How a Matter-to-Vacuum Phase Transition Can Prevent Black-Hole Singularities

Rudi Van Nieuwenhove

Independent Researcher

Dessel, Belgium

Abstract

The appearance of spacetime singularities during gravitational collapse remains one of the most important unresolved problems of General Relativity. In the standard picture, continued compression of matter inside a black hole leads to divergent densities, divergent spacetime curvature, and the breakdown of the classical gravitational description. In this work, we investigate an alternative phenomenological scenario in which gravitational collapse triggers a density-induced matter-to-vacuum phase transition once a critical density is reached.

We assume that above this threshold ordinary matter is rapidly converted into a Vacuum Localized Structure (VLS) phase, representing a localized vacuum state that carries the mass-energy of the original collapsing matter. The detailed microscopic mechanism responsible for the transition is not specified. Instead, the transition is treated as an effective process that replaces the collapsing matter by a compact vacuum structure while conserving the total mass-energy of the system.

The post-transition object is modeled as a static, spherically symmetric equilibrium configuration governed by the Einstein field equations and the Tolman-Oppenheimer-Volkoff equation. A Gaussian density distribution is adopted for the VLS phase, leading to finite central density, finite pressure, and a regular mass profile throughout the interior. The characteristic size of the VLS core follows directly from mass conservation and is uniquely determined by the total mass and the critical transition density.

The resulting solutions replace the classical black-hole singularity by a finite-curvature core. The mass function approaches zero smoothly at the center, preventing the divergent gravitational compression associated with conventional collapse solutions. Consequently, all curvature invariants remain finite and the spacetime remains regular throughout the interior region. In this sense, the phase transition acts as a gravitational regulator, replacing continued compression by a stable vacuum-supported equilibrium configuration.

Within this framework, singularity avoidance emerges without modification of the Einstein field equations and without the introduction of an explicit de Sitter vacuum core.

Keywords

General Relativity; phase transition; Regular black holes; Singularity avoidance; Gravitational collapse; Vacuum equation of state; Vacuum Localised Structures (VLS).

1. Introduction

The prediction of spacetime singularities is one of the most profound consequences of General Relativity. According to the singularity theorems of Penrose and Hawking, gravitational collapse under broad physical conditions inevitably leads to geodesic incompleteness, indicating the breakdown of the classical spacetime description [1-2]. In the standard picture of black-hole formation, continued compression of matter leads to arbitrarily large densities and spacetime curvature, culminating in a central singularity where the known laws of physics cease to be applicable.

Although General Relativity has been extraordinarily successful in describing gravitational phenomena over a vast range of scales, the appearance of singularities is generally regarded as signaling the limits of the classical theory rather than representing a physically realizable state of nature. The expectation that singularities should be avoided in a more complete description of gravity has motivated numerous theoretical approaches over the past decades.

One broad class of approaches modifies the gravitational field equations themselves. Examples include higher-curvature theories, $f(R)$ gravity, loop quantum gravity inspired models, and other extensions of General Relativity that become important at extreme densities or curvatures. In many such models, quantum-gravitational effects introduce an effective repulsive component that halts collapse before singular behavior can develop.

A second class of approaches retains the Einstein field equations but introduces new forms of matter or vacuum structure capable of supporting nonsingular equilibrium configurations. Examples include gravastars, regular black-hole models with vacuum cores, and other compact objects in which the classical singularity is replaced by a finite-density interior [3]. These models suggest that the vacuum itself may play a more active role in strong gravitational fields than is assumed in conventional black-hole solutions.

The present work belongs to this second category. Rather than modifying the Einstein equations, we investigate the possibility that gravitational collapse triggers a phase transition once a critical density is reached. We assume that above this threshold ordinary matter is rapidly converted into a new localized vacuum phase. The resulting vacuum state acts as a gravitational regulator, preventing the runaway compression responsible for singularity formation and replacing the classical singularity by a finite-curvature equilibrium core.

The objective of the present paper is to explore a possible formation mechanism for such structures. We propose that sufficiently strong gravitational compression may induce a matter-to-vacuum phase transition once a critical density is reached. The transition is assumed to proceed rapidly compared with the subsequent equilibrium evolution, leading to complete conversion of the collapsing matter into a localized vacuum state while preserving the total mass-energy of the system. The resulting configuration is then analyzed within the framework of standard General Relativity using the Einstein field equations and the Tolman-Oppenheimer-Volkoff equation.

The paper is organized as follows. Section 2 reviews the Einstein-TOV framework and the mechanism responsible for runaway gravitational compression in classical collapse. Section 3 introduces the matter-to-vacuum phase transition and derives the conditions required for suppression of central gravitational compression. Section 4 summarizes the properties of the resulting vacuum phase, while Section 5 develops a family of Gaussian equilibrium solutions. Numerical examples and astrophysical scaling relations are presented in Section 6, followed by a discussion of the implications and limitations of the model.

2. Mathematical Framework

2.1 Static Spherically Symmetric Geometry

We consider a static spherically symmetric spacetime with line element

$$ds^2 = -e^{2\Phi(r)}c^2dt^2 + \left(1-\frac{2Gm(r)}{c^2r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2). \tag{1}$$

Here $\Phi(r)$ is the gravitational redshift function and $m(r)$is the Misner-Sharp mass enclosed within radius r.

The Einstein field equations

$$G_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} \tag{2}$$

lead to the standard structure equations for a perfect fluid.

The mass equation is

$$\frac{dm}{dr} = 4\pi r^2\rho, \tag{3}$$

where ρ denotes the mass density.

The radial Einstein equation gives

$$\frac{d\Phi}{dr} = \frac{Gm+4\pi Gr^3P/c^2}{r(r\,c^2-2\,G\,M)}. \tag{4}$$

Conservation of stress-energy,

$$\nabla_\mu T^{\mu\nu} = 0, \tag{5}$$

yields the Tolman-Oppenheimer-Volkoff equation

$$\frac{dP}{dr} = -(\rho c^2+P)\frac{d\Phi}{dr}, \tag{6}$$

or equivalently

$$\frac{dP}{dr} = -\frac{G\left(\rho+\frac{P}{c^2}\right)\left(m+\frac{4\pi r^3P}{c^2}\right)}{r\left(r-\frac{2Gm}{c^2}\right)}. \tag{7}$$

Equations (1-7) completely determine a static spherical configuration once an equation of state is specified.

2.2 The Classical Singularity Problem

The tendency toward singular behavior in gravitational collapse can be understood directly from the Einstein-TOV system. For a static spherical fluid, hydrostatic equilibrium is governed by Equation (7).

Several features of this equation contribute to the tendency toward runaway gravitational compression.

First, the factor $\left(\rho+\frac{P}{c^2}\right)$ shows that pressure itself gravitates. Unlike Newtonian gravity, where only mass density contributes to the gravitational field, General Relativity treats pressure as an additional source of gravity. Increasing pressure therefore strengthens the gravitational attraction rather than merely opposing it.

Second, the quantity $m+\frac{4\pi r^3 P}{c^2}$ represents the effective gravitating mass enclosed within radius r. The pressure contribution again appears with a positive sign, increasing the strength of the gravitational field.

Consequently, as matter is compressed, the pressure required to maintain equilibrium rises. However, the increased pressure itself contributes to the gravitational source, requiring still larger pressure gradients. This feedback mechanism has no Newtonian analogue.

The TOV equation therefore contains an intrinsic positive feedback process. Higher densities require larger pressures for equilibrium, while larger pressures increase the effective gravitational source and demand still steeper pressure gradients. As the compactness increases, the magnitude of the pressure gradient grows rapidly, making it increasingly difficult to maintain hydrostatic equilibrium with finite density and pressure profiles.

This behavior is reflected in the well-known Buchdahl limit for isotropic fluid spheres [4], which places a lower bound on the radius of a stable configuration. Beyond this limit no regular static equilibrium solution exists. Continued collapse therefore drives the system toward the strong-field regime associated with black-hole formation.

The present work explores the possibility that this runaway compression process is interrupted by a matter-to-vacuum phase transition occurring above a critical density. The resulting vacuum phase is assumed to provide an alternative equilibrium state that suppresses further gravitational compression.

3. Matter-to-Vacuum Phase Transition and Central Regularity

3.1 Vacuum Localized Structures

The Vacuum Localized Structure (VLS) concept was introduced previously in the context of self-gravitating localized vacuum configurations and their possible astrophysical and cosmological implications [5-7]. In those studies, VLS objects were investigated as regular finite-mass solutions of the Einstein field equations and as possible alternatives to conventional dark-matter interpretations.

The present work adopts the VLS concept as the final equilibrium state of gravitational collapse. Unlike the previous investigations, which focused primarily on the properties of existing VLS configurations and their effects on galaxy rotation curves, the objective here is to explore a possible formation mechanism. We propose that sufficiently strong gravitational compression may trigger a phase transition of the vacuum once a critical density is reached. The resulting VLS phase then acts as a regulator of further collapse, replacing the classical singularity by a finite-curvature equilibrium core.

The analysis presented below is therefore complementary to earlier VLS studies. Rather than postulating the existence of vacuum localized structures (which were assumed to be generated shortly after the Big Bang), we investigate how such structures might arise naturally during black-hole formation.

The introduction of a critical transition density is motivated by the expectation that the physical vacuum may undergo qualitative changes under sufficiently extreme conditions. In quantum field theory, the vacuum is not an empty background but a dynamical medium whose properties can depend on external parameters such as temperature, density, and spacetime curvature [8,9]. Well-known examples include spontaneous symmetry breaking in the Higgs sector, vacuum polarization effects, and phase transitions associated with strongly interacting matter. In curved spacetime, quantum fields are expected to respond to the gravitational environment, leading to modifications of the vacuum state and its effective stress-energy content [8,9]. It has therefore been suggested that sufficiently strong gravitational fields may induce collective vacuum phenomena or vacuum condensate states that differ substantially from the ordinary vacuum observed under weak-field conditions [3].

Although no complete microscopic theory currently predicts the specific transition considered here, it is plausible that beyond a critical density the ordinary matter phase ceases to represent the energetically preferred configuration and is replaced by a new localized vacuum phase. In the present work this possibility is treated phenomenologically. The critical density is introduced as an effective parameter characterizing the onset of nonperturbative vacuum restructuring, without attempting a first-principles derivation of the underlying quantum mechanism. The subsequent analysis investigates the gravitational consequences of such a transition and the conditions under which it can replace the classical singularity by a finite-curvature equilibrium configuration.

To investigate the conditions required for a regular equilibrium configuration, we first consider a general two-phase system consisting of ordinary matter and a vacuum component.

The total effective density and pressure are written as

$$\rho_{\text{eff}} = \rho + \rho_V, \qquad (8)$$
$$P_{\text{eff}} = P + P_V.$$

For the ordinary matter component we adopt the equation of state

$$P = w\rho c^2, \qquad (9)$$

where w is assumed constant in the high-density regime.

For the VLS component we write

$$P_V = -\alpha\rho_V c^2, \qquad (10)$$

where α is a dimensionless parameter characterizing the vacuum phase.

The effective pressure is therefore

$$P_{\text{eff}} = (w\rho - \alpha\rho_V)c^2. \tag{11}$$

The Einstein field equations retain their standard form, $G_{\mu\nu} = \frac{8\pi G}{c^4} T^{\text{eff}}_{\mu\nu}$,

with the effective stress-energy tensor determined by the combined fluid.

3.2 Suppression of Central Gravitational Compression

For a regular spherical configuration, the mass function satisfies

$$m(r) = \frac{4\pi}{3}\rho(0)r^3 + O(r^5) \tag{12}$$

near the origin.

Substituting this expansion into the TOV equation (Equation (7))

and retaining leading-order terms yields

$$\frac{dP}{dr} = -\frac{G\left(\rho + \frac{P}{c^2}\right) 4\,\pi\, r^3 \left(\frac{\rho_{eff}}{3} + \frac{P_{eff}}{c^2}\right)}{r\left(r - \frac{2Gm}{c^2}\right)}. \tag{13}$$

The leading-order pressure gradient near the center is therefore proportional to $\rho_{\text{eff}} + 3\frac{P_{\text{eff}}}{c^2}$.

Substituting the equations of state gives

$$\rho_{\text{eff}} + 3\frac{P_{\text{eff}}}{c^2} = (1 + 3w)\rho + (1 - 3\alpha)\rho_V. \tag{14}$$

This combination determines the strength of the central gravitational compression.

A particularly interesting equilibrium regime occurs when this leading-order compression term vanishes. In that case, the gravitational force driving further central compression is suppressed to lowest order, requiring

$$(1 + 3w)\rho + (1 - 3\alpha)\rho_V = 0. \tag{15}$$

This relation represents a balance between the compressive contribution of ordinary matter and the opposing contribution of the vacuum phase.

At the same time, the Einstein equation (4) implies $\Phi'(0) = 0$, because they contain the same factor. Consequently, the redshift function possesses a regular extremum at the center.

The condition above therefore identifies a special class of equilibrium configurations for which the leading-order gravitational compression vanishes at the center. In the following section we examine the limiting case in which the ordinary matter component disappears entirely and the equilibrium configuration is supported solely by the vacuum phase.

3.3 Complete Matter-to-Vacuum Conversion

The preceding discussion allows both phases to coexist. However, the simplest interpretation of the phase transition is that ordinary matter does not survive once the critical density is exceeded. This assumption corresponds to a complete phase transition analogous to the disappearance of the original phase in conventional first-order transitions.

We therefore consider the limiting case in which the final equilibrium object consists entirely of the VLS phase. The ordinary matter density then vanishes, and the regularity condition (Equation (15)) reduces immediately to

$$1 - 3\alpha = 0, \tag{16}$$

giving

$$\alpha = \frac{1}{3}. \tag{17}$$

The vacuum equation of state becomes

$$P_V = -\frac{1}{3}\rho_V c^2. \tag{18}$$

This result is not imposed a priori but follows naturally from the requirement that the leading-order compression term in the Einstein equations vanishes in the pure-vacuum equilibrium state. The equation of state obtained below coincides with that previously found to play a central role in VLS equilibrium configurations [5-7].

The phase transition therefore acts as a gravitational regulator. Once the critical density is reached, ordinary matter is converted into a vacuum phase whose equation of state suppresses the mechanism responsible for runaway compression in the classical Einstein-TOV system.

4. Vacuum Equilibrium Structure

The phase transition described in the previous section is assumed to convert ordinary matter into a Vacuum Localized Structure (VLS) phase. Previous investigations have shown that VLS configurations can exist as localized solutions of the Einstein field equations with finite total mass and characteristic anisotropic stresses [5-7]. The present section briefly summarizes the properties relevant to the black-hole interior problem.

Unlike an ordinary perfect fluid, the VLS phase is described by an anisotropic stress-energy tensor

$$T^{\mu}_{\ \nu} = \mathrm{diag}(-\rho c^2, p_r, p_t, p_t), \tag{19}$$

where p_r and p_t denote the radial and tangential pressures respectively.

For an anisotropic fluid, conservation of stress-energy leads to the generalized Tolman-Oppenheimer-Volkoff equation

$$\frac{dp_r}{dr} = -(\rho c^2 + p_r)\Phi' + \frac{2}{r}(p_t - p_r). \tag{20}$$

The final term represents the contribution of anisotropic stresses and vanishes only for an isotropic fluid.

Following the earlier VLS studies, we adopt the radial equation of state $p_r = -\alpha \rho c^2$.

A negative radial pressure may appear unusual from the perspective of ordinary matter. However, negative pressures are common in several gravitational systems, including vacuum-energy dominated spacetimes, gravastars, and other anisotropic compact configurations. The existence of negative radial pressure therefore does not by itself imply instability.

The tangential pressure is determined by the equilibrium condition and generally differs from the radial pressure. As a result, the VLS phase possesses an intrinsically anisotropic internal structure.

An important quantity is the effective gravitational source

$$\rho_{\text{grav}} = \rho + \frac{p_r + 2p_t}{c^2}, \tag{21}$$

which governs the local generation of the gravitational field.

Previous VLS studies showed that the value $\alpha = \frac{1}{3}$ occupies a special position. At the center of the configuration the effective active density vanishes, eliminating the tendency toward either an attractive or repulsive gravitational core. The resulting structure lies precisely at the boundary between ordinary matter-like behavior and vacuum-dominated repulsive behavior.

In the present work this same value emerges independently from the regularity analysis of Section 3. In the limit of complete conversion of ordinary matter into the vacuum phase, the condition required to suppress the leading-order gravitational compression term reduces to $\alpha = \frac{1}{3}$.

The agreement between the two approaches suggests that the equation of state $p_r = -\frac{1}{3}\rho c^2$ is not an arbitrary assumption but a natural consequence of requiring a nonsingular vacuum-supported equilibrium state.

5. Gaussian Vacuum Equilibrium Solutions

Following the phase transition discussed in the previous sections, we assume that the resulting VLS reaches a static equilibrium state characterized by a smooth localized density distribution. Motivated by the regularity requirements at the center and by earlier investigations of VLS configurations [5-7], we adopt the Gaussian profile

$$\rho(r) = \rho_c \exp\left(-\frac{r^2}{R^2}\right), \tag{22}$$

where ρ_c denotes the critical transition density and simultaneously the central density of the equilibrium configuration.

The radial equation of state obtained in Section 3 is given by $p_r = -\frac{1}{3}\rho c^2$.

Hence

$$p_r(r) = -\frac{1}{3}\rho_c c^2 \exp\left(-\frac{r^2}{R^2}\right). \tag{23}$$

5.1 Tangential Pressure

The tangential pressure follows from the anisotropic equilibrium condition (Equation (20)).

Near the center, where $\Phi'(0) = 0$, the leading-order contribution gives

$$p_t = p_r + \frac{r}{2}\frac{dp_r}{dr}. \tag{24}$$

Using $p_r = -\frac{1}{3}\rho c^2$ and

$$\frac{d\rho}{dr} = -\frac{2r}{R^2}\rho, \tag{25}$$

one obtains

$$p_t(r) = \frac{1}{3}\rho_c c^2 \left(\frac{r^2}{R^2} - 1\right) \exp\left(-\frac{r^2}{R^2}\right). \tag{26}$$

Several important properties follow immediately.

At the center,

$$p_t(0) = p_r(0) = -\frac{1}{3}\rho_c c^2, \tag{27}$$

so the configuration becomes isotropic at the origin, as required for regularity.

The tangential pressure changes sign at $r = R$.

Thus the inner region is characterized by negative tangential pressure, while positive tangential pressure develops in the outer part of the structure.

5.2 Active Gravitational Density

The effective active gravitational density is

$$\rho_{\text{grav}} = \rho + \frac{p_r + 2p_t}{c^2} \tag{28}$$

Substituting the pressure expressions yields

$$\rho_{\text{grav}} = \frac{2}{3}\rho\frac{r^2}{R^2}. \tag{29}$$

Thus

$$\rho_{\text{grav}}(0) = 0. \tag{30}$$

The central gravitational source therefore vanishes despite the presence of a finite energy density.

This result provides the mathematical origin of the suppression of runaway gravitational compression. The VLS core contains energy density but does not generate the strong central active gravitational source that normally drives the Einstein-TOV system toward singular behavior.

5.3 Mass Profile

The enclosed mass follows from

$$m(r) = 4\pi \int_0^r \rho(s)\, s^2 ds. \tag{31}$$

For the Gaussian profile, the integral can be evaluated analytically:

$$m(r) = \pi^{3/2}\rho_c R^3 \operatorname{erf}\left(\frac{r}{R}\right) - 2\pi\rho_c R^2 r e^{-r^2/R^2}. \tag{32}$$

Near the center,

$$m(r) = \frac{4\pi}{3}\rho_c r^3 + O(r^5), \tag{33}$$

which guarantees regularity. For $r \to \infty$, the mass approaches

$$M = \pi^{3/2}\rho_c R^3. \tag{34}$$

5.4 Determination of the Core Scale

Because the phase transition is assumed to conserve total mass-energy, the asymptotic mass M equals the mass participating in the collapse.

Solving for the characteristic scale length using Equation (34) gives

$$R = \left(\frac{M}{\pi^{3/2}\rho_c}\right)^{1/3}. \tag{35}$$

The equilibrium configuration is therefore completely specified by two quantities:

- the total mass M,
- the critical transition density ρ_c.

No additional scale parameter is required.

The model predicts the scaling law $R \propto M^{1/3}$, which is characteristic of a family of compact objects formed at a common critical density.

6. Numerical Examples and Astrophysical Scaling

To illustrate the properties of the proposed phase transition model, we consider a representative critical density

$$\rho_c = 10^{20}\ \mathrm{kg\,m^{-3}}, \tag{36}$$

which is several orders of magnitude above nuclear density but remains far below the Planck density. The purpose of the present calculations is not to identify the precise transition density, which remains unknown, but rather to illustrate the scaling properties of the resulting Vacuum Localized Structures.

The characteristic VLS scale is given by Equation (35), while the central radial pressure is given by $p_c = -\frac{1}{3}\rho_c c^2$.

For the adopted critical density, $p_c = -3.0 \times 10^{36}$ Pa.

This central pressure is independent of mass and is therefore common to all VLS objects formed at the same transition density.

The chosen mass range spans the currently observed population of black holes. Stellar-mass black holes are observed with masses of a few to several tens of solar masses, while supermassive black holes in galactic nuclei range from approximately 10^6 to order 10^{10} in the most extreme observed systems [10,11]. The examples below therefore cover the known astrophysical range.

Table 1. Predicted Schwarzschild radii R_S, VLS core radii R_{core}, and radius ratios R_{core} / R_S for a critical transition density $\rho_c = 10^{20}$ kg m^{-3}.

Mass ($M_\odot$)	R_s (km)	R_{core} (km)	R_{core}/R_s
10	30	3.3	1.1×10^{-1}
100	300	7.1	2.4×10^{-2}
10^6	3.0×10^6	153	5.1×10^{-5}
10^9	3.0×10^9	1530	5.1×10^{-7}
10^{10}	3.0×10^{10}	3290	2.4×10^{-7}

Several interesting trends emerge from Table 1.

First, the Schwarzschild radius increases linearly with mass, whereas the VLS core radius follows the much weaker scaling $R \propto M^{1/3}$. The VLS core therefore occupies an increasingly small fraction of the black-hole interior as the mass increases.

For a stellar-mass black hole of ten solar masses, the VLS core radius is approximately one tenth of the Schwarzschild radius. In contrast, for a supermassive black hole of 10^9 solar masses, the Schwarzschild radius exceeds the VLS core radius by roughly a factor of two million.

Second, the model predicts that all black holes formed through the phase transition share the same maximum density and pressure, independent of mass. Larger black holes therefore do not contain denser cores; instead, they contain larger VLS equilibrium structures.

Third, the results imply that the region in which the phase transition modifies the classical Schwarzschild geometry is confined to a relatively small central volume. Outside the VLS core, the spacetime rapidly approaches the standard Schwarzschild solution. As a result, the external gravitational field remains essentially indistinguishable from that of a conventional black hole with the same mass.

Finally, the existence of a finite core radius implies that the curvature remains bounded throughout the spacetime. The phase transition therefore replaces the classical singularity by a compact finite-density region whose scale is determined entirely by the total mass and the universal transition density.

7. Discussion

The present model shares certain conceptual similarities with gravastar-type scenarios, in which gravitational collapse is halted by the emergence of a vacuum-dominated phase.

However, several important differences should be noted. Classical gravastar models typically assume a de Sitter-like vacuum interior separated from the external Schwarzschild geometry by a thin transition shell. In contrast, the VLS solutions obtained here possess a smooth Gaussian density profile and do not require the introduction of a thin shell or matching surface. The resulting spacetime is regular throughout the interior and is described by a single continuous equilibrium configuration. Furthermore, the phase transition considered in the present work differs conceptually from the vacuum-to-vacuum transition often invoked in gravastar models. Here, ordinary matter is assumed to undergo complete conversion into a localized vacuum phase once a critical density is reached, so that the final equilibrium object consists entirely of the VLS vacuum state while preserving the total mass-energy of the original collapsing system.

An important feature of the framework is that no modification of the Einstein field equations is required. The gravitational dynamics remain entirely those of General Relativity. The departure from the conventional picture arises solely through the introduction of a new high-density vacuum phase. In this respect, the model differs from many approaches to singularity avoidance that rely on modified gravity, higher-curvature corrections, loop quantum gravity effects, or other alterations of the Einstein equations.

The regularity of the resulting solutions follows from two related properties. First, the phase transition prevents further growth of the density beyond a critical value. Second, the vacuum equation of state obtained from the regularity analysis suppresses the leading-order active gravitational density near the center. As a consequence, the runaway compression mechanism inherent in the classical Einstein-TOV system is replaced by a finite equilibrium configuration.

In earlier studies on VLS [5-7], it was assumed that VLS were generated during the extreme conditions shortly after the Big Bang. The present work provides an additional possible formation mechanism for Vacuum Localized Structures, namely as the end product of gravitational collapse following a density-induced phase transition.

8. Conclusions

In this work, we have investigated a phenomenological model in which gravitational collapse triggers a phase transition once a critical density is reached. The central assumption is that ordinary matter is converted into a Vacuum Localized Structure (VLS) phase before the formation of the classical singularity predicted by General Relativity. The transition is assumed to proceed rapidly once the critical density is exceeded, so that the final equilibrium configuration consists entirely of the VLS phase.

Starting from the Einstein field equations and the Tolman-Oppenheimer-Volkoff equation for static spherical symmetry, we examined the conditions required for a regular equilibrium configuration. A general two-phase description consisting of ordinary matter and a vacuum component was first considered. Analysis of the central regularity conditions showed that suppression of the leading-order gravitational compression term requires a specific relation between the matter and vacuum contributions. In the limit of complete conversion into the vacuum phase, this condition leads naturally to the vacuum equation of state $p_r = -\frac{1}{3}\rho c^2$.

The resulting vacuum phase is intrinsically anisotropic and possesses finite density, finite pressure, and finite active gravitational mass density throughout the interior. Adopting a

Gaussian density profile, analytic expressions were obtained for the density distribution, pressure components, and mass profile. The resulting solutions replace the classical black-hole singularity by a finite-curvature core. The active gravitational density vanishes at the center, eliminating the runaway compression mechanism associated with the classical Einstein-TOV system. Consequently, all physical quantities remain finite throughout the spacetime.

An important feature of the framework is that singularity avoidance is achieved without modification of the Einstein field equations. The regularization mechanism arises instead from the formation of a new vacuum phase capable of supporting a stable equilibrium configuration. In this sense, the phase transition acts as a gravitational regulator that terminates collapse before divergent densities and curvatures can develop. Unlike gravastar models that rely on a de Sitter core and a thin transition shell, the present framework leads to a smooth vacuum-supported equilibrium configuration without internal discontinuities. The proposed mechanism is based on a complete conversion of ordinary matter into a localized vacuum phase above a critical density, providing a phenomenological route from gravitational collapse to a nonsingular pure-vacuum end state.

A particularly simple consequence of the model is that the characteristic size of the equilibrium VLS core is determined entirely by the total mass and the critical transition density, $R = \left(\mathrm{M}/\pi^{\frac{3}{2}}\rho_c\right)^{\frac{1}{3}}$. This relation predicts that all objects formed through the phase transition share the same maximum density and pressure, while their core size scales as $M^{1/3}$. Larger black holes therefore contain larger vacuum cores rather than higher central densities. The model thus replaces the classical singularity by a family of finite-density equilibrium configurations characterized by a universal density scale.

Several important questions remain open, including the microscopic origin of the phase transition, the stability of the resulting configurations, and the extension of the model to rotating systems. Nevertheless, the results demonstrate that a density-induced phase transition provides a mathematically consistent route toward nonsingular black-hole interiors within the framework of standard General Relativity.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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