Identifying Natural Limits to Infinite Algebraic Singularities with Null Algebra

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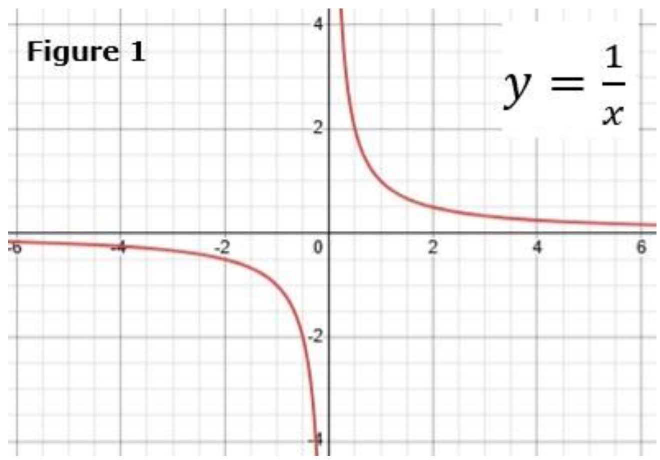
This paper introduces a framework which regularizes the classical algebraic singularity, using the essential hyperbola $y = \frac{1}{x}$ to illustrate its application. Traditional, classical mathematics leaves the behavior at the origin for this function undefined due to divergence toward unachievable infinities. By using the transformational matrix defined in Null Algebra to map $u = -\frac{1}{y}$, the defined subspace of y, we may focus on a rate of information transfer implied by the function, as $x \to 0$. This is achieved by imposing a strict constraint upon $\frac{dy}{du}$ which is required for any function $y = f(x)$ and based upon chosen scale for the system defined by $y = f(x)$. This shall show the singularity cannot actually be achieved due to natural self-limiting properties unique to a given function which emerge from Null Algebra, leaving a function, that is piecewise defined and continuous.

It is assumed the readers has read, and understood **Null Algebra: The Math of Division by Zero and The Negative Radical** (https://vixra.org/abs/2103.0131), Null Algebra Extension I (https://vixra.org/abs/2206.0135), Null Algebra Extension II (https://vixra.org/abs/2304.0205), Null Algebra Extension III (https://vixra.org/abs/2308.0043) as well as basic Algebra and Calculus.

1.1—The space of time and the acceleration of the output value.

Null Algebra provides that the y-axis subspace is defined as the u-axis. The subspace transformation matrix further specifies that $u = -\frac{1}{y}$. The u-axis within Null Algebra is as real as any other axis, but is not a full degree of freedom in the same sense as x, y or z-axes. Instead the u-axis is the defined as the place, where events in the space of time, the output values on the y-axis, occur. Hence the u-axis is a space like time direction for changing outputs of the y-axis.

For the given equation, the essential hyperbola defined by $y = \frac{1}{x}$ we are considering only pure mathematical constructs. When x approaches 0, the output of the function approaches infinity or negative infinity, and thus the limit does not exist due to infinite discontinuity.



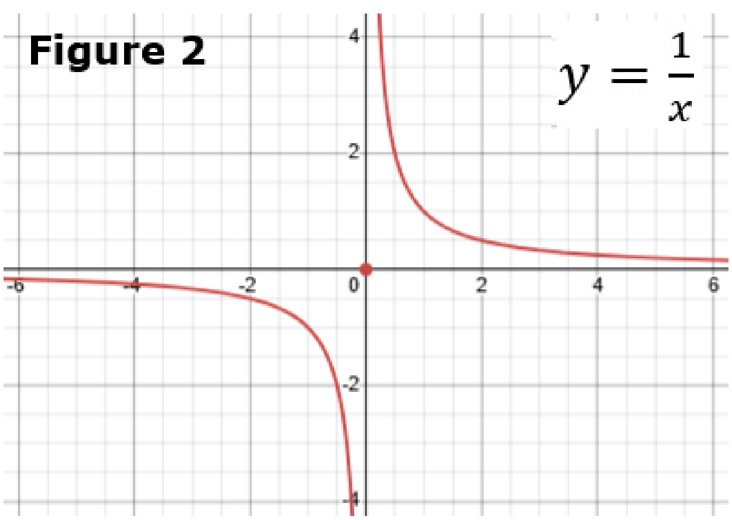
$\lim_{x \to 0^+} \frac{1}{x} = \infty$ Infinite Discontinuity

$\lim_{x \to 0^-} \frac{1}{x} = -\infty$ Infinite Discontinuity

Left and Right Side Limits do not match.
Limit does not exist.

From a Null Algebra perspective we find that infinities which occur at the origin are resolvable to zero.

$\lim_{x \to 0^+} \frac{1}{x} = \infty = \eta_0 \doteq 0$ Resolves to 0 $\lim_{x \to 0^-} \frac{1}{x} = -\infty = -\eta_0 \doteq 0$ Resolves to 0



Left and Right Side Limits are equal. However neither limit approaches this value nor actually reaches a finite value as $x \to 0$.

Here at left in Figure 2 we have the Central Plane dimension of the xy-axes, showing the resolved value for $y = 0$ when $x = 0$.

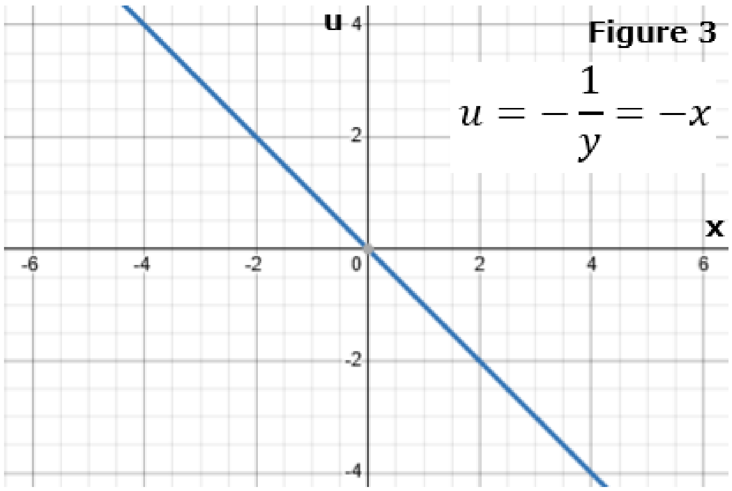


Figure 3

At left in Figure 3 we see Posterior Subspace dimension of the xu-axes. The function $u = -x$ passes through the hyper-origin and is one reason for the resolution to $y = 0$ when $x = 0$ in the Central Plane, as defined in Null Algebra.

Something crucial is happening as x becomes arbitrarily close to 0. If we accept the Null Algebra definition that the u-axis (space like time direction) is the space of time where y-axis points occur, then despite u being a physical place it is still a representation of time. Or more specifically Δu is the time permitted for Δy to occur. Therefore the rate of change of y with respect to u cannot exceed c in a vacuum. As verified by numerous relativity experiments c in a vacuum, is the universal speed limit. The conceptualization of $y = \frac{1}{x}$ from the vantage of Null Algebra is no longer just pure mathematics but rather one of Physics and the following relation must hold true.

$$\frac{dy}{du} < c$$

As shall be seen below it must be this relation we consider. Here, as just discussed, Δu servers the progression of Δt here. Why not use Δt? We don't know what that is with the given information. Given $y = \frac{1}{x}$ in order to determine the rate of change of y with respect to x we need only differentiate. But to find their rate of change with respect to time, we need to implicitly differentiate.

$$\frac{dy}{dt} = \frac{d}{dt}\frac{1}{x} = -\frac{1}{x^2}\cdot\frac{dx}{dt}$$

$\frac{dx}{dt}$ is not definable here unless we have more information to solve for it. The function $y = \frac{1}{x}$ is positional, relating only the output y for chosen inputs of x. There is no consideration for time in this equation from the perspective of the Central Plane, and including it there in any way changes the nature of what we are considering. We need to examine the only part of this equation where a naturally included temporal consideration exists with regard to the singularity at $x = 0$ where y leaps to $\pm\infty$. That is the output value of the u-axis on the Posterior Subspace dimension.

1.2—Conceptualizing Constraints on Δy as $u \to 0$:

Using the transformation Matrices laid out in Null Algebra we find the following relations for the given function $y = \frac{1}{x}$.

$$\lim_{x \to 0^-} \frac{1}{x}$$

x	$y = \frac{1}{x}$	$u = -\frac{1}{y}$
-4	$-\frac{1}{4}$	4
-3	$-\frac{1}{3}$	3
-2	$-\frac{1}{2}$	2
-1	-1	1
$-\frac{1}{2}$	-2	$\frac{1}{2}$
$-\frac{1}{3}$	-3	$\frac{1}{3}$
$-\frac{1}{20}$	-20	$\frac{1}{20}$
$-\frac{1}{5000}$	-5,000	$\frac{1}{5000}$
0	$-\infty \doteq 0$	0

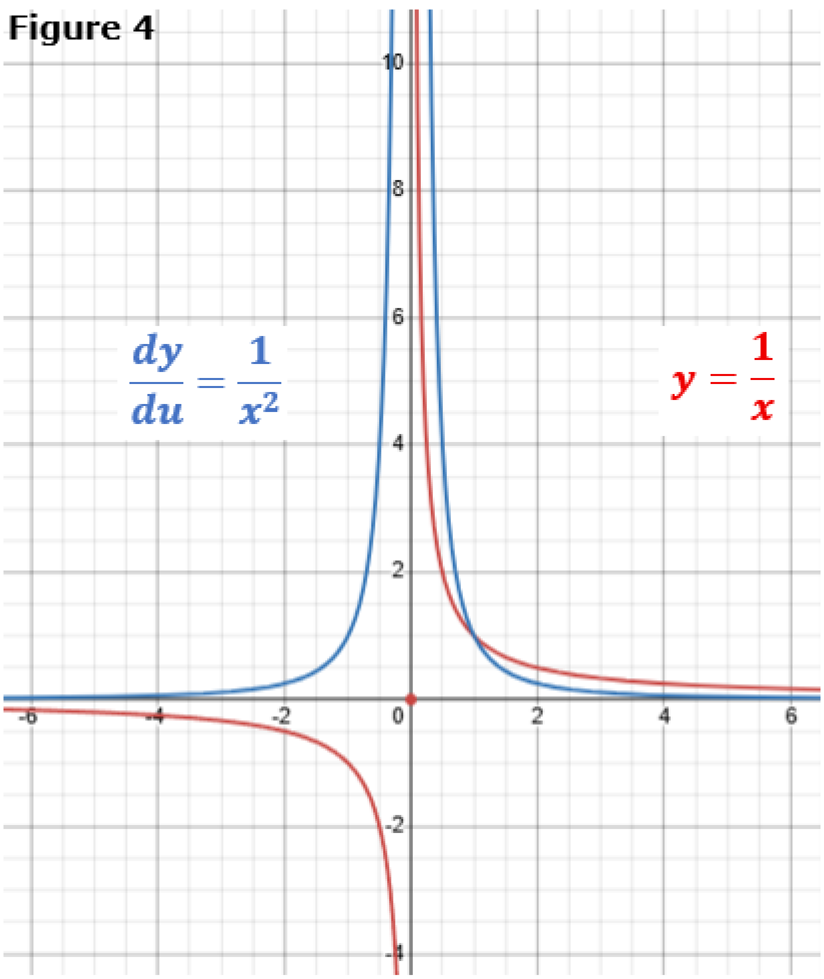
$$\lim_{x \to 0^+} \frac{1}{x}$$

x	$y = \frac{1}{x}$	$u = -\frac{1}{y}$
4	$\frac{1}{4}$	- 4
3	$\frac{1}{3}$	- 3
2	$\frac{1}{2}$	- 2
1	1	- 1
$\frac{1}{2}$	2	$-\frac{1}{2}$
$\frac{1}{3}$	3	$-\frac{1}{3}$
$\frac{1}{20}$	20	$-\frac{1}{20}$
$\frac{1}{5000}$	5,000	$-\frac{1}{5000}$
0	$\infty \doteq 0$	0

It is clear that the as $x \to 0$, for ever smaller shifts in Δx, $\Delta y \to \pm\infty$ depending whether you approach from the left or right. At the same time $\Delta u \to 0$ regardless of from which side $x \to 0$. Because $\Delta u \to 0$ while $\Delta y \to \pm\infty$, the value $\frac{dy}{du} \to \infty$ at rate much faster than will Δy alone. We can see this easily by simply solving for $u = -\frac{1}{y} = -x$.

Without any scale attached to $y = \frac{1}{x}$ on the Central Plane the arguments expressed in Null Algebra stand, with some added clarity. As $x \to 0$, $y \to \pm\infty$. When $x = 0$ the As y outputs jump to infinity but as this value is impossibly far away, unable to be reached, y is ultimately resolved to 0. This is supported by understand that the when $x = 0$, then essentially $y = \nexists$, and $u = 0$. Thus as detailed in Null Alegra when $x = 0$, $y = \eta_0 \doteq 0$. Considered from the that perspective, when $\Delta u = 0$ this subspace value does not so much imply $t = 0$ as but rather $t = \eta_0$. So not only are ever larger values of Δy occurring as $x \to 0$ but they are accelerating, leaping farther and farther ahead as x gets ever closer to 0. At $x = 0$, $\Delta u = 0$ implies a non-existence of time, $t = \eta_0$ for the changes in y meaning the remainder of the y-axis has no boundary limitation, jumping $\pm\infty$. It is then resolved to 0 for reasons just summarized and defined in Null Algebra. Consider the graph of Figure 4 on the following page, which compares the rapidly accelerating changes of $\frac{dy}{du}$ to the given equation $y = \frac{1}{x}$.

Figure 4



Despite how quickly y values either climb or plument, depending on the direction we chose for $x \to 0$ the rate of change $\frac{dy}{du}$, the amount of Δy per ever smaller slice of the space of time for those changes, Δu, increases much faster than $\frac{dy}{dx}$

All of this stands without application of scale but the moment we use any specific scale, whether it be feet, inches, meters, etc., we must consider the existence of boundary conditions which arise due to limitations of energy, time, and velocity in a given medium.

$$\frac{dy}{du}$$

$$\frac{d}{du}y = \frac{d}{du}\frac{1}{x}$$

$$\frac{dy}{du} = \frac{dy}{dx} \cdot \frac{dx}{du}$$

$$\frac{dy}{du} = \left(-\frac{1}{x^2}\right)(-1)$$

$$\frac{dy}{du} = \frac{1}{x^2}$$

x	y	$\frac{dy}{du}$
0.5	2	4
0.1	10	100
0.001	1000	1,000,000
0.00001	100,000	10,000,000,000

The values only continue to increase with the driving power of the squared denominator in the $\frac{dy}{du}$ term as $x \to 0$

<u>1.3—Calculating x and natural limits implied by dy/du</u> :

The necessary place to look for a naturally occurring limit arising from $\frac{dy}{du}$ is the Theory of Relativity. Let y then be representative of some physical quantity such as position, energy, field strength, etc., and let u as space like time, be the literal place in time when y values were recorded. Then $\frac{dy}{du}$ represents a velocity or rate of propagation. Regardless of scale we choose, if we find that $\frac{dy}{du} \geq c$ at any point then we know a limiting factor must be present which will cap the height of y at some input value(s) x. To see if this occurs we need to find an x value such that $\frac{\Delta y}{\Delta u} \geq c$.

Special Relativity states no physical information or mass can travel faster than the speed of light, c. This is shown in Special Relativity by way of the Lorentz Gamma factor.

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The significance of the gamma factor is that in attempt to push a system to meet or exceed c, $\gamma \to \infty$, requiring an infinite amount of energy to push further. The very geometry of spacetime works to provide this upper bound for Special Relativity. In the given equation, $y = \frac{1}{x}$ there is a singularity where $y \to \pm\infty$. We need to examine when $\frac{dy}{dx} \geq c$ for $y = \frac{1}{x}$. When $\frac{dy}{dx} = c$, the velocity it represents will cause the Lorentz Gemma factor equal infinity. This is the point where we would expect to see the energy needed to push values represented by y to require infinite energy to rise higher $x \to 0^+$ and plumet lower as $x \to 0^-$. Both situations requiring infinite energy are not attainable and it is at that point, which implies there will be a some finite cap to the y-axis outputs.

$$\frac{dy}{du} = \frac{1}{x^2}$$

To be as exact as possible we shall agree on the sale of Meters Per Second for $\frac{dy}{du}$. This provides, $c \approx 299{,}792{,}458$ meters per second. Considering that $\frac{dy}{dx} = \frac{1}{x^2}$, then we can set this equation equal to c and solve.

$$299{,}792{,}458 = \frac{1}{x^2}$$

$$x = \sqrt{\frac{1}{299{,}792{,}458}} \approx \pm\frac{1}{17{,}314.51581766} \approx \pm 0.0000577550080$$

The value we seek for this scale is $x \approx \pm 57.755\ \mu m$. This is about the width of a human hair. Plugging in for x in $y = \frac{1}{x}$ we obtain thus:

For: $y = \frac{1}{x}$ with $x \approx \pm 57.755\ \mu m$ we obtain $y = \frac{1}{x} \approx \pm 17{,}314.51582519$

This suggests that the domain for $y = \frac{1}{x}$ according to Null Algebra is

$D: (-\infty < -0.0000577550080) \cup \{0\} \cup (0.0000577550080 < \infty)$

Are we suggesting values $-0.0000577550080 < x < 0.0000577550080$ cannot ever be used? No! This solution only suggests that for some physical system which is itself defined by the equation $y = \frac{1}{x}$, values such that $-0.0000577550080 < x < 0.0000577550080$, except possibly at $x = 0$ due to Null Algebra resolution of $y = \pm\infty \doteq 0$, are either unusable in real world application or governed by some additional as yet unspecified feature of the given equation.

Can we trust $x \approx \pm 0.0000577550080$ is a limiting value, caping y? To begin exploring this we can consider weather an abrupt cutoff of y is how such cap will manifest, or if instead it will take the form of a smooth regularized function. We can apply a weighting function to adjust the advance of y-axis outputs so they arrive at the true upper and lower boundaries of $y = \frac{1}{x}$ instead of exploding to $\pm\infty$ and yet maintain known values of the function which are not very near the singularity.

2.1—Building a Weighting Function:

We need a weighting factor which will not only permit the function to plot out its values as expected within traditional mathematics but also adjust the values y reaches to cap out when $x \to \pm 57.755\ \mu m$ from within boundaries of the defined domain, thereafter dropping to zero outside those defined boundaries.

The weighting factor for the function will take the form of $\mathbf{e}^{\omega}$. For this specific example $y = \frac{1}{x}$, omega, $\omega = {}^{-\kappa}/_{x}$.

$$\mathbf{e}^{\omega} = \mathbf{e}^{-\kappa/x}$$

When functions like $y = \frac{1}{x}$ approach a singularity, like this function does as $x \to 0$, they will grow or decrease at an algebraic rate. Since the projections based on $\frac{dy}{du}$ prevent our function $y = \frac{1}{x}$ from actually leaping to infinity beyond a finite cap, we must apply a weighting factor to act as an emergency brake as x approaches the limits of its defined domain. The weighting factor must decrease toward zero faster than the function is climbing toward infinity and yet have negligible affect on the region of the domain not very near the singularity.

Exponential decay grows faster than any polynomial or algebraic function. A simple polynomial weight like ${}^{\kappa}/_{x}$, is not strong enough dampen the asymptotic climb of $y = \frac{1}{x}$ as $x \to 0$. But the exponential function $\mathbf{e}^{\omega}$ (where ω becomes a massive negative number near the singularity) possesses an aggressive, crushing downward pull. When multiplied by a divergent function, $\mathbf{e}^{\omega}$ completely dominates the algebraic growth. If ω is calculated correctly we safely eliminate the infinity to the function's actual cap and force outputs in the forbidden range to zero, precisely the Null Algebra resolution of $\infty \doteq \eta_0 \doteq 0$.

2.2—Why $\mathbf{e}^{-\kappa/x}$?

The considered function $y = \frac{1}{x}$ explodes at a rate proportional to $\frac{1}{x}$ as $x \to 0$. To properly balance the scale, the weighting factor $\mathbf{e}^{\omega}$ must match the shape and speed of the specific singularity defined by $y = \frac{1}{x}$. Thus ω will be some negative factor of $\frac{1}{x}$.

$$\mathbf{e}^{\omega} = \mathbf{e}^{-\kappa/x}$$

What now remains is choosing the correct value for kappa. If ω is too weak (e.g. $\omega = -\kappa$) then as $x \to 0$, $\mathbf{e}^{\omega}$ is just a constant ($\mathbf{e}^{\kappa}$), contributing too little needed adjustment, failing to dampen $y = \frac{1}{x}$ which still explodes to infinity. If ω is too strong (e.g., $\omega = {}^{-\kappa}/_{x^2}$), then the weighting function weighs in, way too early. It destroys the accuracy of $y = \frac{1}{x}$ long before x approaches its domain boundaries.

Thus the regularized function becomes:

$$y = \frac{1}{x} \cdot \mathbf{e}^{-\kappa/x}$$

2.3—Generalizing ω:

We can specify a generalized ω as a weighting factor for any arbitrary function $f(x)$ which possesses a singularity at some input x_0. Let $f(x)$ be a function that diverges to infinity as $x \to x_0$. Then the weighting exponent ω must:

- Maintain Traditional Values: $\mathbf{e}^{\omega} = 1$ when x is far from x_0.
- Null Algebra Resolution: $\mathbf{e}^{\omega} = 0$ when $x = x_0$.

Thus to perfectly match the severity of the singularity, ω must be proportional to the negative logarithm of the divergent behavior, adjusted by a physical coordinate constraint constant, κ.

$$\omega(x) = -\kappa \cdot |f(x)|^{\alpha}$$

Where α is the scaling power chosen to ensure the exponential expansion outpaces the function's base divergence rate.

3.1—Calculating κ:

From this equation: $y = \frac{1}{x} \cdot \mathbf{e}^{-\kappa/x}$

With known values: $x \approx \pm 0.00005775508$ $\quad c \approx 299{,}792{,}458\ m/s$

The velocity of the weighted function must be such that it too will equal c when $x \approx \pm 0.00005775508$. Then differentiating the weighted function with respect to u:

$$\frac{dy}{du} = \frac{d}{du}\left(\frac{1}{x} \cdot \mathbf{e}^{-\kappa/x}\right) \quad \text{with} \quad u = -x$$

$$\frac{dy}{du} = \left(-\frac{1}{x^2} \cdot (-1) \cdot \mathbf{e}^{-\kappa/x}\right) + \left(\frac{1}{x} \cdot \frac{k}{x^2} \cdot (-1) \cdot \mathbf{e}^{-\kappa/x}\right)$$

$$\frac{dy}{du} = \left(\frac{1}{x^2} \cdot \mathbf{e}^{-\kappa/x}\right) + \left(-\frac{k}{x^3}\mathbf{e}^{-\kappa/x}\right)$$

$$\frac{dy}{du} = \left(\frac{1}{x^2} - \frac{k}{x^3}\right)\mathbf{e}^{-\kappa/x}$$

We may now set $\frac{dy}{du}$ equal to c, the speed of light in Meters Per Second, our chosen scale.

$$c = \left(\frac{1}{x^2} - \frac{k}{x^3}\right)\mathbf{e}^{-\kappa/x}$$

Note that we specified $x \approx \pm 0.00005775508$. This value will be used here because though this is weighted function, it must apply that weight exactly at the same unweighted value for x and reach the same output for y. $c \approx 299{,}792{,}458\ m/s$

For $\frac{1}{x^2}$ with $x \approx \pm 0.00005775508$ we have $\frac{1}{(\pm 0.00005775508)^2} = c$. So we may make this substitution, $\frac{1}{x^2} = c$, wherever it arises in the math.

$$c = \left(c - \frac{k}{x^3}\right)\mathbf{e}^{-\kappa/x}$$

Divide both sides by c

$$1 = \left(1 - \frac{k}{c \cdot x^3}\right)\mathbf{e}^{-\kappa/x}$$

We now re-substitute $c = \frac{1}{x^2}$ as established above.

$$1 = \left(1 - \frac{k}{\frac{1}{x^2} \cdot x^3}\right)\mathbf{e}^{-\kappa/x}$$

$$1 = \left(1 - \frac{k}{x}\right)\mathbf{e}^{-\kappa/x}$$

We want to easily track the behavior of this expression. Now make the substitution that $z = \frac{k}{x}$. We'll use this as a place holder to make the equation easier to solve.

$$1 = (1 - z)\mathbf{e}^{-z}$$

We now seek when this function equals 1. Since we have an exponential multiplied by a constant for some value of z a good start point is to explore the behavior of the function for various sections of its valid domain.

$z = 0$: If zeta equals 0 we plug in and find

$$(1 - 0)\mathbf{e}^{0} \quad = \quad 1 \cdot 1 \quad = \quad 1$$

The statement is true. Since $z = \frac{k}{x}$ it must be that $k = 0$.

$z > 0$: If we start picking some test values here it doesn't take long to realize this part of the domain will not work.

$z = 0.5$ $\quad (1 - 0.5)\mathbf{e}^{0.5} \quad = \quad 0.5 \cdot \mathbf{e}^{0.5} \quad \approx \quad 0.82436063535 \quad \neq \quad 1$

$z = 1$ $\quad (1 - 1)\mathbf{e}^{1} \quad = \quad 0 \cdot \mathbf{e} \quad = \quad 0 \quad \neq \quad 1$

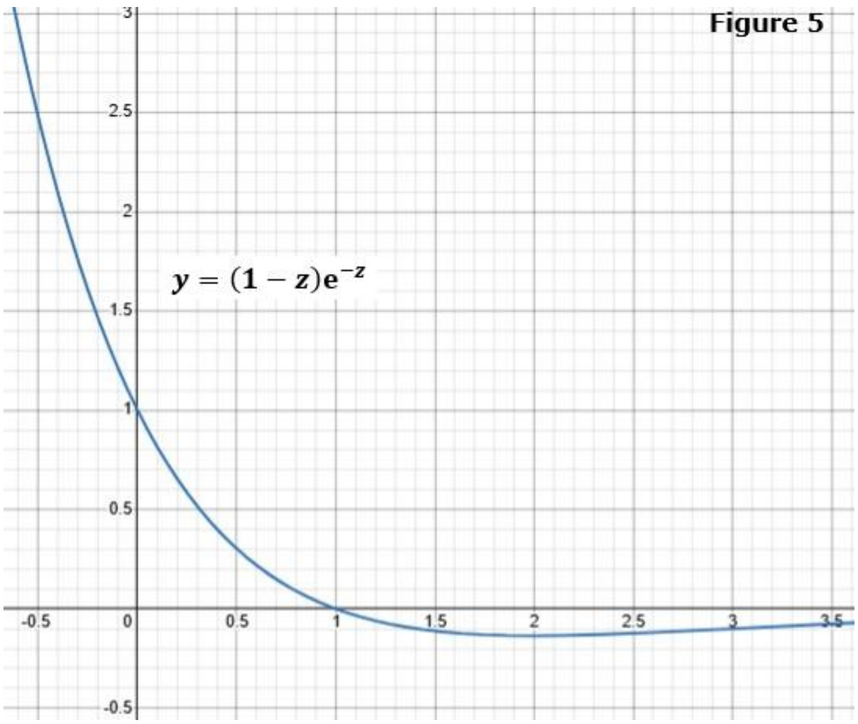
$z = 2$ $\quad (1 - 2)\mathbf{e}^{2} \quad = \quad -1 \cdot \mathbf{e}^{-2} \quad \approx \quad -0.13533 \quad \neq \quad 1$

$z < 0$: The same situation plays out here rather quick. Picking some test values shows this part of the domain does not work either.

$z = -0.5$ $\quad (1 + 0.5)\mathbf{e}^{0.5} \quad = \quad 1.5 \cdot \mathbf{e}^{-0.5} \quad \approx \quad 2.47308 \quad \neq \quad 1$

$z = -1 \qquad (1+1)\mathbf{e}^{-1} = 2 \cdot \frac{1}{e} \approx 5.43656 \neq 1$

$z = -2 \qquad (1+2)\mathbf{e}^{-2} = 3 \cdot \frac{1}{\mathbf{e}^2} \approx 22.16717 \neq 1$



The only place where the equation equals 1 is where $z = 0$. And as stated above since $z = \frac{k}{x}$ it must be that $k = 0$. Further we may define $z = 0$ for $x = 0$ due to Trending as defined in Null Algebra.

<u>3.2—Establishing the upper and lower bound for y, in $y = 1/x$:</u>

We have established for $1 = (1 - z)\mathbf{e}^{-z}$, where $z = \frac{k}{x}$, the only solution is $z = 0$.

Then: $0 = \frac{k}{x} \qquad k = 0 \cdot x \qquad$ And thus $k = 0$

And returning to our weighted equation we find thus:

$y = \frac{1}{x} \cdot \mathbf{e}^{-\kappa/x}$ and $\frac{dy}{du} = \left(\frac{1}{x^2} - \frac{k}{x^3}\right)\mathbf{e}^{-\kappa/x}$ both evaluated at $k = 0$

$y = \frac{1}{x} \cdot \mathbf{e}^{0/x} \qquad \frac{dy}{du} = \left(\frac{1}{x^2} - \frac{0}{x^3}\right)\mathbf{e}^{0/x}$

$y = \frac{1}{x} \cdot \mathbf{e}^{0} \qquad \frac{dy}{du} = \left(\frac{1}{x^2} - 0\right)\mathbf{e}^{0}$

$y = \frac{1}{x} \cdot 1$ $\qquad\qquad\qquad$ $\frac{dy}{du} = \left(\frac{1}{x^2}\right) \cdot 1$

$y = \frac{1}{x}$ $\qquad\qquad\qquad$ $\frac{dy}{du} = \frac{1}{x^2}$

We arrive at the exact same given equation of $y = \frac{1}{x}$, and $\frac{dy}{du} = \frac{1}{x^2}$ from which we calculated the critical values of x, where in the rate of Δy per unit of Δu meets or exceed c, the speed of light in Meters Per Second, our chosen scale.

So we can trust $x \approx \pm 0.00005775508$ are the limiting values where y reaches is absolute maximum and minimum? Yes. The weighed function which tapers toward the upper bound limit when $x \to 0$ is identical to the non-weighted functions we began with and used to find both the critical values of x and the absolute max and min values of y at those inputs. Since was specified a given that the system defined by $y = \frac{1}{x}$ is in meters, and $\frac{dy}{du} = c$ is the source of this limit we should expect the value to reach a sharp stopping value, the value associated with $\frac{dy}{du} = c$, which is exactly what is taking place.

For $y = \frac{1}{x}$ $\qquad$ The Domain may be expressed as

$$D: (-\infty, -0.0000577550080) \cup \{0\} \cup (0.0000577550080, \infty)$$

Then we may further define the sections outside the domain of the given function by the linear equation which intersects those points and the origin. Because we have established this above domain, the inclusion of 0 occurring by way of Null Algebra resolution, the only way the function can be continuous with in confines of algebra is a Piecewise Defined Function. As we shall see this function has unique property which suggests it is correct for inclusion.

Piecewise Defined Function: $\quad y = \frac{1}{x}$

Domain:

$$D = \begin{cases} for \quad (-\infty, -0.0000577550080) \cup \{0\} \cup (0.0000577550080, \infty)\ , & y = \dfrac{1}{x} \\ for \qquad [-0.0000577550080\ , 0.000057755008] & y = cx \end{cases}$$

For verification of this consider the following:

For $\quad P_1 = (x_1\,, y_1) = (-0.00005755008\,, -17{,}314.51582519)$

$$P_2 = (x_2\,, y_2) = (0.00005755008\,, 17{,}314.51582519)$$

$$y - y_2 = m(x - x_2) \qquad\qquad m = \frac{y_2 - y_1}{x_2 - x_1}$$

Using the known values for the critical points and the $y = \frac{1}{x}$ outputs we can solve for the linear equation linking these points.

$$y - 17{,}314.5158251904319708517744446988 = m \cdot (x - 0.00005755008)$$

$$m \quad = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7{,}314.51582519... + 7{,}314.51582519...}{0.00005755008 + 0.00005755008} \quad = \quad \frac{34{,}629.0316503808639417035488893976}{0.000115510016}$$

$$= 299{,}792{,}458.2607699053708342391189 4$$

$$= c$$

$$y - 17{,}314.5158251904319708517744446988 = c \cdot (x - 0.00005755008)$$

$$y - 17{,}314.5158251904319708517744446988 = cx - c \cdot 0.00005755008$$

$$y - 17{,}314.515825190431970851\ ... = cx - 17{,}314.515825190431970851\ ...$$

$$y = cx$$

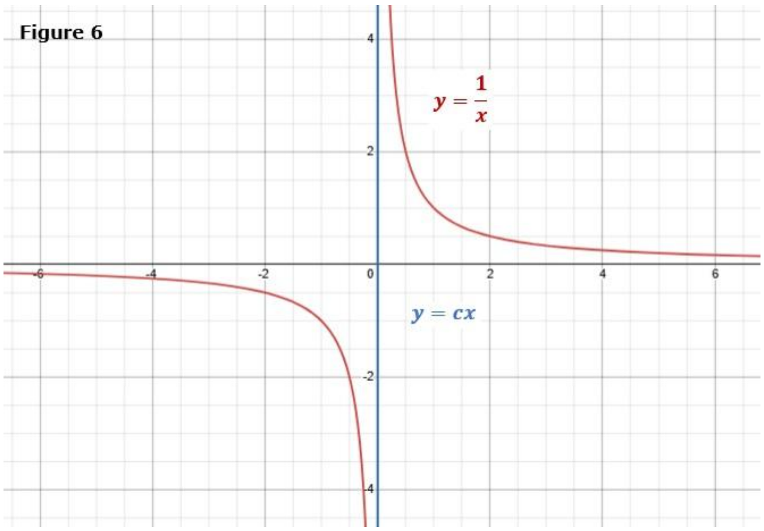
Some things to note is that this connecting piecewise defined function has the rate of change of y with respect to x

$$\frac{dy}{dx} = c\,\frac{d}{dx}x = c$$

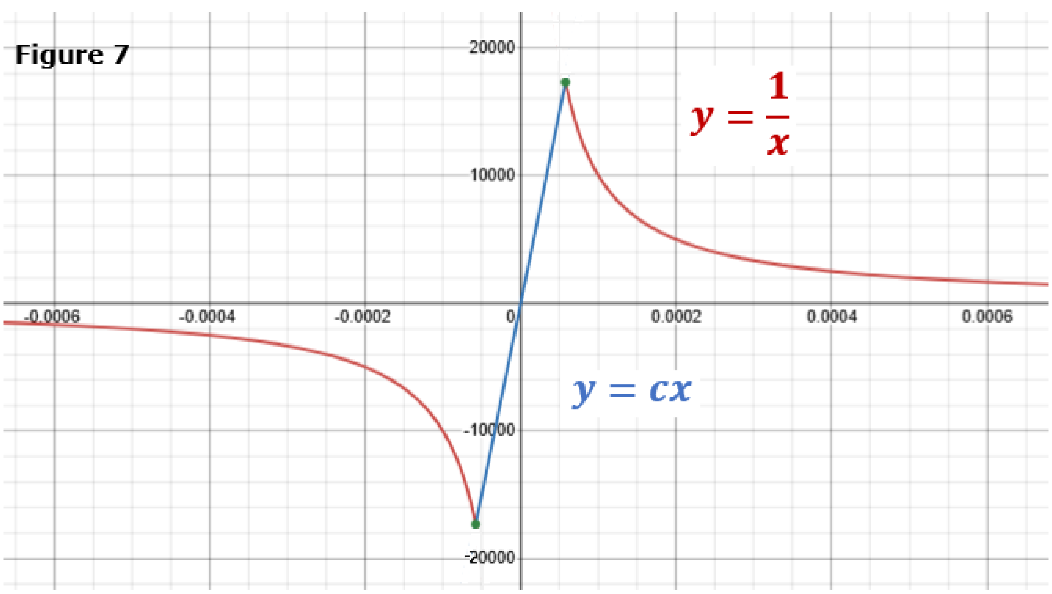
The rate of change of y with respect to x here is at the speed of light. So the y values on this equation actually reach the absolute max and min values, making the critical x value inputs included in its domain. It also naturally includes the value $y = 0$ when $x = 0$ a must as that is the resolved Null Algebra value for the same input in the given equation $y = \frac{1}{x}$. We must include the equation to connect the two halves of $y = \frac{1}{x}$ and the resolution point of (0 , 0) at the origina.

$$D: \begin{cases} for \quad (-\infty, -0.0000577550080) \cup \{0\} \cup (0.0000577550080, \infty) \ , \quad y = \dfrac{1}{x} \\ for \quad [-0.0000577550080, 0.000057755008] \quad y = cx \end{cases}$$

Although $x = 0$ is included in the domain of $y = \frac{1}{x}$ by way of Null Algebra resolutions it is also here included in the domain of $y = cx$ as well.



The graph of $y = cx$ connecting the absolute min -17,314.51582519… when x = -0.00057755008 and the absolute max at +17,314.51582519… is not a straight vertical line. It does have slope *c* and since all points are defined on the graph actually reaching their limits, it is continuous.



We can explain this thus. For the domain of $-0.0000577550080 \leq x \leq 0.000057755008$ in the function $y = \frac{1}{x}$ we are inside the light cone and use $y = cx$. Depending on the direction from which $x \to 0$ the output of the field or physical system defined, in this example, by the equation $y = \frac{1}{x}$ will hit that relativistic boundary within this piecewise defined domain for $y = cx$.

$x \to 0^{+}$ the energy levels of the system defined by $y = \frac{1}{x}$ are increasing and then collapse rapidly at the speed of light when $-0.0000577550080 \leq x \leq 0.000057755008$ plummeting until they stabilize and then begin to recover when $x < -0.0000577550080$. The same could be augured for $x \to 0^{-}$, where values defined by the system plummet rapidly until hitting a relativistic boundary and leaping back into the positive a velocity *c.*

If we consider $\frac{dy}{du}$ for $y = cx$ there is no violation for $\frac{dy}{du}$ actually reaching c, though it still cannot exceed the value. $\frac{dy}{dx} = c$. Here Δy per unit of change Δx is already moving at c. So the space of time Δu available for changes Δy may actually meet c within the defined boundary $y = cx$.

$$y = cx \qquad\qquad u = -\frac{1}{y} = -\frac{1}{cx}$$

$$\frac{dy}{du} = c \cdot \frac{d}{du}x \qquad\qquad \frac{du}{dx} = \frac{1}{c} \cdot \frac{d}{dx}\left(-\frac{1}{x}\right)$$

$$\frac{dy}{du} = c \cdot \frac{dx}{du} \qquad\qquad \frac{du}{dx} = \frac{1}{c}\left(\frac{1}{x^2}\right)$$

$$\frac{dy}{du} = c \cdot \frac{dx}{du} \qquad\qquad \frac{dx}{du} = cx^2$$

$$\frac{dy}{du} = c^2x^2$$

Look at what this gives us. We are again asking when does the change in y per slice of space of time available for those changes to occur within, meet or exceed the speed of light? This easily solved thus,

$$\frac{dy}{du} = c^2x^2 \qquad\qquad c = c^2x^2 \qquad\qquad \frac{1}{c} = x^2 \qquad\qquad x = \sqrt{\frac{1}{299{,}792{,}458}}$$

$$x \approx \pm 0.000577550080$$

This is the exact critical value we found for when $\frac{dy}{du}$ for $y = \frac{1}{x}$ must reach its absolute minimum and maximum values. These values for $\frac{dy}{du} = c^2x^2$ are extreme but finite.

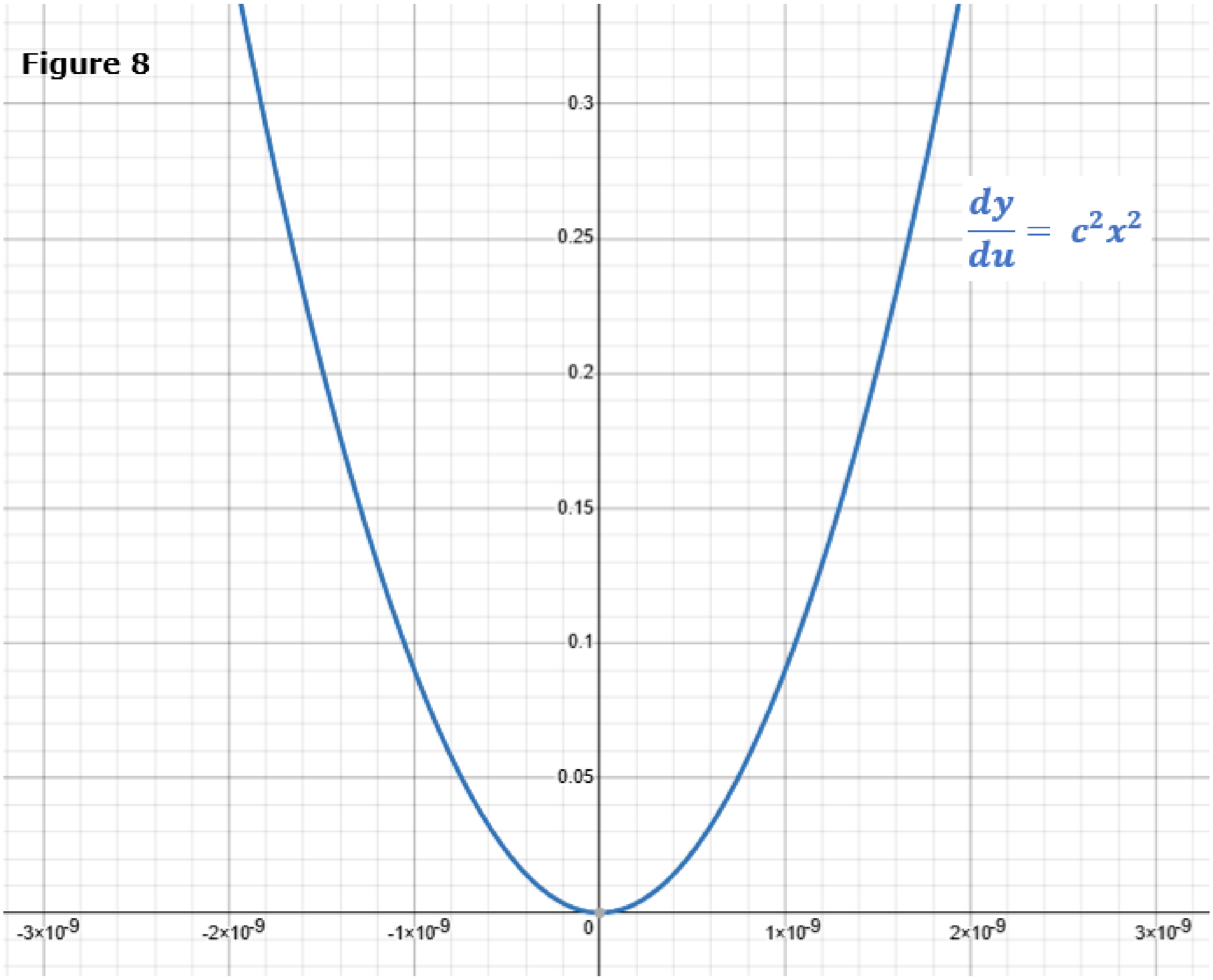


Figure 8 above shows $\frac{dy}{du} = c^2x^2$ is an extremely narrow parabola with an extreme slope. It shows us that, even though $y = cx$ has a slope where y has a rate of change with respect to x which progresses at c this is only maintained under auspices of the original function which allowed us to define that linear graph, $y = cx$, over the domain of $[-0.0000577550080\,, 0.000057755008\,]$. Anywhere out side this defined domain and the function $y = cx$ would propagate with $\frac{dy}{du}$, its output y with respect to the space of time, at a rate $> c$. It is at those very x values, $(-\infty\,, -0.0000577550080) \cup \{0\} \cup$

$(0.0000577550080\,, \infty)$ where we use the given equation $y = \frac{1}{x}$, preventing a violation of law of relativity in the value of $\frac{dy}{du}$.

Thus for a scale of Meters and time in seconds we have following:

For a system defined by the equation $y = \frac{1}{x}$

$$Domain: \begin{cases} for \quad (-\infty\,, -0.0000577550080) \cup \{0\} \cup (0.0000577550080\,, \infty)\ , & y = \dfrac{1}{x} \\ for \quad [-0.0000577550080\,, 0.000057755008] & y = cx \end{cases}$$

$$Range: [-17{,}314.515825190431970851\,, 17{,}314.515825190431970851]$$

4.1—Other Scales :

The values for when $\frac{dy}{du}$, or for that matter any real space axis with respect to its subspace axis, meet or exceed *c* is dependent upon the given equation, as a function of real space axes, and the scale chosen to define distance and time thereon. We would find different values for feet per second or miles per hour but being still based on relativistic limitation of the speed of light, those values will be equivalencies for boundaries we've just explored on the curve defined by $y = \frac{1}{x}$; the places where $\frac{dy}{du}$ meets or exceeds *c*. The methods shown in this paper may be used to locate these natural limits that occur near mathematical singularities for any equation.

The next most extreme scale we might choose to examine is the quantum scale, at Planck length. When a wave packet or, more generally an object, is confined to a small region we define that small region as Δx. Then by way of the Heisenberg uncertainty principle we know that object must have a minimum momentum $\Delta\rho$.

Consider that:

$$\hbar = \frac{h}{2\pi}$$

$$\Delta\rho = mv = \frac{\hbar}{2\Delta x} = \frac{\Delta\mathbf{E}}{c} \qquad\qquad \Delta x = \frac{\hbar}{2\Delta\rho} = \frac{\hbar}{2\Delta\frac{\Delta\mathbf{E}}{c}} = \frac{\hbar c}{2\Delta\mathbf{E}}$$

And, the maximum mass or energy permitted to exist in a space Δx before gravitational influence creates a blackhole, is governed by Quantum Wavelength. To apply relativistic limits to this we need to include the Schwarzschild Radius.

$$\Delta x = R_s = \frac{2 \cdot \mathbf{G} \cdot \Delta\mathbf{E}}{c^4}$$

Then we can resolve Δx as

$$\Delta x \cdot \Delta x = (\Delta x)^2 = \left(\frac{\hbar c}{2\Delta\mathbf{E}}\right) \cdot \left(\frac{2 \cdot \mathbf{G} \cdot \Delta\mathbf{E}}{c^4}\right) = \frac{\hbar\mathbf{G}}{c^3}$$

$$\Delta x = \sqrt{\frac{\hbar\mathbf{G}}{c^3}}$$

These constants are some of the most extreme in physics, of both very big and the very small.

Speed of Light: $c \approx 299{,}792{,}458\ {}^{M}/_{s}$ or $2.99792458 \times 10^8\ {}^{M}/_{s}$

So c^3 is massive: $c^3 \approx 26{,}944{,}002{,}417{,}373{,}989{,}539{,}335{,}912\ {}^{M^3}/_{s^3}$

or

$c^3 \approx 2.6944002417373989539335912 \times 10^{25}\ {}^{M^3}/_{s^3}$

Planck Constant: $h \approx 6.62607015 \times 10^{-34}\ \mathbf{J} \cdot \mathbf{s}$

Reduced Planck Constant: $\hbar = \frac{h}{2\pi} \approx 1.0545718176461565 \times 10^{-34}\ \mathbf{J} \cdot \mathbf{s}$

Newtonian Gravitational Constant: $\mathbf{G} = 6.67430(15) \times 10^{-11}\ {}^{M^3}/_{\mathbf{Kg}\,s^2}$

Plugging these values in for Δx

$$\Delta x = \sqrt{\frac{\hbar\mathbf{G}}{c^3}} = \sqrt{\frac{1.0545718176461565 \times 10^{-34}\mathbf{J} \cdot \mathbf{s} \cdot 6.67430(15) \times 10^{-11}\ {}^{M^3}/_{\mathbf{Kg}\,s^2}}{2.6944002417373989539335912 \times 10^{25}\ {}^{M^3}/_{s^3}}}$$

First, lets look at the units

$$\sqrt{\frac{\mathbf{J}\cdot\mathbf{s}\ \cdot M^3\cdot s^3}{\mathbf{Kg}\cdot\ s^2\ \cdot M^3}}=\sqrt{\frac{\frac{\mathbf{Kg}\,M^2}{s^2}\cdot M^3\cdot s^4}{\mathbf{Kg}\,s^2\,M^3}}=\sqrt{\frac{\mathbf{Kg}\ \cdot M^5\cdot s^4}{\mathbf{Kg}\ \cdot s^4\cdot M^3}}=\sqrt{M^2}=Meters$$

Then looking at the raw numbers:

$$\Delta x=\sqrt{\frac{1.0545718176461565\ \times\ 10^{-34}\ \cdot 6.67430(15)\ \times\ 10^{-11}}{2.6944002417373989539335912\ \times\ 10^{25}}}$$

$$\Delta x=\sqrt{\frac{7.03853026437346879718475\ \times\ 10^{-45}}{2.6944002417373989539335912\ \times\ 10^{25}}}$$

$$\Delta x=\sqrt{2.6122808910657218133511060760102\ \times\ 10^{-70}}$$

$$\Delta x=1.61625520604442967200228528769 11\ \times\ 10^{-35}$$

This is the Plank Length:

$$\Delta x\approx 1.6162552\ \times\ 10^{-35}\ Meters$$

Where Planck length is the idea of the smallest possible space, the smallest unit of time possible to traverse that distance, is the time it takes for light to cross it.

$$\text{Time}=\frac{\text{Distance}}{\text{Velocity}}\qquad t_p=\frac{\sqrt{\frac{\hbar\mathbf{G}}{c^3}}}{c}=\frac{\sqrt{\frac{\hbar\mathbf{G}}{c^3}}}{\sqrt{c^2}}=\sqrt{\frac{\frac{\hbar\mathbf{G}}{c^3}}{c^2}}=\sqrt{\frac{\hbar\mathbf{G}}{c^5}}$$

$$t_p=\sqrt{\frac{1.0545718176461565\ \times\ 10^{-34}\ \cdot 6.67430(15)\ \times\ 10^{-11}}{(2.99792458\ \times\ 10^8)^5}}$$

$$t_p=\sqrt{\frac{7.03853026437346879718475\ \times\ 10^{-45}}{2.4216061708512206534319783561111\ \times\ 10^{42}}}$$

$$t_p = \sqrt{2.9065544798720716810828084242136 \times 10^{-87}}$$

$$t_p = \sqrt{29.065544798720716810828084242136 \times 10^{-88}}$$

$$t_p = \sqrt{29.065544798720716810828084242136 \times 10^{-88}}$$

$$t_p = 5.3912470541351299504748891571 22 \times 10^{-44}$$

This is Planck Time

$$t_p = 5.391247 \times 10^{-44}\ Seconds$$

We can now go back to our given equation $y = \frac{1}{x}$ which we shall have the given, it represents the output of some physical system at quantum, sub-atomic scales. Don't get caught up thinking the closest we can get to origin, is $x = \pm 1.6162552 \times 10^{-35}$ in meters.

For $y = \frac{1}{x}$ Let $x = \pm 1.6162552 \times 10^{-35}$ then $y = \pm 6.1871424 \times 10^{34}$

As we'll see in a moment this value, with out function specific output limits in place, when $x = \pm 1.6162552 \times 10^{-35}$, $y \neq \pm 6.1871424 \times 10^{34}$

We need to examine $y = \frac{1}{x}$ from the perspective of Planck scale when $\frac{\Delta y}{\Delta u} = c$. The speed of light has its own sale of units of distance traveled per unit of time. At the Planck scale we have l_p which is in meters and t_p which is in seconds in our above calculations. So we're are looking for a limiting value of $\frac{dy}{du}$ in $\frac{Meters}{Second}$ and expressed by $\frac{l_p}{t_p}$.

$$\frac{l_p}{t_p} = \frac{\sqrt{\frac{\hbar \mathbf{G}}{c^3}}}{\sqrt{\frac{\hbar \mathbf{G}}{c^5}}} = \sqrt{\frac{\frac{\hbar \mathbf{G}}{c^3}}{\frac{\hbar \mathbf{G}}{c^5}}} = \sqrt{\frac{\hbar \mathbf{G} c^5}{c^3 \hbar \mathbf{G}}} = \sqrt{c^2} = c$$

Thus the limiting value here is again the speed of light even at this scale.

$u = -x$ $\frac{du}{dx} = -\frac{d}{dx}x = -1$	$\frac{dy}{du} = \frac{d}{du}x^{-1} = -x^{-2}\frac{dx}{du}$

$\frac{dx}{du} = \frac{-1}{1} = -1$	$\frac{dy}{du} = -x^{-2}\frac{dx}{du} = -x^{-2}(-1) = \frac{1}{x^2}$

Then setting the limiting condition:

$$\frac{dy}{du} = c = \frac{1}{x^2}$$

$$x \approx \pm 0.0000577550080$$

This is exactly the same limitation for the x value found above when we directly set the $\frac{dy}{du} = c$. Thus we can see that independent of chosen scale, $\frac{dy}{du} = c$ will allow us to calculate the limiting value for the independent variable x and the dependent variable y, limits which are already present in the relationship between x and y defined by the given equation, which in this example is $y = \frac{1}{x}$.

For $y = \frac{1}{x}$ the adjusted Domain and Range, assuming meters for x and y and seconds for u is:

$$Domain: \begin{cases} for \quad (-\infty, -0.0000577550080) \cup \{0\} \cup (0.0000577550080, \infty), & y = \frac{1}{x} \\ for \quad [-0.0000577550080, 0.000057755008] & y = cx \end{cases}$$

$$Range: [-17{,}314.515825190431970851, 17{,}314.515825190431970851]$$