Cosmological Special Theory of Relativity: A Consolidated Review of the Formalism

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Abstract

Over a series of papers published between 2018 and 2025, the author has developed a framework, the Cosmological Special Theory of Relativity (CSTR), that incorporates the time-dependent expansion of the Robertson–Walker universe directly into the coordinate transformations, differential operators, and relativistic wave equations of special relativity, by way of an auxiliary "cosmological inertial frame" in which physical (expanded) coordinates are related to a local, unexpanded coordinate through a cosmological expansion function $\tau(t_0)$ of the cosmological time t_0 (Yi, 2020,?). This paper consolidates the central results of that program into a single, self-contained review. We present the space-time relations that define the cosmological inertial frame, the cosmological Klein-Gordon equation and its free-particle wave function and Yukawa-potential solution (Yi, 2020, 2021), the corresponding non-relativistic Schrödinger limit and cosmological uncertainty relation, the Dirac equation in the cosmological inertial frame and its consistency with the Klein-Gordon equation (Yi, 2021), the Klein-Gordon–Maxwell theory of interacting complex scalar and electromagnetic fields (Yi, 2021), the canonical quantization of the Klein-Gordon scalar field (Yi, 2021), the particle's kinetic energy and force in the cosmological frame, and the extension to a Cosmological General Theory of Relativity (CGTR), including cosmologically embedded Schwarzschild and Kerr-Newman solutions (Yi, 2021, 2025). We close with a discussion of the relationship between this formalism and the standard comoving-coordinate treatment of Robertson-Walker cosmology, and of open questions raised in the literature regarding CSTR's consistency and standing (A Critical Review, 2025).

Keywords: cosmological special theory of relativity; cosmological inertial frame; Klein-Gordon equation; Dirac equation; Yukawa potential; Klein-Gordon-Maxwell theory; cosmological general theory of relativity

1. Introduction

Standard special relativity is formulated on a fixed Minkowski background and does not, by itself, account for the large-scale expansion of the Universe described by the Robertson-Walker metric of general relativity. Beginning with the original formulation of the Cosmological Special Theory of Relativity (CSTR) (Yi, 2020), the author has developed a sequence of papers that instead build the cosmological expansion directly into a modified special-relativistic formalism, by introducing an auxiliary frame – the *cosmological inertial frame* – in which spatial coordinates local to an observer are related to physically expanded coordinates through a time-dependent *cosmological expansion function* $\tau(t_0)$, with t_0 the cosmological time of the background Robertson-Walker geometry. Relativistic wave equations (Klein-Gordon, Schrödinger, Dirac), electrodynamics (Maxwell's equations and the Klein-Gordon-Maxwell theory of a charged scalar field), the canonical quantization of the scalar field, and finally an extension to a full Cosmological General Theory of Relativity (CGTR) encompassing cosmologically embedded black-hole solutions, have each been rederived within this framework across a series of short papers (Yi, 2020, 2021,?,?,?,?). The 2025 paper (Yi, 2025) consolidates this entire program under a single abstract, covering the Maxwell and electromagnetic wave equations, the energy-momentum relation and Klein-Gordon wave function, the Yukawa potential, the Schrödinger equation, the cosmological uncertainty principle, the Dirac equation, Klein-Gordon-Maxwell theory, the quantization of the Klein-Gordon scalar field, the particle's force and kinetic energy, and the Cosmological General Theory of Relativity together with its Schwarzschild and Kerr-Newman solutions.

The program developed incrementally: the original space-time relations and coordinate transformation of CSTR were introduced first (Yi, 2020), followed within the same year by the Klein-Gordon equation and its free-particle wave function (Yi, 2020). The following year extended the framework to the Cosmological General Theory of Relativity and its black-hole solutions (Yi, 2021), the Yukawa potential (Yi, 2021), the Dirac equation (Yi, 2021), the interacting Klein-Gordon-Maxwell theory (Yi, 2021), and finally the canonical quantization of the scalar field (Yi, 2021). Each of these later results depends on the space-time relations introduced in the first paper of the series, so that the 2025 consolidation (Yi, 2025) is best read as an integration of a single, cumulative research program rather than as a collection of independent results.

The physical motivation for this program is that ordinary special relativity, by construction, describes physics on a fixed, non-expanding background, while every laboratory and astrophysical measurement is ultimately made against the backdrop of an expanding universe. In the overwhelming majority of terrestrial and even solar-system contexts, the rate of cosmic expansion is negligible over the relevant length and time scales, and ordinary special relativity is an excellent approximation; CSTR is concerned with the regime – relevant to high-redshift astrophysics and, in principle, to sufficiently long-baseline laboratory tests – in which this approximation is not assumed from the outset, but is instead recovered as a specific limit ($\tau(t_0) \to \mathrm{const}$) of a more general formalism, as discussed further in Section 2.

The purpose of the present paper is to gather these results into a single, consistently notated review, so that the overall structure of CSTR – and the logical dependence of its later results (the Dirac equation, the Klein-Gordon-Maxwell theory, the CGTR solutions) on its earlier ones (the space-time relations of Section 2 and the Klein-Gordon equation of Section 3) – can be seen at a glance. Throughout, we adopt units in which the results are expressed with explicit factors of the speed of light c and the reduced Planck constant $\hbar$, following the convention of the cited original papers.

2. The Cosmological Inertial Frame

2.1. The Robertson-Walker Background

The starting point of CSTR is the spatially flat Robertson-Walker line element of standard cosmology,

$$ds^2 = -c^2 d\tau_{\mathrm{p}}^2 + a(t_0)^2 \left[dr'^2 + r'^2 d\Omega^2\right], \tag{1}$$

where t_0 is the cosmological time, r' is a dimensionless co-moving radial coordinate, $d\Omega^2 = d\theta^2 + \sin^2\theta\, d\phi^2$, and $a(t_0)$ is the scale factor, here taken (following the base ΛCDM model) for a spatially flat universe, $k = 0$ (Yi, 2021). CSTR denotes the scale factor by the cosmological expansion function $\tau(t_0) \equiv a(t_0)$, emphasizing its role, within the theory, not merely as a geometric scale factor but as the quantity that directly enters the coordinate transformations, differential operators, and wave equations of a modified special relativity.

2.2. Wave Function Type A: The Expanded Coordinate

CSTR defines the *cosmological inertial frame* by relating a local, unexpanded spatial coordinate x'^i (or radial coordinate r') to the physically expanded coordinate x^i (or r) through

$$x^i = \tau(t_0)\, x'^i, \qquad t = t_0, \tag{2}$$

which is referred to in the original papers as *Wave function Type A* (Yi, 2020, 2021). Equation (2) is the CSTR counterpart of the usual relation between comoving and physical distance in Robertson-Walker cosmology, but its distinguishing feature is that it is subsequently substituted directly into the differential operators of special-relativistic wave equations, so that an ordinary spatial derivative $\partial/\partial x^i$ is replaced by

$$\frac{\partial}{\partial x^i} \;\longrightarrow\; \frac{1}{\tau(t_0)}\,\frac{\partial}{\partial x'^i}, \tag{3}$$

while the time derivative $\partial/\partial t$ is left referred to the cosmological time t_0. A second convention, *Type B of the expanded distance*, is used in the treatment of the interacting scalar and electromagnetic fields of Section 6, and relates the invariant line element itself, rather than the coordinate operators, to the expansion function (Yi, 2021),

$$ds^2 = -c^2 dt_0^2 + \tau(t_0)^2 \left[dx'^2 + dy'^2 + dz'^2\right], \tag{4}$$

which is simply the spatially flat Robertson-Walker line element of Equation (1) expressed in Cartesian local coordinates. Equations (2), (3), and (4) together constitute the operational definition of the cosmological inertial frame used throughout the remainder of CSTR.

2.3. Cosmological Coordinate and Velocity Transformations

Within the cosmological inertial frame so defined, CSTR retains the Lorentz form of the transformation between two relatively moving frames, but with the ordinary spatial coordinate replaced by its expanded counterpart of Equation (2):

$$c\,t_0' = \gamma\left(c\,t_0 - \frac{v}{c}\,\tau(t_0)\,x'\right), \tag{5}$$

$$\tau(t_0)\,x' = \gamma\left(\tau(t_0)\,x - v\,t_0\right), \tag{6}$$

where $\gamma = (1 - v^2/c^2)^{-1/2}$ is the ordinary Lorentz factor and v is the relative velocity between the two cosmological inertial frames (Yi, 2020). Because the expansion function $\tau(t_0)$ multiplies the spatial part of the transformation, velocities and momenta defined in the cosmological frame acquire an explicit dependence on the epoch t_0 at which they are measured, a feature that propagates through every subsequent wave equation derived in the theory.

2.4. Relation to the Cosmological Redshift

Because $\tau(t_0)$ plays the role of the Robertson-Walker scale factor $a(t_0)$, CSTR identifies it with the standard cosmological redshift z of light emitted at cosmological time t_0^{em} and observed at the present epoch t_0^{obs} through the usual relation

$$1 + z = \frac{\tau(t_0^{\mathrm{obs}})}{\tau(t_0^{\mathrm{em}})}. \tag{7}$$

Equation (7) anchors the otherwise abstract expansion function $\tau(t_0)$ appearing in every wave equation of Sections 3–9 to a directly observable quantity, and it is through this identification that CSTR's modifications to the Klein-Gordon dispersion relation, the Yukawa screening length, and the uncertainty relation are, in principle, open to observational test at different redshifts.

2.5. Consistency Limits

Two limits of the formalism above are useful checks on its internal consistency. First, over any interval of cosmological time short enough that $\tau(t_0)$ may be treated as constant, $\tau(t_0) \to \tau_0 = \mathrm{const}$, Equations (5)–(6) reduce, after an overall rescaling of the spatial coordinate by τ_0, to the ordinary Lorentz transformation, so that CSTR reproduces standard special relativity locally, as required by the equivalence principle. Second, in the same limit, the modified Klein-Gordon equation of Section 3, the Dirac equation of Section 5, and the Klein-Gordon-Maxwell theory of Section 6 all reduce to their standard flat-space forms, since every departure from the ordinary wave equations in this formalism enters exclusively through explicit factors of $\tau(t_0)$ or its time derivatives.

3. The Klein-Gordon Equation and Its Solutions

3.1. The Cosmological Klein-Gordon Equation

Applying the substitution rule of Equation (3) to the ordinary free-particle Klein-Gordon equation yields the Klein-Gordon equation in the cosmological inertial frame (Yi, 2020),

$$\left[-\frac{1}{c^2}\frac{\partial^2}{\partial t_0^2}+\frac{1}{\tau(t_0)^2}\nabla'^2-\left(\frac{mc}{\hbar}\right)^2\right]\psi=0, \tag{8}$$

where ∇'^2 is the Laplacian with respect to the local coordinates x'^i and m is the particle's rest mass. For a free particle, CSTR adopts the plane-wave solution

$$\psi(x',t_0)=\exp\left[\frac{i}{\hbar}\left(\tau(t_0)\,\mathbf{p}'\cdot\mathbf{x}'-E\,t_0\right)\right], \tag{9}$$

which, substituted back into Equation (8), reproduces the relativistic energy-momentum relation of the cosmological frame,

$$E^2=c^2\mathbf{p}'^2+m^2c^4, \tag{10}$$

formally identical to the ordinary relativistic dispersion relation, but with the momentum $\mathbf{p}'$ now understood as conjugate to the local coordinate $\mathbf{x}'$ rather than to the physically expanded coordinate $\mathbf{x}=\tau(t_0)\mathbf{x}'$ (Yi, 2020).

3.2. Normalization and Completeness of the Plane-Wave Basis

At any fixed cosmological time t_0, the plane-wave solutions of Equation (9), labeled by the continuous local-momentum variable $\mathbf{p}'$, form a complete, delta-function-normalized basis for solutions of Equation (8) on the local coordinate space,

$$\int d^3x'\,\psi^*_{\mathbf{p}''}(x',t_0)\,\psi_{\mathbf{p}'}(x',t_0)=(2\pi\hbar)^3\,\delta^3(\mathbf{p}'-\mathbf{p}''), \tag{11}$$

exactly as in ordinary Klein-Gordon theory, because the explicit $\tau(t_0)$ dependence in the exponent of Equation (9) enters only as an overall rescaling of the momentum label and does not affect the orthogonality of the basis at fixed t_0. A general solution of Equation (8) can therefore be written as a superposition $\psi(x',t_0)=\int d^3p'\,c(\mathbf{p}')\,\psi_{\mathbf{p}'}(x',t_0)$ with time-independent expansion coefficients $c(\mathbf{p}')$, providing the basis for the canonical quantization of Section 7 (Yi, 2020).

3.3. The Cosmological Klein-Gordon Current

As in ordinary Klein-Gordon theory, a conserved four-current can be formed from the complex scalar solutions of Equation (8) and its conjugate. Multiplying Equation (8) by ψ^*, subtracting the complex conjugate equation multiplied by ψ, and using the differential substitution of Equation (3), one obtains the continuity equation

$$\frac{\partial\rho}{\partial t_0}+\frac{1}{\tau(t_0)}\nabla'\cdot\mathbf{j}'=0, \tag{12}$$

with density $\rho=(i\hbar/2mc^2)(\psi^*\partial_{t_0}\psi-\psi\partial_{t_0}\psi^*)$ and current $\mathbf{j}'=-(i\hbar/2m\tau(t_0))(\psi^*\nabla'\psi-\psi\nabla'\psi^*)$. As in the ordinary theory, ρ is not positive definite for a general solution of Equation (8), which is one of the motivations, carried over unchanged from the flat-space case, for the first-order Dirac equation of Section 5 (Yi, 2020).

3.4. The Yukawa Potential in the Cosmological Inertial Frame

Introducing a static source term into Equation (8) and solving for the time-independent, spherically symmetric case gives the Yukawa potential as modified by the expansion function (Yi, 2021),

$$V(r')=-\frac{g^2}{r'}\exp\left[-\frac{mc}{\hbar}\,\tau(t_0)\,r'\right], \tag{13}$$

where g is the coupling constant of the source. Equation (13) shows that the effective screening length of the Yukawa interaction, $\hbar/(mc\,\tau(t_0))$ measured in the local coordinate r', contracts as the expansion function $\tau(t_0)$ grows, or equivalently that the physical range of the interaction, $\tau(t_0)$ times the local screening length, remains set by the Compton wavelength $\hbar/(mc)$ of the mediating particle, independent of the epoch t_0 (Yi, 2021).

At early cosmological times, when $\tau(t_0)\ll 1$ relative to its present value (normalized so that $\tau(t_0^{\rm obs})=1$), the exponential factor in Equation (13) approaches unity over a much larger range of the local coordinate r' than it does today, so that the interaction is comparatively long-ranged when measured in the local, unexpanded coordinate, even though its physical range $\hbar/(mc)$ is unchanged. Conversely, at late times with $\tau(t_0)\gg 1$, the same interaction appears strongly localized in the local coordinate. This behavior is a direct consequence of holding the physical Yukawa range fixed at the Compton wavelength while the mapping between local and physical coordinates, Equation (2), stretches with cosmic time (Yi, 2021).

3.5. The Schrödinger Equation as a Non-Relativistic Limit

Following the standard non-relativistic reduction, CSTR writes the Klein-Gordon wave function as $\psi=\exp(-imc^2t_0/\hbar)\,\phi(x',t_0)$ and substitutes into Equation (8), retaining only the leading term in ϕ's time derivative. The result is a cosmological Schrödinger equation (Yi, 2020),

$$i\hbar\frac{\partial\phi}{\partial t_0}=-\frac{\hbar^2}{2m\,\tau(t_0)^2}\nabla'^2\phi, \tag{14}$$

in which the effective kinetic-energy operator is suppressed by a factor of $\tau(t_0)^{-2}$ relative to its ordinary flat-space form, reflecting the dilution of a fixed physical momentum over the growing local coordinate volume as the universe expands. The corresponding Ehrenfest theorem for the expectation value of the local position operator follows in the usual way from Equation (14),

$$\frac{d}{dt_0}\langle\mathbf{x}'\rangle=\frac{1}{m\,\tau(t_0)^2}\langle\mathbf{p}'\rangle, \tag{15}$$

so that a wave packet's local-coordinate velocity is suppressed by $\tau(t_0)^{-2}$ at fixed local momentum, consistent with the interpretation that a fixed physical velocity corresponds to a shrinking rate of change of the local coordinate as the universe expands (Yi, 2020).

3.6. The Cosmological Uncertainty Principle

Because the canonical momentum conjugate to the local coordinate x'^i in Equation (9) is $\tau(t_0)$ times the physical momentum, the ordinary position-momentum commutator, evaluated in terms of the physical coordinate $x^i = \tau(t_0)x'^i$, is rescaled by the same factor. CSTR expresses the resulting cosmological uncertainty relation as (Yi, 2020)

$$\Delta x\,\Delta p \;\geq\; \frac{\hbar}{2}\,\tau(t_0), \tag{16}$$

so that the minimum attainable product of position and momentum uncertainties, referred to physically expanded coordinates, grows in direct proportion to the cosmological expansion function.

A companion time-energy relation follows from the same substitution applied to the ordinary bound $\Delta E\,\Delta t \geq \hbar/2$: because the energy E in Equation (10) is conjugate to the cosmological time t_0 itself, rather than to a rescaled time coordinate, CSTR leaves the time-energy uncertainty relation in its standard form, $\Delta E\,\Delta t_0 \geq \hbar/2$, so that the expansion function enters the cosmological uncertainty principle only through the position-momentum relation of Equation (16) and not through the time-energy relation (Yi, 2020).

4. Maxwell's Equations and Electromagnetic Waves

Before turning to the Dirac equation, it is useful to record the cosmological form of Maxwell's equations in vacuum, since these both motivate the covariant-derivative construction of Section 6 and provide an independent, purely bosonic illustration of the substitution rule of Equation (3). In terms of the electric and magnetic fields $\mathbf{E}'$ and $\mathbf{B}'$ referred to the local coordinate $\mathbf{x}'$, the vacuum Maxwell equations in the cosmological inertial frame read (Yi, 2020)

$$\frac{1}{\tau(t_0)}\nabla'\cdot\mathbf{E}' = 0, \qquad \frac{1}{\tau(t_0)}\nabla'\cdot\mathbf{B}' = 0, \tag{17}$$

$$\frac{1}{\tau(t_0)}\nabla'\times\mathbf{E}' = -\frac{\partial\mathbf{B}'}{\partial t_0}, \quad \frac{1}{\tau(t_0)}\nabla'\times\mathbf{B}' = \frac{1}{c^2}\frac{\partial\mathbf{E}'}{\partial t_0}, \tag{18}$$

each curl and divergence operator having acquired the same $1/\tau(t_0)$ factor as in Equation (3). Taking the curl of the two equations in (18) and using the vector identity $\nabla'\times(\nabla'\times\mathbf{V}') = \nabla'(\nabla'\cdot\mathbf{V}')-\nabla'^2\mathbf{V}'$ together with the divergence-free conditions of Equation (17), each Cartesian component of $\mathbf{E}'$ and $\mathbf{B}'$ is found to obey the cosmological wave equation

$$\left[\frac{\partial^2}{\partial t_0^2} - \frac{c^2}{\tau(t_0)^2}\nabla'^2\right]\{\mathbf{E}', \mathbf{B}'\} = 0, \tag{19}$$

which is identical in structure to the massless limit ($m \to 0$) of the cosmological Klein-Gordon equation, Equation (8). Plane-wave solutions of Equation (19) take the same form as Equation (9) with $m = 0$, so that the local-frame wave vector $\mathbf{k}'$ and angular frequency ω obey the dispersion relation $\omega^2 = c^2\mathbf{k}'^2/\tau(t_0)^2$; equivalently, the physical phase velocity $\omega/(\mathbf{k}'/\tau(t_0))$ of the wave remains equal to c at every cosmological time, while its local-coordinate wavelength $2\pi/|\mathbf{k}'|$ contracts as $1/\tau(t_0)$, consistent with the redshift identification of Equation (7) applied to a light wave rather than to a massive particle (Yi, 2020).

5. The Dirac Equation in the Cosmological Inertial Frame

The ordinary Dirac equation is the first-order wave equation $(i\hbar\gamma^\mu\partial_\mu - mc)\psi = 0$, with $\gamma^0 = \mathrm{diag}(I', -I')$, $\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$, I' the 2×2 identity matrix, and σ^i the Pauli matrices (Yi, 2021). Applying Wave Function Type A of Equation (2) together with the differential substitution of Equation (3), the Dirac equation in the cosmological inertial frame becomes

$$\left[i\hbar\gamma^0\,\frac{\partial}{\partial(ct_0)} + \frac{i\hbar}{\tau(t_0)}\,\gamma^i\,\frac{\partial}{\partial x'^i} - mc\,I\right]\psi = 0, \tag{20}$$

with I the 4×4 identity matrix (Yi, 2021). Multiplying Equation (20) on the left by the conjugate operator $\left[i\hbar\gamma^0\partial/\partial(ct_0) - (i\hbar/\tau(t_0))\gamma^i\partial/\partial x'^i + mc\,I\right]$ and using the anticommutation relation $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}I$ eliminates the cross terms and reduces the result to the matrix form of the cosmological Klein-Gordon equation,

$$\left[-\frac{1}{c^2}\frac{\partial^2}{\partial t_0^2} + \frac{1}{\tau(t_0)^2}\nabla'^2 - \left(\frac{mc}{\hbar}\right)^2\right] I\,\psi = 0, \tag{21}$$

identical, up to the identity matrix acting on the four-component Dirac spinor, to Equation (8). This demonstrates that the Dirac equation in the cosmological inertial frame is consistent with, and implies, the cosmological Klein-Gordon equation of Section 3, in direct analogy with the corresponding property of the ordinary Dirac equation in flat spacetime (Yi, 2021). Writing the Dirac spinor as $\psi = (\psi_1, \psi_2, \psi_3, \psi_4)$ with Hermitian conjugate $\psi^\dagger$ and adjoint spinor $\bar{\psi} \equiv \psi^\dagger\gamma^0$, the associated probability density $j^0 = \bar{\psi}\gamma^0\psi = |\psi_1|^2 + |\psi_2|^2 + |\psi_3|^2 + |\psi_4|^2$ remains positive definite, exactly as in ordinary Dirac theory (Yi, 2021).

5.1. Non-Relativistic Limit: The Cosmological Pauli Equation

Decomposing the four-component spinor into upper and lower two-component blocks $\psi = (\chi, \eta)^T$ and eliminating the small lower component η in the non-relativistic limit, exactly as in the ordinary derivation of the Pauli equation from the Dirac equation, yields a two-component equation for χ in the cosmological inertial frame,

$$i\hbar\,\frac{\partial\chi}{\partial t_0} = \frac{1}{2m\,\tau(t_0)^2}\left(-i\hbar\nabla' - \frac{e}{c}\mathbf{A}'\right)^2\chi - \frac{e\hbar}{2mc\,\tau(t_0)}\,\boldsymbol{\sigma}\cdot\mathbf{B}'\,\chi, \tag{22}$$

where $\mathbf{A}'$ and $\mathbf{B}' = \nabla'\times\mathbf{A}'$ are the vector potential and magnetic field expressed in the local coordinate. Equation (22) shows that the particle's magnetic-moment coupling to $\mathbf{B}'$ is suppressed by one additional power of $\tau(t_0)^{-1}$ relative to its ordinary kinetic term, so that spin- magnetic effects in

CSTR are predicted to be comparatively more sensitive to the cosmological epoch than the orbital kinetic term alone (Yi, 2021).

6. Klein-Gordon-Maxwell Theory in the Cosmological Frame

To describe a charged scalar field interacting with electromagnetism, CSTR combines a complex Klein-Gordon field φ, φ^* with the electromagnetic field tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ in a single Klein-Gordon-Maxwell Lagrangian density (Yi, 2021),

$$\mathcal{L} = (D_\mu\varphi)^*(D^\mu\varphi) - \left(\frac{mc}{\hbar}\right)^2 \varphi^*\varphi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \quad (23)$$

where the covariant derivative $D_\mu = \partial_\mu + ieA_\mu/\hbar$ minimally couples the scalar field to the electromagnetic four-potential A_μ, and e is the scalar field's charge. Using Wave Function Type A and Type B of the expanded distance (Section 2.2), the spatial components of D_μ acquire the same $1/\tau(t_0)$ rescaling as in Equation (3), so that the Euler-Lagrange equations derived from Equation (23) yield a cosmologically modified Klein-Gordon-Maxwell field equation for φ,

$$\left[D_0 D^0 - \frac{1}{\tau(t_0)^2} D_i D^i + \left(\frac{mc}{\hbar}\right)^2\right] \varphi = 0, \quad (24)$$

together with the inhomogeneous Maxwell equation $\partial_\mu F^{\mu\nu} = -ie\left[\varphi^* D^\nu\varphi - \varphi(D^\nu\varphi)^*\right]/\hbar$, sourced by the scalar field's electromagnetic four-current, again with spatial indices carrying the appropriate factor of $\tau(t_0)^{-1}$ inherited from the cosmological inertial frame (Yi, 2021).

6.1. Gauge Invariance

The Lagrangian of Equation (23) is invariant under the local $U(1)$ gauge transformation $\varphi \to e^{-ie\Lambda(x)/\hbar}\varphi$, $A_\mu \to A_\mu + \partial_\mu\Lambda$, for an arbitrary scalar function $\Lambda(x)$, exactly as in ordinary scalar electrodynamics, because the cosmological substitution of Equation (3) acts identically on every spatial derivative in the covariant derivative D_μ and therefore commutes with the gauge transformation. Gauge invariance in turn implies, via Noether's theorem, that the electromagnetic four-current appearing on the right-hand side of the modified Maxwell equation of this section is conserved, $\partial_\mu j^\mu = 0$, with the spatial divergence again carrying the same $1/\tau(t_0)$ factor as in the continuity equation of Equation (12) (Yi, 2021).

6.2. The Cosmological Coulomb Limit

Setting the scalar field's mass to zero and considering a static point charge q at the origin of the local coordinate system, the time component of the modified Maxwell equation of Section 6 reduces to a cosmological Poisson equation for the scalar potential $A_0(r')$,

$$\frac{1}{\tau(t_0)^2} \nabla'^2 A_0(r') = -\frac{q}{\epsilon_0}\, \delta^3(\mathbf{x}'), \quad (25)$$

whose solution is the cosmologically rescaled Coulomb potential

$$A_0(r') = \frac{q\,\tau(t_0)}{4\pi\epsilon_0\, r'}. \quad (26)$$

Expressed in terms of the physical radial coordinate $r_c = \tau(t_0)\, r'$, Equation (26) becomes $A_0 = q/(4\pi\epsilon_0 r_c)$, the ordinary, epoch-independent Coulomb potential, confirming that – as with the Schwarzschild horizon of Section 9 – the physically measured static electric potential of a point charge is unaffected by the cosmological expansion, even though its expression in the local coordinate r' carries an explicit factor of $\tau(t_0)$ (Yi, 2021).

7. Quantization of the Klein-Gordon Scalar Field

CSTR quantizes the free (uncharged) Klein-Gordon field of Equation (8) canonically. Treating $\phi(x', t_0)$ as a field operator with conjugate momentum density $\pi(x', t_0) = \partial\phi/\partial t_0$ / c^2, the equal-time commutation relations are imposed as (Yi, 2021)

$$[\phi(\mathbf{x}', t_0),\, \pi(\mathbf{x}'', t_0)] = i\hbar\, \delta^3(\mathbf{x}' - \mathbf{x}''), \quad (27)$$

with all other equal-time commutators vanishing. The Lagrangian density corresponding to Equation (8) is

$$\mathcal{L} = \frac{1}{2c^2}\left(\frac{\partial\phi}{\partial t_0}\right)^2 - \frac{1}{2\tau(t_0)^2}\,(\nabla'\phi)^2 - \frac{1}{2}\left(\frac{mc}{\hbar}\right)^2 \phi^2, \quad (28)$$

and the corresponding Hamiltonian, obtained by the usual Legendre transform, is

$$H = \int d^3x' \left[\frac{c^2}{2}\pi^2 + \frac{1}{2\tau(t_0)^2}(\nabla'\phi)^2 + \frac{1}{2}\left(\frac{mc}{\hbar}\right)^2 \phi^2\right]. \quad (29)$$

Expanding ϕ in the usual plane-wave mode basis of Equation (9), with time-dependent annihilation and creation operators $a_{\mathbf{p}'}(t_0)$ and $a^\dagger_{\mathbf{p}'}(t_0)$ satisfying $[a_{\mathbf{p}'}, a^\dagger_{\mathbf{p}''}] = \delta^3(\mathbf{p}' - \mathbf{p}'')$, the Hamiltonian of Equation (29) takes the diagonal form $H = \int d^3p'\, E(p', t_0)\, a^\dagger_{\mathbf{p}'} a_{\mathbf{p}'}$ with the single-particle energy $E(p', t_0)$ fixed by Equation (10), so that the quantized theory describes a collection of independent modes whose dispersion relation evolves with the cosmological expansion function (Yi, 2021).

7.1. Vacuum Energy and Particle Number

As in ordinary quantum field theory, the Hamiltonian of Equation (29), prior to normal ordering, contains a zero-point contribution $\frac{1}{2}\int d^3p'\, E(p', t_0)\, \delta^3(\mathbf{0})$ from the non-commutativity of $a_{\mathbf{p}'}$ and $a^\dagger_{\mathbf{p}'}$. CSTR removes this term by normal ordering in the usual way, $H \to : H : = \int d^3p'\, E(p', t_0)\, a^\dagger_{\mathbf{p}'} a_{\mathbf{p}'}$, after which the ground state $|0\rangle$, defined by $a_{\mathbf{p}'}|0\rangle = 0$ for all $\mathbf{p}'$, has zero energy at every cosmological time t_0. The particle-number operator $N = \int d^3p'\, a^\dagger_{\mathbf{p}'} a_{\mathbf{p}'}$ commutes with the normal-ordered Hamiltonian at fixed t_0, so that, exactly as in the flat-space theory, particle number is conserved instantaneously; any change in the total energy of a fixed-particle-number state is attributed entirely to the explicit time dependence of $E(p', t_0)$ through $\tau(t_0)$ in Equation (10), rather than to particle creation or annihilation (Yi, 2021).

7.2. The Feynman Propagator at Fixed Cosmological Time

The two-point function of the quantized field, evaluated between vacuum states at a common cosmological time t_0, is defined in the usual way as

$$G_F(x' - x''; t_0) = \langle 0| \, T\{\phi(x', t_0)\phi(x'', t_0)\} \, |0\rangle, \qquad (30)$$

with T the time-ordering symbol. Inserting the mode expansion of Section 7 and performing the momentum integral in the usual way, G_F takes the same functional form as the ordinary flat-space Feynman propagator, but with the single-particle energy $E(p', t_0)$ of Equation (10) evaluated at the common cosmological time t_0 at which both field insertions are taken. CSTR does not, in the papers reviewed here, extend this construction to a propagator connecting two *different* cosmological times $t_0' \neq t_0''$, which would require specifying how the mode functions of Equation (9) are to be continued across an interval over which $\tau(t_0)$ itself changes; this is identified in Section 10 as an open question for the further development of the quantized theory (Yi, 2021).

8. Particle's Force and Kinetic Energy

From the energy-momentum relation of Equation (10), the particle's kinetic energy in the cosmological inertial frame is

$$T = E - mc^2 = \sqrt{c^2\mathbf{p}'^2 + m^2c^4} - mc^2, \qquad (31)$$

and the force acting on the particle is defined, as in ordinary mechanics, by the rate of change of the momentum $\mathbf{p}'$ with respect to the cosmological time,

$$\mathbf{F} = \frac{d\mathbf{p}'}{dt_0}. \qquad (32)$$

Because the momentum conjugate to the local coordinate $\mathbf{x}'$ is related to the physical momentum by a factor of $\tau(t_0)$ (Section 3), a particle with constant physical momentum experiences a nonzero effective force in the cosmological frame purely as a consequence of the time dependence of $\tau(t_0)$, providing the CSTR analogue of the force required to maintain comoving free-fall in an expanding Robertson-Walker background (Yi, 2025).

In the non-relativistic limit, writing the physical momentum as $p = \tau(t_0)\, p'$ and holding p fixed, Equation (32) gives

$$\mathbf{F} = -\frac{\dot{\tau}(t_0)}{\tau(t_0)}\, \mathbf{p}', \qquad (33)$$

where $\dot{\tau}(t_0) \equiv d\tau/dt_0$. Equation (33) is the CSTR counterpart of the familiar cosmological drag force experienced by a freely moving particle in an expanding Robertson-Walker background, with the usual Hubble parameter $H(t_0) = \dot{\tau}(t_0)/\tau(t_0)$ appearing as the coefficient relating the local-coordinate momentum to the effective force (Yi, 2025).

9. Cosmological General Theory of Relativity

9.1. The Gravity Field Equation in an Expanding Universe

Extending the cosmological inertial frame of Section 2 to curved spacetime, CSTR proposes a Cosmological General Theory of Relativity (CGTR) in which the Einstein field equation is retained in its standard form, $G_{\mu\nu} = (8\pi G/c^4)T_{\mu\nu}$, but is solved on a background that includes the expansion function $\tau(t_0)$ of Equation (1) as a multiplicative factor on the spatial part of the metric, so that the line element of a static, spherically symmetric source embedded in the expanding background takes the general form (Yi, 2021)

$$ds^2 = -c^2 B(r_c)\, dt_0^2 + \tau(t_0)^2 \left[A(r_c)\, dr'^2 + r'^2 d\Omega^2\right], \qquad (34)$$

where $r_c = \tau(t_0)r'$ is the physical radial coordinate and A, B are metric functions to be fixed by the field equation and the nature of the source, analogous in structure to the McVittie-type embedding of a point mass in an expanding Robertson-Walker background found in the standard general-relativity literature.

The stress-energy tensor $T_{\mu\nu}$ sourcing Equation (34) is taken to be the sum of a perfect-fluid background contribution, which alone would reproduce the pure Robertson-Walker cosmology of Equation (1) through the ordinary Friedmann equations for $\tau(t_0)$, and a localized point-mass, point-charge, or rotating-source contribution concentrated at $r_c = 0$, which sources the departure of A and B from unity in Equation (34). Because the background and localized contributions to $T_{\mu\nu}$ are additive, CGTR treats the resulting metric perturbatively: the background expansion function $\tau(t_0)$ is assumed to evolve according to the ordinary Friedmann equations sourced by the cosmological fluid alone, while the metric functions $A(r_c)$ and $B(r_c)$ are then solved for as a correction sourced by the localized mass on top of this fixed background (Yi, 2021). This is the same perturbative logic that underlies the Schwarzschild and Kerr-Newman solutions of the following two subsections.

9.2. The Schwarzschild Solution in an Expanding Universe

For a single point mass M at the origin, solving the CGTR field equation with the ansatz of Equation (34) in the weak-field, vacuum region outside the source reproduces the Schwarzschild metric functions with the radial coordinate replaced by its cosmologically expanded counterpart (Yi, 2021),

$$B(r_c) = A(r_c)^{-1} = 1 - \frac{2GM}{c^2\, \tau(t_0)\, r'}. \qquad (35)$$

Equation (35) reduces to the ordinary Schwarzschild solution when $\tau(t_0)$ is held fixed, and shows that the location of the event horizon in the local coordinate r', $r_{\mathrm{h}}' = 2GM/(c^2\tau(t_0))$, contracts as the universe expands, while the physically expanded horizon radius $r_{\mathrm{h}} = \tau(t_0)r_{\mathrm{h}}' = 2GM/c^2$ remains fixed at its ordinary Schwarzschild value, independent of the cosmological epoch. The gravitational redshift of a photon emitted at physical radius r_c and observed at spatial infinity within a fixed epoch t_0 is correspondingly unchanged from its ordinary Schwarzschild value, $1 + z_{\text{grav}} = B(r_c)^{-1/2}$, with any additional, epoch-dependent redshift contribution entering separately through the cosmological factor of Equation (7) for photons that propagate over a cosmologically significant interval of t_0 between emission and observation (Yi, 2021).

9.3. The Kerr-Newman Solution in an Expanding Universe

CSTR extends the same construction to a rotating, charged source of mass M, angular momentum $J = Mac$, and charge Q, obtaining a cosmologically embedded Kerr-Newman metric whose Boyer-Lindquist-type form is, in the equatorial limit, characterized by the metric function (Yi, 2021)

$$\Delta(r') = \tau(t_0)^2 r'^2 - \frac{2GM}{c^2}\tau(t_0)\, r' + a^2 + \frac{GQ^2}{4\pi\epsilon_0 c^4}, \quad (36)$$

with the same replacement $r_c = \tau(t_0)r'$ used in the Schwarzschild case above. As with the Schwarzschild solution, the physical radii of the inner and outer horizons obtained from $\Delta(r') = 0$ reduce to their standard, epoch-independent Kerr-Newman values once expressed in terms of the physically expanded radial coordinate r_c, while the corresponding local coordinate r' contracts in proportion to $1/\tau(t_0)$. Explicitly, solving $\Delta(r') = 0$ for the physical radius $r_c = \tau(t_0)r'$ gives

$$r_{c,\pm} = \frac{GM}{c^2} \pm \sqrt{\left(\frac{GM}{c^2}\right)^2 - a^2 - \frac{GQ^2}{4\pi\epsilon_0 c^4}}, \quad (37)$$

identical in form to the ordinary Kerr-Newman outer and inner horizon radii, and independent of t_0, exactly as found for the Schwarzschild case in Equation (35). The extremal condition $r_{c,+} = r_{c,-}$, reached when $(GM/c^2)^2 = a^2 + GQ^2/(4\pi\epsilon_0 c^4)$, is likewise unaffected by the cosmological epoch, since it depends only on the physical parameters M, a, and Q and not on $\tau(t_0)$ (Yi, 2021).

9.4. The Homogeneous Limit

Taking $M \to 0$, $Q \to 0$, and $a \to 0$ in Equations (35) and (36) removes the central source entirely, so that $B(r_c) = A(r_c)^{-1} \to 1$ and Equation (34) reduces back to the homogeneous, spatially flat Robertson-Walker line element of Equation (1). This recovery of the background cosmology in the source-free limit is a basic consistency requirement on any embedding of a localized mass within an expanding universe, and CGTR satisfies it by construction, since Equation (34) was built as a perturbation of Equation (1) rather than as an independent ansatz (Yi, 2021).

Table 1 collects the physical (epoch-independent) and local-coordinate (epoch-dependent) horizon radii for the Schwarzschild and Kerr-Newman cases discussed above.

Solution	**Physical horizon** $r_{\text{h}} = \tau(t_0)r'_{\text{h}}$	**Local horizon** r'_{h}
Schwarzschild	$2GM/c^2$ (fixed)	$2GM/(c^2\tau(t_0))$
Kerr-Newman	standard $r_\pm$ (fixed)	$r_\pm/\tau(t_0)$

Table 1: Physical versus local-coordinate horizon radii in the cosmologically embedded Schwarzschild and Kerr-Newman solutions of Section 9. The physical horizon radii are independent of the cosmological epoch t_0, while the local-coordinate radii contract as $\tau(t_0)$ grows.

10. Discussion

The results summarized above show that CSTR is built from a single operational rule – the Wave Function Type A rescaling of Equation (2) together with the differential substitution of Equation (3) – applied systematically to each special-relativistic wave equation and, in Section 9, to the Einstein field equation itself. A recent independent critical review of CSTR has examined this construction in detail, noting that the physical content of Equations (2)–(3) is closely related to, and in several limits reduces to, the standard treatment of physical versus comoving coordinates in Robertson-Walker cosmology, and raising questions about whether treating $\tau(t_0)$ as entering a Lorentz-type transformation, rather than purely as a metric scale factor, is fully consistent with the diffeomorphism invariance of general relativity (A Critical Review, 2025). In particular, the review notes that CSTR's publication history – primarily short notes in a single specialized journal – has limited independent citation and scrutiny of the framework relative to mainstream treatments of relativistic cosmology.

10.1. Dimensional Consistency

As a further internal check, it is worth noting explicitly that every modification introduced by CSTR relative to the corresponding flat-space equation enters through a dimensionless ratio involving $\tau(t_0)$: the expansion function itself is dimensionless in the normalization $\tau(t_0^{\text{obs}}) = 1$ adopted in Section 2, so that Equations (3), (8), (13), (14), (16), (20), (22), (24), and (33) are each dimensionally consistent with their ordinary flat-space counterparts term by term. This is a necessary, though not sufficient, condition for the internal consistency of the formalism, and it is satisfied throughout the program reviewed here.

Table 2 contrasts, for each major result reviewed above, the physically measured (epoch-independent) prediction with the corresponding local-coordinate expression that carries explicit $\tau(t_0)$ dependence, which is the pattern common to essentially every result derived in this formalism.

These are legitimate points for continued development of the theory. The present review has aimed to state the CSTR formalism explicitly and consistently enough that such questions – for instance, whether Equations (5)–(6) compose correctly under successive boosts at different cosmological times, whether the Feynman propagator of Equation (30) can be consistently extended to field insertions at two different cosmological times, or whether the Schwarzschild- and Kerr-

Quantity	Physical value	Local-coordinate value
Yukawa range	$\hbar/(mc)$, fixed	$\hbar/(mc\,\tau(t_0))$
EM wave speed	c, fixed	phase velocity $\times\,\tau(t_0)$
Schwarzschild horizon	$2GM/c^2$, fixed	$2GM/(c^2\tau(t_0))$
Coulomb potential	$q/(4\pi\epsilon_0 r_c)$, fixed	$q\tau(t_0)/(4\pi\epsilon_0 r')$
Position-momentum uncertainty	grows with $\tau(t_0)$	$\hbar/2$ (local)

Table 2: Comparison of physically measured versus local-coordinate expressions for the principal results reviewed in this paper. In every case except the uncertainty relation, the physically measured quantity is independent of the cosmological epoch, and the $\tau(t_0)$-dependence appears only in the local-coordinate expression.

Newman-type solutions of Section 9 solve the full, rather than a linearized or weak-field, CGTR field equation – can be posed and investigated precisely. Clarifying the relationship between the cosmological inertial frame of Section 2 and the standard General-Relativistic treatment of physical distance in an expanding universe, and subjecting the CGTR field equation of Section 9 to the same scrutiny applied to the flat-space wave equations of Sections 3– 7, are natural directions for future work on this program.

10.2. Observational Prospects

Because the redshift identification of Equation (7) ties the abstract expansion function $\tau(t_0)$ appearing throughout Sections 3–9 to a directly measured quantity, several of the modifications derived above are, at least in principle, falsifiable. The redshift-dependent Yukawa screening length of Equation (13), the epoch-dependent single-particle dispersion relation implicit in the quantized theory of Section 7, and the suppressed spin-magnetic coupling of Equation (22) each predict small, $\tau(t_0)$-dependent departures from their standard, epoch-independent counterparts that would need to be constrained against high-redshift spectroscopic or particle-physics data before CSTR could be considered observationally viable as a modification of, rather than merely a reformulation of, standard physics. No such observational comparison is attempted in the papers reviewed here, and it remains an open task for future work to establish realistic bounds on the rate of change of $\tau(t_0)$ implied by these equations against existing atomic-clock, spectroscopic, or collider constraints on time-variation of fundamental constants.

10.3. Relation to Standard Cosmological Perturbation Theory

It is worth emphasizing that the CGTR embedding of Section 9 addresses a question – how a localized mass distorts an otherwise homogeneous, expanding background – that is also addressed within standard general relativity by the McVittie metric and by linear cosmological perturbation theory around a Robertson-Walker background. A detailed, quantitative comparison between the CGTR metric of Equation (34) and these standard constructions, in particular whether Equation (34) solves the exact (rather than linearized) Einstein field equation sourced by a point mass on an expanding background, has not been carried out in the papers reviewed here and would further strengthen the case that CGTR extends, rather than merely reformulates, the standard treatment of localized sources in cosmology.

A further point of comparison concerns the choice of coordinates. Standard cosmological perturbation theory is typically formulated in a specific gauge – for example, the Newtonian or longitudinal gauge, in which the perturbed metric is written directly in terms of physical, rather than comoving, radial distance and a single gravitational potential Φ appears in both the time-time and space-space perturbations under the further assumption of no anisotropic stress. The CGTR ansatz of Equation (34), by contrast, is formulated directly in the local coordinate r' of the cosmological inertial frame, with the physical radius $r_c = \tau(t_0)r'$ appearing only after the metric functions A and B have been solved for. Translating between these two descriptions – in particular, checking whether the weak-field limit of Equation (35) reproduces a single Newtonian-gauge potential $\Phi = -GM/(c^2 r_c)$ consistent with linear perturbation theory, or whether it instead corresponds to a distinct gauge choice particular to the cosmological inertial frame – is a concrete, well-posed calculation that has not been carried out in the papers reviewed here, and doing so would directly address the diffeomorphism- invariance question raised in the independent critical review discussed above (A Critical Review, 2025).

10.4. Summary of Open Questions

Collecting the points raised throughout this section, three concrete calculations stand out as the most direct routes to establishing CSTR's consistency with, and relationship to, standard relativistic cosmology: (i) verifying that the cosmological Lorentz transformation of Equations (5)–(6) composes correctly under successive boosts applied at different cosmological times; (ii) comparing the weak-field limit of the CGTR Schwarzschild solution, Equation (35), directly against the Newtonian-gauge potential of standard linear cosmological perturbation theory; and (iii) extending the Feynman propagator of Equation (30) to field insertions separated in cosmological time, which is a prerequisite for any scattering-amplitude calculation within the quantized theory of Section 7. Each of these is a mathematically well-defined question that could, in principle, be resolved within the existing CSTR formalism without requiring new physical assumptions beyond those already introduced in Sections 2–9.

Table 3 summarizes the principal equations derived across Sections 2–9, together with the original paper in which each first appeared, to serve as a compact index to the consolidated formalism.

Result	Original source
Wave Function Type A, coordinate transf.	Yi (2020)
Cosmological Klein-Gordon equation	Yi (2020)
Schrödinger equation, uncertainty relation	Yi (2020)
Yukawa potential	Yi (2021)
Dirac equation	Yi (2021)
Klein-Gordon-Maxwell theory	Yi (2021)
Scalar-field quantization	Yi (2021)
CGTR field equation, Schwarzschild/Kerr-Newman	Yi (2021)
Full consolidation	Yi (2025)

Table 3: Index of the principal CSTR results reviewed in this paper and their original sources.

11. Conclusion

This paper has consolidated the Cosmological Special Theory of Relativity into a single, uniformly notated review, tracing its development from the basic space-time relations of the cosmological inertial frame (Section 2), through the Klein-Gordon equation, its Yukawa-potential and Schrödinger-equation consequences, and the cosmological uncertainty principle (Section 3), Maxwell's equations and the cosmological electromagnetic wave equation (Section 4), the Dirac equation and its consistency with the Klein-Gordon equation (Section 5), the Klein-Gordon-Maxwell theory of an interacting charged scalar field and its cosmological Coulomb limit (Section 6), the canonical quantization of the free scalar field (Section 7), the particle's force and kinetic energy (Section 8), and finally the Cosmological General Theory of Relativity, with its cosmologically embedded Schwarzschild and Kerr-Newman solutions (Section 9). Taken together, these results constitute a single, internally cross-referenced formalism in which every later equation is obtained from the same Wave Function Type A substitution rule introduced at the outset, and in which, as summarized in Table 2, physically measured quantities are recovered in their ordinary, epoch-independent form even as their expressions in the local coordinate carry an explicit signature of the cosmological expansion function $\tau(t_0)$. Continued engagement with the open questions raised in independent reviews of this framework (A Critical Review, 2025), together with the observational and propagator-extension questions identified in Section 10, is intended to guide the further development and eventual broader evaluation of the theory.

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A. Glossary of Key Symbols

Table 4 collects the principal symbols used throughout this review.

Symbol	Meaning
t_0	Cosmological time
$\tau(t_0)$	Cosmological expansion function (scale factor)
x^i, r	Physically expanded coordinate
x'^i, r'	Local (unexpanded) coordinate, $x^i = \tau(t_0)x'^i$
γ	Ordinary special-relativistic Lorentz factor
ψ	Klein-Gordon or Dirac wave function
m	Particle rest mass
E, $\mathbf{p}'$	Energy and local-frame momentum
g	Yukawa coupling constant
ϕ	Quantized Klein-Gordon field operator
φ	Complex (charged) Klein-Gordon field
e	Scalar field electric charge
A_μ	Electromagnetic four-potential
$F_{\mu\nu}$	Electromagnetic field-strength tensor
$G_{\mu\nu}$	Einstein tensor (CGTR field equation)
M, a, Q	Mass, spin parameter, and charge (CGTR sources)
CSTR	Cosmological Special Theory of Relativity
CGTR	Cosmological General Theory of Relativity

Table 4: Glossary of symbols used in this review.

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