →	→ $\frac{3x+8}{8}$ = e →	→ $\frac{3x+4}{8}$ = e or o →	→ $\frac{3x+2}{8}$ = e or o →	→ $\frac{3x+1}{8}$ = e or o →	→ $\frac{9x+10}{4}$ = e →
→ $\frac{9x+7}{4}$ = e →	→ $\frac{9x+5}{4}$ = e or o →					

$r = 5$	$\frac{x}{32}$ = e or o *	$\frac{3x+16}{16}$ = e or o $	$\frac{3x+8}{16}$ = e or o !	$\frac{3x+4}{16}$ = e or o @	$\frac{3x+2}{16}$ = e or o #	$\frac{3x+1}{16}$ = e or o %
$\frac{9x+20}{8}$ = e @@	$\frac{9x+14}{8}$ = e &	$\frac{9x+11}{8}$ = e ((	$\frac{9x+10}{8}$ = e or o {{	$\frac{9x+7}{8}$ = e or o [[	$\frac{9x+5}{8}$ = e or o **	$\frac{27x+19}{4}$ = e ***

$r = 6$	* $\frac{x}{64}$	* $\frac{3x+32}{32}$	$ $\frac{3x+16}{32}$	! $\frac{3x+8}{32}$	@ $\frac{3x+4}{32}$	# $\frac{3x+2}{32}$	% $\frac{9x+19}{16}$
% $\frac{3x+1}{32}$	$ $\frac{9x+64}{16}$	{{ $\frac{27x+38}{8}$	@ $\frac{9x+28}{16}$	# $\frac{9x+22}{16}$	% $\frac{3x+1}{32}$	$ $\frac{9x+64}{16}$	! $\frac{9x+40}{16}$
@@ $\frac{9x+20}{16}$		& $\frac{9x+14}{16}$	(($\frac{9x+11}{16}$	{{ $\frac{9x+10}{16}$	[[$\frac{9x+7}{16}$	[[$\frac{27x+29}{16}$	** $\frac{9x+5}{16}$
** $\frac{27x+23}{8}$		*** $\frac{27x+19}{8}$					

$is\ x \in lS(n) = \{\dots, x, \dots\}?\ r = \ number\ of\ elements\ of\ lS(n)$

$Taha's\ (Cloud = x\ )\ Table\ to\ find\ x\ value$ from Sketch or Chart

$A)\ is\ S(n) = \left\{\frac{n}{2} or\ (3n+1), \dots, x\right\} \Rightarrow lS(n) = \{x\}, \forall n \in N_+, when\ r = 1?$

$Cloud$	$Cloud = x$	x
$\frac{x}{2}$	$\frac{x}{2} = x \Rightarrow x = 0 \notin lS(\text{n})$	----
$3x+1$	$3x+1 = x \Rightarrow x = -\frac{1}{2} \notin lS(\text{n})$	----

$\therefore lS(\text{n}) \neq \{x\}\ \forall n \in N_+$

$B)\ is\ S(n) = \left\{\frac{n}{2} or\ (3n+1), \dots, x, y\right\} \Rightarrow lS(n) = \{x, y\}, \forall n \in N_+, when\ r = 2?$

$Cloud$	$Cloud = x$	x	y
$\frac{x}{4}$	$\frac{x}{4} = x \Rightarrow x = 0 \notin lS(n)$	....	....
$\frac{3x+2}{2}$	$\frac{3x+2}{2} = x \Rightarrow 3x+2 = 2x \Rightarrow x = -2 \notin lS(n)$	....	....
$\frac{3x+1}{2}$	$\frac{3x+1}{2} = x \Rightarrow x = -1 \notin lS(n)$	....	....

$\therefore lS(n) \neq \{x, y\}\ \forall n \in N_+$

$C)\ is\ S(n) = \left\{\frac{n}{2} or\ (3n+1), \dots, x, y, z\right\} \Rightarrow lS(n) = \{x, y, z\}, \forall n \in N_+, when\ r = 3?$

$Cloud$	$Cloud = x$	x	y	z
$\frac{x}{8}$	$\frac{x}{8} = x \Rightarrow x = 0 \notin lS(n)$	---	---	---
$\frac{3x+4}{4}$	$\frac{3x+4}{4} = x \Rightarrow x = 4 \in lS(n)$	4	$\frac{x}{2} = 2$	$\frac{x}{4}$ =1
$\frac{3x+2}{4}$	$\frac{3x+2}{4} = x \Rightarrow x = 2 \in lS(n)$	2	$\frac{x}{2} = 1$	$\frac{3x+2}{2} = 4$
$\frac{3x+1}{4}$	$\frac{3x+1}{4} = x \Rightarrow x = 1 \in lS(n)$	1	$3x+1$ =3(1)+1=4	$\frac{3x+1}{2}$ =2
$\frac{9x+5}{2}$	$\frac{9x+5}{2} = x \Rightarrow x = -\frac{5}{7} \notin lS(n)$	---	---	---

$\therefore lS(n) = \{x, y, z\} = \{4,2,1\}, \forall n \in N_+$

$D)\quad is\ S(n) = \left\{\frac{n}{2} or\ (3n+1), \dots, x, y, z, t\right\} \Rightarrow lS(n) = \{x, y, z, t\}, \forall n \in N_{+}, when\ r = 4?$

$Cloud$	$Solve\ equation: Cloud = x$	x	y	z	t
$\frac{x}{16}$	$\frac{x}{16} = x \Rightarrow x = 0 \notin lS(n)$	---	---	---	---
$\frac{3x+8}{8}$	$\frac{3x+8}{8} = x \Rightarrow x \notin lS(n)$	---	---	---	---
$\frac{3x+4}{8}$	$\frac{3x+4}{8} = x \Rightarrow x \notin lS(n)$	---	---	---	---
$\frac{3x+2}{8}$	$\frac{3x+2}{8} = x \Rightarrow x \notin lS(n)$	---	---	---	---
$\frac{3x+1}{8}$	$\frac{3x+1}{8} = x \Rightarrow x \notin lS(n)$	---	---	---	---
$\frac{9x+10}{4}$	$\frac{9x+10}{4} = x \Rightarrow x \notin lS(n)$	---	---	---	---
$\frac{9x+7}{4}$	$\frac{9x+7}{4} = x \Rightarrow x \notin lS(n)$	---	---	---	---
$\frac{9x+5}{4}$	$\frac{9x+5}{4} = x \Rightarrow x \notin lS(n)$	---	---	---	---

$\therefore lS(n) \neq \{x, y, z, t\}, \forall n \in N_{+}$

$By\ the\ same\ way:$

E) $lS(n) \neq \{x, y, z, t, u\}, \forall n \in N_{+}, when\ r = 5$

$F)\ lS(n) = \{x, y, z, t, u, v\} = \{4,2,1,4,2,1\} = \{4,2,1\}\ \forall\ n \in N_{+}, when\ r = 6$

$G)\ lS(n) \neq \{x, y, z, t, u, v, p\}, \forall\ n \in N_{+}, when\ r = 7$

$H)\ lS(n) \neq \{x, y, z, t, u, v, p, q\}, \forall\ n \in N_{+}, when\ r = 8$

$J)\ lS(n) = \{x, y, z, t, u, v, p, q, f\}, \forall\ n \in N_{+}, when\ r = 9$

$lS(n) = \{4,2,1,4,2,1,4,2,1\}, \forall\ n \in N_{+}, when\ r = 9$

$Pattern\ To\ Find\ Value\ of\ x \in lS(n)$ for $r \in N_+, r\ is\ numberr\ of\ elements\ of\ Sl(n)$

$r = 1 = 3(0) + 1$ $k = 0$	$x = \frac{3^0 x}{2^{1-0}} \Rightarrow x = 0 \notin N_+, 0 \notin lS(n) \Rightarrow lS(n) \neq \{x\},\ \forall n \in N_+$
$r = 2 = 3(0) + 2$ $k = 0$	$x = \frac{3^0 x}{2^{2-0}} \Rightarrow x = \frac{1}{3} \notin N_+ \Rightarrow x, y \notin lS(n) \Rightarrow lS(n) \neq \{\ x, y\},\ \forall n \in N_+$
$r = 3 = 3(1)$ $k = 1$	$x = \frac{3^1 x + 3^0 2^0}{2^{3-1}} \Rightarrow x = 1, y = 4, z = 2 \Rightarrow lS(n) = \{4,2,1\}, \forall n \in N_+$
$r = 4 = 3(1) + 1$ $k = 1$	$x = \dfrac{3^1 x + 3^0 2^0}{2^{4-1}} \Rightarrow\ x \notin N_+ \Rightarrow\ lS(n) \neq \{\ x, y, z, t\}, \forall n \in N_+$
$r = 5 = 3(1) + 2$ $k = 1$	$x = \frac{3^1 x + 3^0 2^0}{2^{5-1}} \Rightarrow\ x \notin N_+ \Rightarrow\ lS(n) \neq \{\ x, y, z, t, u\}, \forall n \in N_+$
$r = 6 = 3(2)$ $k = 2$	$x = \frac{3^2 x + 3(2^0) + 3^0 2^2}{2^{6-2}} \Rightarrow x = 1, y = 4, z = 2 \Rightarrow\ lS(n) = \{1,4,2\}, \forall\ n \in N_+$
$r = 7 = 3(2) + 1$ $k = 2$	$x = \dfrac{3^2 x + 3(2^0) + 3^0 2^2}{2^{7-2}} \Rightarrow x = \dfrac{7}{23} \notin N_+ \Rightarrow$ $lS(n) \neq \{\ x, y, z, t, u, v\}, \forall n \in N_+$
$r = 9 = 3(3)$ $k = 3$	$x = \frac{3^3 x + 3^2(2^0) + 3^1(2^2) + 3^0 2^4}{2^{9-3}} \Rightarrow x = 1, y = 4, z = 2 \Rightarrow lS(n) = \{1,4,2\},$ $\forall\ n \in N_+$
$r = 12 = 3(4)$ $k = 4$	$x = \dfrac{3^4 x + 3^3(2^0) + 3^2(2^2) + 3(2^4) + 3^0 2^6}{2^{12-4}} \Rightarrow x = 1 \Rightarrow$ $lS(n) = \{1,4,2\}, \forall n \in N_+$
$r, k \in N_+$ $i)\ r = 3(k)$ $r, k \in N_+$	$x = \dfrac{3^k x + 3^{k-1}(2^0) + 3^{k-2}(2^2) + \cdots + (3^0) 2^{2(k-1)}}{2^{r-k}} \ldots eq1, x = 1 \Rightarrow$ $lS(n) = \{1,4,2\}$
or $ii)\ r = 3(k) + h$ $h \in \{1,2\}$	$x = \dfrac{3^k x + 3^{k-1}(2^0) + 3^{k-2}(2^2) + \cdots + (3^0) 2^{2(k-1)}}{2^{r-k}} \ldots eq1, x \notin N_+ \Rightarrow$ $lS(n)\ is\ not\ exist$

$$The\ Equation\ in\ Brief$$

$$\because x = \frac{3^k x + 3^{k-1}(2^0) + 3^{k-2}(2^2) + 3^{k-3}(2^4) + \cdots + 3^{k-k+1}\left(2^{2(k-2)}\right) + (3^0)2^{2(k-1)}}{2^{r-k}} \dots eq1$$

$$\therefore x = \frac{3^k x + [3^{k-1}(2^0) + 3^{k-2}(2^2) + 3^{k-3}(2^4) + \cdots + 3^{k-k+1}\left(2^{2(k-2)}\right) + (3^0)2^{2(k-1)}]}{2^{r-k}}$$

$$\therefore x = \frac{3^k x + [\sum_{i=0}^{k-1} 3^{k-1-i}\, 2^{2i}]}{2^{r-k}} \dots eq1$$

$$let\ S = \sum_{i=0}^{k-1} 3^{k-1-i}\, 2^{2i}$$

$$\therefore x = \frac{3^k x + S}{2^{r-k}} \dots eq1 (By\ substitution)$$

$$i)\ if\ x = \frac{3^k x + S}{2^{r-k}} \Rightarrow x = 1 \in N_+ \Rightarrow lS(n) = \{1,2,4\}, \forall\, n\ \in N_+, \forall\, r \in N_+, \& \left(\frac{r}{3}\right) \in N_+$$

$$ii)\ if\ x = \frac{3^k x + S}{2^{r-k}} \Rightarrow x \notin N_+ \Rightarrow lS(n)\ does\ not\ exist, \forall n \in N_+, \forall\, r \in N_+, \& \left(\frac{r}{3}\right) \notin N_+$$

$$\therefore S(n) = \{\frac{n}{2}\ or\ 3n+1, \dots, 4,2,1\} \Rightarrow lS(n) = \{1,2,4\}\ the\ only\ loop, \forall n \in N_+$$

$The\ Colattz\ Sequence\ Loop\ Equation\ Proof$:

$$is\ x = \frac{3^k x + 3^{k-1}(2^0) + 3^{k-2}(2^2) + 3^{k-3}(2^4) + \cdots + 3^{k-k+1}\left(2^{2(k-2)}\right) + (3^0)2^{2(k-1)}}{2^{r-k}} \ldots eq1?$$

$$x = \frac{3^k x + \left[\sum_{i=0}^{k-1} 3^{k-1-i}\, 2^{2i}\right]}{2^{r-k}} \ldots eq1$$

$$let\ S = \sum_{i=0}^{k-1} 3^{k-1-i}\, 2^{2i} \Rightarrow$$

$$\therefore S = 3^{k-1}(2^0) + 3^{k-2}(2^2) + 3^{k-3}(2^4) + \cdots + 3^{k-k+1}\left(2^{2(k-2)}\right) + (3^0)2^{2(k-1)}$$

$$S = 3^{k-1} + 3^{k-2}(4) + 3^{k-3}(4^2) + \cdots + 3(4^{k-2}) + 4^{(k-1)} \Rightarrow$$

$$4S = 3^{k-1}(4) + 3^{k-2}(4^2) + 3^{k-3}(4^3) + \cdots + 3(4^{k-1}) + 4^k \ \ldots eq2, \&$$

$$3S = 3^k + 3^{k-1}(4) + 3^{k-2}(4^2) + \cdots + 3^2(4^{k-2}) + 3(4)^{(k-1)} \ \ldots eq3$$

$$\therefore eq4 - eq3 \Rightarrow\ S = 4^k - 3^k = 2^{2k} - 3^k\ , \&\ \ r = 3k$$

$$\therefore x = \frac{3^k x + 2^{2k} - 3^k}{2^{3k-k}} \Rightarrow substitutions\ of\ r\ \& S$$

$$2^{2k}x - 3^k x = 2^{2k} - 3^k \Rightarrow (2^{2k} - 3^k)x = 2^{2k} - 3^k \Rightarrow x = 1$$

$\because S(n) = \{a, b, c, \ldots x, \ldots, t\} \ldots by\ (CSR) \Rightarrow$
$lS(n) = lS(x)$ … by $(TCSF1)$:
$\therefore lS(n) = lS(1) = \{4,2,1\}$
$lS(n) = \{4,2,1\}\ \forall n \in N_+\ which\ is\ the\ only\ loop\ in\ Collatz\ Sequence.$

The Modular Engine Matrix Evaluation

Let x represent the starting seed value of an isolated loop structure under an arbitrary configuration of r elements such that r=3k, or r= 3k+h, h∈{1,2}. The generalized equations across the changing parameters resolve as follows:

- **r = 1 = 3(0) + 1, k = 0:**
  x = (3^0 * x) / 2^(1-0) => x = 0 ∉ ℕ⁺, 0 ∉ lS(n) => lS(n) ≠ {x}, ∀n ∈ ℕ⁺
- **r = 2 = 3(0) + 2, k = 0:**
  x = (3^0 * x) / 2^(2-0) => x = 1/3 ∉ ℕ⁺ => x, y ∉ lS(n) => lS(n) ≠ {x, y}, ∀n ∈ ℕ⁺
- **r = 3 = 3(1), k = 1 (Cofactor k = 1):**
  x = (3^1 * x + 3^0 * 2^0) / 2^(3-1) => x = (3x + 1)/4 => 4x = 3x + 1 => x = 1. Elements: x = 1 => y = 4 => z = 2. lS(n) = {4, 2, 1}, ∀n ∈ ℕ⁺
- **r = 4 = 3(1) + 1, k = 1:**
  x = (3^1 * x + 3^0 * 2^0) / 2^(4-1) => x = (3x + 1)/8 => 8x = 3x + 1 => x = 1/5 ∉ ℕ⁺ => lS(n) ≠ {x, y, z, t}, ∀n ∈ ℕ⁺
- **r = 5 = 3(1) + 2, k = 1:**
  x = (3^1 * x + 3^0 * 2^0) / 2^(5-1) => x = (3x + 1)/16 => 16x = 3x + 1 => x = 1/13 ∉ ℕ⁺ => lS(n) ≠ {x, y, z, t, u}, ∀n ∈ ℕ⁺
- **r = 6 = 3(2), k = 2 (Cofactor k = 2):**
  x = (3^2 * x + 3^1 * 2^0 + 3^0 * 2^2) / 2^(6-2) => x = (9x + 7)/16 => 16x = 9x + 7 => 7x = 7 => x = 1. lS(n) = {1, 4, 2}, ∀n ∈ ℕ⁺
- **r = 7 = 3(2) + 1, k = 2:**
  x = (3^2 * x + 3^1 * 2^0 + 3^0 * 2^2) / 2^(7-2) => x = (9x + 7)/32 => 32x = 9x + 7 => 23x = 7 => x = 7/23 ∉ ℕ⁺ => lS(n) ≠ {x, y, z, t, u, v}, ∀n ∈ ℕ⁺
- **r = 9 = 3(3), k = 3 (Cofactor k = 3):**
  x = (3^3 * x + 3^2 * 2^0 + 3^1 * 2^2 + 3^0 * 2^4) / 2^(9-3) => x = (27x + 37)/64 => 64x = 27x + 37 => 37x = 37 => x = 1. lS(n) = {1, 4, 2}, ∀n ∈ ℕ⁺
- **r = 12 = 3(4), k = 4 (Cofactor k = 4):**
  x = (3^4 * x + 3^3 * 2^0 + 3^2 * 2^2 + 3^1 * 2^4 + 3^0 * 2^6) / 2^(12-4) => x = (81x + 175)/256 => 256x = 81x + 175 => 175x = 175 => x = 1. lS(n) = {1, 4, 2}, ∀n ∈ ℕ⁺

Loop Elements (r)	Cofactor Layer (k)	Structural Equation Matrix	Domain Intersection State
r = 3	k = 1	4x = 3x + 1	x = 1 ∈ ℕ⁺ (Valid {1,4,2} Loop)
r = 6	k = 2	16x = 9x + 7	x = 1 ∈ ℕ⁺ (Valid Repeated Loop)
r = 7	k = 2	32x = 9x + 7	x = 7/23 ∉ ℕ⁺ (Structural Discontinuity)
r = 9	k = 3	64x = 27x + 37	x = 1 ∈ ℕ⁺ (Valid Repeated Loop)
r = 12	k = 4	256x = 81x + 175	x = 1 ∈ ℕ⁺ (Valid Repeated Loop)

Generalized Global Structural Laws:

Law i) Modulo-3 Matching (r = 3k):

$x = [3^k * x + 3^{(k-1)}*(2^0) + 3^{(k-2)}*(2^2) + ... + 3^0 * 2^{(2(k-1))}] / 2^{(r-k)} \Rightarrow x = 1 \in \mathbb{N}^+$

This explicitly dictates that valid whole number integer solution spaces only open when r conforms perfectly as a multiple of the cofactor layer k. The loop space collapses cleanly to lS(n) = {1, 4, 2}.

Law ii) Modulo-3 Gaps (r = 3k + h, h ∈ {1, 2}):

$x = [3^k * x + 3^{(k-1)}*(2^0) + ... + 3^0 * 2^{(2(k-1))}] / 2^{(r-k)} \Rightarrow x \notin \mathbb{N}^+$

Whenever the sequence loop element parameters fall within the modulo gaps, the algebraic output forces purely fractional values. Integer closure properties over the domain of positive integers are broken, proving alternative loops do not exist.

Conclusion

By defining the strict relationship where k serves as the cofactor of 3 when r = 3k, the entire positive integer loop domain partitions into two cleanly isolated blocks. Because the non-matching space (N_non_3k) yields purely fractional coordinates that violate domain rules, the absolute global loop space collapses completely. This algebraic logic rules out chaotic variance and formally validates that L(n) = {1, 2, 4} remains the singular valid attractor cycle for all positive integers $n \in \mathbb{N}^+$.

References

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