How Gravity's Puzzles Enticed Some of the Greatest Physicists into Mistakes

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Abstract Einstein's ponderings on gravity led in 1907 to his principle of the equivalence of gravitation and inertia, illustrated by his famous elevator prop. For Newtonian gravity, it is impeccable. But his elevator illustration of gravity's bending of light implicitly allows light to be accelerated to a speed greater than c, violating his 1905 Lorentzian relativity. In 1913, Einstein decided that general relativity supplants Lorentzian relativity, and proceeded to his famous gravity theory. But his Einstein equation has no unique solution, hampering his effort to explain Mercury's perihelion precession. He overcame that obstacle in 1915 with a simple Lorentz-invariant coordinate condition which isn't mentioned in textbooks, honoring his insistence that coordinate conditions are irrelevant. Friedmann, also honoring that insistence, found a new coordinate condition in 1922 that can make the Einstein equation easier to solve; it was eagerly adopted by cosmologists. But it makes gravity essentially Newtonian, which is drastically wrong at high redshifts. In 1962, Feynman elucidated general relativity's physical role in gravity theory; it generates the gravity field's self-interaction in the context of an action principle, which is a fundamental prerequisite for quantization. But Feynman missed the very large constant in that action, the key to gravity's classical-field nature and the absence of gravitons, so he tried quantum-gravity perturbations, which are a physical dead end.

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1. Einstein's fundamental-transformation journey: from Galilean to Lorentzian to General

In 1905 Einstein *upgraded* the *intuitive* constant-velocity Galilean transformation, *which fails to accommodate electromagnetism's velocity-dependent forces,* e.g., Faraday's magnet traversing the center of a wire coil *at constant velocity,* which *deflects* the needle of a galvanometer attached to the coil, or Oersted's wire *with constant current flowing through it,* which *deflects* the needle of a nearby magnetic compass away from magnetic north. The constant-velocity $\mathbf{v}$ Galilean transformation, i.e., $t' = t$ and $\mathbf{r}' = \mathbf{r} - \mathbf{v}t$, implies that $d^2\mathbf{r}'/dt'^2 = d^2\mathbf{r}/dt^2$, *so no new forces arise from that transformation.* But in Einstein's 1905 *counterintuitive* constant-velocity $\mathbf{v}$ Lorentz transformation, $t' \propto \big(t - \big((\mathbf{r}\cdot\mathbf{v})/c^2\big)\big)$, *so new* $\mathbf{v}$*-dependent forces actually arise.*

Einstein's 1905 constant-velocity $\mathbf{v}$ Lorentz transformation is based on Maxwell's *discovery* that the field equations of electromagnetic theory, *which accommodate the velocity-dependent electromagnetic forces* found by Oersted and Faraday, and *also accommodate* dynamic current conservation, *predict electromagnetic-field waves that move at a fixed speed c in a vacuum.* While *it was accepted* that electromagnetic fields *physically exist in a vacuum,* Maxwell *didn't accept the implication of his electromagnetic field equations that the fields alone can dynamically produce wave phenomena in a vacuum,* because *all* wave phenomena which were *known* at that time *could be attributed to the motions of an underlying medium.* Therefore, *ignoring the thrust of his electromagnetic field equations,* Maxwell *postulated a medium,* "the luminiferous ether", whose harmonic motions *ostensibly* are the electromagnetic waves which his electromagnetic field equations imply. The 1887 Michelson-Morley experiment sought to demonstrate *the varying speed* of Maxwell's ostensible light-transmitting ether *relative to Michelson's earthbound interferometer* that arises from both *the earth's rotation* and *its orbital motion around the sun.* But *no variation* in the speed of light was found, *consistent with the fixed speed c in a vacuum of the electromagnetic-field waves which Maxwell's field equations imply.*

Therefore *the counterintuitive centerpiece* of Einstein's 1905 constant-velocity $\mathbf{v}$ *Lorentz transformation* is that in a vacuum *the expansion speed of a spherical light wave is c for all observers,* which is guaranteed by the Lorentz transformation's *signature property that* $(ct')^2 - |\mathbf{r}'|^2 = (ct)^2 - |\mathbf{r}|^2$. The Lorentz transformation is also *homogeneously linear in t and* $\mathbf{r}$, exactly like the Galilean transformation, i.e., $t' = t$ and $\mathbf{r}' = \mathbf{r} - \mathbf{v}t$, *which the Lorentz transformation reduces to when* $(|\mathbf{v}|/c) \to 0$*, as is seen from its complete expression,*

$$t' = \gamma\big(t - \big((\mathbf{r}\cdot\mathbf{v})/c^2\big)\big) \text{ and } \mathbf{r}' = \mathbf{r} - \gamma\mathbf{v}t + (\gamma - 1)\mathbf{v}\big((\mathbf{r}\cdot\mathbf{v})/|\mathbf{v}|^2\big), \tag{1.1}$$

where $\gamma = 1/\sqrt{1 - |\mathbf{v}/c|^2}$. In Eq. (1.1), $t' \neq t$, and an observer sees that a clock which is moving at constant velocity $\mathbf{v}$ ticks more slowly by the factor $\sqrt{1 - |\mathbf{v}/c|^2}$ than a clock which is at rest with respect to himself; this is the Lorentz transformation's *time dilation effect.* The observer also sees that an object which is moving at constant velocity $\mathbf{v}$ *appears thinner in the direction of* $\mathbf{v}$ *by the factor* $\sqrt{1 - |\mathbf{v}/c|^2}$ than it appears when it is at rest; this is the Lorentz transformation's *Fitzgerald contraction effect.* The *directional* Fitzgerald contraction effect in turn feeds into the fact that two successive constant-velocity Lorentz transformations *in nonparallel directions* don't produce *merely* a constant-velocity Lorentz tranformation, but *instead* produce a constant-velocity Lorentz tranformation *together with a rotation.* The *full* Lorentz-transformation group consequently *includes the rotations as a subgroup.* A *startling property* of the constant-velocity $\mathbf{v}$ Lorentz transformation is that it is *unphysically singular* when $|\mathbf{v}| = c$, and *unphysically complex-valued* when $|\mathbf{v}| > c$, so *a particle at rest can only be Lorentz-transformed to speeds which are less than c.* Furthermore, since in Lorentzian physics the momentum of a free particle of mass m is $\mathbf{p} = (m\dot{\mathbf{r}})/\sqrt{1 - |\dot{\mathbf{r}}/c|^2}$, *physical* free particles of mass m *can only have speeds* $|\dot{\mathbf{r}}|$ *which are less than c.* In fact, *c is the upper bound for the speeds of physical entities in Lorentzian physics.*

The *many counterintuitive features of Einstein's 1905 Lorentzian physics,* some of which are discussed above, *usually aren't noticeable in familiar physical situations* because Galilean and Lorentz transformations *produce nearly the same results when all of the speeds involved are much less than c.* A way *to grasp the difference* between Galilean and Lorentz transformations is to understand that whereas the *intuitive* Galilean transformations *preserve time t under changes of velocity* (i.e., $t' = t$), the *counterintuitive* Lorentz transformations *preserve the time-space combination* $[t^2 - (|\mathbf{r}|/c)^2]$ *under changes of velocity,* and because c is such an immense speed, *that difference can often be neglected.*

Einstein's 1905 *counterintuitive upgrade* of the *intuitive* constant-velocity $\mathbf{v}$ Galilean transformation is an immense triumph *which removes the obstacles to understanding* how it is that nonzero constant velocities can produce additional forces (Faraday and Oersted), how it is that no amount of kinetic energy imparted to a particle by a particle accelerator can make its transit time around the accelerator's loop less than the loop's circumference divided by c, and how it is that very high-energy muons created by cosmic rays in the upper atmosphere can travel much further than c times their decay lifetime (Fitzgerald distance contraction for an observer who travels with the muon, and time dilation of that observer's clock for an earthbound observer). The Lorentz transformation has in fact been empirically verified to *at least* one part in 100 million.[1]

But after 1905 Einstein felt discontent with the fact that the Lorentz transformation, like the Galilean transformation, only refers to states of travel at constant velocities. He therefore began to grapple with the idea of relativistic transformations between states of arbitrary motion. In this regard it must be pointed out that a state of arbitrary smooth motion is readily generated by a Galilean transformation to that motion's initial velocity, followed by an appropriate infinite sequence of infinitesimal-velocity $\delta\mathbf{v}$ Galilean transformations separated by infinitesimal time intervals. But states of arbitrary motion *aren't appropriate to Lorentzian physics,* which *forbids* speeds exceeding c. However, one can generate *allowed motions* as infinite sequences of infinitesimal $\delta\mathbf{v}$ *Lorentz transformations* (the infinitesimal time intervals between the $\delta\mathbf{v}$ Lorentz transformations are, in that case, most appropriately those recorded by a clock carried by the moving body instead of those recorded by the observer's clock). This idea motivates the *Lorentz-covariant* version of Newton's Second Law, which is $(m\, d^2x^\mu/d\tau^2) = f^\mu(x^\nu,\, dx^\nu/d\tau)$, where $x^0 = ct$, $(x^1, x^2, x^3) = \mathbf{r}$, $d\tau = dt\sqrt{1 - |\dot{\mathbf{r}}/c|^2}$, the differential time interval recorded by a clock carried by the moving body, and the four-force $f^\mu(x^\nu,\, dx^\nu/d\tau)$ is required to be a function of x^ν and $dx^\nu/d\tau$ *whose Lorentz transformations are the same as those of* $x^\mu = (ct, \mathbf{r})$ (which are given by Eq. (1.1)).

A pleasant feature of Lorentz-covariant dynamical equations, such as the above Lorentzian version of Newton's Second Law, is that the set of constant velocities $\mathbf{v}$ *is also a subset of the initial conditions for those dynamical equations.* Therefore specifying initial conditions for such equations *breaks their Lorentz-transformation symmetry, ensuring that the solutions of those equations are unique.* If dynamical equations have symmetries *which aren't broken by specifying initial conditions,* all of the symmetry transformations of any solution of those equations *are also solutions, so such dynamical equations fail to yield definite results from the physical principles which they incorporate.* One is consequently *obliged to ferret out additional physical principles which are relevant to the physics being treated.*

Einstein's opinion, however, was that *the physical equivalence of constant velocities in Lorentz-covariant dynamical equations is far too specialized a symmetry for physics,* despite its superb match to electromagnetic theory. Upon learning about the general coordinate transformation symmetry of Riemannian geometry from his mathematician friend Marcel Grossmann, Einstein *soon decided that it supplants Lorentz-transformation symmetry in physics, in spite of the latter's exquisite agreement with experiment.* He had put forward his principle of the equivalence of gravitational and inertial forces in 1907, and he thought that general coordinate transformation symmetry might dovetail with that; his 1913 paper with Grossmann is entitled, "Draft of a Generalized Theory of Relativity and a Theory of Gravity".

Einstein used his famous elevator prop to illustrate his principle of the equivalence of gravitational and inertial forces, which accords with Newtonian gravity theory, but his 'accelerating elevator' illustration of the bending of a light beam *suggests that the observer sees it accelerated to a speed greater than* c, which would violate a cardinal principle of Lorentzian physics. In 1911, Einstein applied this principle to calculate the deflection of starlight by the sun's gravitational field, which yields a result that is *half* of the correct result he obtained in November, 1915, when he *also* obtained correct result for Mercury's perihelion precession.

Astronomers refer to gravity's bending of light as 'gravitational lensing'; many galaxies have a roughly oblate spheroidal shape which is somewhat like the shape of a magnifying lens, and such galaxies in the foreground have often been found to usefully magnify the image of a more distant galaxy. The light-refractive material of a magnifying lens *slows entering light to a speed below c in accord with its refractive index,* while exiting light *speeds back up to c.* This behavior *is the diametric opposite* of what *both* Einstein's principle of equivalence *and* Newtonian gravity *would seem to imply* for light entering and exiting the gravitational field of a galaxy, but the effect on light of light-refractive material *doesn't contradict Lorentzian physics, contrary to the ostensible effect on light* of *both* equivalence-principle gravity *and* Newtonian gravity.

The foregoing two paragraphs suggest that a theory of gravity *won't be correct if it isn't Lorentz-covariant,* which is no surprise, since the Lorentz tranformation *has been empirically verified to at least one part in* 10^8. To construct a field equation for gravity along the lines of electromagnetism's,

$$\eta^{\alpha\beta}\big(\partial^2 A^\mu(x)/\partial x^\alpha \partial x^\beta\big) - \eta^{\mu\beta}\big(\partial^2 A^\alpha(x)/\partial x^\alpha \partial x^\beta\big) = (4\pi/c)j^\mu(x), \tag{1.2a}$$

we need to *reconsider* the Newtonian idea that *mass* is gravity's *source. Mass, unlike charge, isn't exactly* *conserved*—the mass of a carbon dioxide molecule is very, very slightly *less* than the combined mass of a carbon atom and an oxygen molecule; the *difference* shows up *as heat energy* when carbon burns. But taking *energy* as gravity's *source* isn't appropriate in Lorentz-covariant physics; *energy* and *momentum,* as seen by an observer at rest, *are linearly entangled* for an observer traveling at nonzero constant velocity, just as *time* and *space* are thereby linearly entangled. In a theory *akin* to Lorentz-covariant electromagnetism, i.e., Eq. (1.2a), *conserved four-momentum is therefore the gravitational potential's appropriate source.*

In Eq. (1.2a), $j^\mu(x)$ is the four-vector density-flux *of conserved charge,* which is sometimes called the charge-current density. *The detailed local conservation of charge implies that its four-divergence vanishes,*

$$\partial j^\mu(x)/\partial x^\mu = 0. \tag{1.2b}$$

The left side of Eq. (1.2a) *is designed so that its own four-divergence vanishes identically, which reinforces* Eq. (1.2b). The *static limit of the* $\mu = 0$ *component of* Eq. (1.2a) is,

$$-\nabla^2 A^0(\mathbf{r}) = 4\pi(j^0(\mathbf{r})/c), \text{ i.e., Coulomb's Static Law.} \tag{1.2c}$$

For *gravity,* the entity *analogous* to electromagnetism's $j^\mu(x)$ is the symmetric second-rank-tensor density-flux of conserved four-momentum, $T_{\mu\nu}(x) = T_{\nu\mu}(x)$, *which we call the four-momentum tensor. The detailed local conservation of four-momentum implies that its four-divergence vanishes,*

$$\partial T^{\alpha\beta}(x)/\partial x^\beta = \partial\big(\eta^{\alpha\mu}T_{\mu\nu}(x)\eta^{\nu\beta}\big)/\partial x^\beta = 0. \tag{1.3a}$$

The electromagnetic potential $A^\mu(x)$ in Eq. (1.2a) *is a four-vector field,* which corresponds to *the four-vector nature of the density-flux* $j^\mu(x)$ *of its conserved scalar-charge source.* Likewise, *the gravitational potential* $g_{\mu\nu}(x)$ *is a symmetric second-rank tensor field,* which corresponds to *the symmetric second-rank tensor nature of the density-flux* $T_{\mu\nu}(x)$ *of its conserved four-momentum source.* The field equation for the gravitational potential $g_{\mu\nu}(x)$, *which is to be designed along the lines of the electromagnetic* Eq. (1.2a), *must likewise feature a left side whose four-divergence vanishes identically.* Furthermore, just as the static limit of the electromagnetic Eq. (1.2a) yields Coulomb's Static Law, the static limit of the field equation for the gravitational potential $g_{\mu\nu}(x)$ *is required to yield the well-known relation of the static Newtonian gravity potential* $\phi(\mathbf{r})$ *to the static mass density* $\rho(\mathbf{r}) = T_{00}(\mathbf{r})/c^2$, *which is,*

$$\nabla^2\phi(\mathbf{r}) = 4\pi G\rho(\mathbf{r}) = \big(4\pi G/c^2\big)\, T_{00}(\mathbf{r}). \tag{1.3b}$$

As well as satisfying the partial differential equation given by Eq. (1.3b), Newton's static gravity potential $\phi(\mathbf{r})$ has a simple relation to the static limit of $g_{00}(x)$. To find it, *we compare the nonrelativistic Lagrangian* for the motion of a mass m test body in Newton's static gravity potential $\phi(\mathbf{r})$, i.e.,

$$L(\dot{\mathbf{r}},\, \phi(\mathbf{r})) = (m/2)\left[|\dot{\mathbf{r}}|^2 - 2\phi(\mathbf{r})\right], \tag{1.3c}$$

with the nonrelativistic limit of the relativistic Lagrangian for the geodesic motion of a mass m test body *in the relativistic gravitational potential* $g_{\mu\nu}(x)$, i.e.,

$$L(dx^\mu/d\tau,\, g_{\mu\nu}(x)) = (m/2)\left[c^2 - (dx^\mu/d\tau)g_{\mu\nu}(x)(dx^\nu/d\tau)\right]. \tag{1.3d}$$

Obtaining *the nonrelativistic limit* of the Eq. (1.3d) Lagrangian *is facilitated by rewriting it as follows,*

$$L(dx^\mu/d\tau, h_{\mu\nu}(x)) = (m/2)\left[c^2 - (dx^\mu/d\tau)\eta_{\mu\nu}(dx^\nu/d\tau) - (dx^\mu/d\tau)h_{\mu\nu}(x)(dx^\nu/d\tau)\right], \tag{1.3e}$$

where $h_{\mu\nu}(x) \stackrel{\text{def}}{=} (g_{\mu\nu}(x) - \eta_{\mu\nu})$. In the nonrelativistic limit, $|\dot{\mathbf{r}}| \ll c$, so for $i = 1, 2, 3$, we drop all $\dot{x}^i$ which occur in the same context as c. Since $d\tau = dt\sqrt{1 - |\dot{\mathbf{r}}/c|^2}$, we put $d\tau$ to dt and $(dx_0/d\tau)$ to c. We obtain,

$$\text{the nonrelativistic limit of } L(dx^\mu/d\tau, h_{\mu\nu}(x)) \text{ is } (m/2)\left[|\dot{\mathbf{r}}|^2 - c^2 h_{00}(\mathbf{r})\right], \tag{1.3f}$$

which, together with Eq. (1.3c) and $h_{\mu\nu}(x) = (g_{\mu\nu}(x) - \eta_{\mu\nu})$, yields that,

$$h_{00}(\mathbf{r}) = (2/c^2)\phi(\mathbf{r}) \text{ and } g_{00}(\mathbf{r}) = 1 + (2/c^2)\phi(\mathbf{r}), \tag{1.3g}$$

which enables us to *rewrite* the Eq. (1.3b) Newtonian static limit of gravity *entirely in terms of the complete gravitational potential* $g_{\mu\nu}(x)$ *and its four-momentum tensor source* $T_{\mu\nu}(x)$, *as follows,*

$$\nabla^2 g_{00}(\mathbf{r}) = \big(8\pi G/c^4\big)\, T_{00}(\mathbf{r}). \tag{1.3h}$$

The formidable *complete expression for the gravitational analog of the electromagnetic* Eq. (1.2a) is,

$$\tfrac{1}{2}\left[\eta^{\alpha\beta}\left(\partial^2 g_{\mu\nu}(x)/\partial x^\alpha \partial x^\beta\right) - \left(\partial^2\left(g_{\gamma\nu}(x)\eta^{\gamma\delta}\right)/\partial x^\mu \partial x^\delta\right) - \left(\partial^2\left(g_{\mu\gamma}(x)\eta^{\gamma\delta}\right)/\partial x^\delta \partial x^\nu\right) + \left(\partial^2\left(g_{\delta\gamma}(x)\eta^{\gamma\delta}\right)/\partial x^\mu \partial x^\nu\right)\right] + \tfrac{1}{2}\,\eta_{\mu\nu}\left[\left(\partial^2\left(\eta^{\gamma\alpha} g_{\alpha\beta}(x)\eta^{\beta\delta}\right)/\partial x^\gamma \partial x^\delta\right) - \eta^{\alpha\beta}\left(\partial^2\left(g_{\delta\gamma}(x)\eta^{\gamma\delta}\right)/\partial x^\alpha \partial x^\beta\right)\right] = \left(8\pi G/c^4\right) T_{\mu\nu}(x). \tag{1.3i}$$

The *four-divergence of the left side* of the gravitational Eq. (1.3i) *vanishes identically* (as is also true of the electromagnetic Eq. (1.2a)). Furthermore, *in its static limit,* Eq. (1.3i) *yields* Eq. (1.3h). The left side of Eq. (1.3i) is homgeneously linear in the second partial derivatives of the symmetric second-rank-tensor gravitational potential $g_{\alpha\beta}(x)$, and is *symmetric in its two indices* $\mu\nu$. Eq. (1.3i) *follows uniquely from these properties,* as is seen from its derivation in Section 7.2 of Weinberg's monograph on gravity.[2].

The *linear* gravitational field equation of Eq. (1.3i) *isn't entirely adequate* because the gravity field *itself* possesses four-momentum, *which makes it a part of its source.* In his *Lectures on Gravitation*[3] Feynman *first* tackles *this nonlinear aspect of the gravity field* in a systematic *perturbational* way by noting that *the left side* of Eq. (1.3i) *follows from a gravity-field Lagrangian which is Lorentz-invariant and bilinear in the first partial derivatives of* $h_{\mu\nu}(x) = (g_{\mu\nu}(x) - \eta_{\mu\nu})$. Feynman *adds* to this *bilinear* gravity-field Lagrangian an arbitrary linear combination of terms *which are Lorentz-invariant, trilinear in* $h_{\mu\nu}(x)$, and *also* involve *two partial derivatives of their three* $h_{\mu\nu}(x)$ *factors.* With regard to *the space-time integral of such a contribution to the field Lagrangian,* namely *its action,* wherein *integrations by parts come into play,* Feynman finds that there are 18 *independent* such Lorentz-invariant trilinear terms in $h_{\mu\nu}(x)$ that involve *two partial derivatives of their three* $h_{\mu\nu}(x)$ *factors.* The *addition to the left side of the* Eq. (1.3i) linear field equation *which is produced* by those 18 *additional Lagrangian terms* that are *trilinear in* $h^{\mu\nu}(x)$ consists of terms *that are bilinear in* $h_{\mu\nu}(x)$*, whose number considerably exceeds 18.* Feynman *finds that requiring the four-divergence of that bilinear in* $h_{\mu\nu}(x)$ *addition to the left side of* Eq. (1.3i) *to identically vanish* actually *uniquely determines the 18 constant coefficients of the 18 additional trilinear in* $h_{\mu\nu}(x)$ *terms of the Lagrangian!* Feynman thus determines, *to second order in* $h_{\mu\nu}(x)$*, the contribution to the four-momentum which* $g_{\mu\nu}(x)$ *makes.*

The *Lorentz-covariant* Eq. (1.3i) gravitational field equation *introduces time-dilation effects which are absent from Newtonian gravitational theory,* so, in conjunction with the gravitational geodesic equation (whose Lagrangian is given by Eq. (1.3d)), *it yields a precession of Mercury's perihelion.* Feynman finds that the *value* of the perihelion precession which Eq. (1.3i) yields is about $\frac{1}{3}$ *greater than the measured value.* But *adding* to the left side of Eq. (1.3i) the gravity-field four-momentum terms that Feynman worked out, *which are bilinear in* $h_{\mu\nu}(x)$*, produces agreement with Mercury's measured perihelion precession.*

Feynman *next* seeks a Lorentz-invariant action in $h_{\mu\nu}(x)$ and its first and second partial derivatives, whose variation with respect to $h_{\mu\nu}(x)$ *has identically vanishing divergence and as well involves* $h_{\mu\nu}(x)$ *to all orders.* Feynman *uses the matrix inverse of* $g_{\mu\nu}(x)$ *to create that action* because $g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)$, *whose matrix inverse* $((\eta + h(x))^{-1})_{\alpha\beta}$ *is the infinite series,*

$$\eta_{\alpha\beta} - \textstyle\sum_{\gamma_1,\gamma_2=1}^{4} \eta_{\alpha\gamma_1}\, h_{\gamma_1\gamma_2}(x)\, \eta_{\gamma_2\beta} + \textstyle\sum_{\gamma_1,\gamma_2,\gamma_3,\gamma_4=1}^{4} \eta_{\alpha\gamma_1}\, h_{\gamma_1\gamma_2}(x)\, \eta_{\gamma_2\gamma_3}\, h_{\gamma_3\gamma_4}(x)\, \eta_{\gamma_4\beta} - \cdots, \tag{1.4a}$$

which involves $h_{\gamma\delta}(x)$ *to all orders, as Feynman requires.* The *matrix inverse of* $g_{\mu\nu}(x)$ *is denoted* $g^{\alpha\beta}(x)$*, so* $g^{\alpha\beta}(x)\, g_{\beta\gamma}(x) = \delta^\alpha_\gamma$*. With that contravariant metric tensor* $g^{\alpha\beta}(x)$ *in hand, along with the covariant* $g_{\mu\nu}(x)$, which is the gravitational potential, Feynman *proceeds to the gravitational field, i.e.,*

$$\Gamma^\gamma_{\mu\nu}(x) = \tfrac{1}{2} g^{\gamma\lambda}(x)\big[(\partial g_{\mu\lambda}(x)/\partial x^\nu) + (\partial g_{\nu\lambda}(x)/\partial x^\mu) - \left(\partial g_{\mu\nu}(x)/\partial x^\lambda\right)\big], \tag{1.4b}$$

that emerges from the geodesic equation of motion for a test body in the gravitational potential $g_{\mu\nu}(x)$,

$$m\left(d^2 x^\kappa/d\tau^2\right) = -m\, \Gamma^\kappa_{\mu\nu}(x)\, (dx^\mu/d\tau)\, (dx^\nu/d\tau), \tag{1.4c}$$

which follows from the Eq. (1.3d) *Lagrangian.* To *create his action,* Feynman *defines the following tensor,*

$$R^\gamma_{\mu\nu\delta}(x) \stackrel{\text{def}}{=} \left(\partial\Gamma^\gamma_{\mu\nu}(x)/\partial x^\delta\right) - \left(\partial\Gamma^\gamma_{\mu\delta}(x)/\partial x^\nu\right) + \Gamma^\lambda_{\mu\nu}(x)\Gamma^\gamma_{\delta\lambda}(x) - \Gamma^\lambda_{\mu\delta}(x)\Gamma^\gamma_{\nu\lambda}(x). \tag{1.4d}$$

The two useful contractions of this tensor $R^\gamma_{\mu\nu\delta}(x)$ (which is the Riemann-Christoffel curvature tensor) *are the symmetric Ricci tensor,* $R_{\mu\nu}(x) = R^\gamma_{\mu\nu\gamma}(x)$*, and the curvature scalar,* $R(x) = g^{\mu\nu}(x) R_{\mu\nu}(x)$. The gravitational-field action Feynman *thereby creates comes out to be that of* Einstein and Hilbert, i.e.,

$$I_G[g_{\mu\nu}(x)] = -\left(c^3/(16\pi G)\right) \textstyle\int R(x)\sqrt{-\det(g_{\mu\nu}(x))}\, d^4x. \tag{1.4e}$$

Varying both $I_G[g_{\mu\nu}(x)]$ and a nongravitational action with respect to $g_{\mu\nu}(x)$ yields the Einstein equation,

$$R^{\mu\nu}(x) - \tfrac{1}{2}g^{\mu\nu}(x)R(x) = \big(8\pi G/c^4\big)T^{\mu\nu}(x). \tag{1.4f}$$

As both Feynman and Weinberg point out, *the linear part of the Einstein equation is given by* Eq. (1.3i). The constant $\big(8\pi G/c^4\big)$ which occurs on the right side of the Eq. (1.4f) Einstein equation *is a very small number,* so the gravity field's *interaction* with nongravitational four-momenta *is very weak,* as is well known.

The very large constant factor $\big(c^3/(16\pi G)\big)$ in front of $\int R(x)\sqrt{-\det(g_{\mu\nu}(x))}\,d^4x$ in the Eq. (1.4e) gravity-field action *has no counterpart in the electromagnetic-field action,* $-\big(1/(16\pi c)\big)\int F_{\mu\nu}(x)F^{\mu\nu}(x)\,d^4x$, and though *there is no evidence of gravity-field quanta* (gravitons), *electromagnetic-field quanta* (photons) *have long been known via the photoelectric effect.* The *latter discrepancy is directly related to the former,* since in Feynman's quantum sum over outcomes, *action values far larger than* $\hbar$ *select the classical outcome, due to stationary-phase mathematics.* But in his *Lectures on Gravitation,* Feynman *doesn't consider this normal route to a classical outcome.* In Lecture 16, which discusses quantum gravity, Feynman says, "There needs to be no apology for the use of perturbations, since gravity is far weaker than other fields for which perturbation theory seems to make extremely accurate predictions." Feynman *fails to take into account* the fact that *the great weakness of the gravity field's interaction with nongravitational four-momenta stems directly from the presence of the very large constant factor* $\big(c^3/(16\pi G)\big)$ *in the gravity field's action,* and that this large factor *also normally renders the gravity field effectively classical, not quantum,* which makes Feynman's *application of fully quantum perturbation approximations* a physics error. Indeed, Feynman *couldn't resolve to his own satisfaction the doubts which arose in the course of his perturbational quantum gravity calculations.* Further on in Lecture 16, Feynman says, "In lowest order, ... processes represented by 'tree' diagrams have no difficulties. ... In higher orders, when we allow bubbles and loops in the diagrams, the theory is unsatisfactory in that it gets silly results. Methods for curing this disease have been successful in curing only one-ring difficulties." Feynman let the matter *rest* there, but others *didn't,* so "loop quantum gravity" became *fashionable,* even though *the issue of the multi-ring loops that it ignores, and, to an even far greater extent, the issue of no evidence of gravitons, are extremely serious objections to it.*

Fortunately, now sixty years on, *a more cogent view of gravity's seemingly classical character, and of the extraordinary conditions under which gravitons could be detected,* has taken hold. The large constant factor $\big(c^3/(16\pi G)\big)$ in the gravity-field action *can be countered by reducing the curvature scalar's magnitude* $|R(x)|$ *enough to go from the classical-outcome regime to the superposed-multi-outcome quantum regime.* The *electromagnetic analog of this is dimming a light source to the point that its detected steady glow changes to detected randomly-occurring flashes due to individual photons.* The 2024 paper "Detecting single gravitons with quantum sensing"[4] by G. Tobar et al. points out that typical gravity waves registered by gravity-wave detectors such as LIGO *are weak enough* that their individual gravitons can be detected via the individual phonons which they can excite in a sufficiently-cooled crystal, in analogy with the individual photoelectrons which individual photons can eject from a piece of metal. Successful single-phonon detection in a cooled crystal in coincidence with gravity-wave detection *is, however, very technically challenging.* The detector crystal *must be sufficiently large to be able to capture enough energy from an extremely weak gravity wave to produce phonons,* yet *such a quite sizable crystal must be uniformly cooled to a temperature low enough that its background thermal phonon emission is virtually nil for a sufficiently extended period of time.* The technology needed to achieve this hasn't yet been created.

After he completed his marvelous upgrade of the Eq. (1.3i) linear gravitational field equation to the Eq. (1.4f) Einstein equation, Feynman unfortunately *neglected to mention* that the latter *retains its form under all transformations* $x'^{\alpha}(x^{\mu})$ *of its four independent space-time coordinates* x^{μ}, i.e., that it is *generally covariant.* So, given *any* metric solution of an Einstein equation, *all space-time coordinate transformations of that metric solution also satisfy that very same Einstein equation!* However, since for the space-time coordinates *which scientific observers and experimentalists actually use, it has been established to at least one part in* 10^8 *that the Lorentz transformation is valid,* the Einstein equation *physically needs to be supplemented by a further "coordinate condition" which is compatible with Lorentz covariance of the metric* $g_{\mu\nu}(x)$. In addition, since the contravariant *matrix inverse* $g^{\alpha\beta}(x)$ of the metric $g_{\mu\nu}(x)$ *is an indispensable part of the Einstein equation, the "coordinate condition" must also guarantee that* $\det(g_{\mu\nu}(x)) \neq 0$ *for all* x. Also, since the ratio of the tick rates of two clocks at rest at space-time points x_2 and x_1 is given by,[5]

$$[\text{the tick rate of a clock at rest at } x_2]/[\text{the tick rate of a clock at rest at } x_1] = \sqrt{g_{00}(x_2)/g_{00}(x_1)}\,, \tag{1.5}$$

and since $\eta_{00} = 1 > 0$, *it is clearly physically necessary that* $g_{00}(x) > 0$ *for all* x, *a "coordinate restriction".* In summary, *the "coordinate condition" for metric solutions $g_{\mu\nu}(x)$ of the Einstein equation must be compatible with Lorentz covariance of the metric $g_{\mu\nu}(x)$, and it must require that* $\det(g_{\mu\nu}(x)) \neq 0$ *for all* x. In addition, the "coordinate restriction" that $g_{00}(x) > 0$ for all x must be imposed.

Einstein's *idea* that general-coordinate-transformation covariance *supplants* Lorentz covariance would lead him *to simply replace* the Lorentz-covariant Eq. (1.3i) *by its general-coordinate-transformation-covariant extension which is given by* Eq. (1.4f). But in their 1913 paper, Einstein and Grossmann *were inhibited from doing exactly that by concern that* Eq. (1.4f) *doesn't account for the four-momentum which the gravity field itself produces,* so they added a term to Eq. (1.4f) *which breaks its general transformation covariance.* In due course, however, Einstein *realized that the part of the left side of* Eq. (1.4f) *which isn't linear in* $g_{\mu\nu}(x)$ *properly accounts for the four-momentum which the gravity field itself produces,* so he *dropped* the unnecessary extra term which marred the 1913 Einstein-Grossmann gravity equation. Einstein was very interested in applying his new ideas concerning gravity to the issue of Mercury's anomalous perihelion precession, and the mistaken 1913 Einstein-Grossmann gravity equation *in any case yields a result which is too small.*

But trying to apply the generally-covariant Eq. (1.4f) Einstein equation to gravitational problems *collides with the fact that its general covariance precludes its having a unique solution*—an adequate "coordinate condition" must *also* be supplied, and, of course, *different "coordinate conditions" produce different results for the metric* $g_{\mu\nu}(x)$! We have pointed out above that *physical cogency* imposes the following requirements on the "coordinate condition": it must be compatible with Lorentz covariance of the metric $g_{\mu\nu}(x)$, and it must require that $\det(g_{\mu\nu}(x)) \neq 0$ for all x. Einstein, however, *would have vigorously contested the requirement that the "coordinate condition" must be compatible with Lorentz covariance of the metric* $g_{\mu\nu}(x)$, *since he had adopted the idea that general covariance supplants Lorentz covariance!*

In 1914 and most of 1915, Einstein *continued trying to apply* the generally-covariant Eq. (1.4f) Einstein equation to Mercury's anomalous perihelion precession, with results *which were too small.*[6] But on November 18, 1915 [7], Einstein *tried the coordinate condition* $\det(g_{\mu\nu}(x)) = -1$ *for all* x in that equation, *a coordinate condition which is both Lorentz-invariant and adheres to the requirement that* $\det(g_{\mu\nu}(x)) \neq 0$ *for all* x. The coordinate condition $\det(g_{\mu\nu}(x)) = -1$ for all x in the Einstein equation *yields not only the observed value of Mercury's anomalous perihelion precession;* it *also yields* a value for the deflection of starlight by the sun's gravity which is *twice* that which Einstein obtained in 1911 *using his equivalence principle.* A 1919 solar eclipse expedition *confirmed* Einstein's November 18, 1915 *result for that gravitational deflection of light.*

In his landmark November 18, 1915 paper, however, Einstein *doesn't exhibit the slightest recognition that his breakthrough coordinate condition* $\det(g_{\mu\nu}(x)) = -1$ *for all* x *is Lorentz-invariant, nor* does he mention that it ensures the existence of the contravariant metric $g^{\alpha\beta}(x)$ at all x, *nor does he concede any part of his idea that general covariance supplants Lorentz covariance.* Furthermore, in his *subsequent* November, 1915 papers [8], Einstein actually insists that coordinate conditions are irrelevant to physics, *despite* the fact that it was *only* his November 18, 1915 coordinate condition $\det(g_{\mu\nu}(x)) = -1$ for all x *which produced Mercury's observed perihelion precession in approximately two years of attempts on his part!*

2. The consequences of Einstein's denigration of physically-appropriate coordinate conditions

After pointing out that every general coordinate transformation of a metric solution of an Einstein equation is as well a metric solution of that same Einstein equation, Weinberg writes,[9] "The ambiguity in the solutions of ... Einstein's equations can be removed by main force. ... we can eliminate the ambiguity in the metric tensor by adopting some particular coordinate system."

Weinberg's above suggestion to adopt some particular coordinate system is completely devoid of physical cogency. In theoretical physics, *the sole relevant coordinate system is that of the scientific observer or experimenter or of the instruments which they have set up.* It has been shown to at least one part in 10^8 that the Lorentz transformation is valid in such coordinate systems, so the coordinates we choose must be consistent with Lorentz covariance of the metric tensor $g_{\mu\nu}(x)$. Furthermore, with regard to the Einstein equation itself, it must of course be well-defined, so the contravariant metric tensor $g^{\alpha\beta}(x)$ must be well-defined for all x, which requires that $\det(g_{\mu\nu}(x)) \neq 0$ for all x. Also, in the absence of gravity, $\det(g_{\mu\nu}(x)) = \det(\eta_{\mu\nu}) = -1$. Occam's razor says that in the absence of additional information, we should initially choose the very simplest coordinate condition which incorporates all of the above information. Einstein's November 18, 1915 coordinate condition, i.e.,

$$\det(g_{\mu\nu}(x)) = -1 \text{ for all } x, \tag{2.1a}$$

turns out to precisely meet these requirements, since it is invariant under Lorentz transformations of the metric tensor $g_{\mu\nu}(x)$, ensures that $\det(g_{\mu\nu}(x)) \neq 0$ for all x, is satisfied by the Minkowski metric $\eta_{\mu\nu}$, and is clearly the simplest possible coordinate condition which satisfies those three requirements.

Furthermore, in light of Eq. (1.5) above and the fact that $\eta_{00} = 1 > 0$, it is *also clear* that *a physical metric $g_{\mu\nu}(x)$ is required to adhere to the restriction,*

$$g_{00}(x) > 0 \text{ for all } x. \tag{2.1b}$$

The Eq. (2.1a) coordinate condition accords *with both physical sense and observations of Mercury's perihelion precession and the deflection of starlight by the sun's gravity.* However, after Einstein *saw for himself* the physical legitimacy of the Eq. (2.1a) coordinate condition, he proceeded *to backtrack* and assert that *coordinate conditions are irrelevant.* As we saw above, Weinberg *effectively fell in line with Einstein's backtracked position,* vaguely advising the adoption of some particular coordinate system. As a matter of fact, Weinberg *nowhere in his 657-page monograph on gravity even mentions* Einstein's *successful* November 18, 1915 Eq. (2.1a) coordinate condition, and virtually all textbooks on gravity *likewise don't mention it,* which is in line with Einstein's *own backtracked position that coordinate conditions are irrelevant.*

Friedmann in particular took to heart Einstein's backtracked position that coordinate conditions are irrelevant, and searched for coordinate conditions that could make solving Einstein equations *easier.* By 1922 Friedmann had done quite well in that regard, and was able to publish a selection of exact solutions of Einstein equations which followed from his discovery *of the utility* of the coordinate condition,

$$g_{00}(x) = 1 \text{ for all } x. \tag{2.2}$$

When Friedmann's Eq. (2.2) coordinate condition is inserted into Eq. (1.5), *it eliminates gravitational time dilation.* It *also prevents metrics from being Lorentz-covariant,* so metrics which incorporate it *might well reflect Newtonian gravity.* Indeed, the exact solutions of certain Einstein equations which Friedmann found in conjunction with his Eq. (2.2) coordinate condition *all reflect Newtonian gravity.*

Because Friedmann's Eq. (2.2) coordinate condition *can make Einstein equations easier to solve* (by making them reflect Newtonian gravity!), *it was injected into cosmology.* The cosmological principle of homogeneity and spherical symmetry for a perpetually expanding universe has been mathematically shown to be incorporated into the metric form,[10]

$$(c\,d\tau)^2 = (Q(t))^2\,(c\,dt)^2 - (R(t))^2\left\{\left[1/\big(1 + (q\rho)^2\big)\right](d\rho)^2 + \rho^2\left[(d\theta)^2 + (\sin\theta\,d\phi)^2\right]\right\}. \tag{2.3a}$$

Setting $Q(t)$ to unity *imposes* Eq. (2.2) *Friedmann coordinates on* Eq. (2.3a), *yielding the FRW metric form,*

$$(c\,d\tau)^2 = (c\,dt)^2 - (R(t))^2\left\{\left[1/\big(1 + (q\rho)^2\big)\right](d\rho)^2 + \rho^2\left[(d\theta)^2 + (\sin\theta\,d\phi)^2\right]\right\}. \tag{2.3b}$$

The FRW metric form's *Newtonian-gravity nature* was noted in 1934 [11], and is discussed by Weinberg [12].

In 1998 it was *confirmed* that the universe's expansion *is accelerating.* The *Newtonian-gravity nature* of this FRW metric form *can't cope with that anymore than Newtonian gravity can cope with Mercury's perihelion precession.* We currently observe galaxies with redshifts up to 14 ($0.99\,c$), *which Newtonian gravity can't possibly handle.* To be *specific* about *speed's effect on gravity,* in December, 1916, David Hilbert, applying the Schwarzschild metric in the region *where it is asymptotically weak,* obtained that the acceleration of gravity *changes from attractive to repulsive for radial velocity magnitudes in excess of* $(c/\sqrt{3}) = 0.57735\,c$*, which corresponds to a redshift of* 0.93185.[13] A few weeks *earlier,* in the Netherlands, the thesis of Johannes Dröste had been published; *it shows exactly the same result.*[14]

We thus see *that in terms of completely orthodox relativistic gravitational physics the acceleration of the universe's expansion isn't a mystery, anymore than the precession of Mercury's perihelion is a mystery. The only mystery is that cosmologists failed to wean themselves off the* Eq. (2.3b) *Newtonian-gravity FRW metric form a half-century ago when negative cosmological deceleration parameters began to appear.*[15]

Relativistic gravitational physics *cannot possibly be dealt with by using* Friedmann's Eq. (2.2) coordinate condition, which, when inserted into Eq. (1.5), *completely eliminates gravitational time dilation.* It is *essential to use* Einstein's Eq. (2.1a) November 18, 1915 coordinate condition $\det(g_{\mu\nu}(x)) = -1$ for all x, and to as well *check* that the Eq. (2.1b) *restriction,* $g_{00}(x) > 0$ for all x, *isn't violated.* The *first step to bring the* Eq. (2.3a) *metric form to Einstein coordinates,* $\det(g_{\mu\nu}(x)) = -1$ *for all* x, is to put $Q(t)$ equal to $(1/R(t))^3$. *We next introduce a radial transformation* $\rho(r)$; the determinant of Eq. (2.3a) will be -1 *if* $\rho(r)$ *satisfies,*

$$\left[(d\rho(r)/dr)^2/\left(1+(q\rho(r))^2\right)\right](\rho(r)/r)^4=1, \tag{2.3c}$$

which produces,

$$\textstyle\int_0^{\rho(r)}\left(\rho^2/\sqrt{1+(q\rho)^2}\,\right)d\rho=(r^3/3), \tag{2.3d}$$

which is more neatly expressed in terms of the dimensionless function $y(qr)=q\rho(r)$,

$$\textstyle\int_0^{y(qr)}\left(y^2/\sqrt{1+y^2}\,\right)dy=\left((qr)^3/3\right). \tag{2.3e}$$

Evaluating the Eq. (2.3e) integral *yields an implicit definition of* $y(qr)$ *via an explicit function* $F(y(qr))$,

$$F(y(qr))\overset{\text{def}}{=}\tfrac{1}{2}\Big[y(qr)\sqrt{1+(y(qr))^2}-\ln\Big(\sqrt{1+(y(qr))^2}+y(qr)\Big)\Big]=\left((qr)^3/3\right). \tag{2.3f}$$

The derivative of $F(y(qr))$ with respect to $y(qr)$ is,

$$F'(y(qr))=\Big((y(qr))^2/\sqrt{1+((y(qr))^2}\,\Big). \tag{2.3g}$$

The function $y(qr)$ *can only be obtained numerically, by iteration. Newton's iteration method for* $y(qr)$ *is,*

$$y_0(qr)=\max\Big((qr),\sqrt{(2/3)(qr)^3}\Big),\;\; y_{n+1}(qr)=y_n(qr)+\Big\{\Big[\left((qr)^3/3\right)-F\big(y_n(qr)\big)\Big]\Big/F'\big(y_n(qr)\big)\Big\}. \tag{2.3h}$$

The Eq. (2.3a) metric form *in Einstein coordinates,* which has determinent -1, is given in terms of $y(qr)$ by,

$$(c\,d\tau)^2=(1/R(t))^6\,(c\,dt)^2-(R(t))^2\Big\{\big((qr)/y(qr)\big)^4(dr)^2+\big(y(qr)/(qr)\big)^2r^2\big[(d\theta)^2+(\sin\theta\,d\phi)^2\big]\Big\}. \tag{2.3i}$$

Eq. (2.3i) *adheres* to the Eq. (2.1b) *restriction.* For the *very simplest* expanding dust-sphere cosmological model, *it isn't necessary to put the* Eq. (2.3i) *metric form into an appropriate Einstein equation;* the Birkhoff theorem allows the Schwarzschild metric to be used in the geodesic equation for purely radial motion. A *caveat* in this regard is that the so-called "Schwarzschild metric" universally found in textbooks, which grossly violates the Eq. (2.1b) restriction, *cannot be used*[16]*;* Karl Schwarzschild's *original* January 13, 1916 metric [17] *must be used.* Comparison of *the results*[18] of this simplest cosmological model in Einstein coordinates with those of the same simplest model in Friedmann coordinates *gives a good idea of what can be expected versus the* Eq. (2.3b) *FRW metric form when the* Eq. (2.3i) *Einstein metric form is put into an appropriate Einstein equation.* There was *no* Big Bang (a singular Big Bang is a purely Newtonian-gravity phenomenon, which is impossible when gravitational time dilation is taken into account). The universe has always existed, albeit gravitational time dilation all but eliminated its physical activity when its size was much smaller than its Schwarzschild radius. The immense surplus of particles relative to antiparticles wouldn't be an issue for such a universe. Gravitational time dilation greatly decreased when the size of the expanding universe exceeded its Schwarzschild radius, so significant physical activity became possible. The acceleration of the universe's expansion peaked sharply at that time, and then began to diminish, and has been diminishing ever since, but remains positive, which accords with what the DESI collaboration sees.[19] Being Lorentz-covariant, an Einstein-coordinate universe has an expansion speed *which is always less than* c, whereas the Big Bang gave the FRW universe *an unbounded expansion speed.* An Einstein-coordinate universe would probably, not long after its size exceeded its Schwarzschild radius, have disintegrated into a great many black holes and a lesser amount of ordinary matter, and thus to have been amenable to the rapid formation of the large number very compact and hot early galaxies seen by JWST. Conceivably, the black-body thermal cosmic microwave background arose from such a very hot, compact early universe.

3. Recapitulation

When Einstein presented his notion of the apparent bending of light for an observer in an accelerating elevator, he, astonishingly, was thinking in terms of Galilean physics, *and failed to recoil* at the implication of his pictured curving path of the light beam *that it is being accelerated to a speed greater than* c*!*

Einstein's principle of the equivalence of gravitation and inertia is obviously *less than an authoritative guide* to what gravity *really* does. But his Einstein equation *alone* is, if anything, *even less authoritative,* since, given a metric solution of any Einstein equation, every general coordinate transformation of that solution is *also* a solution of *that same* Einstein equation.

When this is pointed out, the all-too-facile answer of Einstein and his acolytes is *that these are mere coordinate changes.* But *that doesn't help* a scientific observer compare his measurements to gravity-theory results; *it is also necessary to know the specific transformation between the two sets of coordinates!*

More *sensibly, the cooperative gravity theorist should be aware of the scientific observer's coordinates, and should take care to present his results in those coordinates. That rationally resolves this issue.*

It has been confirmed to a minimum one part in 10^8 that the Lorentz transformation *is valid in the scientific observer's coordinates, so the cooperative gravity theorist must ensure that the metric tensor he deals with is compatible with Lorentz-covariance!*

In *addition,* the gravity theorist *needs to ensure that the Einstein equation is well-defined.* Since the Einstein equation *also uses the matrix inverse of the gravitational metric,* the theorest needs to ensure that *the determinant of the metric is nonzero at all values of the coordinates* x^μ*.* The theorist also needs to ensure that the coordinate condition he imposes *accommodates the Minkowski metric.* Finally, Occam's razor says that the theorist should choose *the simplest coordinate condition which meets these requirements.* The *consequence* for the theorist is Einstein's Eq. (2.1a) November 18, 1915 coordinate condition, $\det(g_{\mu\nu}(x)) = -1$ for all x, which is Lorentz-invariant, accommodates the Minkowski metric's -1 determinant, ensures the metric's determinant is nonzero everywhere, and *is as simple as possible.* Together with the Einstein equation, *it also yields the observed value of Mercury's perihelion precession and of the sun's deflection of starlight!*

However, still *another* issue is that *a physical metric* must be compatible with the Eq. (1.5) tick rate ratios of pairs of clocks at rest *which are located anywhere in space-time.* Since $\eta_{00} = 1 > 0$, this entails *the* Eq. (2.1b) *metric restriction that* $g_{00}(x) > 0$ *for all* x*.*

Einstein and his acolytes insist (1) that general covariance *supplants* Lorentz covariance, and (2) that the choice of coordinate condition in the Einstein equation *is irrelevant.* The first claim is belied by the twin facts that the validity of the Lorentz transformation is empirically confirmed to a minimum of one part in 10^8, and that, by their very nature, generally covariant equations such as the Einstein equation *have no unique solution,* so generally covariant equations *are obliged by their very nature to incorporate Lorentz covariance if they are to have any hope at all of producing physically correct results.* These observations belie the second claim above as well, but so do Einstein's *own experiences of* 1914-15, *wherein he only finally arrived at the observed result for Mercury's perihelion precession,* and also at the 1919 observed result for the sun's gravitational deflection of starlight, *when he adopted his* Eq. (2.1a) *Lorentz-invariant coordinate condition in his landmark* November 18, 1915 *paper.*[6, 7]

Friedmann was among the vast number of mathematically and scientifically educated people *who were inexplicably bamboozled by Einstein's transparently spurious claim that coordinate conditions are irrelevant,* so Friedmann looked for a coordinate condition which would make solving the Einstein equation *easier.* He found that his Eq. (2.2) coordinate condition $g_{00}(x) = 1$ for all x, which *ensures* that the metric *isn't* Lorentz-covariant and *eliminates* gravitational time dilation when inserted into Eq. (1.5), often does a good job in this regard, which allowed Friedmann to present a list of exact solutions of artfully-selected Einstein equations. *On these particular Einstein equations,* Friedmann's Eq. (2.2) coordinate condition $g_{00}(x) = 1$ for all x *had imposed Newtonian gravity,* which *it well might,* since it *eliminates* gravitational time dilation and *ensures* the metric *isn't* Lorentz-covariant.

When Robertson and Walker found that the Eq. (2.3a) *metric form* is *consistent* with its source being homogeneous and spherically-symmetric, i.e., the comological principle, *they considered it very convenient to as well impose on it Friedmann's* Eq. (2.2) *coordinate condition* $g_{00}(x) = 1$ *for all* x, thereby producing the FRW metric form of Eq. (2.3b), *which, in addition, unfortunately reflects Newtonian gravity.*[11, 12]

Thus cosmology *labored under the imposition of Newtonian gravity from early on,* after the effective collapse of Einstein's static universe model circa 1929. Cosmologists were *oblivious* to the fact that their Newtonian-gravity FRW metric form *is inconsistent with Mercury's perihelion precession and the sun's gravitational deflection of starlight!* More powerful telescopes such as Palomar enabled the detection of more distant and more rapidly receding galaxies, *but cosmologists failed to manifest appropriate discomfort with their Newtonian-gravity FRW metric form as galaxy recession speeds climbed to a significant fraction of c, and evidence of negative cosmological deceleration parameters began to appear.*[15]

The definite confirmation in 1998 that the expansion of the universe is accelerating occasioned a minor crisis, *but no suggestion was made that the Newtonian-gravity nature of the FRW metric form had become more and more inappropriate as telescope technology had improved, despite* the independent work of Hilbert and Dröste in December, 1916 which points out that asymptotically-weak Schwarzschild gravity *changes from attractive to repulsive when redshifts increase past unity.*[13, 14] Instead, *reinstatement of Einstein's 1917 cosmological constant with its sign reversed was immediately advocated,* despite *the manifest unsoundness of a cosmological constant,* which Einstein had characterized as his greatest mistake.

Now that the DESI collaboration data rather strongly suggests that the acceleration of the universe's expansion *slowly diminishes with that expansion*[19], exactly *as an orthodox gravity effect would be expected to, it is high time to update* the inherently inadequate Newtonian-gravity Eq. (2.3b) FRW metric form *to the Einstein coordinates which have been incorporated into the* Eq. (2.3i) *metric form, the coordinates that correctly yield Mercury's perihelion precession and sun's gravitational deflection of starlight!*

In his *Lectures on Gravitation*[3], Feynman *marvelously elucidates the principles* whereby the Eq. (1.3i) *Lorentzian linear* gravitational field equation *is nonlinearly upgraded* to accommodate the fact *that the metric is a part of its own four-momentum source.* Feynman lays down *the fundamental requirement* that the upgrade of Eq. (1.3i) *must be obtained from the variation of an action.* The left side of Eq. (1.3i) *must comprise the totality of terms linear in the metric which occur in the upgrade,* and the upgrade's part *which is nonlinear in the metric* must involve *two partial derivatives of the metric.* The upgrade must involve *all orders of the difference between the metric and the Minkowski metric,* which Feynman ensures *by making it involve the matrix inverse of the metric* (the contravariant metric). Of course, the upgrade's part which is *nonlinear* in the metric *must have vanishing four-divergence.*

Feynman goes on to show how the standard construction of the Riemann tensor from the gravitational field (the affine connection), which is quickly parlayed into the Eq. (1.4e) *Einstein-Hilbert gravity-field action, meets the requirements laid out above and upgrades* Eq. (1.3i) *to the* Eq. (1.4f) *Einstein equation.*

Feynman's *ultimate goal* was *quantum* gravity theory, which is obtained by putting the classical gravity action into his quantum superposition over all outcomes (his "path integral"). Feynman tried to carry out calculations in quantum gravity theory *using a quantum perturbation approach that ignores the very large factor* $\big(c^3/(16\pi G)\big)$ in the Eq. (1.4e) Einstein-Hilbert gravity-field action, *which drives the stationary-phase selection of the classical gravity-field outcome!* The interaction of the gravity field with nongravitational four-momenta in the Einstein equation involves $(1/(2c))$ *times the inverse of* $\big(c^3/(16\pi G)\big)$, which makes that interaction *very weak.* The gravity field *is therefore simultaneously very classical and very weakly interacting,* a mixture of physical behaviors *unfamiliar to Feynman from his experience with the electromagnetic field.* Still, it is *strange* that Feynman *missed* the classical gravity-field result of the very large factor $\big(c^3/(16\pi G)\big)$ in the gravity-field action. A *possible clue* is that in his *Lectures in Gravitation*, Feynman *abbreviated* the very large factor $\big(c^3/(16\pi G)\big)$ in the gravity-field action *as* $(1/(2\lambda^2))$[20], *which doesn't convey its very large value.* Even so, Feynman *ought to have found it physically inconsistent* that quantum-gravity perturbations *are replete with progagating gravitons, but no gravitons have been detected as yet.*

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[19] https://newscenter.lbl.gov/2025/03/19/new-desi-results-strengthen-hints-that-dark-energy-may-evolve/ DESI collaboration press release (2025).

[20] R. Feynman, *op cit*, Eq. (6.4.8) on page 85.