Arithmetic Dimensionality of Real Numbers

Robert S. Miller
rmille4612@hotmail.com

This paper will explore the concept of Arithmetic Dimensionality, D_A, for real numbers, whose analog is the fractal dimensionality of geometric constructs, D_G. Beginning with a short review of how the dimension of a geometric object is calculated, it will be shown how this same concept can be applied to real numbers, and that the current understanding of numbers having a geometric dimension of 0 is not accurate. Instead it is show,, that although the point on a number line, like the x-axis is 0 dimensional, the numbers themselves may possess a fractal dimension. It is beyond to scope of this paper to prove the exact Geometric Fractal Dimension of transcendental numbers (like π and **e**) or irrational numbers (like $\sqrt{2}$). The goal however is to show (1) there exists a D_A and D_G for real numbers, (2) that $D_A \to D_G$ as the number of decimal digits considered $n \to \infty$ and (3) where D is a generic stand in for both D_A and D_G, it exists in Information Space or Sub-Hilbert Space of Geometric Dimension such that $0 \leq D < 1$.

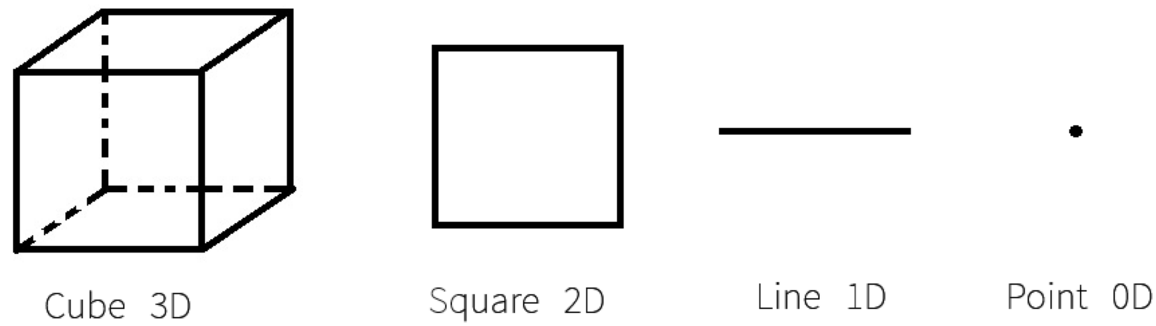
1.0—Defining Dimension:

Britannica online lists the definition of dimension as—

> "**Dimension**, in common parlance, the measure of the size of an object, such as a box, usually given as length, width, and height. In mathematics, the notion of dimension is an extension of the idea that a line is one-dimensional, a plane is two-dimensional, and space is three-dimensional. In mathematics and physics one also considers higher-dimensional spaces, such as four-dimensional space-time, where four numbers are needed to characterize a point…"
>
> https://www.britannica.com/science/dimension-geometry

Essentially when we speak of *dimension* in mathematics we are describing degrees of freedom. Objects having dimensionality less than or equal to the available degrees of freedom can exist within the given space being considered. A 3D space can hold three dimensional volumes defined by cubes, cones, spheres etc. It can also hold planes (2D), lines (1D) and points (0D).

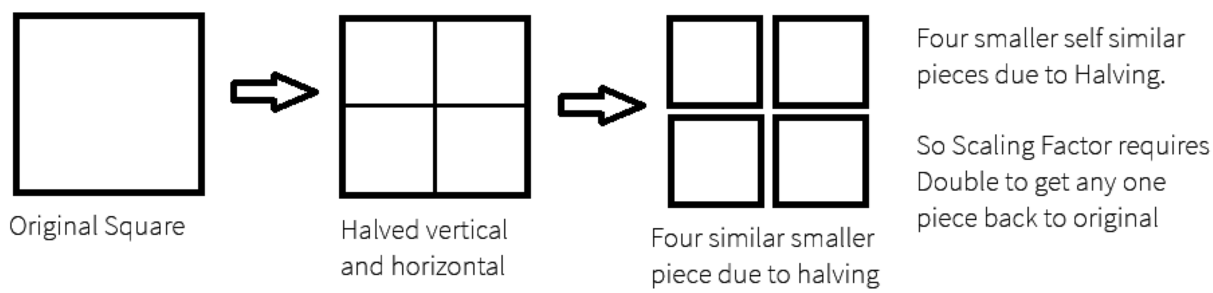
Figure 1:



1.1—Calculating Geometric Dimension:

It is possible to derive the exact dimensionality of a geometric object by examining Scaling Factor and number of Self-Similar parts. Consider the 2D square. How do we know it is 2-dimensional? Dividing the square in half along both degrees of freedom provides four self-similar objects.

Figure 2:



This gives us the equation for Fractal or Self-Similar Dimension

1.1.a—Fractal Dimension Equation

$$Scaling\ Factor^{Dimension} = Number\ of\ Self\ Similar\ Constructs$$

Equation 1:

$$S^D = N$$

Equation 2:

$$D = \frac{\ln N}{\ln S}$$

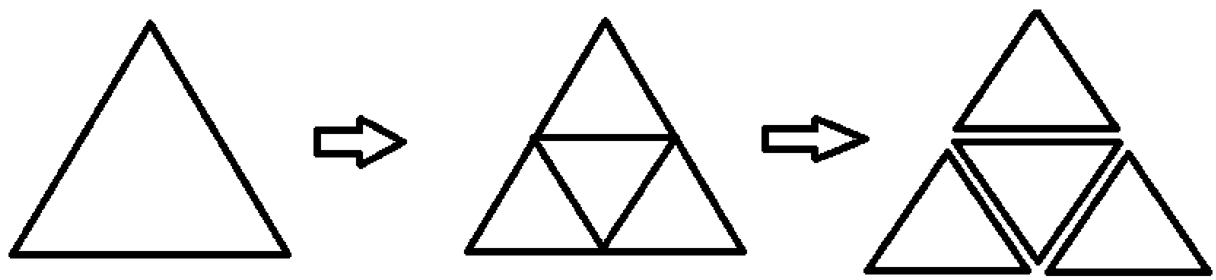
We already established the scaling factor for the Square above is 2, because we must double the size of any one of the four created self-similar parts to create on object the same size as the original. Filling in for S = 2 and N = 4,

$$2^D = 4 \qquad\qquad D = 2$$

It's obvious from this that D = 2, and thus the Square is a 2-dimensional object. Often though it is more difficult to ascertain what D is, which is where the alternate form of the Fractal Dimension Equation (equation 2) is more useful.

Consider how this occurs with an Equilateral Triangle. The difficultly occurs only in attempting to find a way to divide it up. If a line segment is drawn from the midpoint of each side to the midpoint of the two others, we have again divided the original triangle up into self-similar parts. Here are now four self-similar parts, each half the size of the original.

Figure 3:



Thus doubling the size of any one of the smaller pieces will reproduce the original. So Scaling Factor is 2, with four self-similar constructs.

$S = 2$ $\qquad$ $N = 4$

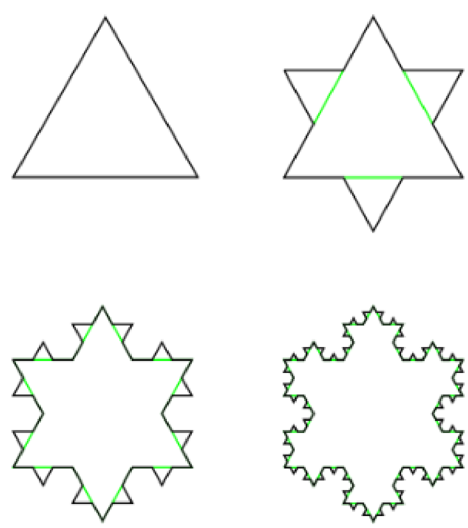
$$2^D = 4$$

$$D = 2$$

The Triangle is a 2-dimensional object.

2.0—Fractal Geometric Dimension:

Figure 4:



The method shown for the square and triangle can be adjusted for objects of higher dimensionality. It can also be used to define the dimensionality of fractal curves, called the Hausdorff Dimension. A classic example of this is the Koch Snowflake.

The construction begins as an equilateral triangle. One-third the length of a side is removed from the middle of each side. That opening is then closed with two more $^1/_3$ side-length lines creating a new, out-pointing, smaller, triangular projection, which would itself be an equilateral triangle if it had the base. This new shape cannot be divided up into equal self-similar parts like actual equilateral triangle could. We need a different approach.

At the first iteration (The six-pointed star in the upper right image of Figure 4), inspection finds this object composed of three self-similar line segments, themselves each composed of four same length pieces. The successive iterations continue infinitely, meaning the Kock snowflake is has an infinite curve length, confined to a finite space. We should reasonably expect the shape to have a dimension greater than one but less than two; it is more than a line but not enough to be a 2D geometric object.

Figure 5

Since each of these self-similar segments is $^1/_3$ the size of the original whole, the scaling factor needed to recreate the original is $S = 3$; we need three of them to build it. And because each self-similar object is composed of four pieces, $N = 4$. Solving for its Hausdorff Dimension provides,

$$S^D = N$$

$$3^D = 4$$

Clearly $1 < D_G < 2$, because $3 < 4 < 9$. This is where Equation 2 is most helpful because logs are used to solve for exponents.

$$D_G = \frac{\ln N}{\ln S}$$

$$D_G = \frac{\ln 4}{\ln 3} \approx 1.2618595$$

We should anticipate a fractal dimension exists for any object possessing properties of an infinite measure along some aspect of its construction and yet confined to a finite space.

3.0—Arithmetic Dimensionality:

Numbers alone are traditionally considered to be of zero geometric dimension. This is of course possible but not a guarantee as work here will show. Methods used for deriving Arithmetical Dimensionality shall parallel those for defining Geometric Dimension. I here hypothesize if points are 0 Dimensional, and yet lines can possess a fractal dimension such that $1 \leq D < 2$, then it is possible numbers represented by points can have fractal dimension such that $0 \leq D < 1$.

3.1—Plotting Numbers:

First consider the placement of several numbers on a the x-axis number line. Below are evaluations for:

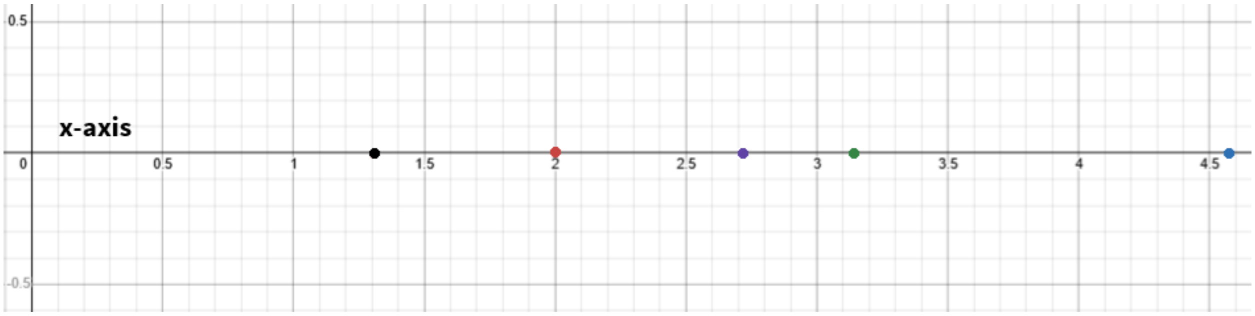
a) 2 b) 2.0 c) 4.57789911102 d) π e) $\sqrt{2}$

f) $\mathbf{e}$

On the number line we plot each of these with a reasonable amount of approximation and error. For instance the place chosen to mark the number defined by $x = 2$ is a completely separate location than the point defined by $x = 2.000000000000000001$. Both are infinitesimally small points. However a human trying to mark $x = 2$ on a paper actually indicates an infinite swath of numbers, members of the uncountably infinite set of numbers, with the pencil lead making the mark, which is far larger the exact location $x = 2$. Similarly, a computer will define the location $x = 2$ precisely but we may not be able to tell the difference between $x = 2$ and $x = 2.000000000000000001$ unless we zoom in quite far, and yet the marked points in pixels on the screen still cover an infinite amount of numbers, belonging to the uncountably infinite number set. The importance sought to convey here is twofold; 1) Each of these example numbers is nonetheless a separate, infinitesimally small point, the very idea of a number having zero geometric dimension, and 2) we are unable to make an infinitesimally small mark is due to there being uncountably infinite values present around, imperceivably close to the denoted single point.

What then of numbers like π , $\mathbf{e}$, (which are transcendental as well as irrational) and $\sqrt{2}$ (which is irrational)? We plot them on the number line in exactly the same fashion as any other number, represented by a single infinitesimally small point. However these numbers, extending in digit length to infinity, never terminate, possessing unique complexity as they also never repeat. It is intuitive to expect numbers like this which are confined to a finite space, the 0D point on the number line, yet have an infinite length of digits will have some analog to geometric fractal dimensionality.

Figure 6



Shown above here in figure six are the numbers

2	2.0	2.000000000000000001	4.57789911102
π	$\sqrt{2}$	$\mathbf{e}$	

Each of the numbers in the first row (the red and blue numbers) actually terminate. It makes sense to consider these as a singular point but are they 0-dimensional? The three numbers shown in red are so close to one another at this scale they all appear as one point in the graph of Figure 6. Further 2 and 2.0 are arithmetically identical.

3.2—Information Space:

Before we attempt to derive the Arithmetic Dimension of a number we need consider what the information content of each number looks like as an analog to a geometric object. To visualize this we can use a Fractal Generating Random Walk, which will create a Cantor Dust.

3.2.a—The Random Walk:

A **Random Walk** is "*…a stochastic process consisting of a sequence of discrete steps… its trajectory and the "clusters" it forms often exhibit fractal geometry—meaning they possess self-similar properties and non-integer dimensions when analyzed at scale… the boundary of the walk forms a fractal object… meaning it forms a ramified, space-filling cluster with holes across many length scales.*"

- **Source:** Kumagai, T. (2014). "Anomalous random walks and diffusions: From fractals to random media." *Proceedings of the International Congress of Mathematicians*. https://t-kumagai.w.waseda.jp/ICM-Proc-TK-new.pdf

To facilitate this process we create an Information Space represented by an axis bar with a numerical scale centered at the origin, indicated by 0. This Information Space, labeled the I-axis, is one dimensional extending infinitely to the negative (left) and to the positive (right), but has no orthogonal axis with which to plot points. Marks only indicate distance from the origin.

The Random Walk is based on the value of each successive integer in the string which comprises a given number. Numbers are often rounded up or down depending on the value of the digit considered to indicate a chosen accuracy. In keeping with this, first plot the point which matches the whole integer value occurring to the left of the decimal point. This is an infinitesimally small point irrespective of the magnitude of the integer value indicated. It is the nature of the decimal components that follow which may indicate a fractal dimension. For following decimal values, digits 0 through 4 will usually indicate rounding down, and digits 5 through 9 indicate rounding up. Respectively, in information space this accounts for moving left and moving right on the I-axis.

In order to consider the entire walk in a limited space we set the following rules:

3.2.a.i—Rule 1—Magnitude of Step

4 = five steps left	5 = five steps right
3 = four steps left	6 = four steps right
2 = three steps left	7 = three steps right
1 = two steps left	8 = two steps right
0 = one step left	9 = one step right

This allows us to make a step in the random walk which accounts for both the magnitude of the number and the direction that value would indicate to move due to rounding. The orientation for the magnitude of step being greatest for digits 4 and 5 reflects the greatest amount of change in position caused by rounding from that point if we were rounding from that digit.

3.2.a.ii—Rule 2—Confinement to Limited Space

The whole value of the step indicated is only taken on the first step toward a given direction. In decimal base each next decimal place is smaller by an additional magnitude of ${}^{1}/_{10}$. Thus we define each subsequent step by a reduction in magnitude, multiplying each digit by 10^{-j}, starting with $j = 0$, incrementing 1 for each next digit. We reset this process back to $j = 0$ whenever we change directions from stepping left to stepping right or vice versa.

3.3—Expressing the Random Walk Mathematically:

Position P on the I-axis of the Information Space after n steps is defined by the sequence of digits of a given number. We label each digit d_n where $d_n \in \{0, 1, \dots, 9\}$. Movement of the steps is defined by the **Raw Step Function** $S(d_n)$, where digits and movement are thus:

3.3.a—Raw Step Function:

$$S(d_n) = \begin{cases} -(d_n + 1) & if\ 0 \leq d_n \leq 4 \\ (9 - d_n + 1) & if\ 5 \leq d_n \leq 9 \end{cases}$$

Digit d_n	Step Value	
0	Left one	-1
1	Left two	-2
2	Left three	-3
3	Left four	-4
4	Left five	-5

Digit d_n	Step Value	
9	Right one	$+1$
8	Right two	$+2$
7	Right three	$+3$
6	Right four	$+4$
5	Right five	$+5$

State and Movement:
The direction of D_n at each step n is determined by the sign of the step. Use the $\mathbf{sgn}(\,)$ function to return the sign of the step, either $-$ or $+$.

3.3.b—**sgn** function:

$$D_n = \operatorname{sgn}\big(S(d_n)\big)$$

$$D_n = Negative \quad = move\ left \qquad \text{if} \qquad 0 \le d_n \le 4$$

$$D_n = Positive \quad = move\ right \qquad \text{if} \qquad 5 \le d_n \le 9$$

3.3.c—Turn—Changing Direction:
A ***Turn*** occurs when the random walk of digits results in movement along the Information Space of the I-axis changing from left to right or from right to left. In other words a turn is being executed whenever $D_{n-1} \neq D_n$.

If $D_{n-1} = D_n$ then $\mathbf{Turn}(D_{n-1}, D_n) = \mathbf{False}$

If $D_{n-1} \neq D_n$ then $\mathbf{Turn}(D_{n-1}, D_n) = \mathbf{True}$

3.3.d—Magnitude of each successive digit:
Because we are attempting to confine the extent of the dust created by the random walk to a smaller finite segment of the Information space, each successive digit d_n where $D_{n-1} = D_n$ (*No Turn has occurred*), we multiply by 10^{-j} where j is the iteration of the digit in the current turn. For example the approximation of $\pi = 3.14159$. The first digit, 3, is handled separately because it is a whole integer value, plotted as the start point for the walk. The next three digits are left moving. Not until arriving at fourth digit is a Turn is executed. So the steps, beginning with the 0th term, $d_0 = 1$, would be thus:

$\pi = 3.14159$

Starting point: $\boldsymbol{I = 3}$

$S(d_0) = S(1) = -2$ $\quad$ $\mathrm{sgn}\big(S(1)\big) = \mathrm{sgn}(-2) = -$ $\quad$ $\mathbf{Turn}(\emptyset\,, D_n) = \boldsymbol{N/A}$

$J = 0$, Move left 2×10^0 $\quad$ $\boldsymbol{I = 1}$

$S(d_1) = S(4) = -5$ $\quad$ $\mathrm{sgn}\big(S(4)\big) = \mathrm{sgn}(-5) = -$ $\quad$ $\mathbf{Turn}(D_{n-1}\,, D_n) = \mathbf{F}$

$J = 1$, Move left $5 \times 10^{-1} = \frac{5}{10} = \frac{1}{2}$ $\quad$ $\boldsymbol{I = 0.5}$

$S(d_2) = S(1) = -2$ $\quad$ $\mathrm{sgn}\big(S(1)\big) = \mathrm{sgn}(-2) = -$ $\quad$ $\mathbf{Turn}(D_{n-1}\,, D_n) = \mathbf{F}$

$J = 2$, Move left $2 \times 10^{-2} = \frac{2}{100} = \frac{1}{50}$ $\quad$ $\boldsymbol{I = 0.48}$

$S(d_4) = S(5) = +5$ $\quad$ $\mathrm{sgn}\big(S(5)\big) = \mathrm{sgn}(+5) = +$ $\quad$ $\mathbf{Turn}(D_{n-1}\,, D_n) = \mathbf{T}$

$\boldsymbol{J = 0}$, Move right $5 \times 10^0 = 5$ $\quad$ $\boldsymbol{I = 5.48}$

$S(d_5) = S(9) = +1$ $\quad$ $\mathrm{sgn}\big(S(9)\big) = \mathrm{sgn}(+1) = +$ $\quad$ $\mathbf{Turn}(D_{n-1}\,, D_n) = \mathbf{F}$

$\boldsymbol{J = 1}$, Move right $1 \times 10^{-1} = \frac{1}{10}$ $\quad$ $\boldsymbol{I = 5.58}$

3.3.e—10^{-J} as a Weighting Factor:

Because we are weighting each digit in a continuous step toward a given direction, we label this with weighting factor W_J

If $D_{n-1} \neq D_n$ $\quad$ then $\quad$ $W_J = W_0 = 10^0 = 1$

If $D_{n-1} = D_n$ $\quad$ then $\quad$ $W_J = 10^{-J}$

This provides a general recursive formula for the position of each point P_n on the I-axis of the Information space:

$$P_n = P_{n-1} + [S(d_n) \cdot W_j]$$

3.3.e.i:--Initial Conditions:
Thus the following initial conditions:

$P_0 = \textbf{Initial Int}$	The whole integer value to left of decimal.
$W_0 = 10^0 = 1$	The weighted function is unitary at d_0 until altered by $S(d_n)$ progression.
$D_0 = \emptyset$	Direction is yet undefined until set by $S(d_n)$ progression beginning with $S(d_0)$

3.3.f—Instructions to conduct the Random Walk

1. Map the Digit
    - Begin at the point representing the whole integer value preceding the decimal.
    - Assign each digit for the decimal portion of the given number a raw step value $S(d_n)$ using the table of values established above.

2. Determine the value of the weighting factor W_j
    - Start with $W_0 = 10^0 = 1$
    - For every step where the direction Left or Right is the same as the previous step $(D_{n-1} = D_n)$ the multiply the weighting factor by 0.1 and increase j by 1
        - In other words $W_j = 10^{-j}$ increasing with each digit d_n.
    - When the Direction $(D_{n-1} \neq D_n)$ changes reset W_j to $W_0 = 10^0 = 1$.

3. Update position P_n for each digit of the given number

$$P_n = P_{n-1} + [S(d_n) \cdot W_j]$$

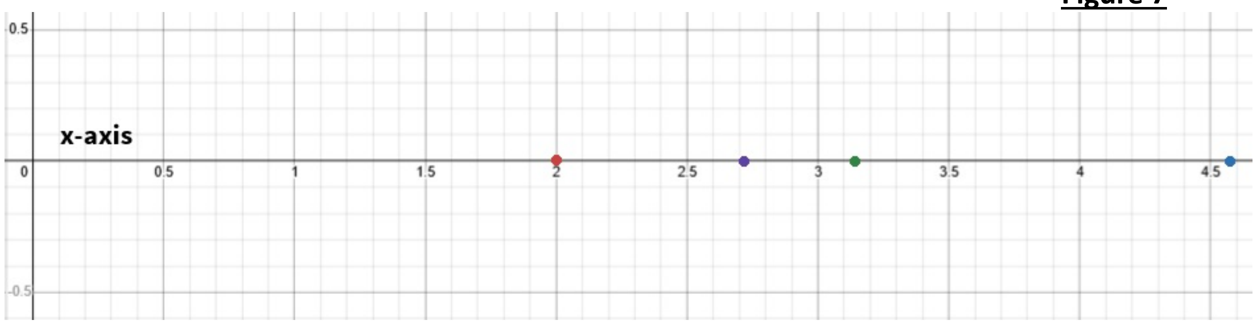
4.0—Examining the Information Space

With this procedure in place we can now plot the points of each digit for various numbers on the information space of the I-axis in images shown below. Due to the condition of $W_j = 10^{-j}$ the values of plotted points P_n are confined to a more limited segment of the I-axis than taking whole steps for each digit. We mark the lower bound limit $-L$, indicating the most negative value plotted, and the upper bound limit $+L$ the most positive value plotted.

$-L$ and $+L$ will drift as more digits are included in consideration. The Line segment has length $L-(-L)$. The more filled with points P_n the I-axis line segment becomes, the closer we expect a valid calculation for Arithmetic Dimension $D_A \to 1$, and yet remain within the boundary $0 \leq D_A < 1$.

Again, below notice that each of the selected test values is listed as nothing more than an infinitesimally small point on the x-axis line, which is a one-dimensional geometric object holding all such zero-dimensional points. On an actual number line, like the x-axis, these numbers are represented only as infinitesimally small points regardless of how spread out their random walk dust is on the Information Space I-axis.

Figure 7



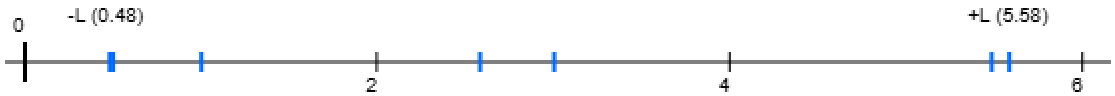
To illustrate the feature of Arithmetic Dimensionality we will use the numbers π and **e**. Because these numbers' decimal components neither terminate nor repeat we can choose to examine any number of digits in their strigs without limit. The process described in section 3.3 will map the digits to the line segment on the I-axis extending from $-L$ to $+L$.

The mapping processing will fill in an amount of I-axis line segment visually consistent with later calculation of the fractal Geometric Dimension D_G for each given number as the number of digits considered approaches infinity. The mapping process provides a visual geometric interpretation of each number's fractal dimension. The appendix of this paper contains the exact single file Index.html document used to manufacture these Dust Images, created with assistance of Google Gemini A.I. Others graphs were created with use of Desmos Graphing Calculator (https://www.desmos.com/calculator). Calculations involving extreme values of n digits were assisted with Google Gemini A.I. (https://gemini.google.com/app).

4.1.a—The first six decimal digits of π and $\mathbf{e}$ as in Random Walk:

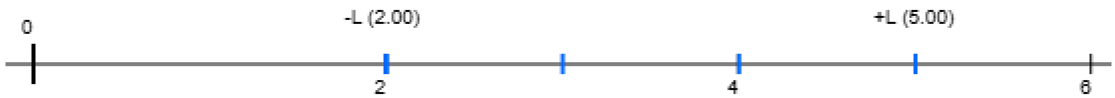
$\pi \approx 3.141592$

Figure 8



$\mathbf{e} \approx 2.718281$

Figure 9

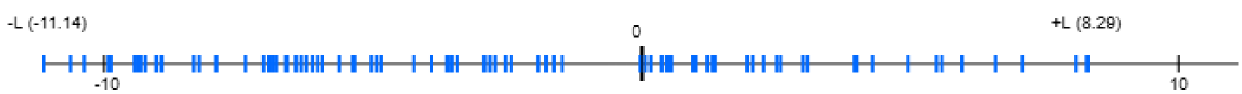


As we begin adding more digits into consideration, we should expect to see more of the specified line segment filled in. For only six digits very little of the Information space is filled. The length of the line segment for π is $L_{length} = 6.06$ units, and for $\mathbf{e}$, $L_{length} = 3$ units.

4.1.b—First 100 decimal digits:

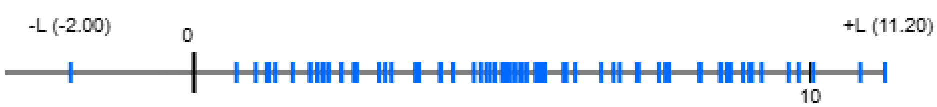
π ≈3.1415926535897932384626433832795028841971693993751058209749445923078164062862089986280348253421170 67

Figure 10



$\mathbf{e} \approx$
2.7182818284590452353602874713526624977572470936999595749669676277240766303535475945713821785251664 27

Figure 11



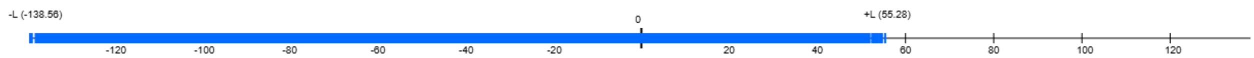
As expected increasing the number of considered digits not only increased the size of the line segment but also the number of points filling in the available space of that line segment. For π the length of the segment is, $L_{length} = 8.29 - (-11.14) = 19.43$ units, and for $\mathbf{e}$, $L_{length} = 11.2 - (-2) = 13.2$ units. Both are certainly more marked with points in the available space.

4.1.c—The first 50,000 digits:

First we explore π. At this scale the line segment extends from $-L$ at -138.56 to $+L$ at 55.28, the length of the entire segment is 193.48 units long. Looking at the point-plots it appears quite full, as shown below in figure 12.

π

Figure 12



In figure 13 we are looking at just the portion which has plotted points. You can see on the edges there are still places of separation, meaning it is not a solid line.

Figure 13

Figure 14 zooms in much closer around $I = -80$ there are numerous spaces which are not marked. This is still a Cantor Dust albeit a very dense one confined to the given line segment.

Figure 14

As we zoon in further into a what appeared to be a very dense section we see again, centered around $I = -120$, the dust nature of the point marks are very clear. What seemed solid zoomed out is still quite porous. Its filling in but is not yet a solid line segment throughout available space.

Figure 15

Next we examine the Cantor Dust generated by the first 50,000 digits of **e.** The line segment for **e** is quite a bit longer. Its extending from $-L$ at -142.35 to $+L$ at 272.83. It spans a length of 415.18 units. Figure 16 is centered at the origin. Figure 17 is slightly more zoomed in looking only at the part of the line segment where points are plotted. On the far left side of Figure 17 we can see the first indication that what otherwise looks like a solid line is still a Cantor Dust.

Figure 16

e

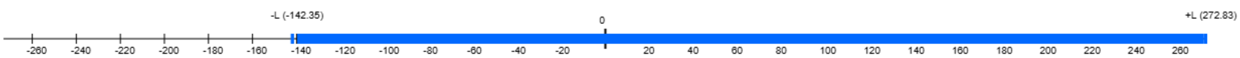
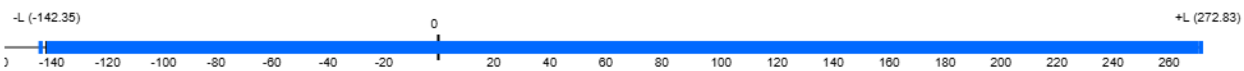


Figure 17



Figures 18 and 19 are zoomed in further around $I = +20$. As we get closer in we see more and more of the fractal nature of the points on the Information Space that comprise this Cantor Dust.

Figure 18

Figure 19

Despite how dense the plots for the 50,000 digits of both π and $\mathbf{e}$, they remain incomplete lines, the very definition of a Cantor Dust. The more digits of these transcendental numbers are considered the more of the I-axis allotted for their line segments are filled in. Additionally, the larger the line segment holding those points will be. Neither will the plotted points ever totally fill the line segment nor can all digits of π and $\mathbf{e}$ ever be exhausted in a plot. Thus we have points that are confined to a finite place, the infinitesimally small points on a number line like the x-axis for each these numbers, π and $\mathbf{e}$. Yet they are of infinite length, never quite able to fill the information space available to them as the number of digits considered increases toward infinity. This concept of infinite and yet confined to a finite space indicates we have a fractal dimension.

<u>5.0—Calculating Fractal Dimension of Arithmetic Values.</u>

To calculate the Arithmetic Dimensionality of real numbers It is necessary to find Arithmetic analogs for the Geometric concepts of Scaling Factor and Number of Self-Similar components.

<u>5.1—Review of Geometric Dimensionality:</u>

For the geometric objects we inspected before like the square, scaling factor was determined by what magnitude of resizing was necessary to make one of the smaller self-similar parts appear again as the original. So for a geometric object we are talking about literal size measurements.

If the sides of the original square were 1 unit length and they were halved to make self-similar pieces then,

$$S = \frac{length\ original\ side}{Factor\ Applied} = \frac{1}{1/_2} = 2$$

<u>5.2—Arithmetic Analog of Scaling Factor:</u>

We don't have this literal side for an arithmetic value in the same way we do for a square. Think about the process.

- We are considering more and more digits of a number's decimal value
- We are making the number longer literally in number of digits
- We are therefore with each digit ever so slightly increasing the arithmetic value.
- Each additional digit
    - Takes up more room in the available Information Space.
    - A physically larger line segment is allotted for its storage on the I-axis.

These digits are the mapping instructions for movement left and right along the Information Space of the I-axis. Yet that same value will be represented as a single infinitesimally small point on a number line like the x-axis. That is all we've done. In 5.1 above we reviewed scaling factor for the geometric square. Now consider a number. Have we made the number considered bigger or smaller? If so, bigger or smaller than what?

If we were calculating the Arithmetic Dimension of the number π then we might choose to list the following approximations:

(a) 3.14

(b) 3.14159

(c.) 3.141592653

For each of these it is accurate to say (a) is an approximation of π. While(b) is a better approximation of π than (a), and (c) is a better approximation of π than both (a) and (b). But for each of these approximations they are just numbers. 3.14 is just 3.14. I've not made it bigger or small than anything else. 3.141592653, has one leading whole integer, +3, and nine additional decimal digits. Yes it has more digits than 3.14159 and than 3.14. Nonetheless, 3.141592653 is both the starting and ending value we have chosen to consider. There is nothing else to compare it to. It has not been made bigger or smaller. It is just 3.141592653.

To put this in context with the geometric square. If the side is of length 1, and then it is divided by 1 units, the resulting side length is 1.

$$S = \frac{length\ original\ side}{Factor\ Applied} = \frac{length\ l}{1} = length\ l$$

This is what we're doing in calculating scaling factor for a number. 3.141592653 has nine decimal digits, its *length*, that will contribute to scaling factor.

<u>5.2.a:</u>

$$S = \frac{Length\ in\ digits}{1} = \frac{n}{1} = n$$

Lets return to the apparent scaling factor of π for several digit expansions. Note we are not counting the integer value. And since we are not considering anything but the number of decimal digits, the calculated scaling factor will apply to any number with the same counted about of decimal digits.

π to six decimal digits:

$$S = \frac{6}{1} = 6$$

π to 50 decimal digits:

$$S = \frac{50}{1} = 50$$

π to 50,000 decimal digits:

$$S = \frac{50{,}000}{1} = 50{,}000$$

The Scaling Factor is a variable dependent on resolution $\frac{n}{1} = n$. This is the simplest possible expression that respects:

<u>The Cantor Dust</u>: As $N \to \infty$,

- The Line Segment on the Information space available to the considered value will become infinitely big
- The percentage the Information Space filled on the Line Segment by plotted points will approach but not reach 100%
- Is exactly how a dust-like fractal maintains its structure
- We are dividing the space into infinitely many, infinitely small pieces.

This maps a symbolic process onto a geometric principle. When we half a square, we partition its spatial extent by 2. When we append a digit to a number like π, we partition its representational extent by adding a new layer of complexity. Consider that for 50,000 digits of π, in addition to the starting point value we plot at $I = 3$ there are 50,000 more small points. But that is a totally different number than had we just plotted an approximation of $\pi = 3.14$. So there is nothing to make as size adjustment back to. Our scaling factor is the number of digits considered, n.

<u>5.3—Calculating Self-Similar pieces:</u>

Now we need the concept of number of self-similar pieces. We have the count, the multiplicity of occurrence, of each digit of the decimal portion of the number. The whole number integer value to the left of the decimal will only ever be viewed as a singular self-similar copy in its entirety. Because each step taken in the information space represents the consideration of another digit in the string of numbers comprising a given value (*from where is derived the scaling factor*) it is the number of instances of each digit that specify the magnitude of the step taken, defining self-similarity.

The following examples give a summary for such counts of digit multiplicity.

2 Has 1 self-similar piece; itself.

2.0 Has two digits. Although 2.0 will take up more room in the information space than 2, it is Arithmetically identical to 2, providing no more usable information than just 2. So it has only 1 self-similar piece, the origin point of the whole integer 2.

2.00 Has one 2 and two 0's. Again 2.00 will take up more room in Information Space than 2 or 2.0. For the same reasons already stated above, this has only 1 self similar piece, the whole integer starting point.

2.01 Here we have one 0, one 1 and one 2. Each has only 1 self-similar copy; itself. Unlike with 2.0 and 2.00, the number 2.01 actually has two digits considered in a calculation, the 0 and the 1.

2.0001 This example has one 1, one 2 and three 0's. The 1 and the 2 each have only one self-similar copy but the 0 now has three self-similar copies.

131.001 131 has only one self-similar copy; itself—the value 131. Recall whole values to the left of the decimal were plotted as the start point for random walks as a single infinitesimally small point. There is only one 1, (shown in red 131.00**1**)and there are two self similar copies of the 0.

Consider the values: **4**.5579**4**8981116. and **444**.5579**4**8981116.

There is only one 4 counted for consideration in the calculation. Physically counting the first number has two 4's present and the next four 4's. However the 4's represented as a whole integer to the left of the decimal are only the start point of the random walk; respectively above $I = 4$ and $I = 444$. Whole integer values will only ever be counted as having 1 instance of self similarity. So for both of these values, **4**.5579**4**8981116 and **444**.5579**4**8981116 there is only one instance counted self-similarity for the calculation in decimal component, the number 4 shown in blue. There are two self-similar copies of 5's, 8's and 9's. There are three 1's. There is one 7. Nothing else is self-similar. The numbers 2 and 3 do not appear at all so we simply do not include any count for them; they have zero self-similar instances.

Because we are counting the number of like digits independently of one another, the number of instances of 1s, 2s, 3s, 4s ect., the numerator of Equation 2 for Arithmetic Dimension must sum the natural logs of each of these counts. Despite the number of digits considered in both the numerator and denominator of our modified Equation 2 are both n they yield different values.

5.4—Modification of Dimension Equation:

If we were talking about the square we have $S^D = N,$ or

$D_G = \frac{\ln N}{\ln S}$ Geometric

$D_A = \frac{\ln(P)}{\ln(n)}$ Arithmetic Here P is number self-similar pieces.

5.4.a:

We now adjust the value for P to account for individual digit multiplicity and their logs, treating the decimal expansion as a frequency distribution of symbols.

We almost have the Arithmetic Dimension formula

$$D_A = \frac{\ln(P)}{\ln(n)} = \frac{\sum_{i=0}^{9} \ln(Count\ P_i)}{\ln(n)}$$

One final necessary adjustment is accounting for the fact that each digit considered, has a 10% chance of occurring at any position in a number. It is not just simply length n. Every considered digit position has ten possible entries, the numbers 0 through 9. Only one of them shows up for each digit entry. If we don't factor for this, we get a final value 10 times too big. So we must also multiply the final answer by 0.1, or $\frac{1}{10}$

$$D_A = \frac{1}{10}\left(\frac{\ln(P)}{\ln(n)}\right) = \frac{1}{10}\left(\frac{\sum_{i=0}^{9} \ln(Count\ P_i)}{\ln(n)}\right)$$

5.4.b:

This is the formula for Arithmetic Dimension of Real Numbers:

$$S^D = N$$

$$(n)^{10\cdot D} = \sum_{i=0}^{9} (Count\ P_i)$$

6.0—Example Calculation:

6.1—Example 1: 4.557948981116

$$D_A = \frac{1}{10}\left(\frac{\sum_{i=0}^{9} \ln(Count\ P_i)}{\ln(n)}\right) = \frac{1}{10}\left(\frac{\sum_{i=0}^{9} \ln(\#|s_k = i)}{\ln(n)}\right)$$

There are 12 contributory digits in the considered decimal string.

$$D_A = \frac{1}{10}\left(\frac{\sum_{i=0}^{9} \ln(\#|s_k = i)}{\ln(12)}\right)$$

Including the 4 to the left of the decimal:

4.557948981116

Number	Int 4	1	4	5	6	7	8	9
Multiplicity	1	3	1	2	1	1	2	2

$$D_A = \frac{1}{10}\left(\frac{\log(1) + \log(3) + \log(1) + \log(2) + \log(1) + \log(1) + \log(2) + \log\,(2)}{\log(12)}\right)$$

$$D_A = \frac{1}{10}\left(\frac{0 + \log(3) + 0 + \log(2) + 0 + 0 + \log(2) + \log\,(2)}{\log(12)}\right)$$

$$D_A = \frac{1}{10}\left(\frac{0.47712125471966 + [3 \cdot (\log\,(2)]}{\log(12)}\right)$$

$$D_A = \frac{1}{10}\left(\frac{0.47712125471966 + 0.90308998699194}{1.0791812460476248}\right)$$

$$D_A = \frac{1}{10}(1.2789429456511)$$

$$D_A \approx 0.1279$$

<u>6.2—Relationship to Geometric Dimension:</u>

Though we have calculated D_A the Arithmetic Dimension of 4.557948981116, we need to find D_G, its Geometric Dimension. As the number of considered digits $n \to \infty$, the dimensional calculation of $D_A \to D_G$.

<u>6.3—Adding More Digits:</u>

For a value like 4.557948981116 we should expect to find it has a $D_G = 0$. The value is an infinitesimally small point on a number line axis which happens to terminate. To arrive at the D_G value from our D_A we simply consider the number of digits,

$$\lim_{n\to\infty} d_n$$

Thus we are modifying 4.557948981116 to

$$4.5579489811160000000000000 \ldots \ldots \ldots$$

The addition of an ever increasing number of zeros appended to the end of the number does nothing to change its value, or alter the number of self-similar components. As already discussed above, both the value of the number and the digits from which we calculate the number of self-similar components will remain 4.557948981116 because the additional 0s don't change its value. However the denominator n will increase without bound; the 0s contribute to n.

$$D_G = \frac{1}{10}\left(\frac{0.47712125471966 + 0.90308998699194}{\log(\infty)}\right) = \frac{1}{10}\left(\frac{1.3802112417116}{\infty}\right) = 0$$

Thus for our first example:

Example 1: 4.557948981116 $\quad D_A \approx 0.1279 \quad D_G = 0$

6.3.a:

Note if we had used the number $4.5579\mathbf{00}48981116\mathbf{000000000000}$ we would have counted only two self-similar copies for the 0s. Only the 0s in the in red here are contributing value to the number. But all of the digits including the 0s attached to the end in blue will contribute to n calculating D_G.

7.0—Examples:

From here it will be shown that any such number which terminates will have this property of $D_G = 0$. This is because they lack an infinite quality within the limited space in which they are confined, both on a number line like the x-axis where they are a point, and on their Information Space line segment, where they take up a much less room than numbers like π.

Numbers like π, which may be a normal number, neither terminating nor repeating will have a larger Geometric Dimension, and an Arithmetic Dimension such that $0 \leq D_A < 1$, as they possess a fractal nature revealed in considering the limit of their digits at infinity. Numbers with repeating digit patters will also have a non-zero Arithmetic Dimension.

7.1—Example 2):

2

This is only an integer. There is no decimal component at all. So there are no digits to consider making $n = 0$. There is only one self similar object, the integer 2. So $P = 1$. As you attempt to calculate the Arithmetic and Geometric Dimensions of any lone integer constant, having no decimal components for consideration, $n = 0$ inevitably results in $D_A = 0$.

$$D_A = \frac{1}{10}\left(\frac{\ln(P)}{\ln(n)}\right) = \frac{1}{10}\left(\frac{\log(1)}{\log(0)}\right) = \frac{1}{10}\left(\frac{0}{-\infty}\right) = 0$$

When we extend $n \to \infty$ we get the value for the Geometric Dimension of 2.

$$D_G = \frac{1}{10} \lim_{n\to\infty} \left(\frac{\ln(P)}{\ln(n)} \right) = \frac{1}{10} \lim_{n\to\infty} \left(\frac{\ln(1)}{\ln(\infty)} \right) = \frac{1}{10} \left(\frac{0}{\infty} \right) = \frac{0}{\infty} = 0$$

So is the case with any lone integer value with no decimal component.

7.2—Example 3):
738,264,421

Using the rules set forth above for calculating D_A, the number 738,264,421 has one self similar part, itself. So $P = 1$. It has no decimal components so $n = 0$.

$$D_A = \frac{1}{10} \left(\frac{\ln(P)}{\ln(n)} \right) = \frac{1}{10} \left(\frac{\ln(1)}{\ln(0)} \right) = \frac{1}{10} \left(\frac{0}{-\infty} \right) = 0$$

The geometric dimension D_G is determined by considering the limit as $n \to \infty$

738,264,421.000000000000000……….

$$D_G = \frac{1}{10} \lim_{n\to\infty} \left(\frac{\ln(P)}{\ln(n)} \right) = \frac{1}{10} \lim_{n\to\infty} \left(\frac{\ln(1)}{\ln(\infty)} \right) = \frac{1}{10} \left(\frac{0}{\infty} \right) = \frac{0}{\infty} = 0$$

7.3—Example 4:
2.01

The whole integer value 2 has only itself as a self-similar component. It has two decimal components so $n = 2$. The decimal components 0 and 1 both only have one self-similar piece, themselves.

$$D_A = \frac{1}{10} \left(\frac{\ln(P)}{\ln(n)} \right) = \frac{1}{10} \left(\frac{\ln(1) + \ln(1) + \ln(1)}{\ln(2)} \right) = \frac{1}{10} \left(\frac{0 + 0 + 0}{0.69314718} \right) = 0$$

$$D_G = \frac{1}{10} \lim_{\substack{\lim \\ n\to\infty}} \left(\frac{\ln(P)}{\ln(n)} \right) = \frac{1}{10} \left(\frac{\ln(1) + \ln(1) + \ln(1)}{\ln(\infty)} \right) = \frac{1}{10} \left(\frac{0}{\infty} \right) = \frac{0}{\infty} = 0$$

7.4—Example 5:

$$\frac{1}{12} = 0.08333333 \ldots \ldots$$

This is our first example with repeating digits. The 3 repeats forever. The Arithmetic Dimension value will be dependent on the number of digits we choose to consider. As we consider an arbitrarily large number of digits we should see the limit as $n \to \infty$ settle on specific value, the limit of the Geometric Dimension. We will see this appears to be the case plugging in $n = \infty$, where there will be an infinite number of 3s. L'Hôpital's rule will give us the value for D_G.

The leading integer is 0, a whole integer having only one self-similar piece. The decimal section has only one 0, one 8 and as many 3s as we choose, by increasing n.

$n = 3$ 0.083

$$D_A = \frac{1}{10}\left(\frac{\ln(P)}{\ln(n)}\right) = \frac{1}{10}\left(\frac{\ln(1) + \ln(1) + \ln(1) + \ln(1)}{\ln(3)}\right) = \frac{1}{10}\left(\frac{0+0+0+0}{1.0986122886681}\right) = 0$$

$n = 10$ 0.0833333333

$$D_A = \frac{1}{10}\left(\frac{\ln(P)}{\ln(n)}\right) = \frac{1}{10}\left(\frac{\ln(1) + \ln(1) + \ln(1) + \ln(8)}{\ln(10)}\right) = \frac{1}{10}\left(\frac{0 + 2.07944154167980}{2.30258509299}\right) = 0.09$$

$n = 100$ One leading integer. One 0, one 8, and ninety-eight 3s.

$$D_A = \frac{1}{10}\left(\frac{\ln(P)}{\ln(n)}\right) = \frac{1}{10}\left(\frac{\ln(1) + \ln(1) + \ln(1) + \ln(98)}{\ln(10)}\right) = \frac{1}{10}\left(\frac{4.584967478670}{4.605170185988}\right) = 0.09956130$$

$n = 100{,}000$

$$D_A = \frac{1}{10}\left(\frac{\ln(P)}{\ln(n)}\right) = \frac{1}{10}\left(\frac{\ln(1) + \ln(1) + \ln(1) + \ln(99998)}{\ln(100000)}\right) = \frac{1}{10}\left(\frac{11.51290546477}{11.51292546497}\right) = 0.0999998$$

$n = 1{,}000{,}000{,}000$

$$D_A = \frac{1}{10}\left(\frac{\ln(1) + \ln(1) + \ln(1) + \ln(999999998)}{\ln(1000000000)}\right) = 0.099999999990349$$

It has a non-zero but low D_G value which appears to be approaching 0.1 as Applying $n = \infty$ we can use L'Hôpital's rule to determine the value it approaches.

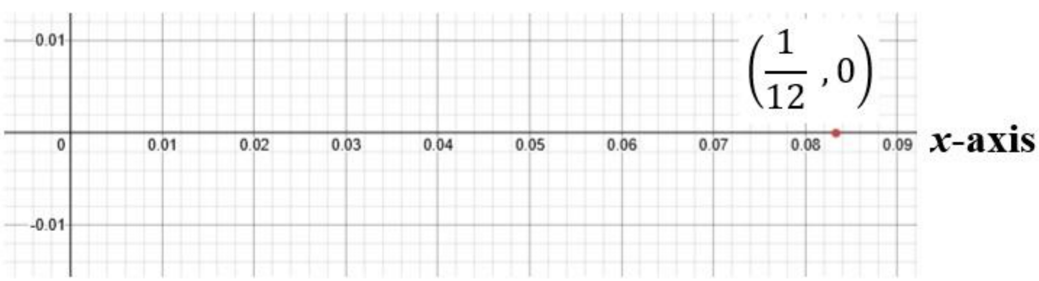
$$D_A = \frac{1}{10}\lim_{n\to\infty}\left(\frac{\ln(P)}{\ln(n)}\right) = \frac{1}{10}\lim_{n\to\infty}\left(\frac{\ln(1) + \ln(1) + \ln(1) + \ln(\infty)}{\ln(\infty)}\right) = \frac{1}{10}\left(\frac{0+0+0+\infty}{\infty}\right) = \frac{\infty}{\infty}$$

We now apply L'Hôpital's rule. Since the number of 3s at $n \to \infty$ will themselves approach infinity, $\#3s \to \infty$ we simply label #3s as n.

$$\frac{1}{10}\lim_{n\to\infty}\left(\frac{\frac{d}{dn}\ln(n)}{\frac{d}{dn}\ln(n)}\right) = \frac{1}{10}\lim_{n\to\infty}\left(\frac{\frac{1}{n}}{\frac{1}{n}}\right) = \frac{1}{10}\lim_{n\to\infty}\left(\frac{n}{n}\right) = \frac{1}{10}(1) = 0.1$$

Both methods show the anticipated limit for D_G of $\frac{1}{12}$ is 0.1, clearly not 0. This is due to a fractal component present in the number, the repeating 3 which extends to infinity and yet confined to a finite space, the infinitesimally small point on a number line like the x-axis marking 1/12.

Figure 20



Cantor Dust *I*-axis. Segment Length 4.44 units.

7.5—Example 6:

$$\frac{5}{11} = 0.\overline{45}$$

Here again we have a repetition. We are just counting occurrences of each. Because the repetition occurs for a set of two digits, we need to consider digit multiples of 2. The value of $D_A \to D_G$, as $n \to \infty$. As we extend the digits $n \to \infty$ we should expect a low value but non-zero value for D_G.

$n = 2$ 0.45

$$D_A = \frac{1}{10}\left(\frac{\ln(P)}{\ln(n)}\right) = \frac{1}{10}\left(\frac{\ln(1) + \ln(1) + \ln(1)}{\ln(2)}\right) = \frac{1}{10}\left(\frac{0}{0.6931471805599453}\right) = 0$$

$n = 10$ 0.4545454545

$$D_A = \frac{1}{10}\left(\frac{\ln(P)}{\ln(n)}\right) = \frac{1}{10}\left(\frac{\ln(1) + \ln(5) + \ln(5)}{\ln(10)}\right) = \frac{1}{10}\left(\frac{3.2188758248682}{2.302585092994}\right) \approx 0.139794$$

$n = 100{,}000$

$$D_A = \frac{1}{10}\left(\frac{\ln(P)}{\ln(n)}\right) = \frac{1}{10}\left(\frac{\ln(1) + \ln(50{,}000) + \ln(50{,}000)}{\ln(100{,}000)}\right) = \frac{1}{10}\left(\frac{21.63955656882}{11.51292546497}\right) \approx 0.1879588$$

$n = 1{,}000{,}000{,}000$

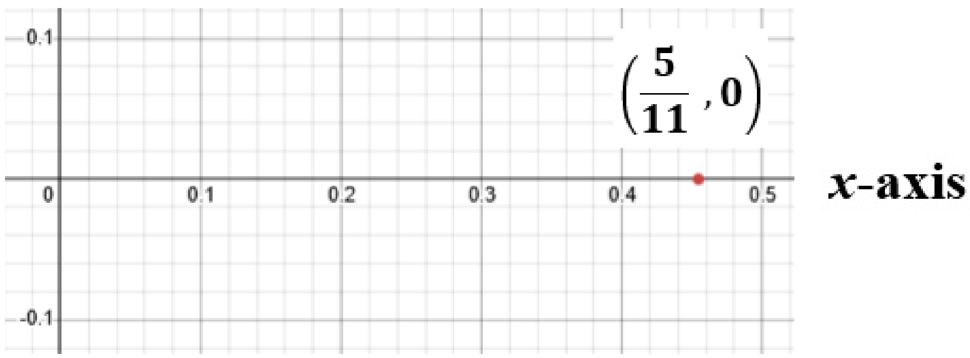
$$D_A = \frac{1}{10}\left(\frac{\ln(P)}{\ln(n)}\right) = \frac{1}{10}\left(\frac{\ln(1) + 2 \cdot [\ln(500{,}000{,}000)]}{\ln(1{,}000{,}000{,}000)}\right) = \frac{1}{10}\left(\frac{40.06023731277293}{20.72326583694641}\right) \approx 0.19331044$$

$n = 1$ quindecillion, $D_A \approx 0.198885$, $D_G \to 0.2$

Now allowing $n \to \infty$ we again use L'Hôpital's rule to find the value the limit approaches. As $n \to \infty$ there will be an infinite number of both 4s and 5s in the considered digits. Thus we may again use $n = \infty$ for the number of 4s and 5s.

$$D_G = \frac{1}{10} \lim_{n\to\infty} \left(\frac{0 + \frac{d}{dn}\,(\ln(n) + \ln(n))}{\frac{d}{dn}\ln(n)} \right)$$

$$D_G = \frac{1}{10} \lim_{n\to\infty} \left(\frac{2\,\frac{d}{dn}\,\ln(n)}{\frac{d}{dn}\ln(n)} \right) = \frac{1}{10} \lim_{n\to\infty} \left(\frac{\frac{2}{n}}{\frac{1}{n}} \right) = \frac{1}{10} \lim_{n\to\infty} (2) = \frac{2}{10} = 0.2$$



<u>7.6—Example 7:</u>

$\sqrt{2}$

The number is irrational and like π neither terminates nor repeats. So not only do we never run out of digits to consider but the multiplicity of those decimal components, their self-similarity, will build and change as n increases.

$n = 3$ $\qquad$ $\sqrt{2} = 1.414$ $\qquad$ One leading coefficient. One 1. Two 4s.

$$D_A = \frac{1}{10}\left(\frac{\ln(P)}{\ln(n)}\right) = \frac{1}{10}\left(\frac{\ln(1)+\ln(1)+\ln(2)}{\ln(3)}\right) = \frac{1}{10}\left(\frac{0.6931471805599}{1.0986122886681}\right) \approx 0.063092975357$$

$n = 100$

Digit	0	1	2	3	4	5	6	7	8	9
Count	10	7	8	11	9	7	10	18	12	8

$$D_A = \frac{1}{10}\left(\frac{\ln(1)+\ln(10)+\ln(7)+\ln(8)+\ln(11)+\ln(9)+\ln(7)+\ln(10)+\ln(18)+\ln(12)+\ln(8)}{\ln(100)}\right)$$

$$D_A = \frac{1}{10}\left(\frac{22.626271825277144764}{4.60517018598809136803}\right) \approx 0.491323249988044$$

$n = 1{,}000{,}000$

Digit	0	1	2	3	4
Count	99,814	98,924	100,436	100,191	100,024

Digit	5	6	7	8	9
Count	100,155	99,886	100,008	100,441	100,121

$$D_A = \frac{1}{10}\left(\frac{\ln(1)+\ln(99814)+\ln(98924)+\ln(100436)+\ln(100191)+\ln(100024)+\ln(100155)+\ln(99886)+\ln(100008)+\ln(100441)+\ln(100121)}{\ln(100)}\right)$$

$$D_A = \frac{1}{10}\left(\frac{115.12917100200984075510999686309}{13.815510557964274104107948728106}\right) = 0.83333272787114579785263143616506$$

The question in calculating D_G requires knowing how many of each digit there is in the considered portion of the number as $n \to \infty$. If $\sqrt{2}$ is a normal number then every single digit 0 through 9 has the same likelihood of occurrence, and at $n \to \infty$, D_G will approach 1. Whether $\sqrt{2}$ is a normal number is not yet known, however at one-million considered digits, it appears $D_G \to 0.83$, not 1.

Figure 22

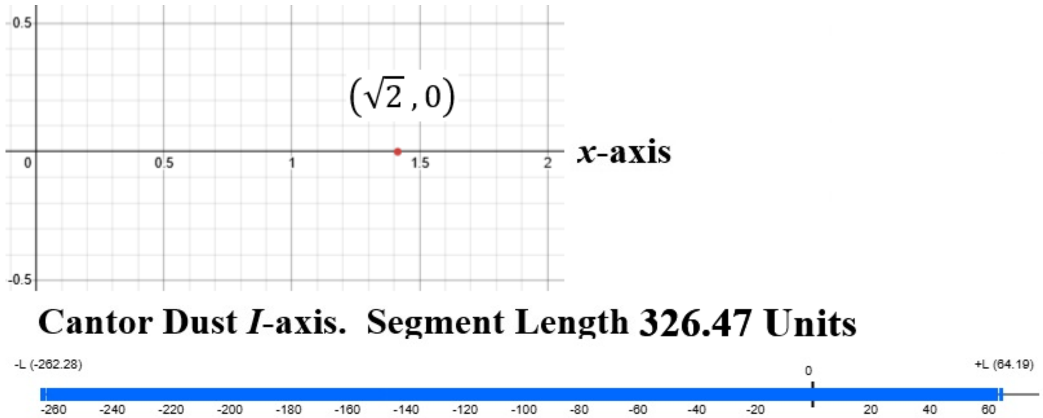


Figure 22 shows the cantor dust for 50,000 digits of $\sqrt{2}$. Even in this scale openings are visible at the edges. Zooming reveals further porousness throughout.

7.7—Example 8:

The last example we'll consider is π, a transcendental and irrational number, with digits that never repeat and never terminate. Like $\sqrt{2}$ its not known for certain whether π is a normal number. For now we can only calculate its D_A to an arbitrarily high number of digits and see what limit it appears D_G is approaching. Because its dust graphs were shown in detail earlier in the paper they are not reintroduced here.

$n = 3$ $\pi \approx 3.141$ Integer 3 has one self-similar piece. There is one 4 and two 1s

$$D_A = \frac{1}{10}\left(\frac{\ln(P)}{\ln(n)}\right) = \frac{1}{10}\left(\frac{\ln(1) + \ln(2) + \ln(1)}{\ln(3)}\right) = \frac{1}{10}\left(\frac{0.6931471805599}{1.0986122886681}\right) \approx 0.063092975357$$

$n = 100$
$\pi \approx$3.141592653589793238462643383279502884197169399375105820974944592307816406
286208998628034825342117067

Digit	0	1	2	3	4
Count	8	8	12	11	10

Digit	5	6	7	8	9
Count	8	9	8	12	14

$$D_A = \frac{1}{10}\left(\frac{\ln(P)}{\ln(n)}\right)$$

$$D_A = \frac{1}{10}\left(\frac{\ln(1)+\ln(8)+\ln(8)+\ln(12)+\ln(11)+\ln(10)+\ln(8)+\ln(9)+\ln(8)+\ln(12)+\ln(14)}{\ln(100)}\right)$$

$$D_A = \frac{1}{10}\left(\frac{22.824341739039}{4.60517018598809}\right) \approx 0.49562428351694$$

$n = 100{,}000$

Digit	0	1	2	3	4
Count	9,999	10,137	9,908	10,025	9,971

Digit	5	6	7	8	9
Count	10,026	10,029	10,025	9,978	9,902

$$D_A = \frac{1}{10}\left(\frac{\ln(1)+\ln(9999)+\ln(10137)+\ln(9908)+\ln(10025)+\ln(9971)+\ln(10026)+\ln(10029)+\ln(10025)+\ln(9978)+\ln(9902)}{\ln(100000)}\right)$$

$$D_A = \frac{1}{10}\left(\frac{92.1031993533998400386201339852 89}{11.51292546497022842}\right) \approx 0.799998224896334$$

$n = 1{,}000{,}000$

Digit	0	1	2	3	4
Count	99,959	99,758	100,026	100,229	100,230

Digit	5	6	7	8	9
Count	100,359	99,548	99,800	99,985	100,106

$$D_A = \frac{1}{10}\left(\frac{115.12923}{13.81551}\right) \approx 0.8333331885685$$

So again though we cannot just plug in infinity at $n = 1{,}000{,}000$ it appears $D_G \to 0.83$. If we continue this out to $n = 10{,}000{,}000$ the value D_G provided by Google Gemini still approaches 0.833333.

Summary:

It is shown that Arithmetic Dimension D_G exists and that its value approaches the Fractal Geometric Dimension D_G as the number of considered digits approaches infinity. Though one cannot always take that number n literally to infinity it has been shown for irrational, and transcendental numbers they do appear to reach a limit for D_G as $n \to \infty$.

Appendix:

Numeric Cantor Dust Creator   Index.html

```
<!DOCTYPE html>
<html lang="en">
<head>
   <meta charset="UTF-8">
   <title>Numeric Cantor Dust Creator</title>
   <style>
      body { background-color: #D6CEB4; font-family: sans-serif; padding: 20px; }
      #controls { margin-bottom: 20px; }
      #scrollWrapper {
         width: 100%; height: 350px; overflow-x: auto; overflow-y: hidden;
         background: white; border: 1px solid #000; margin-top: 10px;
      }
      #axisCanvas { height: 300px; display: block; }
      .input-group { margin: 5px 0; }
      #zoomBar { width: 400px; }
      #statusIndicator { padding: 5px 10px; display: inline-block; border: 1px solid #000; font-
weight: bold; }
   </style>
</head>
<body>

<h1>Numeric Cantor Dust Creator</h1>

<div id="controls">
   <div class="input-group">
      <label>Input Value: <input type="text" id="userInput" placeholder="Enter
number"></label>
      <button onclick="runWalk()">Submit</button>
      <button onclick="clearAll()">Clear</button>
      <button onclick="stopWalk()" style="color:red;">Stop</button>
   </div>

   <div id="statusIndicator" style="background-color: gray;">Not Started</div>
   <span id="timerDisplay"></span>
   <div>Points Plotted: <span id="pointCounter">0</span></div>

   <div class="input-group" style="margin-top:10px;">
      Zoom: <input type="range" id="zoomBar" min="0" max="2" value="0" step="0.01"
oninput="draw(pointsData)">
   </div>
</div>
```

```
<div id="scrollWrapper">
  <canvas id="axisCanvas"></canvas>
</div>

<script>
  let isRunning = false;
  let pointsData = [];
  let startTime;
  const canvas = document.getElementById('axisCanvas');
  const ctx = canvas.getContext('2d');

  function updateStatus(text, color) {
    const el = document.getElementById('statusIndicator');
    el.innerText = text;
    el.style.backgroundColor = color;
  }

  function clearAll() {
    isRunning = false;
    pointsData = [];
    ctx.clearRect(0, 0, canvas.width, canvas.height);
    document.getElementById('userInput').value = '';
    document.getElementById('timerDisplay').innerText = '';
    document.getElementById('pointCounter').innerText = '0';
    updateStatus('Not Started', 'gray');
  }

  function stopWalk() { isRunning = false; }

  async function runWalk() {
    isRunning = true;
    pointsData = [];
    startTime = new Date();
    updateStatus('Running', 'lightyellow');
    document.getElementById('timerDisplay').innerText = '';

    const fullInput = document.getElementById('userInput').value;
    const parts = fullInput.split('.');
    if (parts.length > 2) { alert("Too many decimals"); isRunning = false; updateStatus('Not
Started', 'gray'); return; }

    let currentPos = parseInt(parts[0]) || 0;
    pointsData.push(currentPos);
```

```
    document.getElementById('pointCounter').innerText = pointsData.length;
    draw(pointsData);

    if (parts.length === 2) {
      const digits = parts[1];
      let prevDir = 0;
      let multiplier = 1;

      for (let i = 0; i < digits.length && isRunning; i++) {
        const digit = parseInt(digits[i]);
        if (isNaN(digit)) continue;

        const dir = (digit <= 4) ? 1 : 2;
        const mag = (digit <= 4) ? (digit + 1) : (10 - digit);

        if (dir !== prevDir && prevDir !== 0) {
          multiplier = 1;
        } else if (prevDir !== 0) {
          multiplier *= 0.1;
        }

        currentPos += (dir === 1 ? -1 : 1) * (mag * multiplier);
        pointsData.push(currentPos);
        document.getElementById('pointCounter').innerText = pointsData.length;
        prevDir = dir;
        draw(pointsData);
        await new Promise(r => setTimeout(r, 100));
      }
    }

    if (isRunning) {
      const elapsed = new Date(new Date() - startTime);
      document.getElementById('timerDisplay').innerText = `Elapsed:
${elapsed.getUTCHours()}h ${elapsed.getUTCMinutes()}m ${elapsed.getUTCSeconds()}s`;
      updateStatus('Iteration Complete', 'lightgreen');
    }
  }

  function draw(points) {
    if (points.length === 0) return;
    const zoom = Math.pow(10, parseFloat(document.getElementById('zoomBar').value));
    const minP = Math.min(...points);
    const maxP = Math.max(...points);
    const M = Math.max(Math.abs(minP), Math.abs(maxP), 10);
```

```
        const span = 2 * M;
        const containerWidth = document.getElementById('scrollWrapper').clientWidth;
        const canvasWidth = Math.max(containerWidth, containerWidth * zoom);

        canvas.width = canvasWidth;
        canvas.height = 300;
        const scale = (canvasWidth - 100) / span;
        const originX = canvasWidth / 2;

        ctx.clearRect(0, 0, canvas.width, canvas.height);
        ctx.beginPath();
        ctx.moveTo(50, 150); ctx.lineTo(canvasWidth - 50, 150);
        ctx.stroke();
        ctx.fillRect(originX, 140, 2, 20);
        ctx.fillText("0", originX - 5, 135);

        let step = (span <= 10) ? 1 : (span <= 20) ? 2 : (span <= 100) ? 10 : 20;
        for (let i = -Math.floor(M/step)*step; i <= M; i += step) {
          if (i === 0) continue;
          ctx.fillRect(originX + (i * scale), 145, 1, 10);
          ctx.fillText(i.toString(), originX + (i * scale) - 5, 165);
        }

        ctx.fillStyle = '#0069FF';
        points.forEach(p => ctx.fillRect(originX + (p * scale), 145, 2, 10));
        ctx.fillStyle = 'black';
        ctx.fillText(`-L (${minP.toFixed(2)})`, originX + (minP * scale) - 20, 130);
        ctx.fillText(`+L (${maxP.toFixed(2)})`, originX + (maxP * scale) - 20, 130);
    }
</script>
</body>
</html>
```