

# Charge-dependent equivalence principle violation from kinetic mixing of the photon with a new long-range vector field

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The weak equivalence principle (WEP) has been tested with extraordinary precision over the past decades, but invariably with electrically neutral bodies. This leaves a fundamental blind spot: does gravity feel electric charge? A recent phenomenological analysis introduced a parameter  $\kappa_{\text{fen}}$  to quantify this unexplored sector, but lacked a consistent field-theoretic origin. In this work we show that the simplest mechanism is kinetic mixing between the standard photon and a new massless vector field  $X_\mu$  that mediates a long-range fifth force coupled to the ordinary mass current. The single gauge-invariant dimension-four operator  $\frac{\xi}{2} F_{\mu\nu} X^{\mu\nu}$  preserves causality and unitarity. By adopting the action in which both fields couple to their respective currents with the same sign convention, the two-body potential is fully symmetric and exactly satisfies Newton’s third law, rendering the model free of any mathematical pathology. The fifth force between equal masses is repulsive, a known property of vector-mediated interactions, but its contribution to the neutral gravitational acceleration is absorbed into the measured Newtonian constant. For a static, spherically symmetric source, the mixing induces an electric field proportional to the Newtonian gravitational field, leading to a charge-dependent acceleration from which we identify  $\kappa_{\text{fen}} = -\xi\mu_0 G_X/G_N$ , where  $G_X$  is the fifth-force coupling strength. The symmetric cross term  $\propto q_1 m_2 + q_2 m_1$  in the two-body potential gives rise to dipole radiation from compact binaries, providing a direct link to pulsar-timing constraints. We derive explicit constraints from the Cassini bound on the PPN parameter  $\gamma$ , from lunar laser ranging on  $\alpha_1$ , and from the Hulse–Taylor pulsar, showing that the model is compatible with all existing data while providing a clear roadmap for future experimental tests.

**Keywords:** Equivalence principle; Kinetic mixing; Fifth force; Charge-dependent gravity; Effective field theory.

## I. INTRODUCTION

The universality of free fall—the weak equivalence principle (WEP)—is a cornerstone of general relativity (GR) and a sensitive probe for physics beyond the Standard Model [1–3]. Modern torsion-balance and space-based experiments achieve differential acceleration limits below  $10^{-15}$  by comparing test masses of different composition [4, 5].

In all such experiments the net electric charge of the test bodies is actively minimized to suppress electromagnetic backgrounds [6, 7]. Consequently, the question whether gravitational acceleration depends on a body’s electric charge remains comparatively untested [8–10]. This constitutes a fundamental blind spot: while we test gravity’s coupling to mass and composition with extraordinary rigor, we have never systematically tested its coupling to electric charge.

In a recent companion paper [11], a phenomenological parameter  $\kappa_{\text{fen}}$  was introduced to quantify this unexplored direction:

$$\frac{\Delta a}{g} = \kappa_{\text{fen}} \Delta\left(\frac{q}{m}\right), \quad (1)$$

where  $\Delta a$  is the differential acceleration between two test masses,  $g$  the local gravitational field, and  $\Delta(q/m)$

their charge-to-mass ratio difference. The parameter  $\kappa_{\text{fen}}$  has dimensions of mass per charge,  $[\kappa_{\text{fen}}] = \text{kg C}^{-1}$ . A synthesis of existing experimental data gave the first quantitative limit  $|\kappa_{\text{fen}}| < 2.1 \times 10^{-4} \text{ kg C}^{-1}$  at 95% confidence level [11], an uncertainty roughly eleven orders of magnitude less stringent than composition-dependent bounds.

In this work we construct the simplest consistent field-theoretic origin for  $\kappa_{\text{fen}}$ . We propose a new massless vector field  $X_\mu$  that mediates a long-range fifth force coupled to the ordinary mass current  $J_m^\mu$ . Its dynamics are governed by a Maxwell-like action with coupling strength  $G_X$ . The key ingredient is kinetic mixing with the photon:

$$\mathcal{L}_{\text{mix}} = \frac{\xi}{2} F_{\mu\nu} X^{\mu\nu}, \quad (2)$$

the unique gauge-invariant dimension-four operator connecting the two Abelian sectors. Together, these ingredients define a consistent effective field theory (EFT) for the charge-dependent sector.

Long-range vector fields coupled to the mass current have a long history in gravitational physics, from vector-tensor theories of gravity in the 1970s [12–14] to modern fifth-force models. Independently, kinetic mixing between two Abelian gauge fields was introduced by Holdom [15] and Foot and He [16] as the unique gauge-invariant dimension-four portal between the visible and hidden  $U(1)$  sectors, and has become a cornerstone of dark photon phenomenology [17, 18]. In this work we combine these two ideas: we propose a new massless vec-

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tor field  $X_\mu$  that mediates a long-range fifth force coupled to the ordinary mass current, and connect it to the photon through the kinetic mixing operator  $\frac{\xi}{2}F_{\mu\nu}X^{\mu\nu}$ .

This mixing preserves gauge invariance, Lorentz invariance, causality and unitarity. A consistent choice of signs in the action ensures that the two-body potential is symmetric and exactly satisfies Newton's third law.

In the static, spherically symmetric limit, the mixing causes a mass distribution to source an electric field proportional to the Newtonian acceleration  $\mathbf{g}$ :

$$\mathbf{E} = -\xi\mu_0 \frac{G_X}{G_N} \mathbf{g}. \quad (3)$$

A test particle of charge  $q$  and mass  $m$  then experiences the acceleration

$$\mathbf{a} = \mathbf{g}_{\text{obs}} - \xi\mu_0 \frac{G_X}{G_N} \frac{q}{m} \mathbf{g}_{\text{obs}}, \quad (4)$$

where  $\mathbf{g}_{\text{obs}}$  is the measured local gravitational acceleration (which already includes the fifth-force contribution). From this we identify

$$\kappa_{\text{fen}} = -\xi\mu_0 \frac{G_X}{G_N}. \quad (5)$$

The same parameter combination governs a network of interrelated predictions, including gravitomagnetic induction, tests of the inverse-square law, and dipole radiation from compact binaries. We derive explicit constraints from solar-system parameterized post-Newtonian (PPN) tests and pulsar timing, showing that the model is compatible with all existing data while providing a clear falsifiable roadmap for next-generation experiments.

### Relation to existing fifth-force searches

Long-range fifth forces have been extensively studied in contexts ranging from string-inspired dilaton models [41] to modern dark-sector portals [17, 18]. A particularly powerful probe comes from orbital dynamics: any additional  $1/r$  or Yukawa-like force between massive bodies modifies the Keplerian motion of planets, satellites, and binary systems, producing detectable anomalies in perihelion precession, mean anomaly drift, and radial velocity curves [42, 43, 45–48]. A comprehensive treatment of post-Newtonian orbital effects from modified gravities, including exact expressions for Yukawa-induced perihelion and mean anomaly precessions, can be found in the recent monograph by Iorio [44]. These analyses have placed stringent limits on the strength  $\alpha$  and range  $\lambda$  of a Yukawa-like fifth force using data from Earth's satellites (LAGEOS, LARES), Solar System planets, the S2 star at the Galactic Centre, and exoplanetary transit timing. All such bounds assume that the fifth force couples *universally* to mass (or baryon number) and that the test bodies are electrically neutral. The model presented here

introduces a qualitatively new feature: kinetic mixing with the photon generates a *charge-dependent* component that is invisible to purely mass-based orbital tests. As we shall demonstrate in Sec. VII B, the universal (charge-independent) part of our fifth force is tightly bounded by the same orbital data that constrain generic Yukawa models, forcing  $G_X/G_N \lesssim 10^{-5}$ . This saturation of the universal channel shifts the experimental frontier decisively toward the charge-dependent sector parametrized by  $\kappa_{\text{fen}}$ , which remains unconstrained by any existing orbital or torsion-balance experiment. With this motivation, we now construct the minimal Lagrangian that realizes the charge-dependent WEP violation encoded in  $\kappa_{\text{fen}}$ .

## II. FIFTH-FORCE FIELD WITH KINETIC MIXING

### A. Action and field equations

We consider the standard photon  $A_\mu$  and a new massless vector field  $X_\mu$ , with field strengths  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$  and  $X_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$ . The total action, including matter, is taken as

$$S = \int d^4x \left[ -\frac{1}{4\mu_0} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4\mu_X} X_{\mu\nu} X^{\mu\nu} + \frac{\xi}{2} F_{\mu\nu} X^{\mu\nu} \right] - \sum_i \left( q_i \int A_\mu dx_i^\mu + m_i \int X_\mu dx_i^\mu \right), \quad (6)$$

where  $\mu_0$  is the vacuum permeability,  $\mu_X$  the permeability of the  $X$  sector ( $[\mu_X] = \text{m kg}^{-1}$ ), and  $\xi$  the kinetic mixing parameter ( $[\xi] = \text{C m}^{-1}$ ). The currents are

$$J_e^\mu = \sum_i q_i v_i^\mu \delta^{(3)}(\mathbf{x} - \mathbf{x}_i), \quad J_m^\mu = \sum_i m_i v_i^\mu \delta^{(3)}(\mathbf{x} - \mathbf{x}_i), \quad (7)$$

with  $v_i^\mu = (c, \mathbf{v}_i)$  in the non-relativistic limit. The sign convention in the matter action is the standard one that yields the correct Maxwell equations and Lorentz force. Both fields couple to their respective currents with the same overall sign, which is essential for the consistency of the two-body potential and the reciprocity of forces.

Variation with respect to  $A_\nu$  and  $X_\nu$  yields the coupled field equations

$$\frac{1}{\mu_0} \partial_\mu F^{\mu\nu} - \xi \partial_\mu X^{\mu\nu} = J_e^\nu, \quad (8)$$

$$\frac{1}{\mu_X} \partial_\mu X^{\mu\nu} - \xi \partial_\mu F^{\mu\nu} = J_m^\nu. \quad (9)$$

The factor  $\xi$  (without an extra factor of  $1/2$ ) follows directly from the variation of the action, and the source terms appear with the same positive sign.

### B. Static limit and gravitoelectric field

In the static, non-relativistic limit we define

$$\mathbf{E}^i = cF^{i0}, \quad \mathbf{E}_X^i = cX^{i0}, \quad (10)$$

which are related to the scalar potentials  $\phi$  and  $\phi_X$  (with  $A^\mu = (\phi/c, \mathbf{A})$ ,  $X^\mu = (\phi_X/c, \mathbf{X})$ ) by

$$\mathbf{E} = -\nabla\phi, \quad \mathbf{E}_X = -\nabla\phi_X. \quad (11)$$

The time components of Eqs. (8)–(9) reduce to

$$\nabla \cdot \mathbf{E} - \xi\mu_0 \nabla \cdot \mathbf{E}_X = \frac{\rho_e}{\varepsilon_0}, \quad (12)$$

$$\nabla \cdot \mathbf{E}_X - \xi\mu_X \nabla \cdot \mathbf{E} = 4\pi G_X \rho_m, \quad (13)$$

where we introduced the fifth-force coupling constant

$$G_X \equiv \frac{\mu_X c^2}{4\pi}. \quad (14)$$

The positive sign of the source term in Eq. (13) implies  $\nabla \cdot \mathbf{E}_X = 4\pi G_X \rho_m$  at leading order in  $\xi$ , which has the opposite sign to the Newtonian gravitational field  $\nabla \cdot \mathbf{g} = -4\pi G_N \rho_{\text{ext}}$ .

### C. Diagonalisation and formal properties

In the Lorenz gauge ( $\partial_\mu A^\mu = \partial_\mu X^\mu = 0$ ), the field equations become

$$K \square \begin{pmatrix} A^\nu \\ X^\nu \end{pmatrix} = \begin{pmatrix} J_e^\nu \\ J_m^\nu \end{pmatrix}, \quad K = \begin{pmatrix} 1/\mu_0 & -\xi \\ -\xi & 1/\mu_X \end{pmatrix}. \quad (15)$$

The matrix  $K$  is symmetric and positive definite for  $|\xi| < 1/\sqrt{\mu_0\mu_X}$ , guaranteeing a ghost-free theory with bounded Hamiltonian. The diagonalization proceeds by a field redefinition to canonical fields  $\tilde{A}_\mu, \tilde{X}_\mu$ ; details are given in Appendix B. The theory is invariant under  $U(1)_{\text{EM}} \times U(1)_X$  gauge transformations and is causal.

### III. STATIC SOLUTION AND INDUCED ELECTRIC FIELD

Consider a static, spherically symmetric mass distribution  $\rho_{\text{ext}}(\mathbf{r})$  with no macroscopic electric charge ( $\rho_e = 0$ ). Solving Eqs. (12)–(13) to first order in  $\xi$  gives

$$\nabla \cdot \mathbf{E}_X \approx 4\pi G_X \rho_{\text{ext}}, \quad (16)$$

$$\nabla \cdot \mathbf{E} \approx 4\pi G_X \xi \mu_0 \rho_{\text{ext}}. \quad (17)$$

Comparing with the Newtonian field  $\nabla \cdot \mathbf{g} = -4\pi G_N \rho_{\text{ext}}$ , we obtain for a uniform sphere of mass  $M_\oplus$  the exterior solutions

$$\begin{aligned} \mathbf{g}(\mathbf{r}) &= -\frac{G_N M_\oplus}{r^2} \hat{\mathbf{r}}, \\ \mathbf{E}_X(\mathbf{r}) &= -\frac{G_X}{G_N} \mathbf{g}(\mathbf{r}), \\ \mathbf{E}(\mathbf{r}) &= -\xi \mu_0 \frac{G_X}{G_N} \mathbf{g}(\mathbf{r}). \end{aligned} \quad (18)$$

The electric field is sourced by the mass distribution through kinetic mixing; both  $\mathbf{E}_X$  and  $\mathbf{E}$  point radially outward (repulsive direction), while  $\mathbf{g}$  points inward.

*Electrical shielding by the conducting Earth.* Although Eq. (18) predicts an electric field outside a static mass distribution, the Earth is a conducting body. Free charges rearrange to cancel electrostatic fields in equilibrium. In an isolated neutral conductor the total charge is zero, and the external electric field far from the Earth will have a multipolar character. Consequently, the simple identification  $\mathbf{E} \propto \mathbf{g}$  is not directly applicable to experiments on the Earth's surface. This does not invalidate the theoretical parameter  $\kappa_{\text{fen}}$ ; it implies that terrestrial WEP tests with charged particles must operate in space, use artificially charged source masses, or account for the detailed electrostatic environment.

### IV. ACCELERATION OF A TEST PARTICLE

The force on a test particle is obtained from the matter action in (6). In the non-relativistic limit, the Lagrangian for a particle with charge  $q$  and mass  $m$  reads

$$L = \frac{1}{2}mv^2 + q\mathbf{v} \cdot \mathbf{A} - q\phi + m\mathbf{v} \cdot \mathbf{X} - m\phi_X, \quad (19)$$

so that the potential energy is  $V = q\phi + m\phi_X$  and the force is  $\mathbf{F} = -\nabla V = -q\nabla\phi - m\nabla\phi_X = q\mathbf{E} + m\mathbf{E}_X$ . Hence the total acceleration is

$$\mathbf{a} = \mathbf{g} + \mathbf{E}_X + \frac{q}{m}\mathbf{E}. \quad (20)$$

Substituting the fields (18) gives

$$\mathbf{a} = \mathbf{g} \left( 1 - \frac{G_X}{G_N} \right) - \frac{q}{m} \xi \mu_0 \frac{G_X}{G_N} \mathbf{g}. \quad (21)$$

The universal term  $\mathbf{g}(1 - G_X/G_N)$  is the total acceleration of a neutral test body and is absorbed into the measured local gravitational acceleration

$$\mathbf{g}_{\text{obs}} \equiv \mathbf{g} \left( 1 - \frac{G_X}{G_N} \right). \quad (22)$$

This absorption is standard in effective field theories; the measured Newtonian constant is already the renormalized coupling  $G_{\text{eff}} = G_N - G_X$ , analogous to how electromagnetic corrections renormalize the fine-structure constant. Expressing the acceleration in terms of  $\mathbf{g}_{\text{obs}}$  yields

$$\mathbf{a} = \mathbf{g}_{\text{obs}} - \xi \mu_0 \frac{G_X}{G_N} \frac{q}{m} \mathbf{g}_{\text{obs}} + \mathcal{O}\left(\frac{G_X^2}{G_N^2}\right), \quad (23)$$

and comparison with the phenomenological definition (1) gives

$$\boxed{\kappa_{\text{fen}} = -\xi \mu_0 \frac{G_X}{G_N}}. \quad (24)$$

The current limit  $|\kappa_{\text{fen}}| < 2.1 \times 10^{-4} \text{ kg/C}$  [11] bounds the combination  $|\xi|G_X/G_N$ .

## V. TWO-BODY POTENTIAL, RECIPROCITY, AND DIPOLE RADIATION

### A. Symmetric two-body potential

The static potential between two point particles is obtained from the inverse of the kinetic matrix in Sec. II C. Solving  $K \square (A^\nu, X^\nu)^T = (J_e^\nu, J_m^\nu)^T$  with point sources and expanding to leading order in  $\xi$  gives

$$\phi(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \frac{q}{r} + \xi\mu_0 G_X \frac{m}{r}, \quad \phi_X(\mathbf{r}) = \frac{\xi\mu_0 G_X q + G_X m}{r}, \quad (25)$$

where we used  $\mu_X c^2 = 4\pi G_X$  and  $c^2 = 1/(\mu_0\epsilon_0)$ . The interaction energy of a second particle is  $V = q_2\phi_1 + m_2\phi_{X1}$  (as required by the Lagrangian), yielding the exact two-body potential

$$V_{\text{int}}(d) = \frac{1}{1 - \xi^2\mu_0\mu_X} \left[ \frac{q_1 q_2}{4\pi\epsilon_0 d} + \xi\mu_0 G_X \frac{q_1 m_2 + q_2 m_1}{d} + G_X \frac{m_1 m_2}{d} \right]. \quad (26)$$

All terms are manifestly symmetric under  $1 \leftrightarrow 2$ . The force  $\mathbf{F}_{12} = -\nabla_1 V_{\text{int}}$  exactly obeys  $\mathbf{F}_{12} = -\mathbf{F}_{21}$ , guaranteeing momentum conservation. The positive sign of the mass-mass term produces a repulsive force between equal masses, as expected for a vector-mediated interaction.

### B. Dipole radiation and pulsar timing

The cross term in the potential reveals that a particle's charge contributes to its effective mass as seen by the  $X$  field. From the diagonal-basis field equations one finds the  $X$ -field sources the effective current

$$J_{\text{eff}}^\mu = J_m^\mu + \xi\mu_0 J_e^\mu, \quad (27)$$

so the mass dipole moment that couples to the  $X$ -radiation is

$$\mathbf{d}_{\text{eff}} = \sum_i (m_i + \xi\mu_0 q_i) \mathbf{r}_i. \quad (28)$$

The power radiated in the dipole channel follows from the vector-field Lagrangian and is analogous to electric dipole radiation with the replacement  $\mu_0 \rightarrow \mu_X$ . For a dipole moment  $\mathbf{d}_{\text{eff}}$  the orbit-averaged energy loss is

$$\dot{E}_X = \frac{\mu_X}{6\pi c} \langle \ddot{\mathbf{d}}_{\text{eff}}^2 \rangle = \frac{2G_X}{3c^3} \langle \ddot{\mathbf{d}}_{\text{eff}}^2 \rangle, \quad (29)$$

where we used  $\mu_X c^2 = 4\pi G_X$ .

For a binary with masses  $m_1, m_2$  and charges  $q_1, q_2$ ,  $\mathbf{d}_{\text{eff}} = \mu \Delta \left( \frac{m + \xi\mu_0 q}{m} \right) \mathbf{r}$ , where  $\mu = m_1 m_2 / (m_1 + m_2)$  is the reduced mass,  $\mathbf{r}$  the relative separation, and  $\Delta$  the difference between the two bodies. In the Newtonian limit

$\ddot{\mathbf{r}} = -G_N M \mathbf{r} / r^3$  ( $M = m_1 + m_2$ ). The ratio of  $\dot{E}_X$  to the general-relativistic quadrupole loss  $\dot{E}_{\text{GR}}$  then scales as

$$\frac{\dot{E}_X}{\dot{E}_{\text{GR}}} \sim \frac{G_X}{G_N} \left( \frac{\xi\mu_0 \Delta q}{\mu} \right)^2 f(e), \quad (30)$$

with  $f(e)$  an eccentricity factor of order unity.

For the Hulse–Taylor pulsar PSR B1913+16, the observed orbital decay matches general relativity to within  $\sim 0.2\%$  [19]. Attributing any excess to  $\dot{E}_X$  yields the constraint

$$\frac{G_X}{G_N} \left( \frac{\xi\mu_0 \Delta q}{\mu} \right)^2 \lesssim 2 \times 10^{-3}. \quad (31)$$

A pulsar in vacuum can develop a Goldreich–Julian charge  $q \sim 10^{12} \text{ C}$  [20]. Even for this extreme value,  $q \ll mc\sqrt{\epsilon_0/G_N}$ , hence  $\Delta q/\mu \ll 1 \text{ C/kg}$ . Using the identification  $\kappa_{\text{fen}} = -\xi\mu_0 G_X/G_N$ , the constraint (31) can be rewritten as

$$\kappa_{\text{fen}}^2 \frac{G_N}{G_X} \left( \frac{\Delta q}{\mu} \right)^2 \lesssim 2 \times 10^{-3}. \quad (32)$$

This intertwines the laboratory parameter  $\kappa_{\text{fen}}$  with the astrophysical coupling ratio  $G_X/G_N$  and does not, by itself, yield a direct bound on either. However, for any given  $G_X/G_N$  consistent with the PPN limits derived below ( $G_X/G_N \lesssim 10^{-5}$ ), Eq. (32) implies

$$|\kappa_{\text{fen}}| \lesssim 10^{-2} \left( \frac{G_X/G_N}{10^{-5}} \right)^{1/2} \left( \frac{1 \text{ C/kg}}{\Delta q/\mu} \right), \quad (33)$$

which is compatible with the laboratory limit  $|\kappa_{\text{fen}}| < 2.1 \times 10^{-4} \text{ kg/C}$  for  $G_X/G_N \lesssim 10^{-5}$  and typical pulsar charges.

## VI. FURTHER PHENOMENOLOGICAL CONSEQUENCES

### A. Effective fifth-force mass and gravitomagnetic induction

Equation (26) shows that a point charge  $q$  acquires an effective fifth-force mass

$$\delta m_X = \xi\mu_0 q, \quad (34)$$

which is a direct consequence of the  $q$ - $m$  cross term in the potential. Thus a neutral test particle of mass  $m_t$  would feel, in addition to Newtonian gravity, a short-range fifth-force acceleration toward a nearby charged body of order

$$a_X \sim \frac{G_X \delta m_X}{r^2}. \quad (35)$$

For a macroscopic charge of 1 C and  $G_X/G_N \sim 10^{-5}$ , the acceleration is  $\sim 10^{-12} \text{ m/s}^2$  at 1 m, far below current torsion-balance sensitivity [2, 7]. Nevertheless, the

effective-mass concept unifies the phenomenology: every electric charge carries a tiny “fifth-force weight”.

The stationary limit of Eq. (9) yields the gravitomagnetic induction law

$$\nabla \times \mathbf{B}_X = \xi \mu_X \nabla \times \mathbf{B}, \quad (36)$$

where  $\mathbf{B}_X$  is the gravitomagnetic field (frame-dragging analogue of the  $X$  sector) and  $\mathbf{B}$  the ordinary magnetic field. In integral form,

$$\oint \mathbf{B}_X \cdot d\boldsymbol{\ell} = \xi \mu_X \oint \mathbf{B} \cdot d\boldsymbol{\ell} = \xi \mu_X \mu_0 I_{\text{enc}}, \quad (37)$$

so that an electric current directly sources a gravitomagnetic circulation. This is the exact analogue of the linear gravitomagnetic effect discussed by Braginsky, Caves and Thorne [21] and later by Peng [22, 23]. For a typical laboratory solenoid with  $I = 10^4$  A and turn density  $n = 10^3 \text{ m}^{-1}$ , the induced  $B_X$  is of order  $10^{-19} \text{ s}^{-1}$ , corresponding to an acceleration gradient  $\nabla a \sim 10^{-11} \text{ s}^{-2}$ , far below current sensitivities [24, 25].

### B. Superconductors and anomalous claims

Kinetic mixing modifies the Maxwell–London equations inside a superconductor, leading to a renormalized London penetration depth and a small gravitomagnetic Meissner effect. Through the London moment  $\mathbf{B} = (2m_e/e)\boldsymbol{\omega}$ , a rotating superconductor generates

$$\mathbf{B}_X = \xi \mu_X \frac{2m_e}{e} \boldsymbol{\omega}. \quad (38)$$

For  $\omega \sim 10^3 \text{ rad/s}$ , the induced gravitomagnetic flux through a  $1 \text{ cm}^2$  pickup loop is  $\Phi_B \sim 10^{-23} \text{ Wb}$ , more than ten orders of magnitude below SQUID noise floors [26].

Likewise, high-voltage discharge experiments claiming anomalous weight reductions [e.g., [52, 53]] can be assessed within the EFT. The radiated  $X$ -wave power from a discharging capacitor is bounded by  $E_{\text{rad}} \lesssim 10^{-36} \text{ J}$  for typical parameters (using the maximal laboratory value  $|\kappa_{\text{fen}}| < 2.1 \times 10^{-4} \text{ kg/C}$ ), some forty orders of magnitude below the discharge energy. This mechanism therefore cannot explain such anomalies. Any reproducible signal of macroscopic magnitude would falsify this minimal model, pointing instead to more exotic physics.

### C. Implications for precision tests of the $1/r^2$ law and short-range gravity

Although the  $X$  field is massless in this minimal model, a tiny mass  $m_X$  could easily be introduced via a Higgs or Stueckelberg mechanism [18]. Even for an eV-scale mass, the fifth force acquires a Yukawa suppression  $\propto e^{-m_X r}$  on laboratory scales. Precision torsion-balance experiments testing the Newtonian inverse-square law [27, 28] then

bound the product of mixing strength and  $G_X$  at micron to millimeter distances. In terms of the parameterization commonly used for short-range deviations,

$$V(r) = -G_N \frac{m_1 m_2}{r} [1 + \alpha e^{-r/\lambda}], \quad (39)$$

our model yields  $\alpha = G_X/G_N$  (for equal masses) and  $\lambda = 1/m_X$ . Current constraints  $\alpha \lesssim 10^{-3}$  for  $\lambda \sim 10^{-3} \text{ m}$  [27] are automatically satisfied if  $G_X/G_N \lesssim 10^{-5}$ . Moreover, the charge-dependent component introduces a new signature: the effective coupling strength between two bodies depends on their electrical charging state, offering a unique tool to discriminate a genuine fifth force from systematic electromagnetic backgrounds.

## VII. GRAVITATIONAL TESTS AND THEORETICAL CONSISTENCY

### A. Post-Newtonian constraints

The long-range vector field modifies the parametrized post-Newtonian (PPN) parameters of the theory. To derive these constraints, we note that the total Newtonian potential sourced by a mass  $M$  is

$$U_{\text{tot}} = -\frac{G_{\text{eff}} M}{r}, \quad G_{\text{eff}} \equiv G_N - G_X. \quad (40)$$

The temporal metric component is sourced by all forms of energy and acquires the standard post-Newtonian correction  $g_{00} = -1 + 2U_{\text{tot}}$ . The spatial metric, however, receives contributions only from the tensor sector (general relativity), since the vector field  $X_\mu$  does not couple directly to the photon in the geometric optics limit (kinetic mixing is a derivative coupling and does not alter the null geodesics of light). Thus,

$$g_{ij} = \delta_{ij} \left( 1 + 2G_N \frac{M}{r} \right). \quad (41)$$

The PPN parameter  $\gamma$  is defined by  $g_{ij} = \delta_{ij}(1 + 2\gamma U_{\text{tot}})$ , hence

$$\gamma = \frac{G_N}{G_N - G_X} = 1 + \frac{G_X}{G_N} + \mathcal{O}\left(\frac{G_X^2}{G_N^2}\right). \quad (42)$$

This result is specific to models where the vector field does not directly modify the spatial metric; in alternative constructions where  $X_\mu$  couples directly to the matter stress-energy tensor, the expression for  $\gamma$  would differ. The Cassini measurement  $|\gamma - 1| < 2.3 \times 10^{-5}$  [29] therefore imposes

$$\frac{G_X}{G_N} \lesssim 2.3 \times 10^{-5}. \quad (43)$$

For vector-tensor theories, the preferred-frame parameter  $\alpha_1$  is of order  $G_X/G_N$  [12]. A detailed match to the standard PPN formalism yields

$$\alpha_1 \simeq 4 \frac{G_X}{G_N}. \quad (44)$$

Lunar laser ranging gives  $|\alpha_1| < 4 \times 10^{-5}$  [30], while binary pulsar timing provides a similar constraint [31]. These imply

$$\frac{G_X}{G_N} \lesssim 10^{-5}, \quad (45)$$

which is compatible with the Cassini bound. Importantly, none of these PPN constraints forbid a non-zero  $\kappa_{\text{fen}}$ ; they only restrict the ratio  $G_X/G_N$ , while  $\kappa_{\text{fen}} = -\xi\mu_0 G_X/G_N$  can be enhanced by a large  $\xi$  within the perturbative regime  $|\xi| < 1/\sqrt{\mu_0\mu_X}$ .

## B. Complementary orbital constraints on $G_X/G_N$

Before focusing on the charge-dependent observable  $\kappa_{\text{fen}}$ , it is necessary to verify that the universal (charge-independent) sector of the model is consistent with existing orbital bounds. As we show below, these bounds confine  $G_X/G_N$  to the  $10^{-5}$  level, making the universal force invisible in all present-day experiments. This conclusion holds regardless of the value of the kinetic mixing parameter  $\xi$ , which drops out of every observable involving only neutral bodies. The immediate consequence is that the *only* window left open by current data is the charge-dependent channel, to which the rest of this paper is devoted.

The universal (charge-independent) part of the fifth force corresponds to a repulsive  $1/r$  potential between two masses,  $\Delta V = +G_X m_1 m_2 / r$ . In the Newtonian limit, this is equivalent to replacing the Newtonian constant by  $G_{\text{eff}} = G_N - G_X$ . A test body orbiting a central mass  $M$  therefore follows a Keplerian orbit with effective gravitational parameter  $\mu_{\text{eff}} = (G_N - G_X)M$ . This produces a shift in the perihelion precession per orbit compared to the general-relativistic prediction. Using the formalism of [44], for a  $1/r$  correction the extra precession per orbit is

$$\Delta\varpi = -2\pi \frac{G_X}{G_N}, \quad (46)$$

where the negative sign indicates a reduction relative to the GR value.

Modern planetary ephemerides constrain anomalous perihelion precessions with exquisite precision. For Mercury, the residual after subtracting the known GR precession is  $|\Delta\dot{\varpi}| < 2.4 \times 10^{-4}$  arcsec/cy [49], which translates into

$$\frac{G_X}{G_N} \lesssim 5 \times 10^{-6}. \quad (47)$$

Similar bounds are obtained from the Earth and Mars ranging data. Satellite laser ranging to LAGEOS and LARES provides an independent constraint: the measured precession of the orbital plane (Lense–Thirring effect) and the pericenter advance are consistent with GR within  $10^{-3}$ , yielding  $G_X/G_N \lesssim 10^{-4}$  [50].

At the Galactic Centre, the GRAVITY collaboration has measured the Schwarzschild precession of the S2 star’s orbit around Sgr A\* [51]. The observed precession  $f_{\text{SP}} = 1.10 \pm 0.19$  times the GR value directly constrains an additional  $1/r$  potential. Adapting the Yukawa bounds of [46, 48] to the massless limit, one finds

$$\frac{G_X}{G_N} \lesssim 3 \times 10^{-5}, \quad (48)$$

fully consistent with the solar-system limits.

Taken together, these orbital bounds reinforce the PPN constraints derived in Sec. VII A and confirm that any long-range fifth force universally coupled to mass must have  $G_X/G_N \lesssim 10^{-5}$ . This conclusion is robust across scales spanning from Earth’s satellites to the Galactic Centre.

**Why  $\kappa_{\text{fen}}$  remains unconstrained.** All the orbital tests discussed above—and indeed all classical fifth-force searches with neutral bodies—constrain only the combination  $G_X/G_N$  that governs the universal force. The charge-dependent parameter  $\kappa_{\text{fen}} = -\xi\mu_0 G_X/G_N$  contains the additional factor  $\xi\mu_0$ , which drops out of every neutral-body observable. Even with  $G_X/G_N \sim 10^{-5}$ , a mixing parameter  $\xi\mu_0$  of order 10 kg/C would give  $|\kappa_{\text{fen}}| \sim 10^{-4}$  kg/C, right at the current laboratory bound. Larger values of  $\xi$  (still within the perturbative regime  $|\xi| < 1/\sqrt{\mu_0\mu_X}$ ) yield even larger  $\kappa_{\text{fen}}$  without conflicting with any orbital data. This underscores the necessity of dedicated experiments using *charged* test masses, as the companion paper [11] discusses in detail.

## C. Mass of the $X$ field and effective field theory validity

Throughout this work we have taken the  $X$  field to be exactly massless. However, a small mass  $m_X$  can be naturally introduced via a Stueckelberg mechanism or a Higgs portal without spoiling gauge invariance. In the presence of a mass term, the static potential acquires a Yukawa suppression:

$$V_X(r) = -G_X \frac{m_1 m_2}{r} e^{-m_X r}. \quad (49)$$

For distances  $r \gg m_X^{-1}$ , the fifth force is exponentially suppressed, relaxing the PPN bounds on  $G_X/G_N$ . If  $m_X \gtrsim 10^{-3}$  eV, corresponding to a Compton wavelength  $\lambda_X \lesssim 10^{-4}$  m, laboratory tests of the inverse-square law become the dominant constraints [27]. This provides a direct connection between our parameter  $\kappa_{\text{fen}}$  and tabletop experiments searching for Yukawa-like deviations.

The effective field theory remains valid as long as the dimensionless mixing parameter

$$\varepsilon \equiv \xi\sqrt{\mu_0\mu_X} \quad (50)$$

satisfies  $|\varepsilon| < 1$  (see Appendix B). This ensures the kinetic matrix is positive definite and the Hamiltonian is

bounded from below. For values of  $\xi$  required to produce a detectable  $\kappa_{\text{fen}}$  while respecting  $G_X/G_N \lesssim 10^{-5}$ , one has  $\varepsilon \sim \xi\mu_0\sqrt{G_X/(G_N)} \ll 1$  for  $\xi\mu_0 \sim \kappa_{\text{fen}}G_N/G_X \sim 10^{-4} \times 10^5 \sim 10$ , safely within the perturbative regime.

A natural question is whether the large hierarchy  $G_X \ll G_N$  is technically natural. In the context of effective field theory, a small dimensionless parameter is technically natural if the theory acquires an enhanced symmetry when that parameter is set to zero [32]. Here, in the limit  $\mu_X \rightarrow 0$  (equivalently  $G_X \rightarrow 0$ ), the  $X$  field decouples from the mass current and the free sector enjoys a shift symmetry  $X_\mu \rightarrow X_\mu + c_\mu$ , which leaves the field strength invariant. This symmetry protects  $G_X$  against large quantum corrections: any radiative contribution to  $G_X$  must be proportional to  $G_X$  itself, ensuring  $\delta G_X \sim G_X \times (\text{loop factor})$ . The hierarchy  $G_X/G_N \ll 1$  is therefore radiatively stable, much like the smallness of the electron Yukawa coupling in the Standard Model is protected by chiral symmetry. Additional protection can arise if the  $X$  field is embedded in a Stueckelberg mechanism, which preserves a softly broken  $U(1)_X$  gauge symmetry and guarantees that corrections to  $m_X$  and  $G_X$  remain under control even in the presence of ultraviolet physics [33].

#### D. Implications for dark matter

Although the present work focuses on the implications of the  $X$  field for equivalence principle tests, any massive particle—including dark matter candidates—would also experience the fifth force mediated by  $X_\mu$ . If dark matter is composed of massive particles, the  $X$  field provides an additional attractive or repulsive interaction in the dark sector, effectively acting as a self-interaction channel. Models of self-interacting dark matter (SIDM) mediated by light vector bosons have been extensively studied as a solution to the small-scale structure problems of collisionless cold dark matter, such as the core-cusp and missing-satellite tensions [34]. The present model shares key structural features with these SIDM frameworks, with the coupling strength  $G_X$  setting the self-interaction cross section. Furthermore, the kinetic mixing portal provides a natural bridge between the visible and dark sectors: if dark matter particles carry a charge under a new  $U(1)_D$  gauge symmetry, the  $X$  field (or the photon through mixing) can mediate interactions between ordinary and dark matter [15, 18]. This connection opens the possibility that the same parameter combination  $\xi G_X/G_N$  that governs charge-dependent WEP violation also controls the strength of visible-dark interactions, offering a unified framework for laboratory tests of the equivalence principle and dark matter phenomenology.

## VIII. DISCUSSION

### A. Effective field theory and the double-counting problem

The model presented in this work is a consistent effective field theory (EFT) of a long-range vector fifth force coupled to the ordinary mass current, with the unique gauge-invariant dimension-four kinetic mixing to the photon as the portal between the two Abelian sectors. As such, it respects all fundamental requirements: Lorentz invariance, unitarity, causality and a stable vacuum (the kinetic matrix is positive definite for  $|\xi| < 1/\sqrt{\mu_0\mu_X}$ ). In this respect it contrasts sharply with earlier attempts to identify a new massless vector field with the “graviphoton” that appears in  $N = 2$  supergravity [35, 36]. In those constructions the vector field is part of the metric multiplet and its effects are already included in the tensor gravitational interaction; coupling it directly to matter leads to the well-known “double-counting problem”—an overcounting of the Newtonian force [12]. Here,  $X_\mu$  is an independent degree of freedom, not a component of the spacetime metric, and its long-range force is simply an additional interaction superimposed on general relativity.

The absorption of the neutral-body fifth-force acceleration into the renormalized local gravitational field  $\mathbf{g}_{\text{obs}}$  formalizes the fact that laboratory measurements of  $G_N$  are, in reality, measurements of an effective coupling

$$G_{\text{eff}} = G_N - G_X, \quad (51)$$

where  $G_X$  is the strength of the  $X$  interaction. The Newtonian constant extracted from torsion-balance experiments [37] is therefore  $G_{\text{eff}}$ , not the bare  $G_N$  of the Einstein–Hilbert action. The charge-dependent term proportional to  $\kappa_{\text{fen}}$  survives this renormalization and constitutes a genuinely new observable, free from the double-counting ambiguity that plagued earlier “vector gravity” proposals [13, 14].

### B. Experimental prospects

The charge-dependent acceleration anomaly quantified by  $\kappa_{\text{fen}}$  defines a clear experimental target. A detailed discussion of experimental strategies, sensitivity projections, and the challenges posed by the Schiff–Barnhill effect and electrostatic shielding is presented in the companion paper [11]. In brief, the optimal approach is to maximize the charge-to-mass ratio difference  $\Delta(q/m)$  rather than minimize it, using platforms such as optically levitated nanoparticles [38, 39], repurposed drop-tower facilities [10], or atom interferometry with charged species [40]. The theoretical identification  $\kappa_{\text{fen}} = -\xi\mu_0 G_X/G_N$  derived here provides the quantitative link between such laboratory searches and the fundamental parameters  $\xi$  and  $G_X$ , and is complementary to the astrophysical bounds from pulsar timing discussed in Sec. VB. A dedicated

analysis of the double pulsar PSR J0737–3039 may already be sensitive to  $\kappa_{\text{fen}}$  values well below the current laboratory limit.

In summary, the model is fully predictive and falsifiable across an unusually wide range of scales: from laboratory Eötvös-type experiments, through gravitomagnetic induction and superconductor tests, to binary pulsar timing. The identification  $\kappa_{\text{fen}} = -\xi\mu_0 G_X/G_N$  transforms a phenomenological parameter into a measurable combination of fundamental couplings.

## IX. CONCLUSION

In this work we have constructed the minimal and mathematically consistent field-theoretic origin for the charge-dependent equivalence principle violation parametrized by  $\kappa_{\text{fen}}$ . The mechanism is kinetic mixing between the standard photon and a new massless vector field  $X_\mu$  that mediates a long-range fifth force coupled to the ordinary mass current  $J_m^\mu$ . By adopting the action in which both gauge fields couple to their respective currents with identical sign conventions, the two-body potential is fully symmetric and Newton’s third law is exactly satisfied, eliminating any concern of self-acceleration or momentum non-conservation. This consistent sign convention is what distinguishes our construction from earlier inconsistent vector-gravity proposals. The central identification is

$$\kappa_{\text{fen}} = -\xi\mu_0 \frac{G_X}{G_N}, \quad (52)$$

linking the phenomenological parameter directly to the kinetic mixing strength  $\xi$  and the fifth-force coupling  $G_X$ .

This single parameter combination governs a remarkably broad network of interrelated predictions. In the laboratory, it dictates the acceleration anomaly of charged test masses relative to neutral ones, providing a direct target for next-generation WEP experiments with levitated nanoparticles [38, 39], cold ions [40], and space-based platforms [10]. In superconductors, it governs the strength of the gravitomagnetic Meissner effect and the induced magnetic flux in pickup loops—effects that are unmeasurably small with current SQUID technology [26] but sharply defined for future magnetometers. In compact binaries, the same combination controls the dipole radiation rate of  $X$ -waves, enabling a complementary astrophysical bound from pulsar timing [19] that we derived as  $|\xi| G_X/G_N \lesssim 10^{-3}$ .

The model is fully compatible with general relativity once  $G_X/G_N$  is constrained by PPN and orbital tests. Cassini ranging [29], planetary ephemerides [49], and the S2 star at the Galactic Centre [51] all constrain  $G_X/G_N$  to the  $10^{-5}$  level, a bound that is further tightened by lunar laser ranging [30] and binary pulsar limits [31] on  $\alpha_1$ , effectively saturating the experimental sensitivity to the universal fifth force. These bounds leave the charge-dependent observable  $\kappa_{\text{fen}}$  as the sole unconstrained window into the  $X$ -sector at low energies. They do not forbid

a detectable  $\kappa_{\text{fen}}$ ; they merely require the mixing  $\xi$  to be sufficiently large, a condition easily satisfied within the perturbative regime  $|\xi| < 1/\sqrt{\mu_0\mu_X}$ .

The charge-dependent equivalence principle thus emerges not as a speculative addendum to gravitational physics, but as a well-posed EFT prediction that can be tested with existing and near-term technology. The parameter  $\kappa_{\text{fen}}$  unifies laboratory torsion-balance experiments, gravitomagnetic induction, superconductor magnetometry, and binary pulsar timing into a single quantitative, falsifiable framework. We conclude that the question “does gravity feel electric charge?” is both physically meaningful and experimentally decidable. The dedicated experimental program outlined in the companion paper [11] provides a roadmap to explore this largely uncharted sector of fundamental physics.

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### Appendix A: Dimensions and unit conversions

The model employs SI units throughout, which is the natural choice when connecting to laboratory experiments that measure forces in newtons and charges in coulombs. For reference we collect here the dimensions of all fundamental quantities.

#### 1. Dimensions of fields and couplings

In SI, the electric field has dimensions  $[E] = \text{N C}^{-1} = \text{kg m C}^{-1} \text{s}^{-2}$ . The electromagnetic field strength  $F_{\mu\nu}$  has components  $F_{0i} = E_i/c$  and  $F_{ij} = \epsilon_{ijk} B_k$ . Since  $[E/c] = \text{kg C}^{-1} \text{s}^{-1}$ , the dimension of  $F_{\mu\nu}$  is

$$[F_{\mu\nu}] = \text{kg C}^{-1} \text{s}^{-1} \equiv \text{T}. \quad (\text{A1})$$

The vector potential  $A_\mu$ , defined through  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ , therefore has  $[A_\mu] = \text{kg m C}^{-1} \text{s}^{-1}$ .

The new field  $X_\mu$  couples to the mass current via the matter action term  $\sum_i m_i \int X_\mu dx_i^\mu$ . In SI the action  $S$  has dimensions of  $\text{J s} = \text{kg m}^2 \text{s}^{-1}$ . For a point particle the interaction term is  $-m \int X_\mu dx^\mu = -m \int X_\mu v^\mu dt$ , where  $v^\mu = (c, \mathbf{v})$  has dimensions of velocity ( $\text{m s}^{-1}$ ). Dimensional consistency requires  $[mX_\mu v^\mu] = \text{J} (\text{kg m}^2 \text{s}^{-2})$ , hence

$$[X_\mu] = \frac{\text{kg m}^2 \text{s}^{-2}}{\text{kg} \times \text{m s}^{-1}} = \text{m s}^{-1}. \quad (\text{A2})$$

Thus  $X_\mu$  has dimensions of velocity. Its field strength  $X_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$  therefore has dimensions of fre-

quency:

$$[X_{\mu\nu}] = \frac{[X_\mu]}{\text{m}} = \text{s}^{-1}. \quad (\text{A3})$$

The gravitoelectric field  $\mathbf{E}_X = cX^{i0}$  inherits dimensions of acceleration,  $[\mathbf{E}_X] = \text{m s}^{-2}$ , consistent with its interpretation as a fifth-force acceleration.

The kinetic mixing parameter  $\xi$  appears in the Lagrangian density through the term  $\frac{\xi}{2}F_{\mu\nu}X^{\mu\nu}$ . The action  $S = \int d^4x \mathcal{L}$  has  $[S] = \text{kg m}^2 \text{s}^{-1}$  and the four-volume element  $[d^4x] = \text{m}^3 \text{s}$  (since  $dx^0 = c dt$ ). Therefore the Lagrangian density has  $[\mathcal{L}] = \text{kg m}^{-1} \text{s}^{-2}$ . The kinetic mixing term must have the same dimension, yielding

$$\begin{aligned} [\xi] [F_{\mu\nu}] [X_{\mu\nu}] &= \text{kg m}^{-1} \text{s}^{-2} \\ \Rightarrow [\xi] &= \frac{\text{kg m}^{-1} \text{s}^{-2}}{\text{kg C}^{-1} \text{s}^{-1} \times \text{s}^{-1}} = \text{C m}^{-1}. \end{aligned} \quad (\text{A4})$$

Equivalently,  $\xi\mu_0$  has dimensions

$$[\xi\mu_0] = \text{C m}^{-1} \times \text{H m}^{-1} = \text{kg C}^{-1}, \quad (\text{A5})$$

which matches the dimensions of the phenomenological parameter  $\kappa_{\text{fen}}$ .

## 2. Fifth-force coupling

The permeability  $\mu_X$  of the  $X$  sector is defined through the kinetic term  $-\frac{1}{4\mu_X}X_{\mu\nu}X^{\mu\nu}$ . Dimensional analysis of the Lagrangian density gives

$$\frac{1}{[\mu_X]} [X_{\mu\nu}]^2 = [\mathcal{L}] \quad \Rightarrow \quad \frac{1}{[\mu_X]} \text{s}^{-2} = \text{kg m}^{-1} \text{s}^{-2}, \quad (\text{A6})$$

hence

$$[\mu_X] = \text{m kg}^{-1}. \quad (\text{A7})$$

This agrees with the dimension quoted in the main text ( $[\mu_X] = \text{m kg}^{-1}$ ) and differs from the dimensions of  $\mu_0$  ( $[\mu_0] = \text{H m}^{-1} = \text{kg m C}^{-2}$ ).

The coupling strength  $G_X$  is defined by the Poisson-like equation  $\nabla \cdot \mathbf{E}_X = 4\pi G_X \rho_m$ , which mirrors  $\nabla \cdot \mathbf{g} = -4\pi G_N \rho_m$  for Newtonian gravity. Using  $\mathbf{E}_X = -\nabla\phi_X$  and the static limit of the  $X$ -field equation  $\nabla^2\phi_X = -\mu_X c^2 \rho_m$  (in Lorenz gauge), we obtain

$$G_X = \frac{\mu_X c^2}{4\pi}. \quad (\text{A8})$$

Dimensional check:  $[\mu_X c^2] = \text{m kg}^{-1} \times \text{m}^2 \text{s}^{-2} = \text{m}^3 \text{kg}^{-1} \text{s}^{-2}$ , the same as the Newtonian constant  $[G_N] = \text{M}^{-1} \text{L}^3 \text{T}^{-2}$ .

## 3. Dimensionless mixing parameter

In the dark-photon literature the mixing is often parametrized by the dimensionless quantity

$$\varepsilon \equiv \xi \sqrt{\mu_0 \mu_X}. \quad (\text{A9})$$

Using  $[\mu_0] = \text{H m}^{-1} = \text{kg m C}^{-2}$  and  $[\mu_X] = \text{m kg}^{-1}$ , one finds  $[\xi^2 \mu_0 \mu_X] = \text{C}^2 \text{m}^{-2} \times \text{kg m C}^{-2} \times \text{m kg}^{-1} = 1$ , confirming that  $\varepsilon$  is dimensionless. The perturbative condition  $|\varepsilon| < 1$  (ghost-free theory) translates to  $|\xi| < 1/\sqrt{\mu_0 \mu_X}$ .

## 4. Summary table

TABLE I. Dimensions of main quantities in SI units. Mass (M), length (L), time (T), electric current (I).

| Quantity              | Symbol       | Dimensions                             |
|-----------------------|--------------|----------------------------------------|
| Photon field strength | $F_{\mu\nu}$ | $\text{MT}^{-1}\text{I}^{-1}$          |
| $X$ field strength    | $X_{\mu\nu}$ | $\text{T}^{-1}$                        |
| $X$ potential         | $X_\mu$      | $\text{LT}^{-1}$                       |
| Kinetic mixing        | $\xi$        | $\text{TIL}^{-1}$                      |
| Vacuum permeability   | $\mu_0$      | $\text{MLT}^{-2}\text{I}^{-2}$         |
| $X$ permeability      | $\mu_X$      | $\text{M}^{-1}\text{L}$                |
| Fifth-force coupling  | $G_X$        | $\text{M}^{-1}\text{L}^3\text{T}^{-2}$ |
| $\xi\mu_0$            |              | $\text{MI}^{-1}$                       |
| $\kappa_{\text{fen}}$ |              | $\text{MI}^{-1}$                       |
| $\varepsilon$         |              | dimensionless                          |

## 5. Connection to natural units

In natural units ( $\hbar = c = 1$ ) one sets  $\mu_0 = 1/(\varepsilon_0 c^2) \rightarrow 1$ , so that the photon kinetic term becomes  $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ . The  $X$  kinetic term is then  $-\frac{1}{4g_X^2}X_{\mu\nu}X^{\mu\nu}$ , with the dimensionless gauge coupling  $g_X = \sqrt{4\pi G_X}$ . The kinetic mixing is written as  $-\frac{\varepsilon}{2}F_{\mu\nu}X^{\mu\nu}$ . In this convention our results read

$$\kappa_{\text{fen}} = -\frac{\varepsilon g_X}{\sqrt{4\pi G_N}}, \quad (\text{A10})$$

which is equivalent to Eq. (52) upon restoring  $\mu_0$  and  $G_X$ . This translation facilitates comparison with the high-energy literature on dark photons and extra  $U(1)$  gauge bosons [15, 18].

## Appendix B: Diagonalisation of the kinetic mixing

The kinetic Lagrangian

$$\mathcal{L}_{\text{kin}} = -\frac{1}{4\mu_0}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4\mu_X}X_{\mu\nu}X^{\mu\nu} + \frac{\xi}{2}F_{\mu\nu}X^{\mu\nu} \quad (\text{B1})$$

is first rescaled by defining

$$A'_\mu = \frac{A_\mu}{\sqrt{\mu_0}}, \quad X'_\mu = \frac{X_\mu}{\sqrt{\mu_X}}, \quad (\text{B2})$$

with field strengths  $F'_{\mu\nu} = \partial_\mu A'_\nu - \partial_\nu A'_\mu$  and  $X'_{\mu\nu} = \partial_\mu X'_\nu - \partial_\nu X'_\mu$ . The kinetic Lagrangian becomes

$$\mathcal{L}_{\text{kin}} = -\frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} - \frac{1}{4}X'_{\mu\nu}X'^{\mu\nu} + \frac{\varepsilon}{2}F'_{\mu\nu}X'^{\mu\nu}, \quad (\text{B3})$$

with the dimensionless mixing parameter

$$\varepsilon \equiv \xi \sqrt{\mu_0 \mu_X}. \quad (\text{B4})$$

In the  $(A', X')$  basis the kinetic matrix is

$$K' = \begin{pmatrix} 1 & -\varepsilon \\ -\varepsilon & 1 \end{pmatrix}. \quad (\text{B5})$$

Because the diagonal entries are equal, the orthogonal matrix that diagonalizes  $K'$  is a rotation by exactly  $\theta = \pi/4$ :

$$\begin{pmatrix} \tilde{A}_\mu \\ \tilde{X}_\mu \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} A'_\mu \\ X'_\mu \end{pmatrix}. \quad (\text{B6})$$

In the  $(\tilde{A}, \tilde{X})$  basis the kinetic Lagrangian reads

$$\mathcal{L}_{\text{kin}} = -\frac{1+\varepsilon}{4} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} - \frac{1-\varepsilon}{4} \tilde{X}_{\mu\nu} \tilde{X}^{\mu\nu}. \quad (\text{B7})$$

A final rescaling

$$\tilde{A}_\mu \rightarrow \frac{\tilde{A}_\mu}{\sqrt{1+\varepsilon}}, \quad \tilde{X}_\mu \rightarrow \frac{\tilde{X}_\mu}{\sqrt{1-\varepsilon}} \quad (\text{B8})$$

brings the kinetic terms to the canonical form  $-\frac{1}{4}\tilde{F}^2 - \frac{1}{4}\tilde{X}^2$ . The theory is free of ghosts and tachyons as long as  $|\varepsilon| < 1$ , i.e.  $|\xi| < 1/\sqrt{\mu_0 \mu_X}$ .

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