# The Missing Electric Aharonov-Bohm Test and the Search for the Electromagnetic Scalar Mode


Renato Vieira dos Santos[1, *]

[1] *Instituto de Ciência, Tecnologia e Inovação – ICTIN, Universidade Federal de Lavras – UFLA, Campus Paraíso, MG 37950-000, Brazil*



(Dated: July 9, 2026)



The electromagnetic four-potential $A^\mu = (\Phi/c, \mathbf{A})$ comprises a scalar potential $\Phi$ and a vector potential $\mathbf{A}$. In gauge-invariant electrodynamics, gauge symmetry together with the Lorenz condition $\partial_\mu A^\mu = 0$ ensure that only the two transverse photon polarizations are physical, while the scalar degree of freedom decouples. Whether this elimination holds in all physical regimes is an empirical question that has never been tested in configurations where the electric field vanishes but the potentials vary in time. We ask whether the scalar mode, described by the quantity $\mathcal{B} = \partial_\mu A^\mu$, can leave a physical imprint on matter that is detectable in the electric Aharonov-Bohm effect. The original configuration proposed in 1959—time-dependent potentials applied to shielded conducting cylinders—has never been experimentally realized. We show that a phenomenological coupling $\propto \mathcal{B}\,\bar{\psi}\psi$ yields a phase shift proportional to $1-\cos(\omega T)$, orthogonal to the standard $\sin(\omega T)$ prediction. A frequency sweep can cleanly separate the two contributions. We outline a realistic electron interferometry experiment, analyze the dominant systematic (fringe fields), and argue that such a test is within reach of existing technology. The question is testable: a null result would provide the first empirical bound on a physical scalar mode in a field-free regime; a positive signal would indicate that the gauge-fixing procedure discards a genuine degree of freedom. Either outcome illuminates the foundations of gauge theory.


## I. INTRODUCTION

In 1959, Aharonov and Bohm [1] predicted that electrons passing through regions free of electric and magnetic fields can accumulate measurable phase shifts due to electromagnetic potentials. The magnetic version has been confirmed with remarkable precision [3–5] and its topological protection [6] places it among the most secure predictions of quantum mechanics. The electric counterpart, in its original formulation with time-dependent shielded potentials, remains experimentally unexplored. Standard quantum mechanics predicts a phase $\Delta\phi_{\mathrm{QM}} = -(e/\hbar)\int \Delta\Phi(t)\,dt$, but the functional dependence on the temporal profile of the potential has never been systematically measured. The only related experiment, the steady-state Matteucci-Pozzi effect [7], was later shown to be force-based [8]. The genuine time-dependent electric Aharonov-Bohm (AB) configuration remains an open empirical question.

This gap is also an opportunity to interrogate a foundational tenet of electrodynamics: whether the scalar degree of freedom removed by the Lorenz gauge condition can leave a physical imprint. The four-potential $A^\mu = (\Phi/c, \mathbf{A})$ carries four components. In standard gauge-invariant electrodynamics, local $U(1)$ symmetry requires that only gauge-invariant quantities are physical; the Lorenz condition $\partial_\mu A^\mu = 0$ is the usual gauge choice that ensures the scalar mode decouples from observables [13, 21]. (We use "scalar mode" to refer to the divergence $\mathcal{B} = \partial_\mu A^\mu$, not merely the $A_0$ component.) Yet gauge symmetry itself has never been subjected to direct experimental verification in the regime where the electromagnetic fields vanish while the potentials vary in time. The present article argues that such a test is both possible and overdue.

The history of electrodynamics contains a persistent minority view that the scalar mode may be more than a gauge artifact. Already in the 1930s, Fock and Podolsky [10] and Dirac, Fock, and Podolsky [11] treated the scalar sector as an independent dynamical variable. Stueckelberg [9] showed that a compensating scalar field $\phi$ can restore gauge invariance while making the combination $\tilde{A}_\mu = A_\mu + \partial_\mu\phi$ gauge invariant and $\mathcal{B} = \partial_\mu \tilde{A}^\mu$ a physical field. Ohmura [12] proposed a scalar electrodynamics governed by $\Box\mathcal{B} = 0$. More recently, Modanese [14, 15] developed a generalized electrodynamics in which the Lorenz condition becomes a dynamical equation for gauge waves. Even within the AB literature, the possibility of additional scalar-field contributions was noted early: in their 1963 sequel, Aharonov and Bohm [2] discussed scenarios where the phase might depend on more than the line integral of $A_\mu$. These works share the conviction that the scalar sector of the four-potential may contain observable physics that the standard gauge-fixing procedure discards.

If the scalar mode $\mathcal{B}$ couples to matter, the shielded electric AB configuration is uniquely suited to detect it, because any phase beyond the standard $\sin(\omega T)$ must arise from the potentials themselves, not from forces. We consider a phenomenological coupling

$$\mathcal{L}_{\mathrm{int}} \supset -\frac{g}{\Lambda}\bar{\psi}\psi\,\mathcal{B}, \tag{1}$$

where $\psi$ is the electron Dirac field, $\Lambda$ a large mass scale, and $g$ a dimensionless coefficient. This term is not gauge


* renato.santos@ufla.br

invariant; whether such a term is forbidden or permitted is precisely the question we wish to test. As shown in Appendix B, it can be consistently formulated within an electrodynamics without gauge invariance, and it finds natural motivation in emergent gauge theories where gauge symmetry is only approximate [27, 28]. In the shielded AB configuration, $\mathcal{B} = \dot{\Phi}/c^2$, and the resulting phase shift is a boundary term: $\Delta\phi_{\mathcal{B}} \propto \Delta\Phi(T) - \Delta\Phi(0)$. For sinusoidal modulation, this yields a $1 - \cos(\omega T)$ signature, which is linearly independent of the standard $\sin(\omega T)$ and can be separated by a frequency sweep.

The article is organized as follows. Section II examines the Lorenz condition, its historical challengers, and the consistency of theories without gauge invariance. Section III reviews the shielded electric AB configuration. Section IV derives the scalar coupling phase signature. Section V outlines the experimental concept and addresses the key systematic of fringe fields. Section VI discusses implications of either outcome, compatibility with existing limits (including the Josephson effect), and the broader epistemological stakes. Appendices provide quantitative fringe-field estimates, a detailed analysis of electrodynamics without gauge invariance, and an explicit derivation of the scalar phase.

## II. THE GAUGE PARADIGM AND ITS ALTERNATIVES

### A. The Lorenz condition: convenience or necessity?

In classical electrodynamics, the field strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is invariant under local gauge transformations $A_\mu \to A_\mu + \partial_\mu \chi$. The four components of $A^\mu$ are therefore not independent. The Lorenz condition,[35]

$$\mathcal{B} \equiv \partial_\mu A^\mu = \nabla \cdot \mathbf{A} + \frac{1}{c^2}\frac{\partial \Phi}{\partial t} = 0, \tag{2}$$

removes the redundant degree of freedom in a Lorentz-covariant manner. Under a gauge transformation, $\mathcal{B} \to \mathcal{B} + \Box\chi$, so the condition restricts the gauge to residual transformations satisfying $\Box\chi = 0$. In the standard formulation of quantum electrodynamics, the Lorenz condition is enforced as an operator identity on the physical Hilbert space, ensuring that longitudinal and scalar modes decouple from all observables [13, 21].

This procedure, while mathematically elegant, implements a symmetry principle—local $U(1)$ gauge invariance—whose empirical validity in the field-free, time-dependent regime has never been verified. The homogeneous Maxwell equations are gauge invariant, but the inhomogeneous ones are typically expressed in terms of potentials and require a gauge choice to yield unique solutions. The Lorenz condition selects a convenient representative from each gauge orbit. The crucial question is not whether gauge invariance provides a consistent description of nature where it has been tested, but whether it remains exact in configurations where the fields vanish and only the potentials are operative.

The very arbitrariness of gauge fixing raises a natural question: would it be possible to construct an electrodynamics where such arbitrariness is absent from the outset, where the potentials are physical fields with no redundancy? If the answer is affirmative, the next question is whether nature makes use of that possibility. The experiment proposed here is designed to test precisely this.

### B. Historical challenges to the gauge paradigm

The idea that the scalar mode may be physical has a long and distinguished history. The earliest manifestation was Proca's massive photon theory [20], which explicitly breaks gauge invariance and introduces three physical polarization states; the condition $\partial_\mu A^\mu = 0$ emerges as a dynamical consequence. Stueckelberg [9] showed that gauge invariance can be restored by introducing a compensating scalar field $\phi$, with the combination $\tilde{A}_\mu = A_\mu + \partial_\mu \phi$ behaving as a gauge-invariant vector and $\partial_\mu A^\mu + \Box\phi$ becoming the physical scalar mode. This mechanism underpins the Higgs mechanism and demonstrates that the scalar sector can be given a gauge-invariant formulation while retaining physical content.

In the early days of quantum field theory, Fock and Podolsky [10] and Dirac, Fock, and Podolsky [11] treated the scalar sector as an independent dynamical variable before gauge fixing. Ohmura [12] explicitly proposed a scalar electrodynamics governed by $\Box\mathcal{B} = 0$. More recently, Modanese [14, 15] has developed a generalized electrodynamics in which the Lorenz condition becomes a wave equation for gauge waves that carry energy and momentum. Dirac's work on magnetic monopoles [16] provided a complementary impetus: the non-integrable phase factor $\exp(i \oint A_\mu dx^\mu)$ acquires physical reality, reinforcing the idea that the potentials, rather than the fields, are the fundamental objects.

Even within the AB literature, the possibility of additional scalar contributions was noted early. In their 1963 paper, Aharonov and Bohm [2] discussed scenarios in which the phase might depend on more than the line integral of $A_\mu$. These works, spanning nearly a century, demonstrate that the question of the scalar mode's physicality is not a modern contrivance but a persistent undercurrent in the foundations of electrodynamics.

The epistemological status of the Lorenz condition becomes clearer when compared with other gauge choices. The Coulomb gauge, $\nabla \cdot \mathbf{A} = 0$, breaks manifest Lorentz invariance but is useful in non-relativistic quantum mechanics. The temporal gauge, $A_0 = 0$, is convenient for Hamiltonian formulations. Each choice selects a different representative from the gauge orbit, and physical observables must be independent of the choice. What elevates the Lorenz condition above others is its Lorentz covariance, which has made it almost synonymous with the definition of electrodynamics itself. The elimination of the scalar mode by imposing a gauge condition—however convenient and Lorentz-covariant that choice may be—has never been subjected to empirical verification in the regime where fields vanish and only potentials are operative.

### C. Are theories without gauge invariance inconsistent?

A common objection is that abandoning gauge invariance produces inconsistencies: negative-norm states, acausal propagation, or violations of charge conservation. This objection conflates the specific quantization procedure of massless quantum electrodynamics with a general logical requirement. The Proca theory [20] of a massive vector field,

$$\mathcal{L}_{\text{Proca}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu - A_\mu J^\mu, \tag{3}$$

is a perfectly consistent, Lorentz-invariant, causal field theory without gauge invariance. Its spectrum contains three positive-norm spin-1 states and no ghosts. In this theory, the condition $\partial_\mu A^\mu = 0$ is enforced dynamically and the scalar mode is not an independent degree of freedom. The massless limit $m \to 0$ is smooth and recovers standard electrodynamics with the Lorenz condition as a dynamical constraint. Thus Proca theory demonstrates that gauge symmetry is not a logical necessity for a consistent electrodynamics, but it does not itself provide a dynamical scalar mode of the type we wish to probe.

Our specific phenomenological model, with the coupling $\delta\mathcal{L} = -(g/\Lambda)\bar{\psi}\psi\,\partial_\mu A^\mu$, requires a different mechanism to make $\mathcal{B}$ an observable degree of freedom. As detailed in Appendix B, such a mechanism is provided by emergent gauge theories [27, 28], where exact gauge invariance is only approximate and gauge-violating operators naturally appear suppressed by the microscopic scale. Alternatively, the theories of Ohmura [12] and Modanese [14, 15] explicitly endow $\mathcal{B}$ with dynamics. In either formulation, the internal consistency of the theory is guaranteed.

### D. Why test the apparently untestable?

Principles that are considered inviolable are rarely subjected to direct empirical scrutiny. The history of physics provides cautionary examples: parity conservation was considered fundamental until Lee, Yang, and Wu demonstrated its violation in weak interactions [25, 26]. The lesson is not that all symmetries must be broken, but that their validity should be checked in every accessible regime.

Gauge invariance has never been directly tested in a configuration where the scalar mode could manifest. The statement "only gauge-invariant quantities are physical" is a pragmatic principle, not an empirical result. The shielded electric AB effect provides the first feasible configuration to test it. If the scalar mode couples to matter, the experiment will detect it. If not, gauge invariance will have passed a test in a new regime. Neither outcome is trivial, and both advance our understanding of the foundations of electrodynamics.

## III. THE SHIELDED ELECTRIC AHARONOV-BOHM CONFIGURATION

Consider an electron beam split into two coherent paths. Along each path the electron passes through a cylindrical conducting tube of length $L$. Potentials $\Phi_1(t)$ and $\Phi_2(t)$ are applied to the respective tubes. The inner surfaces are ideal conductors, and the electron travels along the axis. Under these conditions the electric field inside each tube vanishes: the Laplace equation $\nabla^2\Phi = 0$ with Dirichlet boundary condition $\Phi|_{\text{surface}} = \Phi_j(t)$ has the unique solution $\Phi(\mathbf{r}, t) = \Phi_j(t)$ throughout the interior, giving $\nabla\Phi = 0$. Maxwell's equations in vacuum then imply $\Box\mathbf{A} = 0$. With the boundary condition $\mathbf{A} = 0$ on the inner surface (the tangential component vanishes for a perfect conductor, and the normal component can be gauged to zero), the unique solution is $\mathbf{A} = 0$ everywhere inside [22]. Hence $\mathbf{E} = \mathbf{B} = 0$ in the region traversed by the electron.

In standard quantum mechanics, the phase accumulated along each path is

$$\phi_j = -\frac{e}{\hbar}\int_0^T \Phi_j(t)\,dt, \tag{4}$$

where $T = L/v$ is the transit time and $-e$ the electron charge. The measurable phase difference between the two paths is

$$\Delta\phi_{\text{QM}} = -\frac{e}{\hbar}\int_0^T \big[\Phi_1(t) - \Phi_2(t)\big]\,dt. \tag{5}$$

For a sinusoidal modulation of the potential difference,

$$\Delta\Phi(t) = \Phi_1(t) - \Phi_2(t) = \Delta\Phi_0\cos(\omega t), \tag{6}$$

the phase becomes

$$\Delta\phi_{\text{QM}} = -\frac{e\Delta\Phi_0}{\hbar\omega}\sin(\omega T). \tag{7}$$

In the low-frequency limit $\omega T \ll 1$ this reduces to $\Delta\phi_{\text{QM}} \approx -(e\Delta\Phi_0/\hbar)T$, linear in the transit time. The standard prediction has zero-crossings at $\omega T = n\pi$ and maxima at $\omega T = (2n+1)\pi/2$.

The crucial element is the shielding. Without the conducting cylinders, a time-varying potential would generate an electric field via $\mathbf{E} = -\nabla\Phi - \partial_t\mathbf{A}$. The cylinders enforce $\nabla\Phi = 0$ and $\mathbf{A} = 0$ inside, so that $\mathbf{E} = 0$ and $\mathbf{B} = 0$ exactly, even though $\Phi(t)$ varies on the cylinder surfaces. This is fundamentally different from the Matteucci-Pozzi experiment [7], where unshielded biprism wires produced non-zero fields that dominated the observed phase shifts [8]. The field-free condition is what makes the electric AB effect a genuine quantum phenomenon and the ideal setting for testing contributions that would otherwise be masked by classical forces.

## IV. SCALAR MODE COUPLING AND ITS SIGNATURE

### A. Phenomenological coupling

If the elimination of the scalar mode by gauge fixing is not the last word, the scalar $\mathcal{B} = \partial_\mu A^\mu$ can appear in effective interactions. The most general Lorentz-invariant interaction Lagrangian between a Dirac field $\psi$ and the electromagnetic sector, to leading order in a derivative expansion, includes

$$\mathcal{L}_{\text{int}} = -e\bar{\psi}\gamma^\mu\psi A_\mu - \frac{g}{\Lambda}\bar{\psi}\psi\,\mathcal{B}, \tag{8}$$

where $\Lambda$ is a heavy mass scale and $g$ a dimensionless coefficient. The first term is the standard minimal coupling. The second is a phenomenological Pauli-like scalar term. It is not invariant under the full $U(1)$ gauge group; if gauge symmetry is a fundamental principle, such a term is forbidden. If, however, the redundancy of description is not the final word, then $\mathcal{B}$ is not compelled to decouple completely, and its coupling to matter is a legitimate phenomenological possibility.

The coupling is suppressed by $\Lambda$, ensuring that in the static limit ($\mathcal{B} = 0$ for DC potentials) the standard model is recovered. It does not violate Lorentz symmetry: $\mathcal{B}$ is a Lorentz scalar. As shown in Appendix B, the same effective coupling can be obtained from a fully consistent electrodynamics without gauge invariance, so the absence of gauge symmetry at the effective level does not signal a fundamental inconsistency.

The same effective coupling can also be obtained in a gauge-invariant framework by introducing a Stueckelberg scalar field $\phi$ transforming as $\phi \to \phi + \chi$ and writing $\mathcal{B} = \partial_\mu(A^\mu + \partial^\mu\phi)$. If $\phi$ carries no kinetic term, it is a purely auxiliary field that can be eliminated by a gauge transformation, reducing the theory to the present form; the resulting gauge invariance is a "paper symmetry" with no physical consequences. If $\phi$ is endowed with dynamics, it becomes a new particle, a physical scalar whose existence would be equally revolutionary. In either scenario, the detection of a scalar-mode coupling would signal that the Lorenz condition, and the standard elimination of the longitudinal mode, are a matter of convenience rather than a physical necessity.

### B. Phase shift in the shielded AB configuration and gauge choice

Because the scalar coupling explicitly breaks gauge invariance, the predicted phase shift depends on the choice of gauge. To obtain a definite experimental prediction, we must specify the gauge in which the calculation is performed. The natural choice is the gauge physically realized by the experimental apparatus: inside each ideal conducting cylinder, the vector potential vanishes ($\mathbf{A} = 0$) and the scalar potential equals the applied voltage on the inner surface. This is the unique solution satisfying the boundary conditions of the conductors and the requirement of zero fields inside. In this gauge, the potentials are operationally defined by the experimental voltages, and the predicted phase shift is tied directly to measurable quantities.

Working in this physical gauge and in natural units ($\hbar = c = 1$), the effective single-particle Hamiltonian from the scalar term is $V_{\mathcal{B}} = (g/\Lambda)\mathcal{B}$. In the shielded AB configuration, $\mathbf{A} = 0$ and $\mathbf{E} = 0$ inside the tubes, which gives $\mathcal{B} = \dot{\Phi}$. The accumulated scalar phase is

$$\Delta\phi_{\mathcal{B}} = -\frac{g}{\Lambda}\int_0^T \Delta\dot{\Phi}(t)\,dt = -\frac{g}{\Lambda}\big[\Delta\Phi(T) - \Delta\Phi(0)\big], \tag{9}$$

where $\Delta\Phi$ is in energy units. To express the result in SI, we convert the potential to Volts via $\Delta\Phi_{(\text{energy})} = e\Delta V$. The scalar phase becomes

$$\Delta\phi_{\mathcal{B}} = -\frac{ge}{\Lambda}\big[\Delta V(T) - \Delta V(0)\big]. \tag{10}$$

It is convenient to introduce the phenomenological coupling constant

$$\kappa \equiv \frac{ge}{\Lambda} \quad \text{(in natural units)}, \tag{11}$$

which has dimensions of $\mathrm{V}^{-1}$ when expressed in SI. We emphasize that $\kappa$ is defined directly as the coefficient in the phase shift formula

$$\Delta\phi_{\mathcal{B}} = -\kappa\left[\Delta\Phi(T) - \Delta\Phi(0)\right], \tag{12}$$

where $\Delta\Phi$ is now understood to be in Volts. The minus sign is a consequence of the convention in Eq. (8); the experimentally measurable coefficient $C$ in the fit is $C = -\kappa\Delta\Phi_0$.

The relationship between $\kappa$ and the microscopic parameters $g$ and $\Lambda$ depends on the ultraviolet completion and the unit system; in natural units $\kappa = ge/\Lambda$, while in SI additional factors of $\hbar$ and $c$ may appear (see Appendix C). For the purpose of the experimental analysis, we simply treat $\kappa$ as an unknown constant of dimension $\mathrm{V}^{-1}$ to be constrained by data. The important point is that the phase shift is operationally defined by the applied voltages and the coupling $\kappa$, independent of any theoretical interpretation.

### C. Sinusoidal modulation and functional orthogonality

For $\Delta\Phi(t) = \Delta\Phi_0 \cos(\omega t)$, the total phase (standard plus scalar) is

$$\Delta\phi(\omega T) = A \sin(\omega T) + C\big[1 - \cos(\omega T)\big], \tag{13}$$

with $A = -e\Delta\Phi_0/(\hbar\omega)$ and $C = \kappa\Delta\Phi_0$. The functions $\sin(\omega T)$ and $1 - \cos(\omega T)$ are linearly independent, with interleaved zero-crossings and distinct low-frequency scaling (linear vs. quadratic), as detailed in Table I.

TABLE I. Comparison of phase shift predictions for sinusoidal modulation $\Delta\Phi(t) = \Delta\Phi_0 \cos(\omega t)$.

| **Feature** | **Standard QM** | **Scalar coupling** |
|---|---|---|
| Static limit ($\omega = 0$) | $\propto T$ | 0 |
| Low-$\omega T$ scaling | Linear in $T$ | Quadratic in $\omega T$ |
| Functional form | $\sin(\omega T)$ | $1 - \cos(\omega T)$ |
| Zeroes | $\omega T = n\pi$ | $\omega T = 2n\pi$ |
| Maxima | $\omega T = (2n+1)\pi/2$ | $\omega T = (2n+1)\pi$ |

## V. OUTLINE OF THE PROPOSED EXPERIMENTAL TEST

The experiment measures the electron interference phase as a function of $\omega T$ and fits it to Eq. (13). The standard amplitude $A$ is fixed by independently known quantities; the scalar amplitude $C$ (equivalently $\kappa$) is the single free parameter. Sweeping $\omega T$ over $[0.2\pi, 10\pi]$ and varying the electron energy to change $T$ provides a two-dimensional dataset that breaks the correlation between $A$ and $C$.

For $L = 1$ cm, typical transit times (Table II) correspond to RF frequencies of 30 MHz to 1 GHz. With $\Delta\Phi_0 \sim 1$ mV, standard phases of $10^3$–$10^4$ rad are well within the dynamic range of phase detection.

TABLE II. Electron transit times for $L = 1$ cm.

| $E$ (eV) | $v$ (m/s) | $T$ (ns) |
|---|---|---|
| 10 | $1.88 \times 10^6$ | 5.33 |
| 100 | $5.93 \times 10^6$ | 1.69 |
| $10^3$ | $1.88 \times 10^7$ | 0.533 |
| $10^4$ | $5.93 \times 10^7$ | 0.169 |

The interferometer (Fig. 1) uses a field-emission source, a biprism beam splitter, two shielded cylindrical electrodes driven by phase-locked RF sources, and a 2D detector. To prevent the fringe fields of one cylinder from affecting the other, the two arms are separated by a distance much larger than the tube radius, and grounded guard electrodes are placed between them. The cylinders can be driven via coaxial cables and enclosed in a common Faraday cage to suppress stray electromagnetic pickup; similar shielding techniques are standard in electron interferometry [18].

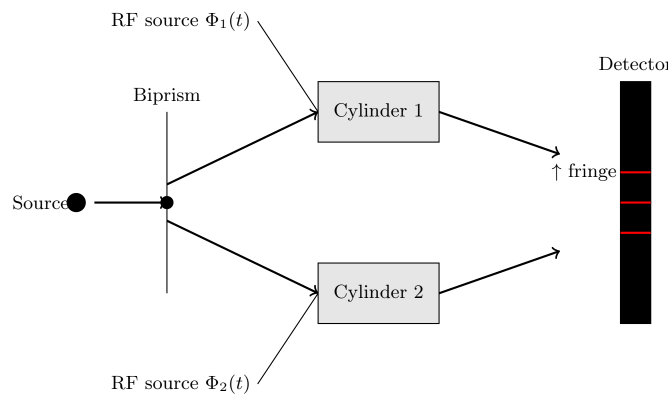


FIG. 1. Schematic of the proposed electron interferometer. The two arms are individually shielded. Guard electrodes (not shown) placed between the cylinders ensure that the fringe fields of one arm do not reach the other.

### A. Fringe fields and systematics

At the cylinder ends, fringe electric fields modify the relation $\mathcal{B} = \dot{\Phi}/c^2$. Appendix A treats this analytically. For cylinder radii $R \ll L$, the spurious phase is suppressed by $\sim (R/L)^2$, and its functional form $(1+\cos\omega T)$ is orthogonal to both the standard and scalar signals. The analysis assumes $\omega R/v \ll 1$; for the proposed parameters ($v \gtrsim 5.9 \times 10^6$ m/s, $R \lesssim 100$ $\mu$m, $\omega/2\pi \lesssim 1$ GHz), the worst-case value $\omega R/v$ at the highest $\omega T$ point of the sweep is below 0.1, safely within the adiabatic regime. Even for the extreme parameter combination ($E = 10$ keV, $\omega T = 10\pi$, giving $\omega R/v \approx 0.31$ for $R = 100$ $\mu$m), the adiabatic approximation is still reasonable, and the tube radius can be reduced to 10 $\mu$m if needed, which brings $\omega R/v$ down to 0.03. Additional systematics (finite conductivity, cavity resonances, beam monochromaticity) are at the $10^{-3}$ rad level.

### B. Feasibility

The required technology is available: single-electron interferometry with sub-nanosecond resolution [19], free-space biprism interferometers with energy spreads below 0.1 eV [18], and commercial GHz RF sources. A full sweep of $(\omega, T)$ points requires a few hours, limited by shot noise. The projected sensitivity to $C$ is $\sim 10^{-3}$ rad, corresponding to $\kappa \sim 10^{-3}$ V$^{-1}$ for $\Delta\Phi_0 \sim 1$ mV.

## VI. DISCUSSION

### A. What either outcome would mean

A null result would place the first direct bound on a scalar coupling in the field-free regime. A positive result would indicate that the standard gauge-fixing procedure discards a genuine physical degree of freedom. The $1-\cos(\omega T)$ signature is also predicted by massive Stueckelberg electrodynamics, so the experiment tests the scalar sector regardless of the underlying mechanism.

### B. Compatibility with existing precision tests

The scalar coupling vanishes for static potentials; hence DC precision measurements—electron $g-2$ [17], Lamb shift, DC voltage standards—are insensitive. The magnetic AB effect [6] has $\Phi = 0$ and $\nabla \cdot \mathbf{A} = 0$ outside the solenoid, so $\mathcal{B} = 0$.

The AC Josephson effect deserves special attention. In a Josephson junction biased by a time-dependent voltage $V(t)$, the phase evolves as $(2e/\hbar)\int V(t)\,dt$ plus a possible scalar contribution $\propto V(t_1) - V(t_0)$. For the Josephson voltage standard, the junction is irradiated with microwaves and the resulting constant-voltage steps are compared to a DC reference. The relevant time scale is the inverse of the microwave frequency (typically $\sim 10^{-10}$ s). Over one period of the microwave drive, the net change $\Delta V$ is zero, so the scalar term averages to zero. Rapid turn-on transients of the bias circuit could produce a net $\Delta V \sim 1$ V over millisecond times. The standard Josephson phase accumulated during this transient is $(2e/\hbar)\int V dt \sim 10^{15}$ rad, while the scalar phase is $\sim \kappa \cdot 1$ V. For the scalar term to be comparable, $\kappa$ would need to be $\sim 10^{15}$ V$^{-1}$, which is many orders of magnitude larger than the projected sensitivity of $\kappa \sim 10^{-3}$ V$^{-1}$. Hence the Josephson effect places no meaningful constraint on $\kappa$ at the level probed by the proposed experiment [34].

Torsion-balance tests [23, 24] probe static, gradient-driven forces and are insensitive to the four-divergence of $A^\mu$. The AB experiment proposed here explores a genuinely new regime.

### C. Epistemological stakes

The history of physics teaches that principles, however successful, should be tested when new regimes become accessible. Gauge invariance has never been verified in the regime of field-free, time-dependent potentials. The proposed experiment rectifies this.

## VII. CONCLUSION

We have presented the historical, theoretical, and experimental basis for a test of the electromagnetic scalar mode using the time-dependent electric Aharonov-Bohm effect. A phenomenological coupling of $\mathcal{B} = \partial_\mu A^\mu$ to electrons yields a $1 - \cos(\omega T)$ phase orthogonal to the standard $\sin(\omega T)$ and separable by a frequency sweep. The proposed electron interferometry experiment is within reach of existing technology. We emphasize that this is not a proposal of a specific alternative to gauge theory, but an agnostic experimental test of whether local gauge invariance—a foundational principle of electrodynamics—holds in a regime where it has never been directly verified. Such a test would subject gauge invariance to its first empirical scrutiny in the field-free, time-dependent regime.

## CONFLICT OF INTEREST

The author declares no conflicts of interest.

## DATA AVAILABILITY STATEMENT

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

### Appendix A: Analytical estimate of fringe-field phase shifts

Consider a cylindrical tube of inner radius $R$ and length $L$, with end caps at ground. Near the ends, the potential decays smoothly to zero over a distance $\sim R$. For $\omega R/v \ll 1$, the potential at entry ($t = 0$) and exit ($t = T$) is constant during the fringe traversal. Writing the fringe potential as $V(z,t) \approx \Phi(t) f(z/R)$ with $f(-\infty) = 1$, $f(\infty) = 0$ (and the reverse at the exit), the fringe phase for path $j$ is

$$\phi_j^{\text{fringe}} \approx -\frac{e}{\hbar}\Phi_j(0)\frac{R}{v}I_f \;-\; \frac{e}{\hbar}\Phi_j(T)\frac{R}{v}I_f, \tag{A1}$$

with $I_f = \int_{-\infty}^{\infty} f(u)\,du \sim 1$. The difference between arms is

$$\Delta\phi_{\text{fringe}} = -\alpha\frac{e}{\hbar}\frac{R}{v}\big[\Delta\Phi(0) + \Delta\Phi(T)\big], \tag{A2}$$

where $\alpha \equiv I_f \sim 1$.

For $\Delta\Phi(t) = \Delta\Phi_0 \cos\omega t$,

$$\Delta\phi_{\text{fringe}} = -\alpha\frac{e\Delta\Phi_0}{\hbar}\frac{R}{v}\big[1 + \cos(\omega T)\big]. \tag{A3}$$

The relative amplitude is $|\Delta\phi_{\text{fringe}}|/|\Delta\phi_{\text{QM}}| \sim \omega T\, R/L$. With $\omega T \lesssim 3$ and $R/L \lesssim 0.05$, this is below 0.15. The functional form $1 + \cos\omega T$ is independent of $\sin\omega T$ and $1 - \cos\omega T$, enabling a three-component fit.

The adiabatic condition $\omega R/v \ll 1$ holds for the proposed parameters. The worst-case combination within the proposed sweep ($E = 10^4$ eV, $\omega T = 10\pi$) gives $\omega R/v \approx 0.31$ for $R = 100$ $\mu$m. While still moderate, this value can be reduced to 0.03 by using a smaller tube radius of 10 $\mu$m, which is feasible with focused ion-beam machining. For the majority of the parameter range, $\omega R/v$ remains well below 0.1.

## Appendix B: Consistency of electrodynamics without gauge invariance

The scalar coupling $\delta\mathcal{L} = -(g/\Lambda)\bar{\psi}\psi\,\partial_\mu A^\mu$ is not gauge invariant, but it can be embedded in a consistent theory. The Proca massive vector field [20] demonstrates that causal, ghost-free electrodynamics exists without gauge symmetry, although in that theory $\partial_\mu A^\mu = 0$ holds on-shell and the scalar mode is not independently dynamical. Extensions that do provide a dynamical scalar mode include the theories of Ohmura [12] and Modanese [14, 15], where $\mathcal{B}$ carries its own wave equation and couples to matter, with no classical instabilities.

A particularly compelling ultraviolet motivation comes from emergent gauge theories [27, 28]. In strongly correlated systems, gauge bosons arise as collective excitations of an underlying lattice or spin model. The low-energy gauge symmetry is exact only in the scaling limit; away from it, corrections appear suppressed by the microscopic scale $E_g$. The effective Lagrangian generically contains gauge-non-invariant operators:

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \sum_n \frac{c_n}{E_g^{d_n-4}}\mathcal{O}_n(A_\mu, \partial_\nu A_\rho, \dots). \quad \text{(B1)}$$

Among these, the scalar operator $\mathcal{B} = \partial_\mu A^\mu$ is the lowest-dimension gauge-non-invariant term respecting Lorentz invariance. Its coupling to the electron density,

$$\mathcal{L}_{\text{eff}} \supset -\frac{\xi}{E_g}\,\bar{\psi}\psi\,\partial_\mu A^\mu, \quad \text{(B2)}$$

is precisely of the form (8), with $g/\Lambda \sim \xi/E_g$, where $\xi \sim \mathcal{O}(1)$.

The scale $\Lambda \sim E_g$ can range from meV (narrow-gap correlated insulators) to TeV (composite-Higgs or extradimensional models). In natural units, the experimentally accessible coupling is $\kappa = ge/\Lambda$. For $E_g \sim$ 1–10 eV, $\kappa \sim$ 1–0.1 V$^{-1}$; with $\Delta\Phi_0 \sim 1$ mV, the scalar phase amplitude $|C| = \kappa\Delta\Phi_0$ is in the range $10^{-3}$–$10^{-4}$ rad, within experimental reach. For $E_g \gtrsim 1$ TeV, $\kappa \lesssim 10^{-16}$ V$^{-1}$, and the signal is far below sensitivity; a null result would then place no constraint on such high-scale emergence, but would still provide the first direct laboratory bound on $\kappa$ in the field-free regime.

The coupling can also be embedded in a gauge-invariant Stueckelberg framework with an auxiliary or dynamical scalar $\phi$; the phenomenology in the shielded AB configuration is unchanged. The emergent gauge paradigm thus provides a natural, concrete motivation for the operator we seek to test.

Regarding unitarity, we note that the scalar coupling term $-\frac{g}{\Lambda}\,\bar{\psi}\psi\,\partial_\mu A^\mu$ appears in an effective low-energy description. Any apparent violation of unitarity is suppressed by the heavy scale $\Lambda$ and would manifest only at energies comparable to $\Lambda$, far above the regime probed by the proposed experiment. The underlying fundamental theory can be perfectly unitary, with the gauge-non-invariant term emerging as a residual effect of approximate gauge symmetry. Hence the phenomenology explored here is consistent with standard quantum mechanical principles.

## Appendix C: Notes on unit systems and dimensional analysis

The derivation of the scalar phase can be performed in any consistent unit system; the final result for the experimental observable is independent of the choice. Here we collect the relevant expressions in natural units ($\hbar = c = 1$) and in SI for clarity.

**Natural units.** In units where $\hbar = c = 1$, the Lagrangian has dimension [energy]$^4$. The coupling $g/\Lambda$ has dimension [energy]$^{-1}$, and the scalar mode $\mathcal{B} = \partial_\mu A^\mu$ has dimension [energy]. The electron field $\psi$ has dimension [energy]$^{3/2}$. The scalar phase accumulated by an electron is

$$\Delta\phi_{\mathcal{B}} = -\frac{g}{\Lambda}\big[\Delta\Phi(T) - \Delta\Phi(0)\big],$$

where $\Delta\Phi$ is an energy. To convert to SI, replace $\Delta\Phi_{\text{(energy)}} = e\Delta V$ with $\Delta V$ in Volts. The phase becomes

$$\Delta\phi_{\mathcal{B}} = -\frac{ge}{\Lambda}\big[\Delta V(T) - \Delta V(0)\big],$$

yielding the SI coupling $\kappa = ge/\Lambda$ with dimension V$^{-1}$.

**SI units.** Starting directly from the SI Lagrangian,

$$\mathcal{L} = \bar{\psi}(i\hbar\gamma^\mu\partial_\mu - mc)\psi - e\bar{\psi}\gamma^\mu\psi A_\mu - \frac{g}{\Lambda}\bar{\psi}\psi\,\partial_\mu A^\mu,$$

with $A^\mu = (\Phi/c, \mathbf{A})$. In the shielded configuration, $\partial_\mu A^\mu = \dot{\Phi}/c^2$. The interaction Hamiltonian density is $\mathcal{H}_{\text{int}} = (g/\Lambda c^2)\psi^\dagger\psi\,\dot{\Phi}$. The phase is

$$\Delta\phi_{\mathcal{B}} = -\frac{1}{\hbar}\int_0^T \frac{g}{\Lambda c^2}\,\dot{\Phi}(t)\,dt = -\frac{g}{\hbar\Lambda c^2}\big[\Phi(T) - \Phi(0)\big].$$

The combination $g/(\hbar\Lambda c^2)$ does not have the required dimension V$^{-1}$ because the SI Lagrangian as written is not dimensionally consistent without additional factors. This is a known subtlety when transcribing effective operators from natural units to SI [13]. The resolution is to recognize that the parameter $\Lambda$ appearing in the effective Lagrangian (1) is defined in natural units; when converted to SI, the dimensionally consistent coupling is $\kappa \sim ge/\Lambda$ (up to factors of $\hbar$ and $c$ that depend on the ultraviolet completion). For the purposes of the present experimental proposal, we simply treat $\kappa$ as a free parameter with dimensions $\mathrm{V}^{-1}$, as defined in Eq. (11) and Eq. (12).


[1] Y. Aharonov and D. Bohm, "Significance of electromagnetic potentials in the quantum theory," *Phys. Rev.* **115**, 485–491 (1959).

[2] Y. Aharonov and D. Bohm, "Further considerations on electromagnetic potentials in the quantum theory," *Phys. Rev.* **130**, 1625–1632 (1963).

[3] A. Tonomura, T. Matsuda, R. Suzuki, A. Fukuhara, N. Osakabe, H. Umezaki, J. Endo, K. Shinagawa, Y. Sugita, and H. Fujiwara, "Observation of Aharonov-Bohm effect by electron holography," *Phys. Rev. Lett.* **48**, 1443–1446 (1982).

[4] R. A. Webb, S. Washburn, C. P. Umbach, and R. B. Laibowitz, "Observation of $h/e$ Aharonov-Bohm oscillations in normal-metal rings," *Phys. Rev. Lett.* **54**, 2696–2699 (1985).

[5] H. Batelaan and A. Tonomura, "The Aharonov-Bohm effects: Variations on a subtle theme," *Phys. Today* **62**(9), 38–43 (2009).

[6] T. T. Wu and C. N. Yang, "Concept of nonintegrable phase factors and global formulation of gauge fields," *Phys. Rev. D* **12**, 3845–3857 (1975).

[7] G. Matteucci and G. Pozzi, "New diffraction experiment on the electrostatic Aharonov-Bohm effect," *Phys. Rev. Lett.* **54**, 2469–2472 (1985).

[8] S. A. Hilbert, B. Barwick, M. Fabrikant, C. J. G. J. Uiterwaal, and H. Batelaan, "Experimental observation of time-delays associated with electric Matteucci-Pozzi phase shifts," *New J. Phys.* **13**, 093025 (2011).

[9] E. C. G. Stueckelberg, "Die Wechselwirkungskräfte in der Elektrodynamik und in der Feldtheorie der Kräfte," *Helv. Phys. Acta* **11**, 225–244 (1938).

[10] V. Fock and E. Podolsky, "On the quantization of electromagnetic waves and the interaction of charges," *Phys. Z. Sowjetunion* **1**, 801–817 (1932), reprinted in V. A. Fock, *Selected Work: Quantum Mechanics and Quantum Field Theory* (Chapman & Hall/CRC, New York, 2004), pp. 225–241.

[11] P. A. M. Dirac, V. A. Fock, and B. Podolsky, "On quantum electrodynamics," *Phys. Z. Sowjetunion* **2**, 468–479 (1932), reprinted in V. A. Fock, *Selected Work: Quantum Mechanics and Quantum Field Theory* (Chapman & Hall/CRC, New York, 2004), pp. 243–255.

[12] T. Ohmura, "A new formulation on the electromagnetic field," *Prog. Theor. Phys.* **16**, 684–685 (1956).

[13] S. Weinberg, *The Quantum Theory of Fields, Vol. 1* (Cambridge University Press, Cambridge, 1995).

[14] G. Modanese, "Generalized Maxwell equations and charge conservation censorship," *Mod. Phys. Lett. B* **31**, 1750052 (2017).

[15] G. Modanese, "Gauge waves generation and detection in Aharonov-Bohm electrodynamics," *Eur. Phys. J. C* **83**, 1067 (2023).

[16] P. A. M. Dirac, "Quantised singularities in the electromagnetic field," *Proc. R. Soc. Lond. A* **133**, 60–72 (1931).

[17] D. Hanneke, S. Fogwell, and G. Gabrielse, "New measurement of the electron magnetic moment and the fine structure constant," *Phys. Rev. Lett.* **100**, 120801 (2008).

[18] F. Hasselbach, "Progress in electron- and ion-interferometry," *Rep. Prog. Phys.* **73**, 016101 (2010).

[19] H. Bartolomei, E. Frigerio, M. Ruelle, G. Rebora, Y. Jin, U. Gennser, A. Cavanna, E. Baudin, J.-M. Berroir, I. Safi, P. Degiovanni, G. C. Ménard, and G. Fève, "Time-resolved sensing of electromagnetic fields with single-electron interferometry," *Nat. Nanotechnol.* (2025), doi:10.1038/s41565-025-01899-3.

[20] A. Proca, "Sur la théorie du positron," *C. R. Acad. Sci. Paris* **202**, 1362–1364 (1936).

[21] N. Nakanishi, "Covariant quantization of the electromagnetic field in the Landau gauge," *Prog. Theor. Phys.* **35**, 1111–1116 (1966).

[22] M. Peshkin and A. Tonomura, *The Aharonov-Bohm Effect* (Lecture Notes in Physics, Vol. 340), Springer, Berlin (1989).

[23] E. G. Adelberger, B. R. Heckel, and A. E. Nelson, "Tests of the gravitational inverse-square law," *Annu. Rev. Nucl. Part. Sci.* **53**, 77–121 (2003).

[24] E. Fischbach and C. L. Talmadge, *The Search for Non-Newtonian Gravity* (Springer, New York, 1999).

[25] T. D. Lee and C. N. Yang, "Question of Parity Conservation in Weak Interactions," *Phys. Rev.* **104**, 254–258 (1956).

[26] C. S. Wu, E. Ambler, R. W. Hayward, D. D. Hoppes, and R. P. Hudson, "Experimental Test of Parity Conservation in Beta Decay," *Phys. Rev.* **105**, 1413–1415 (1957).

[27] X.-G. Wen, *Quantum Field Theory of Many-Body Systems* (Oxford University Press, Oxford, 2004).

[28] M. Levin and X.-G. Wen, "Colloquium: Photons and electrons as emergent phenomena," *Rev. Mod. Phys.* **77**, 871–879 (2005).

[29] R. Healey, *Gauging What's Real: The Conceptual Foundations of Contemporary Gauge Theories* (Oxford University Press, Oxford, 2007).

[30] C. Rovelli, "Why Gauge?," *Found. Phys.* **44**, 91–104 (2014).

[31] T. Maudlin, *Philosophy of Physics: Quantum Theory* (Princeton University Press, Princeton, 2019).

[32] C. Patrignani *et al.* (Particle Data Group), "Review of Particle Physics," *Chin. Phys. C* **40**, 100001 (2016).

[33] A. Strominger, "Lectures on the infrared structure of gravity and gauge theory," arXiv:1703.05448 (2017).

[34] M. Tinkham, *Introduction to Superconductivity*, 2nd ed. (Dover, New York, 2004).

[35] Following historical usage, the gauge condition $\partial_\mu A^\mu = 0$ is named after the Danish physicist Ludvig Lorenz (1829–1891), not H. A. Lorentz.