LINEAR TRANSFORMATIONS IN THE RECIPROCAL GAMMA FUNCTION QUADRATURE

RUIGUO HE

Abstract. As an extension from my previous work, the paper attempts to generalize the Reciprocal Gamma Quadrature under the curve to general linear transformations on the argument itself. It extends further into the territory of Volterra Functions and Ramanujan's Log-Gamma integrals related to the Fransèn-Robinson Constant.

1. Introduction

Prior to this manuscript, we have derived a clean series representation for the Reciprocal Gamma Quadrature. In this paper, we ask for the extension where the argument is replaced by an affine transformation $ax + b$ instead of x. The entire process is much simpler and more straightforward than the original process itself and serves as a formalization purpose.

2. Evaluation of Integrals

The first mathematical section offers a light-weight extension of the original series. The linear objective initiates with:

$$\int_0^\infty \frac{\mathrm{dx}}{\Gamma(ax+c)} \tag{1}$$

We immediately apply parametrization $t = ax$. This way, the argument inside the Gamma Function becomes $\Gamma(t + c)$. The derivative of the new substitution is:

$$\frac{\mathrm{dt}}{\mathrm{dx}} = a \Rightarrow \mathrm{dx} = \frac{\mathrm{dt}}{a} \tag{2}$$

The limits stay invariant. As established, multiplying either infinity or 0 by a finite constant is still infinity and 0. We may move the inverse constant $1/a$ out of the integral:

$$\frac{1}{a}\int_0^\infty \frac{\mathrm{dt}}{\Gamma(t+c)} \tag{3}$$

A dilation factor actually directly translates to a physical constant scaling outside the integral. Now use a 2nd substitution $u = t + c$. The derivative is simply 1. The Gamma Function collapses to an isolated argument $1/\Gamma(u)$. The upper limit of integration is invariant, while the lower limit becomes $0 + c = c$. Thus:

$$\frac{1}{a}\int_c^\infty \frac{\mathrm{du}}{\Gamma(u)} \tag{4}$$

Date: June 19th, 2026.

Because the function inside is continuous and integratable, a standard rule in calculus allows us to split the larger integral into two smaller pieces using the formula:

$$\int_a^b f(x)\,\mathrm{dx} = \int_0^b f(x)\,\mathrm{dx} - \int_0^a f(x)\,\mathrm{dx} \tag{5}$$

In our case:

$$\int_c^\infty \frac{\mathrm{du}}{\Gamma(u)} = \int_0^\infty \frac{\mathrm{du}}{\Gamma(u)} - \int_0^c \frac{\mathrm{du}}{\Gamma(u)} \tag{6}$$

The first integral simplifies to the Fransèn-Robinson Constant definition as established:

$$\int_c^\infty \frac{\mathrm{du}}{\Gamma(u)} = F - \int_0^c \frac{\mathrm{du}}{\Gamma(u)} \tag{7}$$

To solve the 2nd integral, we apply a known series representation for the reciprocal Gamma Function [1]:

$$\frac{1}{\Gamma(z)} = \sum_{n=1}^{\infty} b_n z^n \tag{8}$$

This integration process is different. In the original improper evaluation of the constant, a swap between the series and integral is not allowed and causes asymptotic divergence. Here our integration bounds are finite. To integrate a series term-by-term over a finite interval $[a, b]$, the series must converge uniformly on that interval. By standard power series properties, any power series converges uniformly and absolutely on any closed disk D_R strictly inside its radius of convergence. Because $\frac{1}{\Gamma(z)}$ is entire, its radius of convergence is ∞. It converges absolutely over the entire real axis. Therefore with the finite bounds, we may perform a swapping:

$$\int_0^c \sum_{n=1}^{\infty} b_n u^n\,\mathrm{du} = \sum_{n=1}^{\infty} b_n \int_0^c u^n\,\mathrm{du} = \tag{9}$$

Now take the definite integral. Using the power rule,

$$\int_0^c u^n\,\mathrm{du} = \left[\frac{u^{n+1}}{n+1}\right]_0^c = \frac{c^{n+1}}{n+1} - \frac{0^{n+1}}{n+1} = \frac{c^{n+1}}{n+1} \tag{10}$$

The integral is perfectly derived. Therefore combining with the series grants:

$$\sum_{n=1}^{\infty} \frac{b_n c^{n+1}}{n+1} \tag{11}$$

Now subtract this from the Fransèn-Robinson Constant:

$$\int_c^\infty \frac{\mathrm{du}}{\Gamma(u)} = F - \sum_{n=1}^{\infty} \frac{b_n c^{n+1}}{n+1} \tag{12}$$

Multiply both sides by the original factor $1/a$ to proceed:

$$\frac{1}{a}\int_c^\infty \frac{\mathrm{du}}{\Gamma(u)} = F/a - \frac{1}{a}\sum_{n=1}^{\infty} \frac{b_n c^{n+1}}{n+1} \tag{13}$$

The left-hand side is equivalent to the original linearly transformed integral:

$$\int_0^\infty \frac{\mathrm{dx}}{\Gamma(ax+c)} = F/a - \frac{1}{a}\sum_{n=1}^{\infty} \frac{b_n c^{n+1}}{n+1} \tag{14}$$

Apply the identity derived in the previous paper on the Fransèn-Robinson Constant F [2]:

$$F = \mathrm{e} + \frac{1}{2}\sum_{n=0}^{\infty}\frac{b_{2n+2}B_{2n+2}}{n+1} \tag{15}$$

Substitute this series into the relation above:

$$\int_0^{\infty}\frac{\mathrm{dx}}{\Gamma(ax+c)} = \frac{\mathrm{e}}{a} + \frac{1}{2a}\sum_{n=0}^{\infty}\frac{b_{2n+2}B_{2n+2}}{n+1} - \frac{1}{a}\sum_{n=1}^{\infty}\frac{b_n c^{n+1}}{n+1} \tag{16}$$

In the 2nd series, set the index to $n = 0$ by adding one to each n in the coefficients:

$$-\frac{1}{a}\sum_{n=1}^{\infty}\frac{b_n c^{n+1}}{n+1} = -\frac{1}{a}\sum_{n=1}^{\infty}\frac{b_{n+1}c^{n+2}}{n+2} \tag{17}$$

Now both series terms begin with the same index. We are permitted to combine them into one larger series:

$$\int_0^{\infty}\frac{\mathrm{dx}}{\Gamma(ax+c)} = \frac{\mathrm{e}}{a} + \frac{1}{a}\sum_{n=0}^{\infty}\frac{b_{2n+1}B_{2n+2}}{2n+2} - \frac{b_{n+1}c^{n+2}}{n+2} \tag{18}$$

This is the generalized series formula in the case of a linear transformation. When $a = 1$, there is no scaling. When $b = 0$ there is no extra reduction term in the back because $b^{n+2} = 0^{n+2} = 0$. This is a perfect extended evaluation.

3. Generalization on Ramanujan's Eccentric Integrals

In the previous paper, the log-gamma integral

$$\int_0^{\infty}\frac{\mathrm{e}^{-t}}{\pi^2 + \ln^2(t)}\,\mathrm{dt} \tag{19}$$

was merely a special case scenario. There was for instance, a shifted Gamma integral that Ramanujan created for a different case [3]:

$$\int_0^{\infty}\frac{\mathrm{dx}}{\Gamma(x+1)} = \mathrm{e} - \int_0^{\infty}\frac{\mathrm{e}^{-t}}{t(\pi^2 + \ln^2(t))}\,\mathrm{dt} \tag{20}$$

There is yet another slightly different representation for shifted values. Of course if one may see, all shifted values with the argument $x + b$ can be formulated into a similar integral prospect. That's the target integral in this section. We begin by invoking Hankel's loop integral representation of the reciprocal Gamma function. This definition states that for any complex number

$$z$$

, the reciprocal function can be expressed as

$$\frac{1}{\Gamma(z)} = \frac{1}{2\pi i}\int_{\mathcal{C}}\mathrm{e}^t t^{-z}\,\mathrm{dt} \tag{21}$$

where the integration path

$$\mathcal{C}$$

is a standard keyhole contour enclosing the branch cut along the negative real axis. This contour begins at

$$-\infty - i\epsilon$$

, moves parallel to the negative real axis toward the origin, circles counter-clockwise around it, and then retreats back to

$$-\infty + i\epsilon$$

parallel to the upper edge of the cut. We now introduce a real shift parameter

$$c > 0$$

by setting

$$z = x + c$$

. Substituting this argument into our primary integral over the positive real axis yields the nested expression

$$\int_0^\infty \frac{1}{\Gamma(x+c)}\,dx = \int_0^\infty \left(\frac{1}{2\pi i} \int_{\mathcal{C}} \mathrm{e}^t t^{-(x+c)}\,\mathrm{dt} \right) \mathrm{dx} \tag{22}$$

At this step, we need to rigorously justify the application of Fubini's theorem for swapping the order of integration, we must demonstrate that the double integral converges absolutely. This requires verifying that the absolute value of the integrand over the combined domains is finite, which is stated as the condition

$$\int_0^\infty \left(\frac{1}{2\pi} \int_{\mathcal{C}} \left| \mathrm{e}^t t^{-(x+c)} \right| |\mathrm{dt}| \right) \mathrm{dx} < \infty \tag{23}$$

We analyze this condition by parameterizing the geometric components of our deformed Hankel contour

$$\mathcal{C}$$

, where the circular loop centered at the origin is fixed at a radius

$$R > 1$$

and the linear paths lie along the negative real axis from

$$-R$$

to

$$-\infty$$

. For the linear paths, we use the parameterization

$$t = u\mathrm{e}^{\pm i\pi} = -u$$

, where

$$u$$

ranges from

$$R$$

to

$$\infty$$

. This yields the absolute values

$$\left|\mathrm{e}^t\right| = \mathrm{e}^{-u}$$

and

$$\left| t^{-(x+c)} \right| = \left| (-u)^{-(x+c)} \right| = u^{-(x+c)}$$

, while the differential element is

$$|\mathrm{dt}| = \mathrm{du}$$

. For the circular loop parameterized by

$$t = R\mathrm{e}^{i\theta}$$

for

$$\theta \in [-\pi, \pi]$$

, the absolute values are bounded by

$$\left|\mathrm{e}^{t}\right| \leq \mathrm{e}^{R}$$

and

$$\left|t^{-(x+c)}\right| = R^{-(x+c)}$$

, with

$$|\mathrm{dt}| = R\,\mathrm{d}\theta$$

. Substituting these geometric bounds into the absolute double integral allows us to separate the domain into the linear branch cut components and the circular path component. Evaluating the inner integral with respect to

$$x$$

over the domain

$$[0, \infty)$$

for the linear paths yields

$$\int_0^\infty u^{-(x+c)}\,\mathrm{dx} = u^{-c}\int_0^\infty \mathrm{e}^{-x\ln u}\,\mathrm{dx} = \left[\frac{u^{-c}\mathrm{e}^{-x\ln u}}{-\ln u}\right]_0^\infty = \frac{u^{-c}}{\ln u} \tag{24}$$

Because the contour radius was explicitly chosen such that

$$R > 1$$

, the logarithmic term

$$\ln u \geq \ln R > 0$$

remains strictly positive, bounded away from zero, and safe from singularities across the entire path. The outer integration along the linear paths then reduces to

$$\int_R^\infty \frac{\mathrm{e}^{-u}u^{-c}}{\ln u}\,\mathrm{du} \tag{25}$$

This integral is guaranteed to converge to a finite value for any finite real parameter

$$c$$

. If

$$c \leq 0$$

, the term

$$u^{-c}$$

becomes a positive power of

$$u$$

, representing polynomial growth. However, because exponential decay dominates any polynomial growth at infinity, the integrand vanishes rapidly as

$$u \to \infty$$

, ensuring absolute convergence for all real finite values of

$$c$$

. Similarly, evaluating the inner integral over

$$x$$

for the circular component gives

$$\int_0^\infty R^{-(x+c)}\,\mathrm{dx} = R^{-c}\int_0^\infty \mathrm{e}^{-x\ln R}\,\mathrm{dx} = \frac{R^{-c}}{\ln R} \tag{26}$$

Since the path length of the circular loop is a finite constant

$$2\pi R$$

, multiplying it by the bounded function value produces a strictly finite contribution:

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} \mathrm{e}^R\left(\frac{R^{-c}}{\ln R}\right)R\,\mathrm{d}\theta = \frac{\mathrm{e}^R R^{1-c}}{\ln R} < \infty \tag{27}$$

Because both the linear branch components and the circular loop components evaluate to finite scalar values for any real finite

$$c$$

, the absolute double integral is bounded. This completes the proof of absolute convergence, validating the use of Fubini's theorem to legally exchange the integration order across the entire real spectrum of the parameter. Since absolute convergence holds, Fubini's theorem permits us to exchange the order of integration, which isolates the variable

$$x$$

within the inner integral as follows:

$$\int_0^\infty \frac{1}{\Gamma(x+c)}\,dx = \frac{1}{2\pi i}\int_{\mathcal{C}} \mathrm{e}^t t^{-c}\left(\int_0^\infty t^{-x}\,\mathrm{dx}\right)\mathrm{dt} \tag{28}$$

The inner integral with respect to

$$x$$

can be calculated directly by treating

$$t^{-x}$$

as an exponential function with a base of

$$\mathrm{e}$$

, which results in the evaluation

$$\int_0^\infty t^{-x}\,\mathrm{dx} = \int_0^\infty \mathrm{e}^{-x\ln t}\,\mathrm{dx} = \left[\frac{\mathrm{e}^{-x\ln t}}{-\ln t}\right]_0^\infty = \frac{1}{\ln t} \tag{29}$$

For this evaluation to converge at its upper boundary, the real part of

$$\ln t$$

must be strictly positive, which dictates that

$$|t| > 1$$

. To satisfy this requirement, we deform our Hankel contour

$$\mathcal{C}$$

outward so that its path safely stays completely outside the unit circle

$$|t| = 1$$

. Because the contour has been expanded outward past

$$|t| > 1$$

, it now wraps around the point

$$t = 1$$

. Looking at our modified integrand, a simple pole exists at

$$t = 1$$

due to the singularity where

$$\ln(1) = 0$$

. By applying the Residue Theorem, the complete contour integral can be split into the isolated residue contribution at this simple pole plus the remaining path elements traversing the branch cut:

$$\frac{1}{2\pi i}\int_{\mathcal{C}} \frac{\mathrm{e}^t t^{-c}}{\ln t}\,\mathrm{dt} = \mathrm{Res}\left(\frac{\mathrm{e}^t t^{-c}}{\ln t}, t = 1\right) + \frac{1}{2\pi i}\int_{\text{branch cut}} \frac{\mathrm{e}^t t^{-c}}{\ln t}\,\mathrm{dt} \tag{30}$$

Evaluating the residue at the simple pole

$$t = 1$$

produces Euler's number

$$\mathrm{e}$$

, as shown by the limit computation

$$\mathrm{Res}\left(\frac{\mathrm{e}^t t^{-c}}{\ln t}, t = 1\right) = \lim_{t\to 1}(t-1)\frac{\mathrm{e}^t t^{-c}}{\ln t} = \mathrm{e}^1 \cdot 1^{-c} \cdot 1 = \mathrm{e} \tag{31}$$

Next, we parameterize the remaining segments of the contour running along the branch cut. The path consists of a lower edge and an upper edge running parallel to the negative real axis, which we map using the real variable

$$u$$

as it spans from

$$0$$

to

$$\infty$$

. On the lower edge, the phase is given by

$$\theta = -\pi$$

, meaning

$$t = u\mathrm{e}^{-i\pi}$$

,

$$\ln t = \ln u - i\pi$$

, and

$$t^{-c} = u^{-c}\mathrm{e}^{i\pi c}$$

. On the upper edge, the path returns in the opposite direction with a phase of

$$\theta = +\pi$$

, meaning

$$t = u\mathrm{e}^{i\pi}$$

,

$$\ln t = \ln u + i\pi$$

, and

$$t^{-c} = u^{-c}\mathrm{e}^{-i\pi c}$$

. Substituting these definitions into the branch cut integral components gives

$$\frac{1}{2\pi i}\int_{\text{branch cut}} \frac{\mathrm{e}^t t^{-c}}{\ln t}\,\mathrm{dt} = \frac{1}{2\pi i}\left[\int_0^\infty \frac{\mathrm{e}^{-u}(u\mathrm{e}^{-i\pi})^{-c}}{\ln u - i\pi}\,\mathrm{du} + \int_\infty^0 \frac{\mathrm{e}^{-u}(u\mathrm{e}^{i\pi})^{-c}}{\ln u + i\pi}\,\mathrm{du}\right] \tag{32}$$

To combine these two integrals under uniform integration boundaries, we flip the limits of integration on the upper edge component, which changes

$$\int_\infty^0$$

into

$$-\int_0^\infty$$

and introduces a net subtraction to the second term:

$$\frac{1}{2\pi i}\int_{\text{branch cut}} \frac{\mathrm{e}^t t^{-c}}{\ln t}\,\mathrm{dt} = \frac{1}{2\pi i}\int_0^\infty \mathrm{e}^{-u}u^{-c}\left[\frac{\mathrm{e}^{i\pi c}}{\ln u - i\pi} - \frac{\mathrm{e}^{-i\pi c}}{\ln u + i\pi}\right]\mathrm{du} \tag{33}$$

We combine the terms enclosed within the brackets by finding a common denominator, which expands the expression into the form

$$\frac{\mathrm{e}^{i\pi c}(\ln u + i\pi) - \mathrm{e}^{-i\pi c}(\ln u - i\pi)}{(\ln u - i\pi)(\ln u + i\pi)} = \frac{\ln u\left(\mathrm{e}^{i\pi c} - \mathrm{e}^{-i\pi c}\right) + i\pi\left(\mathrm{e}^{i\pi c} + \mathrm{e}^{-i\pi c}\right)}{\ln^2 u + \pi^2} \tag{34}$$

By applying Euler's canonical identities for sine and cosine, we can transform these complex exponential terms back into standard real-valued trigonometric functions since

$$\mathrm{e}^{i\pi c} - \mathrm{e}^{-i\pi c} = 2i\sin(\pi c)$$

and

$$\mathrm{e}^{i\pi c} + \mathrm{e}^{-i\pi c} = 2\cos(\pi c)$$

. Substituting these identities back into our numerator gives

$$\frac{\ln u \cdot [2i\sin(\pi c)] + i\pi \cdot [2\cos(\pi c)]}{\ln^2 u + \pi^2} \tag{35}$$

Factoring out a common value of

$$2i$$

from the entire numerator block simplifies the bracketed structure to

$$2i \cdot \frac{\ln u \sin(\pi c) + \pi\cos(\pi c)}{\ln^2 u + \pi^2} \tag{36}$$

Finally, we multiply this simplified fractional expression by the leading prefactor

$$\frac{1}{2\pi i}$$

that sits outside our branch cut integral:

$$\frac{1}{2\pi i} \cdot 2i \cdot \left(\frac{\ln u \sin(\pi c) + \pi\cos(\pi c)}{\ln^2 u + \pi^2}\right) = +\frac{1}{\pi} \cdot \left(\frac{\ln u \sin(\pi c) + \pi\cos(\pi c)}{\ln^2 u + \pi^2}\right) \tag{37}$$

The imaginary terms and the coefficient

$$2$$

cancel out completely, leaving a positive

$$\frac{1}{\pi}$$

out front. Swapping our dummy variable

$$u$$

back to

$$t$$

and combining this evaluated branch cut path with our isolated pole residue

$$\mathrm{e}$$

yields the final, generalized identity:

$$\int_0^\infty \frac{1}{\Gamma(x+c)}\,\mathrm{dx} = \mathrm{e} + \frac{1}{\pi}\int_0^\infty \frac{\mathrm{e}^{-t}t^{-c}\left[\ln(t)\sin(\pi c) + \pi\cos(\pi c)\right]}{\pi^2 + \ln^2 t}\,\mathrm{dt} \tag{38}$$

Correlate this result to the previous series identity obtained in the previous section:

$$\int_0^\infty \frac{1}{\Gamma(x+c)}\,\mathrm{dx} = \mathrm{e} + \sum_{n=0}^{\infty} \frac{b_{2n+1}B_{2n+2}}{2n+2} - \frac{b_{n+1}c^{n+2}}{n+2} \tag{39}$$

These two expressions are equal:

$$\mathrm{e} + \sum_{n=0}^{\infty} \frac{b_{2n+1}B_{2n+2}}{2n+2} - \frac{b_{n+1}c^{n+2}}{n+2} = \mathrm{e} + \frac{1}{\pi}\int_0^\infty \frac{\mathrm{e}^{-t}t^{-c}\left[\ln(t)\sin(\pi c) + \pi\cos(\pi c)\right]}{\pi^2 + \ln^2 t}\,\mathrm{dt} \tag{40}$$

Subtract e from both sides:

$$\frac{1}{\pi}\int_0^\infty \frac{\mathrm{e}^{-t}t^{-c}\left[\ln(t)\sin(\pi c) + \pi\cos(\pi c)\right]}{\pi^2 + \ln^2 t}\,\mathrm{dt} = \sum_{n=0}^{\infty} \frac{b_{2n+1}B_{2n+2}}{2n+2} - \frac{b_{n+1}c^{n+2}}{n+2} \tag{41}$$

Now multiply both sides by π:

$$\int_0^\infty \frac{\mathrm{e}^{-t}t^{-c}\left[\ln(t)\sin(\pi c) + \pi\cos(\pi c)\right]}{\pi^2 + \ln^2 t}\,\mathrm{dt} = \pi\sum_{n=0}^{\infty} \frac{b_{2n+1}B_{2n+2}}{2n+2} - \frac{b_{n+1}c^{n+2}}{n+2} \tag{42}$$

Use the fraction rule to combine the coefficients:

$$\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd} \tag{43}$$

$$\frac{b_{2n+1}B_{2n+2}}{2n+2} - \frac{b_{n+1}c^{n+2}}{n+2} = \frac{b_{2n+1}B_{2n+2}(n+2) - 2b_{n+1}c^{n+2}(n+1)}{2(n+1)(n+2)} \tag{44}$$

Therefore we solve for the final transformed relation:

$$\int_0^\infty \frac{\mathrm{e}^{-t}t^{-c}\left[\ln(t)\sin(\pi c) + \pi\cos(\pi c)\right]}{\pi^2 + \ln^2 t}\,\mathrm{dt} = \pi\sum_{n=0}^{\infty} \frac{b_{2n+1}B_{2n+2}(n+2) - 2b_{n+1}c^{n+2}(n+1)}{2(n+1)(n+2)} \tag{45}$$

Hence this is the exact numerical series for a shifted Ramanujan-style integral without using special functions or nested sums at all. As discussed, the Volterra Function has a shift. The previous paper on this topic included only a single case of the function where the variables were $1, 1, 0$. This time we expand the case a little bit wider to a single-variate function:

$$\mu(x, \alpha, \beta) = \frac{1}{\Gamma(1+\beta)} \int_0^\infty \frac{x^t t^\beta}{\Gamma(t+\alpha+1)} \, \mathrm{dt} \tag{46}$$

Here α is the variable shift while $x = 1$ and $\beta = 0$. This indicates to:

$$\mu(1, \alpha, 0) = \int_0^\infty \frac{\mathrm{dt}}{\Gamma(t+\alpha+1)} \tag{47}$$

Now to match our specified form we require $\alpha - 1$ as a substitution. Thus:

$$\mu(1, \alpha, 0) = \int_0^\infty \frac{\mathrm{dt}}{\Gamma(t+\alpha)} \tag{48}$$

This is exactly our shifted integral. Now apply the original linear transform series to solve:

$$\mu(1, \alpha, 0) = \int_0^\infty \frac{\mathrm{dt}}{\Gamma(t+\alpha)} = \mathrm{e} + \sum_{n=0}^{\infty} \frac{b_{2n+1} B_{2n+2}}{2n+2} - \frac{b_{n+1} \alpha^{n+2}}{n+2} \tag{49}$$

We apply the combining of fractions to posit the final VOlterra Function identity:

$$\mu(1, \alpha - 1, 0) = \mathrm{e} + \frac{1}{2} \sum_{n=0}^{\infty} \frac{b_{2n+1} B_{2n+2}(n+2) - 2b_{n+1}\alpha^{n+2}(n+1)}{(n+1)(n+2)} \tag{50}$$

Through utilizing standard integration techniques, we were able to successfully derive the series form for both a generalized shifting Ramanujan integral and its corresponding Volterra Function. This technique is a resultant extension of the initial thesis on the Fransèn-Robinson Constant. It is clear that many other similar examples involving the reciprocal Gamma Function can be solved in similar flexible ways.

Conclusion

In this paper, we are introduced to a transformed extension of the previous manuscript on the Fransèn-Robinson Constant. By scaling the total quadrature and shifting the curve sideways, we construct a general series that applies to both the generalized integral from Ramanujan's perception as well as a expanded case of the Volterra Function. The entire purpose of the extended work is to show the direct application shown in my previous papers. The linear transformation is the first general case possible out of that scenario.

References

[1] M. Abramowitz and I. A. Stegun (eds.), *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Applied Mathematics Series, vol. 55, National Bureau of Standards, Washington, DC, 1972.

[2] He, R. (2026). *New Interpretation on the Fransén-Robinson Constant.* viXra:2605.0073. https://vixra.org

[3] Ramanujan, S. (1915). Some definite integrals. *Messenger of Mathematics*, *44*, 10–18.

Independent Researcher, Allen, Texas, 75013

Email address: ruiguo.he.physicsresearch@gmail.com