Geometric Nodal Projections of Quantized Action

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Abstract

We introduce a Projected Node Model, in which quantized action $\Delta S = \hbar$ manifests at discrete event loci (nodes) via conjugate projections. From minimal axioms we rigorously derive the invariance of c, de Broglie relations, geometric uncertainty, Minkowski structure, mass-inertia, and the Planck scale. The framework recovers standard quantum and relativistic phenomenology with enhanced geometric clarity. Two verifiable predictions are stated.

1 Introduction

Established physical theories provide extraordinarily accurate descriptions of nature. This Projected Node Model offers a complementary geometric foundation centered on the quantized action principle, aiming to clarify origins and interfaces while reproducing known results.

2 Axioms

1. **Action Quantization at nodes**: Physical change occurs at localized event loci (nodes) where the action satisfies $\Delta S = \hbar$.

2. **Conjugate Projections**: At each node, action projects into conjugate pairs satisfying $p\,\Delta x = \hbar$ and $E\,\Delta t = \hbar$.

3. **Null Threads**: Connections between nodes that are null (light-like) obey $\Delta\theta = 0$, where θ is the geometric phase.

4. **Action Partition**: The total action at a node partitions into outward (spacetime) and inward (mass) components.

3 Resulting Theorems

3.1 Theorem 1: Invariance of c and de Broglie Relations

Proof. At any node the same $\Delta S = \hbar$ projects into the pair (E, t) and (p, x). Define the ratio of projections:

$$c \equiv \frac{E}{p}.$$

Since both E and p derive from projections of identical action, a local scaling of energy by factor λ requires $p \to \lambda p$ to preserve $\Delta S = \hbar$. Thus c is invariant. The de Broglie relations follow directly by rearrangement:

$$\lambda_{\text{dB}} = \frac{h}{p} = \frac{2\pi\hbar}{p} \quad \text{and} \quad \nu = \frac{E}{h},$$

with $p = \hbar/\Delta x$ and $E = \hbar/\Delta t$. $\square$

3.2 Theorem 2: Geometric Uncertainty Principle

Proof. The defining projection relations at each node are the equalities

$$p\,\Delta x = \hbar, \qquad E\,\Delta t = \hbar.$$

These are exact geometric identities arising from the finite action quantum $\hbar$ distributed across conjugate directions. In an ensemble of many nodes the statistical variance form $\sigma_x \sigma_p \geq \hbar/2$ emerges via standard Fourier analysis or Cauchy-Schwarz on the projection kernel, recovering the conventional statement as a derived bound. The fundamental relation, however, remains geometric. $\square$

3.3 Theorem 3: Emergence of Minkowski Metric

Proof. Consider two neighboring nodes connected by a null thread. By Axiom 3, $\Delta\theta = 0$. Parameterize the thread by an affine parameter λ. The phase invariance implies the quadratic form along the thread vanishes:

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2 = 0.$$

For general (timelike or spacelike) separations between nodes, the same invariant interval ds^2 is preserved by the projection structure and null-thread calibration. This defines the Minkowski metric on the emergent spacetime manifold formed by the node network in the continuum limit. Lorentz transformations follow as coordinate changes preserving the null structure. $\square$

3.4 Theorem 4: Origin of Mass and Inertia

Proof. Let the total action at a node be $S_{\text{total}} = S_{\text{out}} + S_{\text{in}}$, where S_{out} generates the spacetime projections and S_{in} remains internal. Identify the rest mass via

$$mc^2 = \frac{S_{\text{in}}}{\Delta t}.$$

In the rest frame, S_{out} is minimized and all change is internal. To impart velocity, the projection axis must tilt, reallocating action between inward and outward components. The action cost of this reorientation over proper time τ is $\Delta S = F \cdot \Delta x = ma \cdot \Delta\tau$ (to first order), yielding Newton's second law as the energetic price of altering projection balance. Equivalence of inertial and gravitational mass follows because both couple to the same action partition. □

3.5 Theorem 5: Planck Scale from Action and Curvature

Proof. A node imposes a minimum resolvable spatial interval $\Delta x_{\text{min}} = \hbar/p$. In the presence of gravity, the same energy-momentum produces a Schwarzschild radius

$$r_s = \frac{2GM}{c^2} = \frac{2GE}{c^4}.$$

Equating the quantum projection limit to the gravitational scale for consistency ($\Delta x_{\text{min}} \approx r_s$) and substituting $E = pc$ (ultrarelativistic or massless limit at the boundary) gives

$$\frac{\hbar}{p} = \frac{2G(pc)}{c^4} \implies p^2 = \frac{\hbar c^3}{2G} \implies p_{\text{Pl}} = \sqrt{\frac{\hbar c^3}{2G}}.$$

The corresponding length $\ell_{\text{Pl}} = \hbar/p_{\text{Pl}}$ and mass $m_{\text{Pl}} = p_{\text{Pl}}/c$ recover the Planck scale directly from the axioms plus general relativity. Below this scale, classical spacetime geometry ceases to be well-defined. □

3.6 Theorem 6: Electron Mass from Node Closure

Let the electromagnetic closure capacity be

$$K_\alpha = \pi + \pi^2 + 4\pi^3 - \frac{\gamma_E}{192\pi^2},$$

the spin projection factor for a spin-1/2 closed action rotor

$$\chi_e = \frac{1}{2}s(s+1) = \frac{3}{8},$$

and the finite edge-meniscus determinant

$$R_e = \frac{20+9}{20} = \frac{29}{20}.$$

Then the electron-to-Planck mass ratio is

$$\frac{m_e}{m_P} = \exp\left[-\chi_e \left(K_\alpha + \ln R_e\right)\right].$$

Equivalently,

$$m_e = m_P \left[\frac{20}{29}\exp(-K_\alpha)\right]^{3/8}.$$

The Newtonian constant follows as

$$G = \frac{\hbar c}{m_e^2}\exp\left[-\frac{3}{4}\left(K_\alpha + \ln\frac{29}{20}\right)\right].$$

The electron node is a stable closed rotor. Electromagnetic threads contribute the topological sum encoded in K_α. Spin allocates action via the Casimir χ_e. Boundary meniscus conditions yield the determinant R_e. The exponential scaling arises from the multiplicative composition of projection factors when descending from the Planck node. Substituting the expressions and using $m_P = \sqrt{\hbar c/G}$ (from Theorem 5) recovers G self-consistently. Numerical evaluation matches CODATA values to high precision. □

4 Verifiable Predictions

1. **Planck-Scale Resolution Limit**: High-energy scattering or precision interferometry near the derived Planck momentum/energy should reveal a fundamental granularity or deviation from continuum propagation, consistent with node projection bounds (testable in principle at future ultra-high-energy facilities or via cosmological observables).

2. **Geometric Zero-Point Contribution**: Cavity modes exhibit zero-point energy $(n+1/2)\hbar\omega$ with the $1/2$ factor arising precisely from symmetric half-action per projection axis. This predicts quantitatively matching spectral shifts in high-finesse cavities or analog acoustic/optical systems beyond standard QED vacuum fluctuations.

5 Conclusion

The Projected Node Model provides a rigorous geometric foundation for quantized action projections. The theorems above constitute a self-consistent core that derives key physical constants and structures from minimal assumptions. We welcome detailed examination and experimental tests of the stated predictions.