

Gravity as an Emergent Force Via the Uncertainty Principle

Morgan W. Weinberg
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The core concept of this paper is the following relation between an accelerating field and time uncertainty: $a = 2c/\Delta t_u$. This relation—which distills from the derivation of Newton’s Law of Gravitation via the uncertainty principle—suggests that the force of gravity is carried by virtual gravitons. It’s shown, further, that the relation can be used to derive the Coulomb Force, perhaps indicating that it may be a general principle in nature. Taking this concept further, this paper draws a connection between expanding space and acceleration—offering an explanation for the missing-mass problem in galaxies. The paper goes on to draw a connection between the expansion of space and time uncertainty, leading to a startling conclusion: The expansion of space is not a function of the energy contained therein, but conversely, the mass of the universe is created *in concert* with the expansion of space. Further, because this expansion is a function of time, the mass of the universe is a function of time and increases in lockstep therewith. It’s deemed that the model of the universe presented herein obviates the need for dark matter and dark energy to explain our observations of the cosmos.

I. DERIVATION OF NEWTON’S LAW OF GRAVITATION

Why Newton? Why not general relativity? Einstein’s field equations reduce to the Newtonian limit for relatively low-energy gravitational systems—which constitute the bulk of the observable universe. Planetary motions in the Solar System are predicted by Newton’s Laws to a high degree of precision. This includes the perihelion precession of Mercury, which Newtonian mechanics predicts with roughly 93% accuracy. With the relativistic effect of 43 (arcsec cy⁻¹) [1] factored in, the predicted value is brought to within .2% of the measured precession. To put this in perspective, Mercury’s precession is measured in arc-seconds per *century*.

Considerably more interesting are the motions of bodies far away from massive objects, where accelerations are measured at 1E-9 m s⁻² and below. Importantly, this is the low-acceleration regime where galactic rotational-velocity curves start to diverge markedly from those predicted by Newtonian gravity (and general relativity). Observations repeatedly yield rotation curves that flatten out, which is in stark contrast to the expected profile—a Keplerian decline as seen in our Solar System. If Keplerian decline is expected in all gravitational systems, what’s so special about galaxies?

The prevailing explanation for this discrepancy is that the mass needed to resolve flat rotation curves must exist but has yet to be directly detected because it doesn’t interact with light—hence the term “dark matter”; but is this a problem of missing mass or missing physics?

The present thesis asserts that missing physics is at issue here, but before venturing down that road, a new take on gravity is presented that is based on the quantum principle of uncertainty. The aim is not to quantize gravity but to examine it in the context of the quantum realm and possibly gain new insights. Because we’re ultimately interested in low-energy systems and low-acceleration regimes, the following analysis is non-relativistic.

$$E_f = \oint \vec{g}_f \cdot d\vec{A} = mc^2 \quad (1)$$

It is posited that a mass m has a gravitational field $\vec{g}_f$ that, when integrated over a closed surface, is equivalent to the energy contained within the surface. In the rest frame of m , the net energy of this field E_f —as shown by Eq. (1)—is irrespective of distance. If m is a point mass, and the closed surface is a sphere with radius r , Eq. (1) simplifies to:

$$g_f = \frac{mc^2}{4\pi r^2} \quad (2)$$

It is further suggested that the field energy E_f cannot be infinitesimally divided. There is a lower bound to the elements that comprise E_f , which is found at the Planck scale. Multiplying field g_f times the Planck area a_p gives the smallest measurable unit of energy ΔE in the field at a given offset r from the point mass:

$$l_p = \sqrt{\frac{\hbar G}{c^3}} \quad (3)$$

$$a_p = \pi l_p^2 \quad (4)$$

$$a_p = \frac{\hbar G}{2c^3} \quad (5)$$

$$\Delta E = \frac{m \cancel{c^2}}{4\pi r^2} \frac{\hbar G}{2c^3} \quad (6)$$

$$\Delta E = \frac{m \hbar G}{8\pi r^2 c} \quad (7)$$

Eq. (3) defines the Planck length l_p [2], which, in turn, is used to define the Planck area a_p as given by Eq. (4). The definition of Planck area a_p here—with the inclusion of π as a scalar—is slightly different from that typically found in publications. With the inclusion of π , the reduced Planck constant $\hbar$ in Eq. (3) distills to its non-reduced form h in Eq. (5). If a_p is defined as a point, then ΔE represents the energy of gravitational field g_f at that

point. It is imagined that particles in a gravitational field interact with the field at these points.

With ΔE defined as the smallest measurable unit of energy in a field, the time needed to measure this energy—which is to say, the time Δt_u needed for a particle to sense this energy at a point in the field—is given by the uncertainty principle:

$$\Delta E \Delta t_u \geq \frac{h}{4\pi} . \quad (8)$$

The time uncertainty Δt_u is deemed to be an inherent property of a gravitational field—and possibly accelerating fields in general.

Plugging Eq. (7) into Eq. (8) gives the following relations:

$$\frac{m\hbar G}{8\pi r^2 c} \Delta t_u \geq \frac{\hbar}{4\pi} , \quad (9)$$

$$\frac{mG}{2r^2 c} \Delta t_u = 1 . \quad (10)$$

Notice that as Eq. (9) progresses to Eq. (10), the $\geq$ sign has been replaced with an equals sign. This will pin Δt_u to its minimal value—with the understanding that ΔE has also been defined as a minimum.

It's well established that a gravitational field—and information, in general—travels at c for all observers. This is embodied by Eq. (11) where t_f is the time it takes for the field to travel a distance of Δr :

$$\frac{\Delta r}{\Delta t_f} = c . \quad (11)$$

Multiplying Eq. (10) by Eq. (11) gives this:

$$\frac{mG}{2r^2 c} \Delta t_u c = \frac{\Delta r}{\Delta t_f} , \quad (12)$$

$$\frac{mG}{2r^2} = \frac{\Delta r}{\Delta t_f \Delta t_u} . \quad (13)$$

Non-relativistic acceleration is described by the familiar equation:

$$\Delta r = \frac{1}{2} a \Delta t_p^2 , \quad (14)$$

$$\Delta t_p^2 = \frac{2\Delta r}{a} . \quad (15)$$

In Eq. (15), Δt_p relates to a particle p accelerating at rate a . Substituting Δt_p^2 in Eq. (15) for $\Delta t_f \Delta t_u$ in Eq. (13) gives Newton's equation:

$$\frac{mG}{2r^2} = \frac{\Delta r a}{2\Delta r} , \quad (16)$$

$$a = \frac{mG}{r^2} . \quad (17)$$

With this substitution, an equivalence between the terms Δt_p^2 and $\Delta t_f \Delta t_u$ is being asserted that clearly needs to be unpacked. Fig. 1 shows a particle accelerat-

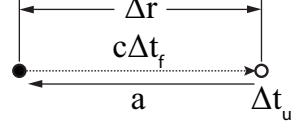


FIG. 1. Acceleration between discrete points in a field.

ing leftward through a minute distance Δr from one point in space to another. The points may be considered to be points in a field with the right point having a time uncertainty of Δt_u . The field, in the figure, is propagating left to right between the points—opposite the direction of acceleration—at c . These concepts, taken together, yield the following relations:

$$\begin{aligned} \Delta t_f &= \frac{\Delta r}{c} , \\ \Delta t_p^2 &= \frac{2\Delta r}{a} , \\ \Delta t_p^2 &= \Delta t_f \Delta t_u , \end{aligned} \quad (18)$$

$$\frac{2\Delta r}{a} = \frac{\Delta r}{c} \Delta t_u , \quad (19)$$

$$a = \frac{2c}{\Delta t_u} . \quad (20)$$

Plugging Eqs. (11) and (15) into Eq. (18) gives Eq. (19), which reduces to Eq. (20).

Applying direction to the magnitudes of acceleration and field speed—as shown in Fig. 1—gives the vector form of Eq. (20):

$$\vec{a} = -\frac{2\vec{c}}{\Delta t_u} . \quad (21)$$

In essence, Eqs. (18) through (21) relate the acceleration of a particle in a field to the field's speed of propagation and the time uncertainty of the field at the point coincident with the particle—where acceleration is opposite the direction of field propagation. The equivalence asserted in Eq. (18) is the finger in the dam that holds the above derivation of Newton's equation together. It's thus necessary and asserted to be correct. Eq. (15) relates Δt_p^2 to acceleration, which, along with Eq. (11) allow the Δr terms to cancel, leaving Eqs. (20) and (21).

Notably, Eq. (21) has a large numerator. Plugging Earth's near-surface gravity acceleration of 9.81 m s^{-2} into Eq. (21) gives a time uncertainty of 1.94 years.

Playing around with Eq. (10) also yields an interesting result. When the estimated mass of the universe— $1.5\text{E}53 \text{ kg}$ [3] and the estimated age of the universe— $13.8 \text{ billion years}$ [4] ($4.36\text{E}17 \text{ s}$) are plugged into Eq. (10), the diameter— $2r$ calculates to $1.71\text{E}26 \text{ m}$, which is less than an order of magnitude from the estimated diameter of the universe— $8.8\text{E}26 \text{ m}$.

In Eq. (21), as acceleration goes to zero, time uncer-

tainty goes to infinity. With Δt_u set equal to the age of the universe, acceleration a calculates to $1.38\text{E}-9 \text{ m s}^{-2}$. Coincidentally, this value marks the beginning of the low-acceleration regime mentioned above, where galactic-rotation curves start to become anomalous.

1. A General Relation?

Looking more closely at Eq. (21), Δt_u is a property of the field, and $\vec{c}$ is the field's propagation velocity relative to the point in the field where Δt_u is measured. It's thus posited that $\vec{a}$ is also a property of the field—specifically, a measure of the field's strength at this point. Particles interacting with the field will gain or lose energy according to $\vec{a}$ depending on whether they are moving with or against the field.

It just so happens that a particle initially at rest in the field—upon the transfer of energy from the field to the particle described by $\vec{a}$ —will *also* be kinematically accelerated according to $\vec{a}$. However, particles moving at higher velocities—approaching relativistic velocities—will experience noticeably less kinematic acceleration for a given quanta of energy transfer—as relativistic effects are now apparent. For example, the acceleration of light manifests as blueshift (or redshift depending on the direction relative to the field) vs. a change in speed.

However, Eq. (21) was derived via Eq. (14)—the non-relativistic relation for acceleration. Here Eq. (21) is appropriate because it correctly describes the motion of a particle—initially at rest relative to the point where Δt_u is measured—as it experiences constant acceleration across a **minute**, non-differential distance.

Eq. (21) also derives from the concept that Δt_u is sensed at points—where each point has an area on the Planck scale as given by Eq. (4). It's imagined that when a particle at rest interacts with the field at a point, energy is transferred to or from the particle in a quantum such that its acceleration can be considered constant across the minute distance between the point and the next point.

As mentioned, in the specific case where a particle is accelerated from rest relative to a point in the field, the kinematic acceleration is equivalent to the field strength $\vec{a}$. Thus, with Eq. (14) correctly describing the kinematic acceleration in this case, Eq. (21) gives the field strength. Further, Eq. (21) gives the field strength in general because this is a property of the field irrespective of the motions of the particles interacting with it.

Also worth noting, there is no reference to the field source in Eq. (21). In the derivation, the source of the field was a point mass, but in Eq. (21), there are no parameters that refer to this mass. Eq. (21) only has parameters that describe a field at the point of interaction with a particle. This implies that Eq. (21) is possibly a deeper, more fundamental relation from which gravity is just one manifestation. In the following sections, this relation will be used to derive the Coulomb force, modified

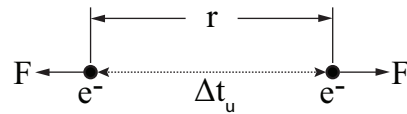


FIG. 2.

gravity, and the mass of the universe!

II. COULOMB FORCE

Fig. 2 shows two electrons separated by a distance r . To illustrate how Eq. (21) could be applied in general, this equation is applied to the electron pair in the figure to determine the Coulomb force urging the electrons apart. Using the model that the electrostatic force is carried by virtual photons, a virtual photon emitted between the electrons would have an energy given by the uncertainty principle:

$$\begin{aligned} \Delta E \Delta t &\geq \frac{\hbar}{2}, \\ \Delta E &\geq \frac{\hbar}{2\Delta t}. \end{aligned} \quad (22)$$

The “effective mass” m_e of this photon is as follows:

$$\begin{aligned} m_e &= \frac{\Delta E}{c^2}, \\ m_e &= \frac{\hbar}{2c^2\Delta t}. \end{aligned} \quad (23)$$

Because ΔE is a minimum, the $\geq$ sign in Eq. (22) can be replaced with an equals sign. With the application of Eq. (21), the reactive force on an electron emitting the virtual photon is thus:

$$\begin{aligned} F_p &= ma, \\ F_p &= m_e \frac{2c}{\Delta t_u}, \\ F_p &= \frac{\hbar}{2c\Delta t \Delta t_u}. \end{aligned} \quad (24)$$

The lifespan Δt and the uncertainty Δt_u of the virtual photon are equivalent and equate to r/c . Plugging this result into Eq. (24) gives the following:

$$\begin{aligned} F_p &= \frac{\hbar}{c\Delta t \Delta t_u}, \\ F_p &= \frac{\hbar c}{r^2}, \\ F_p &= \frac{\hbar c}{r^2}. \end{aligned} \quad (25)$$

Eq. (25) gives the reactive force F_p on an electron from the emission of a virtual photon. To translate this reactive force to the Coulomb force between the electrons, F_p is multiplied by the Sommerfeld constant α . This

constant relates the potential energy PE_c between two electrons at a distance r and the energy E_p of a photon of angular wavelength r where:

$$\begin{aligned}\alpha E_p &= PE_c, \\ \alpha \frac{\partial E_p}{\partial r} &= \frac{\partial PE_c}{\partial r}, \\ \alpha &= \frac{F_c}{F_p}.\end{aligned}$$

Taking the partial derivative of these energies with respect to r gives a relation between the Coulomb force and the force of a photon of energy E_p . With this relation, the Coulomb force is given by multiplying Eq. (25) by α —the latter defined below:

$$\begin{aligned}\alpha &= \frac{e^2}{4\pi\epsilon_0\hbar c}, \\ F_c &= \alpha \frac{\hbar c}{r^2}, \\ F_c &= \frac{e^2\hbar c}{4\pi\epsilon_0\hbar c r^2}, \\ F_c &= \frac{e^2}{4\pi\epsilon_0 r^2}.\end{aligned}\tag{26}$$

Eq. (26) is the familiar equation for the Coulomb force, where e is the charge of an electron and r is the distance between charges.

1. The Application of Eq. (21)

In the rest frame of an electron, a virtual photon with time uncertainty Δt_u and velocity c is emitted by the electron. With Eq. (21), the electron, at the time of emission, is accelerated by a —opposite in direction to c . In contrast to the case with gravity, Δt_u here is associated with a virtual photon—which is a quantum of the electromagnetic field—vs. the field itself. This expands the scope of Eq. (21) to include the quanta of fields that travel at c and have time uncertainty.

This raises a question: In a gravitational field, is the acceleration of a particle more precisely a consequence of fluctuations in the field? In other words, in the context of gravity, does Δt_u , in Eq. (21), refer to a virtual particle—perhaps, a virtual graviton? If the mechanism is similar to that of the virtual photon, it's imagined that, in the presence of a particle, a fluctuation in the gravitational field yields a massless graviton (that, by definition, travels at c) with a lifespan of Δt_u . The graviton travels in the direction of field propagation—causing the particle to accelerate in the opposite direction.

In Section I, the time uncertainty of Earth's near-surface gravitational field was calculated to be 1.94 years. If this is the lifespan of a virtual graviton—as described by the uncertainty principle—this would be an extremely low-energy particle.

If Eq. (21) can describe both gravitational and elec-

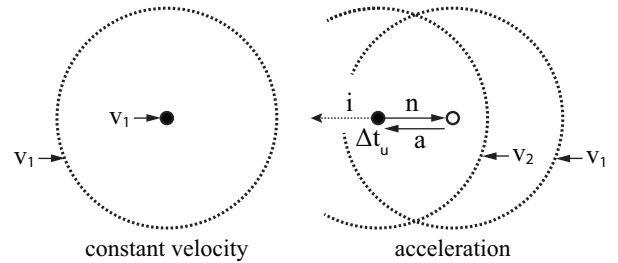


FIG. 3.

tromagnetic acceleration—which, on the surface, seems to be the case—the mechanism described by Eq. (21) could be fundamental—possibly bridging the gap between gravity and the other known forces.

Further, if, fundamentally, virtual gravitons mediate gravitational interactions, then the curvature of space-time, it seems, would have to emerge from quantum principles. Well beyond the scope of this paper—the unification of general relativity and quantum mechanics is an ongoing decades-long quest.

III. INERTIA

If Eq. (21) can describe acceleration from an external field, can it also elucidate the mystery of inertia?

Imagine two capsules in space, separated by a certain distance, at rest with respect to each other. The second capsule fires its engine and starts accelerating. From the vantage point of an observer in the second capsule, the first capsule looks to be moving, and its relative velocity seems to be increasing. Conversely, an observer in the first capsule sees the second capsule moving at an increasing velocity. From the perspective of either observer, the other is accelerating.

There is, however, additional information that these observers can use to determine who, in fact, is accelerating. The second observer, looking at the accelerometers in the cabin, could measure his rate of acceleration. The second observer—assuming that he was facing the direction of acceleration—would be pressed into his seat, which would be pushing against the inertia of his body. The first observer, looking at his instruments, would detect no such acceleration—remaining essentially weightless in his seat. Here, inertia is the source of truth. It informs which observer is *actually* accelerating.

At a certain point, the second observer looks out of his window and sees the first capsule moving at a velocity v_1 . Assuming that the first capsule and its gravitational field are at rest with respect to each other, the second observer would see the first capsule *and its field* moving at v_1 —as illustrated by the left diagram in Fig. 3. As the relative velocity increases, the first capsule and its field, which continue to remain at rest with respect to each other—from the perspective of the second observer, move in concert.

Does this hold in the opposite perspective—where the first observer is looking at the second capsule? If the engine of the second capsule were to be cut off, and it stopped accelerating, the capsule wouldn't have detached from its field. The second capsule and its field would be at rest with respect to each other. Therefore, even during acceleration, the second capsule's field must be accelerating along with the capsule.

It's posited that accelerating this field requires information flow—which, in effect, is how the second observer comes to realize that *he* is the one who is accelerating. Looking at Eq. (21), if this equation is run in reverse, acceleration coupled with time uncertainty—which is related to the measurable energy at a point in the field—can drive information flow in the field. Here, the acceleration of particles of and within the second capsule is informing the fields around those particles. Further, it's surmised that the inertia—the resistance to acceleration—experienced by the second observer as he is pressed into his seat is a result of that observer interacting with his *own* gravitational field. This interaction, during acceleration, is continuously re-informing his field.

In short, if gravity can be described as information flow driving acceleration, inertia can be thought of as acceleration driving information flow. Here, inertia is the price of re-informing the field around an accelerating mass. Note that Eq. (21) has no mass terms, and implicitly makes no distinction between gravitational mass and inertial mass. Further, there's no distinction between gravitational information flow and inertial information flow. Thus, it's imagined that Eq. (21) is compatible with the weak equivalence principle.

The right diagram in Fig. 3 shows a particle of the second capsule accelerating its own field from v_1 to v_2 . Acceleration $\vec{a}$ drives information flow $\vec{i}$ leftward into the field—in the direction of acceleration. This information flow, as measured by Δt_u , is such that the inertial acceleration—represented by $\vec{n}$ in the diagram—is equal and opposite to the applied acceleration $\vec{a}$. As described by Eq. (21), the direction of $\vec{n}$ is opposite that of information flow $\vec{i}$.

Δt_u , in this case, is not fine tuned. It's precisely what this parameter would evaluate to with $\vec{a}$ plugged into Eq. (21)—with the negative sign in the equation ignored. While seemingly a tautology, if acceleration drives information flow, it's logical that the same information flow would result in counter-acceleration of equal magnitude.

Also notice that time uncertainty in Eq. (21) is inversely proportional to acceleration, which means that as acceleration approaches infinity, time uncertainty approaches zero. Perhaps time uncertainty, here, can be thought of as the urgency with which information must flow, where higher rates of acceleration require higher rates of information transfer. As acceleration goes to zero, time uncertainty goes to infinity, indicating that information flow is no longer necessary.

1. Virtual Gravitons?

In the virtual-graviton scenario, it's imagined that the mechanism of inertia would be as follows: The acceleration of a particle in its own gravitational field causes a fluctuation in this field—that manifests as a virtual graviton with a lifespan of Δt_u . The graviton (with a lifespan of Δt_u)—traveling in the direction of $\vec{a}$, which is to say in the direction of information flow—causes inertia $\vec{n}$ equivalent in magnitude to $\vec{a}$ and opposite in direction.

IV. A GRAVITATIONAL FIELD IN EXPANDING SPACE

If photons are redshifted over time by expanding space as they propagate, can gravitational fields also be redshifted as they propagate? Are all propagating fields in expanding space subject to redshift? With respect to photons, cosmological redshift is literally the shifting of a photon's frequency to the red end of the spectrum as a consequence of its wavelength being stretched by expanding space. An observer of a redshifted photon will have noticed its energy having been decreased relative to its emitted frequency.

Gravitational fields, however, show no signs of dilution over time—with gravitational systems remaining stable over billions of years. Even at the outermost edges of galaxies, 1000s of light-years from their centers (that comprise the bulk of the galactic mass), there is no indication of a weakening field. In fact, the opposite is true; this field typically seems to have more strength than the mass of the galaxy can account for—as evidenced by flat rotation curves.

This is actually a clue. Unlike light, a gravitational field is intrinsically connected to its source. As the source translates or accelerates, the field must move in concert. If the source loses or gains energy, the field strength changes in proportion thereto. In short, the field requires a constant flow of information to maintain its connection with its source. With this in mind, it may be more correct to imagine a gravitational field in expanding space as being simultaneously stretched and re-informed, with the latter immediately undoing any sign of dilution. Furthermore, it's imagined that the tension between stretching and re-informing the field adds time uncertainty that, in turn, yields an additional acceleration component on top of the Newtonian term.

Eq. (7), restated below, gives the energy uncertainty at a point in a field. Taking the derivative with respect to time gives the following:

$$\begin{aligned} \Delta E &= \frac{mhG}{8\pi r^2 c}, \\ \frac{d\Delta E}{dt} &= \frac{-mhG}{4\pi r^3 c} \frac{dr}{dt}. \end{aligned} \quad (27)$$

The expansion of space is given by:

$$\vec{v} = \vec{r}H_l, \quad (28)$$

$$\frac{dr}{dt} = rH_l. \quad (29)$$

Eq. (28) is recognizable as Hubble’s law, which is usually written with the Hubble parameter H_0 —a measure of intergalactic expansion. Notice that Eqs. (28) and (29) use the parameter H_l , which is proffered as a local measure of expansion (or “expansion parameter”) at a specific point in a frame of reference. The case for the idea—that the expansion parameter measured by an observer is dependent on that observer’s frame of reference—will be built slowly over the course of the following sections. This will, hopefully, lead to a reconciliation of H_l and H_0 .

Plugging Eq. (29) into Eq. (27) gives the following:

$$\frac{d\Delta E}{dt} = \frac{-mhG}{4\pi r^2 c} H_l, \quad (30)$$

$$\frac{\Delta(\Delta E)}{\Delta t_u} = \frac{-mhGH_l}{4\pi r^2 c}, \quad (31)$$

$$\Delta E = \frac{mhGH_l}{4\pi r^2 c} \Delta t_u. \quad (32)$$

There’s a lot going on between Eq. (30) and Eq. (32) that needs to be deconstructed. Eq. (30) shows the energy at a point in a gravitational field that is decreasing over time as a result of being stretched in expanding space. In Eq. (31), $d(\Delta E)$ and dt are quantized to $\Delta(\Delta E)$ and Δt_u , with the presumption that any change in energy, at this scale, would be discrete over a finite period of time. In Eq. (32), $\Delta(\Delta E)$ is simplified to ΔE with the observation that the latter *is* the change in energy in question.

Also, notice that the negative sign has been stripped from the definition of ΔE . As discussed, the strength of this field, being a function of the source mass, does not dilute over time. If a mass is stable, its field must remain stable in concert. This means that a negative ΔE must be balanced with a positive ΔE as the field is re-informed. The ΔE s cancel, which means there is no net energy transfer to or from the field as a consequence of expanding space. It may be more correct to imagine ΔE here as a potential change of energy that never actually takes place—where expansion creates a certain tension in the field, so to speak, that manifests as energy and time uncertainty, which are positive numbers.

Plugging Eq. (32) into the uncertainty equation (Eq. (8)) gives the following:

$$\Delta E \Delta t_u = \frac{h}{4\pi}, \quad (33)$$

$$\frac{mhGH_l}{4\pi r^2 c} \Delta t_u^2 = \frac{h}{4\pi}, \quad (34)$$

$$\Delta t_u = \frac{r\sqrt{c}}{\sqrt{mGH_l}}. \quad (34)$$

As before, with ΔE and Δt_u defined as minima, the $\geq$ sign in the uncertainty equation has been replaced with an equals sign. Note that the time uncertainty Δt_u in Eq. (32) corresponds to Δt_u in Eq. (8), as they are both measuring the same energy. This is reflected in the Δt_u^2 term of Eq. (33). Solving for Δt_u gives Eq. (34)—a relation between time uncertainty Δt_u and local-frame expansion H_l .

Eq. (20) may then be used to convert this time uncertainty to acceleration:

$$a = \frac{2c}{\Delta t_u}, \quad (35)$$

$$a = \frac{2c\sqrt{mGH_l}}{r\sqrt{c}}, \quad (35)$$

$$a = \frac{2\sqrt{cH_l mG}}{r}. \quad (36)$$

The acceleration term defined by Eq. (36) is designated as the non-Newtonian or “expansion” component and is additive to the Newtonian term given by Eq. (17). Because the direction of field propagation is the same for both components, both accelerations point toward the field source. Combining these terms gives a complete equation for acceleration in the Newtonian limit:

$$a_n = \frac{mG}{r^2} + \frac{2\sqrt{cH_l mG}}{r}. \quad (37)$$

1. Virtual Gravitons?

The energy ΔE at a point in the gravitational field can be imagined as fluctuating between the redshifted value and the baseline—in Δt_u time—as the field is re-informed. In this framing of the case, it’s easy to imagine that this fluctuation could manifest as a virtual graviton with a lifespan of Δt_u , which would impart an additional component of acceleration to any interacting particle on top of baseline gravity. Consistency, here, would also necessitate that baseline gravity be carried by gravitons.

V. THE RELATION TO MOND

In Section VI, a more general form of Eq. (37) will be derived in the context of a system of particles. Before moving on, however, the similarity between Eq. (36) and the deep-MOND equation for acceleration should be discussed.

MOND (Modified Newtonian Dynamics), proposed by Mordehai Milgrom in 1983 [5] [6] [7], offers an alternative to dark matter as an explanation for the flattening of galactic-rotation curves. MOND is an algorithm that separates regimes of acceleration according to an empirically-derived constant a_0 . Gravitational accelerations far above a_0 remain Newtonian, whereas accelerations far below a_0 , termed the “deep-MOND regime”,

taper to the following relation:

$$a = \frac{\sqrt{a_0 m G}}{r}. \quad (38)$$

An interpolation function mathematically bridges the Newtonian and deep-MOND regimes. Milgrom's initial estimate of a_0 was slightly higher than the accepted value of $1.2\text{E}-10 \text{ m s}^{-2}$, originally estimated by Begeman et al. in 1991 [8]. Interestingly, a_0 has not materially wavered from this value in the ensuing decades since the early 1990s, even as the catalog of galaxies with measurable rotation curves has vastly expanded.

Note the congruence between Eqs. (38) and (36), with both having an acceleration $a \propto \sqrt{mG}/r$. Both of these equations satisfy the Baryonic Tully-Fisher Relation (BTFR), which holds that $v^4 \propto m$ [9] [10] [11]:

$$\frac{v^2}{\chi} = a = \frac{\sqrt{a_0 m G}}{\chi}, \quad (39)$$

$$v^4 = a_0 m G.$$

In this relation, v is the speed of a particle in circular orbit around a galaxy of mass m —at the outer edge where the Newtonian component is a small fraction of the net acceleration. The rotation curve, given by this relation, mathematically flattens out—as speed is independent of distance.

Three predictions stem from MOND: The rotation curves of isolated galaxies flatten out. The speed at the flat end of rotation curves raised to the power of four is proportional to the galaxy mass, and the constant a_0 demarks the transition to the non-Newtonian acceleration regime. These are specific predictions that leave MOND and the BTFR very exposed to falsification. Extensive galaxy surveys, however, repeatedly find MOND to be consistent with observations, and this holds for low-mass and high-mass galaxies with varying levels of surface brightness [12] [13] [14] [15] [16].

Another striking confirmation of the flatness prediction comes from a recent paper where the authors used weak-gravitational-lensing data from the KiDS survey to construct the extended rotation curves of isolated galaxies [17], which seem to remain flat even out to 1 Mpc with no sign of decline! This may prove to be disruptive to the mainstream understanding of the cosmos, which has traditionally been informed by rotation curves that typically extend out to tens of kpc.

Put succinctly, “If the universe is made of cold dark matter, why does MOND get any predictions right?” [18] It seems highly unlikely that the predictive successes of MOND are mere coincidences, but on the contrary, an admonition that our understanding of gravity in the low-acceleration regime is likely incomplete. In this respect, the BTFR and MOND serve as important guidestones along the path to a more complete comprehension of nature—on par with Kepler's discovery that the planets have elliptical orbits.

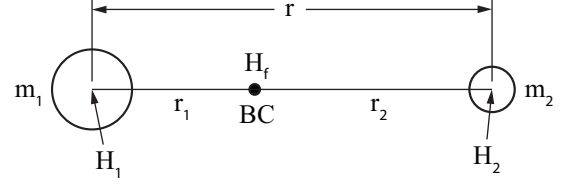


FIG. 4.

VI. TWO-BODY SYSTEM

Fig. 4 shows a two-body system with masses m_1 and m_2 , separated by distance r , with barycenter BC; and where m_1 is of larger mass than m_2 . With Eq. (37), the attractive force between the two masses is thus:

$$F_1 = m_1 \left(\frac{m_2 G}{r^2} + \frac{2\sqrt{cH_2 m_2 G}}{r} \right), \quad (40)$$

$$F_2 = m_2 \left(\frac{m_1 G}{r^2} + \frac{2\sqrt{cH_1 m_1 G}}{r} \right), \quad (41)$$

$$F_1 = F_2. \quad (42)$$

F_1 is the magnitude of the force on m_1 by m_2 's gravitational field, and vice versa for F_2 . F_1 and F_2 must be equivalent, as vectorially, they are opposed and must cancel according to Newton's Third Law of Motion—which is to say, the momentum of the system must be conserved.

H_2 , in Eq. (40), refers to the local expansion parameter at m_2 ; and conversely, H_1 , in Eq. (41), refers to the local expansion parameter at m_1 . For F_1 to be equivalent to F_2 , H_1 and H_2 must differ, which will shortly be clear.

Here, it's posited that the frame of reference comprising the m_1 – m_2 system has an expansion parameter H_f , where H_1 and H_2 are linear functions of H_f as follows:

$$H_1 = k_1 H_f, \quad (43)$$

$$H_2 = k_2 H_f, \quad (44)$$

where k_1 and k_2 are constants.

With these relations, equating F_1 and F_2 , gives the following:

$$\begin{aligned} m_1 \left(\frac{m_2 G}{r^2} + \frac{2\sqrt{ck_2 H_f m_2 G}}{r} \right) &= \\ m_2 \left(\frac{m_1 G}{r^2} + \frac{2\sqrt{ck_1 H_f m_1 G}}{r} \right) &, \\ \cancel{\frac{m_1 m_2 G}{r^2}} + \frac{2m_1 \sqrt{ck_2 H_f m_2 G}}{r} &= \\ \cancel{\frac{m_2 m_1 G}{r^2}} + \frac{2m_2 \sqrt{ck_1 H_f m_1 G}}{r} &, \end{aligned}$$

which, in turn, reduces to:

$$\frac{\frac{\mathfrak{Z}m_1\sqrt{\mathfrak{K}k_2H_f m_2\mathfrak{G}}}{\mathfrak{K}}}{m_1\sqrt{k_2m_2}} = \frac{\frac{\mathfrak{Z}m_2\sqrt{\mathfrak{K}k_1H_f m_1\mathfrak{G}}}{\mathfrak{K}}}{m_2\sqrt{k_1m_1}},$$

$$\sqrt{m_1k_2} = \sqrt{m_2k_1},$$

$$\frac{k_1}{k_2} = \frac{m_1}{m_2}. \quad (45)$$

Taking the ratio of Eqs. (43) and (44) and plugging Eq. (45) into the result, yields this:

$$\frac{H_1}{H_2} = \frac{m_1H_f}{m_2H_f}, \quad (46)$$

$$H_1 = km_1H_f, \quad (47)$$

$$H_2 = km_2H_f. \quad (48)$$

Eqs. (47) and (48) show that the expansion parameter at each mass is proportional to that mass. In separating these two equations from Eq. (46), a new constant k is introduced—noting that the k s would cancel upon retaking the ratio of H_1 and H_2 . Adding Eqs. (47) and (48) together gives this:

$$H_1 + H_2 = k(m_1 + m_2)H_f. \quad (49)$$

If the sum of the local expansion parameters in the frame is set equal to the expansion parameter of the frame H_f , Eq. (49) can be solved for k :

$$H_1 + H_2 = H_f, \quad (50)$$

$$1 = k(m_1 + m_2), \quad (51)$$

$$k = \frac{1}{m_1 + m_2}. \quad (52)$$

The correctness of Eq. (50) will become more evident as the discussion continues. With this relation for k , Eqs. (47) and (48) can be rewritten as functions of the frame's expansion parameter H_f and the masses in that frame:

$$H_1 = \frac{m_1}{m_1 + m_2}H_f, \quad (53)$$

$$H_2 = \frac{m_2}{m_1 + m_2}H_f. \quad (54)$$

What these relations are revealing is that the masses in a system govern the rates of expansion seen by the particles in that system. The expansion rate, however, given by Eq. (28)—which underpins the acceleration and force equations that follow—has no mass terms and is a function of distance! The next step is to reconcile Eqs. (53) and (54) with Eq. (28).

In Fig. 4, distances r_1 and r_2 are related to m_1 and m_2 as follows:

$$\frac{r_1}{r_2} = \frac{m_2}{m_1}. \quad (55)$$

This ratio holds regardless of the motions of these masses. Whether they are in circular orbits, eccentric orbits, or accelerating toward each other in a straight line, the barycenter of m_1 and m_2 maintains the ratio given by Eq. (55). Versions of this equation with r in the denominator of the distance ratio are deduced as follows:

$$\frac{r_1}{r_2} + 1 = \frac{m_2}{m_1} + 1,$$

$$\frac{r_1}{r_2} + \frac{r_2}{r_2} = \frac{m_2}{m_1} + \frac{m_1}{m_1},$$

$$\frac{r_1 + r_2}{r_2} = \frac{m_1 + m_2}{m_1},$$

$$\frac{r_2}{r} = \frac{m_1}{m_1 + m_2}, \quad (56)$$

$$\frac{r_1}{r} = \frac{m_2}{m_1 + m_2}. \quad (57)$$

Applying Eq. (28) to the two-body system of Fig. 4, gives the expansion rates at m_1 and m_2 :

$$v_1 = rH_2, \quad (58)$$

$$v_2 = rH_1. \quad (59)$$

With m_2 as an origin, space expanding from this point has an expansion rate of v_1 as measured at m_1 . This is embodied by Eq. (58). Likewise, Eq. (59) gives the expansion rate v_2 relative to m_1 —as seen by m_2 .

Here, it's postulated that the expansion rate v_1 measured at m_1 should be consistent at a given point in time, regardless of the chosen origin along the line of expansion. This would likewise hold for v_2 as measured at m_2 . In other words, **the expansion rate measured at a point in a reference frame—in a particular direction at a point in time—should be invariant with respect to an arbitrary origin of expansion along the line of measurement.**

Further, referring back to Eq. (27), the idea—that dr/dt (v_1 and v_2 in the present example) relative to a reference mass should be consistent—is required by this equation, which relates a physical effect $d\Delta E/dt$ to an expansion rate dr/dt . It doesn't make sense that this physical effect should change as a result of choosing different coordinates.

Thus, with the barycenter set as the origin of expansion, the following relations can be written:

$$v_1 = r_1H_f, \quad (60)$$

$$v_2 = r_2H_f. \quad (61)$$

Notice that Eqs. (60) and (61) are functions of H_f , the expansion parameter at the barycenter, which is the same origin for both observers m_1 and m_2 . Contrast this with the case above for Eqs. (58) and (59), where the observers see expansion rates from different perspectives and origins.

Equating Eqs. (58) and (59) with Eqs. (60) and (61)

distills as follows:

$$rH_2 = r_1H_f, \quad (62)$$

$$rH_1 = r_2H_f, \quad (63)$$

$$H_1 = \frac{r_2}{r}H_f, \quad (64)$$

$$H_2 = \frac{r_1}{r}H_f, \quad (65)$$

$$H_1 = \frac{r_2}{r_1}H_2. \quad (66)$$

Plugging Eqs. (56) and (57) into Eqs. (64) and (65), recovers Eqs. (53) and (54):

$$H_1 = \frac{m_1}{m_1 + m_2}H_f, \quad (67)$$

$$H_2 = \frac{m_2}{m_1 + m_2}H_f. \quad (68)$$

Thus, given a two-body system of masses m_1 and m_2 , with a frame of reference at the barycenter—with the frame having an expansion parameter of H_f , Eqs. (53) and (54) give the local expansion parameters at m_1 and m_2 , respectively. As demonstrated, these local expansion parameters are functions of barycenter distances, which in turn are functions of mass. Importantly, these relations allow the force vectors—on m_1 by m_2 and conversely, on m_2 by m_1 —to cancel, avoiding a possible inconsistency.

In somewhat of a leap, the above equations for a two-body system are generalized for an n-body system. Evidence to support this extrapolation will build in the following sections. The general form of Eqs. (53) and (54) is thus:

$$H_r = \frac{m_r}{m_t}H_f. \quad (69)$$

Eq. (69) relates the expansion parameter H_r at the barycenter of a reference mass m_r (or collection of masses) to that of another mass or collection of masses m_t , where H_f is the expansion parameter at the barycenter of m_t . m_t may include m_r , in which case, H_f is the expansion parameter at the barycenter of the entire system.

With Eqs. (37) and (69), a general relation for the magnitude of acceleration relative to a reference mass m_r , in an n-body system of total mass m_t , is as follows:

$$\begin{aligned} a_n &= \frac{m_r G}{r^2} + \frac{2\sqrt{cH_r m_r G}}{r}, \\ a_n &= \frac{m_r G}{r^2} + \frac{2m_r \sqrt{cH_f G}}{r\sqrt{m_t}}, \\ a_n &= \frac{m_r G}{r} \left(\frac{1}{r} + \frac{2\sqrt{cH_f}}{\sqrt{m_t G}} \right). \end{aligned} \quad (70)$$

With the arguments surrounding Eqs. (60)–(66), Eq. (50) is assumed to have been correct. Extending

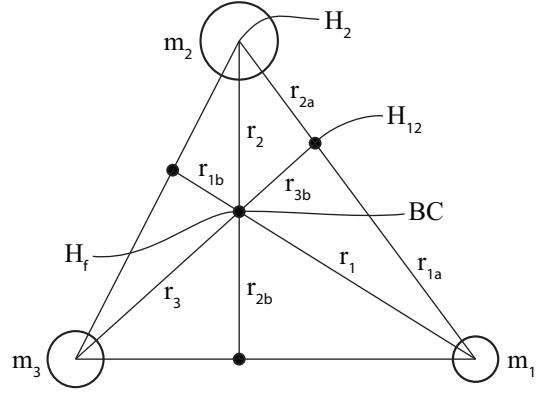


FIG. 5.

this relation to the general case gives this:

$$H_f = \sum_{i=1}^n H_i. \quad (71)$$

Eq. (71) shows that in a system of n bodies with an expansion parameter of H_f at the barycenter, the sum of local expansion parameters for these bodies is equivalent to H_f .

VII. THREE-BODY SYSTEM

As a quick sanity check of the n-body-system equations, the concepts from the last section are applied to a three-body system and checked for consistency. Fig. 5 shows a three-body system of dissimilar masses. The center dot in the triangle marks the barycenter BC of the system, and the dots on the edges denote the centers of mass of the two bodies along each edge. The relations between distances and masses in this figure can be written as follows:

$$\frac{r_3}{r_{3b}} = \frac{m_1 + m_2}{m_3}, \quad (72)$$

$$r_{3c} = r_3 + r_{3b}, \quad (73)$$

$$r_{3c} = \left(\frac{m_1 + m_2}{m_3} \right) r_{3b} + r_{3b}, \quad (74)$$

$$r_{3c} = \left(\frac{m_1 + m_2}{m_3} + \frac{m_3}{m_3} \right) r_{3b}, \quad (75)$$

$$\frac{r_{3b}}{r_{3c}} = \frac{m_3}{m_1 + m_2 + m_3}, \quad (76)$$

$$\frac{r_3}{r_{3c}} = \frac{m_1 + m_2}{m_1 + m_2 + m_3}. \quad (77)$$

Equating rates of expansion at m_3 along the line ex-

tending from H_{12} gives this:

$$v_3 = (r_3 + r_{3b})H_{12} , \quad (78)$$

$$v_3 = r_3 H_f , \quad (79)$$

$$H_{12} = \frac{r_3}{r_3 + r_{3b}} H_f . \quad (80)$$

Plugging Eq. (77) into Eq. (80) yields this:

$$H_{12} = \frac{m_1 + m_2}{m_1 + m_2 + m_3} H_f . \quad (81)$$

With H_{12} marking the barycenter of m_1 and m_2 , distance is related to mass as follows:

$$\frac{r_{1a}}{r_{2a}} = \frac{m_2}{m_1} , \quad (82)$$

$$\frac{r_{1a}}{r_{1a} + r_{2a}} = \frac{m_2}{m_1 + m_2} . \quad (83)$$

With a definition for H_{12} , the expansion parameter at m_2 can also be determined via the expansion rate at m_1 :

$$v_1 = (r_{1a} + r_{2a})H_2 , \quad (84)$$

$$v_1 = r_{1a}H_{12} , \quad (85)$$

$$H_2 = \frac{r_{1a}}{r_{1a} + r_{2a}} H_{12} . \quad (86)$$

Plugging Eqs. (81) and (83) into Eq. (86) is thus:

$$H_2 = \frac{m_2}{\cancel{m_1} + m_2} \frac{\cancel{m_1} + m_2}{m_1 + m_2 + m_3} H_f ,$$

$$H_2 = \frac{m_2}{m_1 + m_2 + m_3} H_f . \quad (87)$$

Note the congruency between this equation and the general local-expansion relation given by Eq. (69), where H_2 is the expansion parameter at reference mass m_2 . With Eq. (69) satisfied, agreement with Eq. (70) follows when H_2 is plugged into Eq. (37):

$$m_t = m_1 + m_2 + m_3 ,$$

$$H_r = \frac{m_2}{m_t} H_f ,$$

$$a_n = \frac{m_2 G}{r^2} + \frac{2\sqrt{cm_2 H_f m_2 G}}{r\sqrt{m_t}} ,$$

$$a_n = \frac{m_2 G}{r} \left(\frac{1}{r} + \frac{2\sqrt{cH_f}}{\sqrt{m_t G}} \right) .$$

Agreement with Eq. (71) is also observed upon taking the sum of the expansion parameters at the three masses. Note, the relations H_1 and H_3 below are readily derived

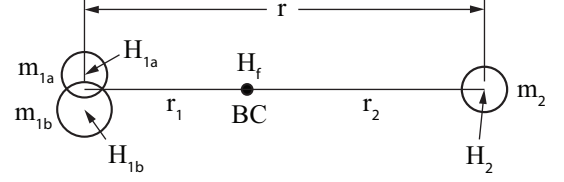


FIG. 6.

with the same approach used for H_2 :

$$H_1 = \frac{m_1}{m_t} H_f ,$$

$$H_2 = \frac{m_2}{m_t} H_f ,$$

$$H_3 = \frac{m_3}{m_t} H_f ,$$

$$H_1 + H_2 + H_3 = \frac{\cancel{m_t}}{\cancel{m_t}} H_f = H_f .$$

Here, the above n-body relations are recovered.

VIII. SPLIT TWO-BODY SYSTEM

Fig. 6 has the same two-body system of Fig. 4 with mass m_1 having been split into two unequal parts, m_{1a} and m_{1b} . In the figure, visually, the two parts are slightly offset. Mathematically, however, assume they are coincident. Using Eq. (70), the magnitude of the forces between m_{1a} , m_{1b} , and m_2 is thus:

$$m_t = m_{1a} + m_{1b} + m_2 ,$$

$$F_{1a2} = \frac{m_{1a} m_2 G}{r} \left(\frac{1}{r} + \frac{2\sqrt{cH_f}}{\sqrt{m_t G}} \right) ,$$

$$F_{1b2} = \frac{m_{1b} m_2 G}{r} \left(\frac{1}{r} + \frac{2\sqrt{cH_f}}{\sqrt{m_t G}} \right) ,$$

$$F_{12} = \frac{(m_{1a} + m_{1b}) m_2 G}{r} \left(\frac{1}{r} + \frac{2\sqrt{cH_f}}{\sqrt{m_t G}} \right) .$$

Looking at the above equations, the individual force on an element of m_1 is proportional to its mass, and the sum of those forces equates to the net force F_{12} between m_1 and m_2 , as consistency would demand.

IX. RESTRICTED THREE-BODY SYSTEM

Fig. 7 shows a restricted three-body system with an isolated two-body system orbiting a relatively large mass. This is the relationship between the Earth, Moon, and the Sun. The force between the Earth and the Moon as calculated in the three-body reference frame, which has an expansion parameter of H_{f1} at the Sun–Earth–Moon

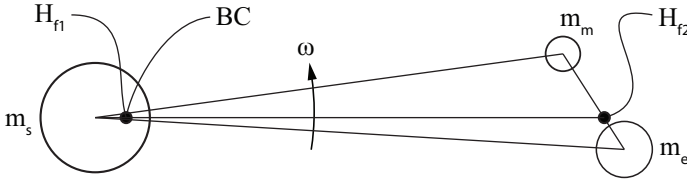


FIG. 7.

barycenter, is thus:

$$F_{em} = \frac{m_e m_m G}{r} \left(\frac{1}{r} + \frac{2\sqrt{cH_{f1}}}{\sqrt{(m_s + m_e + m_m)G}} \right). \quad (88)$$

In the Earth–Moon reference frame, with an expansion parameter of H_{f2} , the force between the Earth and the Moon is this:

$$F_{em} = \frac{m_e m_m G}{r} \left(\frac{1}{r} + \frac{2\sqrt{cH_{f2}}}{\sqrt{(m_e + m_m)G}} \right). \quad (89)$$

It should be necessary that F_{em} , as calculated in either reference frame, should be equivalent. Referring back to Fig. 5, which shows a three-body system with no clear subsystem, the force between masses m_1 and m_2 —as calculated via Eq. (70)—is consistent in both the two-body and three-body frames:

$$\begin{aligned} H_{12} &= \frac{m_1 + m_2}{m_1 + m_2 + m_3} H_f, \\ F_{12a} &= \frac{m_1 m_2 G}{r} \left(\frac{1}{r} + \frac{2\sqrt{cH_{12}}}{\sqrt{(m_1 + m_2)G}} \right), \\ F_{12b} &= \frac{m_1 m_2 G}{r} \left(\frac{1}{r} + \frac{2\sqrt{cH_f}}{\sqrt{(m_1 + m_2 + m_3)G}} \right), \\ F_{12a} &= F_{12b}. \end{aligned}$$

Recall that this derives mathematically by enforcing consistency in the measurement of expansion rates with respect to arbitrary origins of expansion along a given vector. There doesn't seem to be any reason why this consistency should not be applicable in the case where two bodies of Fig. 5 happen to comprise a subsystem. Said another way, as shown by the above equations, the force between two bodies—as calculated in either the two- or three-body frames—is a function of all three masses and doesn't care how these bodies are moving relative to each other or whether they're part of a subsystem or not. Imagine a three-body system with chaotic motions where two bodies split off to form a subsystem. The forces between these bodies should be consistent immediately before and after the formation of this subsystem.

With this understanding, Eq. (88)—the relation for F_{em} in the three-body reference frame—and Eq. (89)—the relation for F_{em} in the two-body subsystem—can be considered equivalent. This, in turn, allows the follow-

ing relation between the expansion parameter H_{f1} of the three-body frame and the parameter H_{f2} of the two-body frame to be written:

$$\begin{aligned} \frac{H_{f2}}{m_e + m_m} &= \frac{H_{f1}}{m_s + m_e + m_m}, \\ H_{f2} &= \frac{m_e + m_m}{m_s + m_e + m_m} H_{f1}. \end{aligned} \quad (90)$$

As expected, Eq. (90) parallels Eq. (81)—the equation above for H_{12} . Further, both Eqs. (81) and (90) are congruent with Eq. (69), restated below, when the reference mass m_r , in this equation, comprises two of the three bodies.

$$H_r = \frac{m_r}{m_t} H_f.$$

The observation that m_r in Eq. (69) can refer to a subsystem may not seem that interesting, but this actually has an important ramification—external mass can affect the gravitational forces within subsystems! Looking at Eq. (88), the presence of the Sun's mass m_s affects the force *between* the Earth and the Moon. It doesn't affect the Newtonian component of acceleration, which is purely a function of the Earth's and Moon's masses. It does, however, affect the expansion component, which is muted by the presence of the Sun. In actuality, it's muted by the total mass of the Solar System, which comprises the Sun, the planets, and the asteroid belt!

But the Solar System is a subsystem of the Milky Way, and therefore, the mass of the Milky Way must affect the forces within the Solar System—drastically reducing the expansion component relative to the Newtonian component. Traversing further up the hierarchy, the Milky Way is a member of the Local Group, which is, in turn, a member of the Virgo Supercluster, which is part of the Laniakea Supercluster complex.

To what extent does external mass affect internal systems? The universe has a lot of mass. Does all of this mass count? As the total system mass m_t in Eq. (70) goes to infinity, the expansion component asymptotically goes to zero. If m_t were set to the net mass of the universe, this effectively would be the case—any non-Newtonian acceleration components in galaxies and clusters of galaxies would be reduced to insignificance. However, we know that galaxies have flat rotation curves, and there are clusters where galaxies are moving too quickly to be bound by Newtonian gravity. If non-Newtonian motion is to be explained by modified gravity—specifically, as it's described by Eq. (70), it's not possible that the expansion parameters of galaxies and clusters are diluted by the entire mass of the universe.

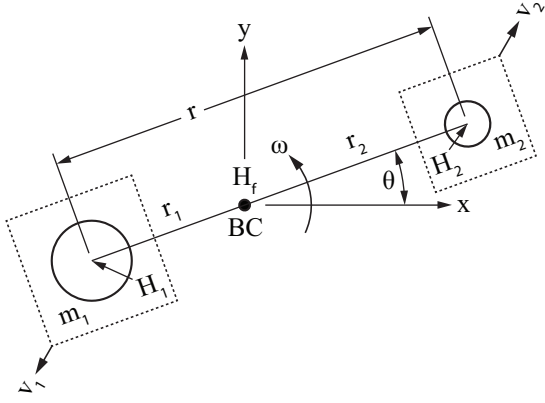


FIG. 8.

X. EXPANSION PARAMETERS OF MOVING FRAMES

In the above relations, expansion parameters only ever equate to other expansion parameters—via distance or mass. There is no relation that describes expansion in a frame purely as a function of other metrics. So far, the mechanism by which expansion in a frame is manifested is unknown. Understanding this mechanism and how it relates to the expansion of the universe will be necessary to delve deeper into the mystery of flat galactic-rotation curves and the motions of galaxies with respect to each other.

Fig. 8 shows the same two-body system from Fig. 4 with each body having a coincident frame that moves relative to the parent frame—the latter fixed at the barycenter BC of the system. The x and y coordinates of frame two—which is coincident with the barycenter of m_2 —relative to the BC frame are thus:

$$\begin{aligned} x &= r_2 \cos \theta, \\ y &= r_2 \sin \theta. \end{aligned}$$

Taking the derivative of x and y with respect to time gives the following:

$$\begin{aligned} \dot{x} &= \dot{r}_2 \cos \theta - r_2 \omega \sin \theta, \\ \dot{y} &= \dot{r}_2 \sin \theta + r_2 \omega \cos \theta, \end{aligned}$$

where

$$\omega = \dot{\theta}.$$

Taking the sum of the squares of $\dot{x}$ and $\dot{y}$ gives the square of the speed of frame two relative to BC:

$$v_2^2 = \dot{x}^2 + \dot{y}^2, \quad (91)$$

$$v_2^2 = \dot{r}_2^2 + (r_2 \omega)^2. \quad (92)$$

Notice here, in Eq. (92), that speed v_2 squared is simply the sum of the squares of the radial and transverse components of vector $\vec{v}_2$.

Recall Eq. (55), which gives a relation between distance and mass for this two-body system. Taking the derivative of this equation with respect to time is thus:

$$\begin{aligned} r_2 &= \frac{m_1}{m_2} r_1, \\ \dot{r}_2 &= \frac{m_1}{m_2} \dot{r}_1. \end{aligned}$$

Plugging these equations into Eq. (92) gives this:

$$v_2 = \frac{m_1}{m_2} \sqrt{\dot{r}_1^2 + (r_1 \omega)^2}.$$

With r_1 substituted for r_2 , the speed of frame one—which is coincident with the barycenter of m_1 —relative to the BC frame is similar to v_2 :

$$v_1 = \sqrt{\dot{r}_1^2 + (r_1 \omega)^2}.$$

Taking the ratio of the above two speed equations gives a result that intuitively parallels Eq. (55):

$$\frac{v_2}{v_1} = \frac{m_1}{m_2} \frac{\sqrt{\dot{r}_1^2 + (r_1 \omega)^2}}{\sqrt{\dot{r}_1^2 + (r_1 \omega)^2}}, \quad (93)$$

$$\frac{v_1}{v_2} = \frac{m_2}{m_1}. \quad (94)$$

Akin to Eq. (55), Eq. (93)—wherein the radial and transverse components of $\vec{v}_1$ and $\vec{v}_2$ cancel—shows that the ratio of v_2 to v_1 is irrespective of the paths of motion. Eq. (93) distills to Eq. (94), which shows that the relative speed between bodies one and two (and thus frames one and two) is purely a function of their masses.

Again, Eq. (94) is a natural extension of Eq. (55). If the distances relative to the barycenter of a system are a function of mass, then it follows that the derivatives of those distances—specifically, the derivatives of the distance vectors—with respect to time should obey the same relation. With this in mind, in general, if expansion parameters can be related via distance, they should also be related via speed.

Eq. (69) can be used to derive the ratio between the expansion parameters at m_1 and m_2 :

$$H_1 = \frac{m_1}{m_t} H_f, \quad (95)$$

$$H_2 = \frac{m_2}{m_t} H_f, \quad (96)$$

$$\frac{H_1}{H_2} = \frac{m_1}{m_2}, \quad (97)$$

$$H_1 = \frac{m_1}{m_2} H_2. \quad (98)$$

Similar to Eq. (69), Eqs. (97) and (98) can be regarded as general relations that relate the expansion parameter H_1 at the barycenter of m_1 —representing a mass or collection of masses—to the expansion parameter H_2 at the

barycenter of m_2 —representing a second mass or collection of masses.

Plugging Eq. (94) into Eq. (98), relates expansion parameters to relative speeds:

$$\frac{H_1}{H_2} = \frac{v_2}{v_1}, \quad (99)$$

$$H_1 = \frac{v_2}{v_1} H_2. \quad (100)$$

The interpretation of Eq. (100) is thus: An observer in reference frame two, moving with respect to a common frame at BC, would measure a different expansion parameter relative to m_1 than would an observer in reference frame one relative to m_2 —when their speeds differ—with the ratio of measured expansion parameters being equivalent to the ratio of their relative speeds.

Recognizing that speeds v_1 and v_2 are relative to BC—which is to say, relative to this common frame, Eq. (99) can be extended as follows:

$$\frac{H_1}{H_2} \frac{r_f}{r_f} = \frac{v_2}{v_1}, \quad (101)$$

$$H_1 = \frac{v_2}{r_f}, \quad (102)$$

$$H_2 = \frac{v_1}{r_f}. \quad (103)$$

Here, an abstract parameter r_f , termed the “frame size”, with the units of distance, is introduced. As will become evident, this factor is dynamic and is a function of speed and mass. However, for a given collection of bodies, r_f is a function of speed with respect to the barycenter of that collection. When the speeds in Eqs. (102) and (103) are taken with respect to a common frame, the frame size r_f is equivalent for both observers. This isn’t to say r_f necessarily remains constant over time. It can vary, but it varies by the same amount for both observers such that they always agree on its value.

If, for the sake of argument, the speed of frame one were taken directly relative to frame two and vice versa, an observer in each frame would measure the same relative speeds. These observers, however, would not be moving relative to a common frame, and thus, r_f in Eqs. (102) and (103) would not be equivalent.

Because H_1 and H_2 are also functions of mass and are constant with respect to the expansion parameter of the common frame— H_f , r_f , in Eqs. (102) and (103) must vary with speed. This is illustrated by equating Eqs. (95) and (96) with Eqs. (102) and (103), respectively:

$$\frac{m_1}{m_t} H_f = \frac{v_2}{r_f}, \quad (104)$$

$$\frac{m_2}{m_t} H_f = \frac{v_1}{r_f}, \quad (105)$$

$$r_f = \frac{m_t}{H_f} \frac{v_2}{m_1} = \frac{m_t}{H_f} \frac{v_1}{m_2}. \quad (106)$$

As demonstrated by Eq. (106), for the left side of Eqs. (104) and (105) to remain constant, the frame size

r_f must increase and decrease in concert with speed. The greater the speed, the larger the frame size. This concept will be revisited below when the redshift of light is considered from the perspective of a moving reference frame.

Eq. (106) also describes r_f in terms of the parent frame’s expansion parameter H_f and mass. Notice that as H_f goes to zero, the frame size r_f goes to infinity, which results in local expansion parameters H_1 and H_2 going to zero in concert with H_f .

With the above concepts in mind, a general set of relations for moving frames is as follows:

$$H_f = \frac{v_f}{r_f}, \quad (107)$$

$$r_f = \frac{v_f}{k_f m_r}, \quad (108)$$

$$k_f = \frac{H_i}{m_i} = \frac{H_f}{m_t}. \quad (109)$$

Eq. (107) can be interpreted as thus: For an observer moving relative to a parent frame at a speed of v_f , there exists a frame size r_f such that the ratio of v_f to r_f gives the expansion parameter H_f of the moving frame. Another observer, moving relative to the same parent frame with a speed of v_{f2} , for example, would see a different expansion parameter equal to the ratio of v_{f2} to r_f .

Recognize that Eq. (107) is essentially Hubble’s law (Eq. (28)) with the parameters having been rearranged, but these two equations and their parameters have different interpretations. In Eq. (28), $\vec{v}$ is the velocity at which space—with an expansion parameter of H_t —expands at a distance of $\vec{r}$. In contrast, under Eq. (107), expansion parameters are functions of speed and frame size. Also, in contrast to Eq. (28)—where $\vec{v}$ and $\vec{r}$ are vectors, all of the parameters in Eq. (107) are scalars. Speed v_f and frame size r_f , in this equation, have no direction.

All of the parameters in Eq. (28) are measured in the same reference frame, and the same situation applies to Eq. (107), where all parameters are measured in the parent frame. Speed v_f of the moving frame can be thought of as a scalar property of the moving frame relative to the parent frame. Likewise, H_f is also a scalar property of the moving frame that relates to the parent frame through frame size r_f . Inasmuch as every point in the moving frame will have the same speed v_f relative to the parent frame, any given point in the moving frame will have the same expansion parameter H_f . Interestingly, however, with a point established in the moving frame as having an expansion parameter of H_f , a second point at a certain distance will *not* have the same expansion parameter. This phenomenon will be discussed in detail in Section XVIII.

Generally, in the case where there is a frame moving relative to a rest frame, the relativistic effects of time dilation and length contraction need to be considered—especially as speeds approach c . However, because all of the parameters in Eq. (107) are measured in the rest frame (or parent frame), relativistic effects are not of

concern here.

Eq. (108) defines r_f as a function of the frame speed v_f , a reference mass m_r , and a constant k_f , termed the “frame constant”. For a given frame constant and reference mass, **the frame size r_f with respect to an observer is dynamic and linearly proportional to the observer’s speed v_f relative to the frame.** In the two-body system of Fig. 8, for example, from the perspective of either moving frame, r_f changes in proportion to any change in speed relative to BC. As shown by Eq. (106), both frames always agree on the value of r_f .

Eq. (109) defines frame constant k_f —which has the units of $(\text{kg}^{-1}\text{s}^{-1})$ —as expansion parameter per unit mass. As evidenced by Eqs. (69) and (98), **no matter how the mass in a system is divided, this ratio holds.** It’s postulated that this applies to all systems—from a simple two-body system to a massive spiral galaxy. For example, a two-body child system within a galaxy would share the same frame constant as that of the parent galaxy. In essence, k_f can be thought of as a state that applies equally to the system as a whole or any subsystem or element thereof.

This state is a function of mass and expansion parameter, and one metric can vary independently of the other. It’s concluded, therefore, that **the expansion parameter of a frame is a property of the frame irrespective of the mass contained therein.**

Further, looking at the relation for k_f , the internal dynamics of the system within the frame are immaterial. It’s concluded, therefore, that **the positions of particles in a system within a parent frame and the motions of those particles relative to that frame have no bearing on the frame’s expansion parameter.**

Interestingly, $1/k_f$ has the units of mass-time (kg-s), which seems to be somewhat unique. The Planck constant, which has the units of action, is similar—with units of (J-s). The uncertainty principle, which employs this constant, can be rearranged to give units of mass-time as follows:

$$\Delta E \Delta t \geq \frac{\hbar}{2},$$

$$\Delta m \Delta t \geq \frac{\hbar}{2c^2 \Delta t}.$$

One possible deduction is that the frame constant k_f , based on its units, is related to action.

Eqs. (107)–(109) align with the observation made at the beginning of the section that expansion parameters only ever relate to other expansion parameters—as k_f is a function of expansion parameter. While earlier discussions focused on relating expansion parameters via mass and distance, they may also be related through speed—an important concept that will prove to be integral in the following sections.

XI. THE EXPANSION PARAMETER—AS SEEN BY LIGHT

Continuing the line of reasoning from the previous section, the following question is posed: Can the expansion parameter seen by light be examined in the context of a photon traveling in a frame moving relative to a parent frame?

The standard interpretation of redshift is as follows: As a photon travels through space, over time, the expansion of space stretches the photon—increasing its wavelength. Increasing distance (and time) correlates with increasing redshift as described here:

$$v = \dot{r} = rH_0, \quad (110)$$

$$\dot{\lambda} = \lambda H_0. \quad (111)$$

Eq. (110) is Hubble’s law, and Eq. (111) is Hubble’s law applied to light, where λ is the emitted wavelength.

Given the arguments of the previous section, an alternative interpretation is that H_0 is the expansion parameter of a photon in a moving frame as a *result* of moving at c relative to a parent frame. In either point of view, the photon is stretched over time *and distance* according to H_0 , which is a literal description of observed redshift that is agnostic of an underlying mechanism. Under this alternative interpretation, Eq. (110) is rearranged akin to Eq. (107) such that H_0 is a function of speed c and frame size r_0 :

$$H_0 = \frac{c}{r_0}. \quad (112)$$

The interpretation of Eq. (112) is thus: A photon in a moving frame—traveling at c relative to a parent frame with a frame size of r_0 —has an expansion parameter of H_0 . In the previous section, with regard to Eq. (107), the frame size was defined as an abstract parameter. Is this still the case for Eq. (112)? What does r_0 actually represent?

This will be covered in detail in Section XV, but in the case where v_f , in Eq. (107), goes to the extent of speed in the universe— c , H_f goes to the extent of expansion parameter— H_0 , and r_f goes to the extent of distance— r_0 . At the other extreme—where v_f goes to zero, both H_f , and r_0 go to zero. While r_f is abstract, at these two extents, it corresponds to a physical distance.

Thus, if H_0 demarks the expansion parameter of the universe, it’s surmised that r_0 represents the size of the universe. When terms of Eq. (112) are arranged according to Eq. (110), its interpretation changes: c is the rate at which space expands at a distance of r_0 —where the universe has an expansion parameter of H_0 . Both interpretations are deemed to be correct. Therefore, it’s concluded that the universe is expanding at c .

Eqs. (107) and (112) further suggest that a frame moving at a subluminal speed relative to the same parent frame in which light travels (which is the frame of the universe) would also have an expansion parameter—and

one that would be less than H_0 . Because—as discussed in the previous section—both r_f and H_f vary with speed, the expansion parameter H_f as seen by a frame moving at a subluminal speed v_f in the universe is unclear. And what is meant by a speed “relative to the universe”? Does the universe have a frame? This topic and the reconciliation of H_f with H_0 will be addressed in the following sections.

1. Hubble time

Taking the inverse of H_0 gives the age of the universe. The estimated age of the universe is 13.787 billion years [4], which, working backwards, translates to a Hubble parameter equal to:

$$\begin{aligned} t &= 13.787 \times 10^9 \text{ yr} \Rightarrow 435.075 \times 10^{15} \text{ s} , \\ H_0 &= 2.298 \times 10^{-18} \text{ m s}^{-1} \text{ m}^{-1} , \\ H_0 &= 70.92 \text{ km s}^{-1} \text{ Mpc}^{-1} . \end{aligned}$$

This relationship is well known, and the inverse of the Hubble parameter is termed the “Hubble time”. Recent measurements of the Hubble parameter vary based on the chosen method, but they tend to fall within this range: 67–74 $\text{km s}^{-1} \text{ Mpc}^{-1}$ [19] [20], which averages to 70.5 $\text{km s}^{-1} \text{ Mpc}^{-1}$. Early-universe techniques place H_0 at the low end of the range, whereas late-universe analyses pin H_0 towards the upper end. The discrepancy between these results is termed the “Hubble tension”. Combining a variety of late-universe methods, places H_0 towards the upper-middle of the range— $72 \pm 8 \text{ km s}^{-1} \text{ Mpc}^{-1}$ [21].

The definition of H_0 —and the derivative thereof with respect to time—as a function of Hubble time t_0 is as follows:

$$H_0 = \frac{1}{t_0} , \quad (113)$$

$$\frac{dH_0}{dt} = \frac{-1}{t_0^2} \frac{dt_0}{dt} , \quad (114)$$

$$\dot{H}_0 = -H_0^2 . \quad (115)$$

Here, t_0 is the proper time in an inertial rest frame. Both dt_0 and dt are measured in the same frame and are thus equivalent. Notice that the subscript 0 in t_0 , H_0 , and r_0 refers to the current values of each parameter in the inertial reference frame.

It seems highly unlikely that the agreement between the Hubble time and the estimated age of the universe is *simply* an interesting coincidence. The most straightforward explanation is that time and the expansion of the universe are directly related—as described by Eq. (113). Otherwise, it must be presumed that the trajectory of time happens to have intersected the trajectory of the universe’s expansion parameter—where one is the inverse of the other—at precisely the point where humanity has the capability of measuring both quantities.

Returning to the discussion of r_0 in Eq. (112), taking the derivative of Eq. (112) vs. time gives the following:

$$\frac{d}{dt}(H_0) = \frac{d}{dt} \left(\frac{c}{r_0} \right) , \quad (116)$$

$$-H_0^2 = -\frac{c}{r_0^2} \dot{r}_0 , \quad (117)$$

$$(r_0 H_0)^2 = c \dot{r}_0 , \quad (118)$$

$$c^2 = c \dot{r}_0 , \quad (119)$$

$$\dot{r}_0 = c . \quad (120)$$

The first derivative of H_0 with respect to time—which was defined by Eq. (115)—is plugged into Eq. (116), which thus distills to Eq. (120). This latter equation shows that r_0 expands at c ; and further, because the rate is constant, r_0 has *always* been expanding at c . As mentioned, it’s posited that r_0 defines the size of the universe. It’s further imagined that all observers, in their rest frames, see r_0 as expanding at c ; and from the perspective of these observers, a universe expanding at c would appear to be of infinite size.

XII. TIME-DRIVEN COSMOLOGY

The idea that the universe is describable by constant, linear expansion at c is referred to as “coasting cosmology”. This is not a new idea [22][23].

Constant, linear expansion at c is a stark contrast to the evolution of the universe prescribed by Λ CDM—the current standard model of cosmology, which holds that there was a brief period of exponential expansion—termed “inflation”—immediately after the Big Bang. Inflation ostensibly solves the “flatness problem”.

The flatness problem is as follows: In order for the present universe to be spatially flat—which has been confirmed by numerous experiments [4][24][25]—the ratio of the total energy density to the critical density at the birth of the universe must have been equal to one—out to 59 decimal places [26]. Any deviation from this initial condition would’ve been magnified many times over as the universe expanded—either resulting in the universe quickly collapsing in on itself or expanding exponentially—and the flat universe we know wouldn’t exist. Moreover, without the critical density of the early universe having been fine-tuned, *we* wouldn’t exist to be able to contemplate it.

Flatness is typically gauged by the spatial-curvature density, or Ω_K , where $\Omega_K = 1 - \Omega$ and Ω is the total energy density parameter—the ratio of total density to the critical density. The Planck results, for example, measure Ω_K at 0.001 ± 0.002 [4]. During inflation, which lasted from 10E-36 to 10E-32 seconds, the universe—expanding many times the speed of light—grew from the size of a proton to the size of a grapefruit, which, in effect, drives Ω_K to zero [27][28].

Within that first nanosecond after the birth of the universe, inflation ends as abruptly as it starts, and

for 10 billion years—according to Λ CDM—the universe expanded roughly linearly but started to accelerate in growth thereafter as the total density transitioned from being dominated by matter—comprising dark and baryonic matter—to dark energy. Current estimates hold that the energy of the universe is comprised of: mostly dark energy—at 70%, dark matter at 25%, and ordinary “baryonic” matter at only 5% [29].

In contrast to Λ CDM, coasting cosmology solves the flatness problem by keeping the density parameter $\Omega(t)$ constant over time—which happens mathematically—preventing any initial deviations from flatness from being amplified. Similarly, it solves the “horizon problem” by equating the gravitational horizon with r_0 —the distance that light has traveled since the Big Bang [30]. In a universe that expands at c , there are no disconnected regions. All coordinates are close enough to be in causal communication—allowing thermal equilibrium *before* the cosmic microwave background (CMB) release. However, while addressing many critical issues with Λ CDM, coasting models typically struggle with describing an early-universe thermal evolution that aligns with observations. This will be addressed in Section XXI.

With this in mind, an alternative coasting model called “Time-Driven Cosmology” is proposed—where, in every reference frame, the universe expands linearly at the speed of light. The universe expands forever at c while the expansion parameter of the universe, H_0 , asymptotically tapers to zero—as shown by Eq. (115) above. This happens *not* because the universe contains the exact amount of mass needed to slow the expansion parameter to zero after an infinite amount of time, but because the inverse of time *mathematically* tapers to zero.

Similarly, space doesn’t expand as a consequence of energy density. The opposite is true: Energy is created as a consequence of the expansion of space, which, in turn, is purely a function of time. As space expands, energy is borrowed from the vacuum in concert, which ultimately manifests as the mass that comprises the material structure of the universe. The good news is: Because the universe will never stop expanding, the loan never has to be repaid!

Special relativity sets a speed limit of c to all motion in the universe that conveys information. The implication is that in a universe that obeys this speed limit, information can never be lost or destroyed. A given particle, regardless of distance, can never travel faster than the information flow from a second particle. Space—as the medium for fields and light, which is to say information—should also be subject to this limit. Photons traveling from one point in space to another—if the space between those points were expanding faster than c —would be redshifted to an infinitely long wavelength, which would effectively destroy the information carried by those photons.

It’s posited that our universe is describable by special relativity—where all events that have led to the present structure of the universe fall within the light cone that extends from the initial event that gave birth to the

universe—ostensibly, the Big Bang. All events are time-like related and can be traced back to this initial event.

XIII. THE HUBBLE FRAME

Under the principle that there should be no preferred frame, all observers in their respective rest frames should see the most distant coordinate—which demarks the size of the universe r_0 —receding at c . More generally, in all frames, an observer should see a given coordinate receding at a speed between 0 and c that varies linearly with distance. In addition, the recessional speed v between any two coordinates should be seen as a constant over time as shown here:

$$v = rH_0 ,$$

$$\frac{dv}{dt} = \frac{dr}{dt}H_0 + r\frac{dH_0}{dt} , \quad (121)$$

$$\dot{v} = rH_0^2 - rH_0^2 , \quad (122)$$

$$\dot{v} = 0 . \quad (123)$$

At a given distance r between any two points in space, that distance expands according to Eq. (110), which is Hubble’s law. Taking the derivative of Eq. (110) with respect to time gives Eq. (121). Plugging in Eq. (115) gives Eq. (122), which shows that the rate of speed increase at a given H_0 is canceled by the rate of H_0 decrease at a given distance—leaving the recessional speed constant over time. It stands to reason that if the most distant coordinate recedes at the constant rate of c , intermediate coordinates should recede at constant rates as well.

Bodies that move in sync with these receding coordinates—also referred to as “comoving” coordinates—are said to be moving in the “Hubble flow”. This system of receding coordinates can be imagined as the cosmic rest frame or the “Hubble frame” for short. A body at rest in the Hubble frame would be seen as receding from an observer—also at rest in the frame—such that there was no relative speed or “peculiar speed” between the body and the coordinate coincident with that body—which is receding. Typically, however, bodies such as galaxies are found having peculiar speeds relative to the Hubble frame. Why, in nature, there would be an expectation to find bodies moving in the Hubble flow will be explored in Section XXI.

This system of moving coordinates can be visualized in Fig. 9, which is a spacetime diagram with time axis ct and space axis vt —where the rest frame is a coordinate in the Hubble frame. The rest frame, by definition, is only moving in time—along ct —while v_1 , v_2 , and c describe coordinates moving at varying speeds relative to the rest frame according to Eq. (110). These are termed “Hubble coordinates”. It can also be imagined that v_1 , v_2 , and c represent the world lines of moving frames coincident with these coordinates. The inverse slope of a world line in the diagram ($vt/ct = v/c$) gives its speed relative to the rest frame. Thus, a line where $v = c$ is drawn at 45°.

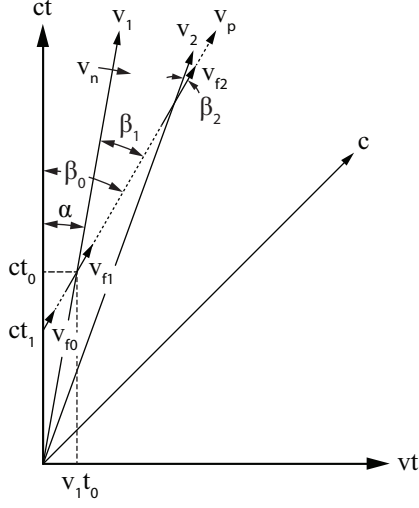


FIG. 9.

Note, Fig. 9 only shows the right side of the complete spacetime diagram, which is mirrored about the ct axis. Imagine coordinates v_1 , v_2 , and c as all having mirrored counterparts—with the left and right c world lines establishing a light cone.

As demonstrated by Eq. (123), the velocity of any given Hubble coordinate remains constant over time; and thus, in the diagram, the world lines for these coordinates are drawn with straight lines. Notice that, when the clocks of these world lines are all run in reverse, the world lines all converge at a point in spacetime that demarks the universe's initial event. Accordingly, an observer in the rest frame of any Hubble coordinate, moving backwards in time along the ct axis, would see bodies in the Hubble flow converging on his location, making it appear to the observer that he is at the center of the universe.

The world line of an inertial frame moving at speed v_p relative to the rest frame of a Hubble coordinate is also drawn in the diagram. Notice that the v_p world line does not intersect the point of convergence of Hubble coordinates. However, an observer in the v_p rest frame would see the universe as existing in its own light cone—expanding at c in all directions—with a point of convergence at the intersection of these world lines at a Hubble time of zero in the frame. Thus, an observer in this rest frame, and, by extension, any inertial observer, can imagine himself as being located at the center of the universe.

Nonetheless, it's reasoned that the Universe had only one initial event—demarked by a single point in spacetime—this event establishing the Hubble frame. The implication is that the Hubble frame constitutes a *preferred* frame—where motion with respect to this frame is discernible. While it may seem outlandish that there would be a preferred frame in the universe, there is evidence for this case. For example, an observer moving along the v_p world line would see blueshift in light observed from the right side of the ct axis and redshift from the opposite direction. We see this effect—based on our

motion in the Milky Way and the Milky Way's peculiar speed—as a dipole in the CMB where one side of a spherical survey is slightly blueshifted relative to the other.

Notice that the v_p world line crosses coordinates in the Hubble frame at angles that decrease over time starting with β_0 —relative to the rest frame, followed by β_1 relative to v_1 , and finally β_2 with respect to v_2 . Decreasing angles translate to decreasing relative speeds, and thus, the frame moving along the v_p world line is decelerating with respect to coordinates in the Hubble frame—which is to say decelerating in the Hubble frame.

As the frame moving at v_p intersects a Hubble coordinate, its speed relative to that coordinate is defined as v_f . Specifically, v_f is defined as the peculiar speed relative to the coincident Hubble coordinate. More broadly, v_f can be understood as the peculiar speed relative to the overall Hubble flow (or Hubble frame). This is in contrast to v_p , which is defined as the speed with respect to a fixed Hubble coordinate.

This can be visualized in the diagram. Notice that the v_p world line is broken into a series of segments: v_{f0} , v_{f1} , and v_{f2} —where each segment represents the peculiar speed v_f at the coincident Hubble coordinate. At time t_1 —where the frame moving at v_p crosses the ct axis of the rest frame, v_{f0} is equivalent to v_p —as evidenced by both segments having the same angle β_0 with respect to the Hubble coordinate. At the point where the moving frame intersects coordinate v_1 , relative to v_1 , v_f has decelerated to v_{f1} ; and at v_2 , relative to v_2 , v_f has further slowed to v_{f2} .

As the moving frame continues along the v_p world line, it will asymptotically approach the slope of the Hubble-frame coordinate of the same slope—which is to say the same speed. After an infinite amount of time, the moving frame will come to rest in the Hubble frame. Light, with a $v_p = c$, will asymptotically approach the edge of the light cone—defined by the c coordinate in the diagram. Keeping in mind that the diagram is mirrored, there is no direction where a frame with a non-zero peculiar speed will not decelerate relative to the Hubble frame.

As the v_p world line crosses the v_1 coordinate, v_1 becomes the rest frame, which would look identical to the frame of Fig. 9—with the most distant coordinate receding at c . Relative to this frame, the peculiar speed would not be v_{f0} but v_{f1} —and the blueshift in the dipole would be slightly less than before.

From the diagram, the following equation can be written:

$$v_n t_0 = v_p (t_0 - t_1) . \quad (124)$$

As time progresses, coordinate v_n rotates clockwise about the origin—tracking the v_p world line as it intersects Hubble coordinates. At time t_0 , v_n has rotated an angle α relative to time axis ct and is coincident with v_1 —which is to say, at t_0 , v_n is equivalent to v_1 . Looking at the diagram, in the timespan $t_0 - t_1$, a particle moving at speed v_p will have traveled the same distance as a parti-

cle moving at the current value of v_n —which is v_1 . With t_0 defined as the Hubble time, v_n and t_0 in Eq. (124) are dynamic with respect to time, while v_p and t_1 are constant.

Taking the derivative of Eq. (124) with respect to time gives the following:

$$\frac{dv_n}{dt}t_0 + v_n \frac{dt_0}{dt} = \frac{dv_p}{dt}(t_0 - t_1) + v_p \left(\frac{dt_0}{dt} - \frac{dt_1}{dt} \right), \quad (125)$$

$$\dot{v}_n t_0 + v_n = v_p. \quad (126)$$

As before, both dt_0 and dt refer to changes in time in the rest frame and thus cancel. Speed v_p is constant in the rest frame over time, and thus, dv_p/dt is zero. With these considerations, Eq. (125) reduces to Eq. (126).

Plugging Eq. (113)—the relation between Hubble time t_0 and the Hubble parameter H_0 —into Eq. (126), gives this:

$$\dot{v}_n \frac{1}{H_0} + v_n = v_p, \quad (127)$$

$$\dot{v}_n = (v_p - v_n)H_0. \quad (128)$$

In the diagram, at time t_1 , the coincident coordinate v_n is in line with the ct axis of the rest frame and thus has a speed of zero in the frame. With this in mind, the following equations are written:

$$v_f = v_p - v_n, \quad (129)$$

$$\dot{v}_n = (v_f - v_n)H_0 = v_f H_0, \quad (130)$$

$$v_f H_0 = (v_p - v_n)H_0, \quad (131)$$

$$v_f = v_p - v_n, \quad (132)$$

$$\dot{v}_f = \dot{v}_p - \dot{v}_n, \quad (133)$$

$$a_f = \dot{v}_f = -v_f H_0. \quad (134)$$

In Eq. (129), at a v_n of zero, v_f is equivalent to v_p . This is shown in the diagram where v_{f0} and v_p have the same slope relative to the ct axis—and thus the same speed. With Eq. (129), Eq. (128) can be rewritten as Eq. (130), which gives the acceleration of the floating coordinate v_n as it traces the moving frame in the reference frame of the coincident Hubble coordinate—which is to say the Hubble frame.

Plugging Eq. (130) into Eq. (128) gives Eq. (131), which reduces to Eq. (132). Notice the inclusion of the v_n term in Eq. (132), which correctly indicates that after an infinitesimal amount of time, v_f will be slightly smaller than v_p . Importantly, because v_f is always relative to the reference frame of the coincident coordinate, which changes over time, this equation is only valid within this infinitesimal time period. Taking the derivative of Eq. (132) with respect to time gives Eq. (133), where, as before, $\dot{v}_p$ is zero. Like Eq. (132), Eq. (133) is only correct for an infinitesimal span of time.

Eq. (134), which follows from Eqs. (130) and (133), gives the deceleration a_f of a frame moving relative to the coincident Hubble coordinate—which is to say deceleration in the Hubble frame—as a function of its peculiar speed v_f in the frame. Because both v_f and a_f are always measured in the frame of the coincident coordinate, Eq. (134) is deemed to be valid at all times. Notice that v_f is a scalar—with no direction, and thus, so is deceleration a_f . In the specific case where the peculiar speed of a given frame is zero, that frame is considered to be at rest in the Hubble frame.

Notice further that in static—non-expanding—space, which is described by an H_0 of zero in Eq. (134), there is no deceleration. A static universe, with an H_0 of zero, would be of infinite age and size—and *frameless*. In contrast, the present model holds that the universe is finite and perpetually expanding at c . It's surmised that this constitutes a frame against which peculiar speed v_f and acceleration a_f can be defined.

However, even our finite universe does look to be limitless. It's only finite on paper! All inertial observers see themselves as being at the center of the universe. Further, a given inertial observer can also have a peculiar speed relative to the universe—while curiously, always seeing himself at its center. This observer can certainly travel to other regions of the universe and see other galaxies, but in his rest frame, the universe will always be expanding at c in every direction.

XIV. MASS OF THE UNIVERSE

Eq. (134), when applied to light, holds that a photon is decelerated in the Hubble frame as follows:

$$a_f = -cH_0. \quad (135)$$

This deceleration, which can be described as a photon losing energy over time as it travels through expanding space, manifests as redshift.

The following equations explore the idea that the redshift of a photon can be equivalently imagined in two different contexts: traveling through expanding space and

traveling against a decelerating field:

$$\dot{\lambda} = \lambda H_0 ,$$

$$p = \frac{h}{\lambda} , \quad (136)$$

$$\dot{p} = \frac{-h}{\lambda^2} \dot{\lambda} , \quad (137)$$

$$\dot{p} = \frac{-h}{\lambda^2} \lambda H_0 , \quad (138)$$

$$m_e a_f = \dot{p} = \frac{-h}{\lambda} H_0 , \quad (139)$$

$$m_e = \frac{E}{c^2} = \frac{p}{c} = \frac{h}{c\lambda} , \quad (140)$$

$$\frac{\hbar}{\lambda} \frac{1}{c} a_f = -\frac{\hbar}{\lambda} H_0 , \quad (141)$$

$$a_f = -cH_0 .$$

Eq. (111), which gives Hubble's law applied to the wavelength of light, is restated above. The momentum of a photon as a function of wavelength is given by Eq. (136) and the change in momentum over time $\dot{p}$ by Eq. (137). Plugging Eq. (111) into Eq. (137) gives Eq. (138), which reduces to Eq. (139)—the latter relating the effective mass of a photon m_e and deceleration a_f to the change in momentum. With the definition of m_e in Eq. (140), Eq. (139) distills to Eq. (135).

Referring back to Eq. (21), acceleration implies time uncertainty. Equating this relation with the deceleration of a photon in the Hubble frame—as given by Eq. (135)—yields the following:

$$\hbar H_0 = \frac{2\hbar}{\Delta t_u} , \quad (142)$$

$$H_0 = \frac{1}{t_0} = \frac{2}{\Delta t_u} , \quad (143)$$

$$\Delta t_u = 2t_0 . \quad (144)$$

To clarify the applicability of Eq. (21) in this case—space itself is modeled as a field with time uncertainty Δt_u at all points therein. A photon moving at c relative to space—which is to say the Hubble frame—sees acceleration opposite the direction of travel. The photon here is the carrier of information (vs. the field in previous cases), and in keeping with the general principle of Eq. (21), acceleration is opposite the direction of information flow. As both accelerations in Eq. (142) are relative to the Hubble frame (and are redshifting accelerations), they may be directly equated.

Plugging Eq. (114) into Eq. (142) gives Eq. (144). Eq. (144) asserts that the time uncertainty of the universe t_u is double the Hubble time t_0 , which, as argued, is the age of the universe. Notably, this time uncertainty will *always* be double the age of the universe. Is the time and energy of the universe perpetually on loan—one that never expires?

In previous sections, it was proffered that t_u in Eq. (21) corresponds to the lifespan of a vacuum fluctuation. Is the universe a vacuum fluctuation? In 1973, this question

was originally posed by Edward Tryon in a paper of the same title [31].

With the relation between H_0 and time uncertainty t_u given by Eq. (143), the energy uncertainty of the universe E_u is derived via the uncertainty principle:

$$\Delta E_u \Delta t_u \geq \frac{\hbar}{2} ,$$

$$\Delta t_u = \frac{2}{H_0} ,$$

$$\Delta E_u \frac{2}{H_0} \geq \frac{\hbar}{2} , \quad (145)$$

$$\Delta E_u = \frac{\hbar H_0}{4} . \quad (146)$$

As before, replacing the $\geq$ sign with an equals sign in Eq. (146) pins ΔE_u at its minimum.

To recap, each observer in his respective rest frame will see the most distant coordinate of the universe receding at c . The distance to this coordinate r_0 —as given by Eq. (112)—is equivalent in all directions. This establishes a sphere where points on that sphere have time and energy uncertainties as described by Eqs. (144) and (146), respectively. As such, Eq. (7) may be used to calculate the mass within that sphere—which is to say the mass of the universe:

$$\Delta E_u = \frac{m_0 \hbar G}{8\pi r_0^2 c} , \quad (147)$$

$$\Delta E_u = \frac{m_0 \hbar G}{4r_0^2 c} , \quad (148)$$

$$\frac{m_0 \hbar G}{4r_0^2 c} = \frac{\hbar H_0}{4} , \quad (149)$$

$$m_0 = \frac{c H_0 r_0^2}{G} , \quad (150)$$

$$r_0 = c t_0 = \frac{c}{H_0} , \quad (151)$$

$$m_0 = \frac{c \hbar G c^2}{G H_0^2} , \quad (152)$$

$$m_0 = \frac{c^3}{G H_0} = \frac{c^3 t_0}{G} . \quad (153)$$

Eq. (7) is restated as Eq. (147) with m_0 and r_0 corresponding to the mass and radius of the universe in a given frame. In Eq. (149), Eq. (148) is equated with Eq. (146), which, via Eq. (112), reduces to Eq. (153). Eq. (153) shows that the mass of the universe m_0 is directly proportional to the Hubble time t_0 and thus inversely proportional to the Hubble parameter H_0 .

Specifically, m_0 —as described by Eq. (153)—is the rest mass of the universe in the rest frame of an inertial observer at a Hubble time of t_0 , which is the proper time in the frame. It's surmised that all inertial observers agree on the value of t_0 in their own frames, and thus all inertial observers should agree on the mass of the universe m_0 .

But other questions remain: Do observers in other

frames on different world lines agree on the *mass density* of the universe? Does the cosmological principle still hold? By what reasoning does m_0 only reflect rest mass? What accounts for the kinetic and potential energies of bodies in the Hubble flow? These questions will be addressed in Sections XX and XXI.

With an H_0 of $70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, m_0 calculates to:

$$\begin{aligned} H_0 &= 70 \text{ km s}^{-1} \text{ Mpc}^{-1}, \\ H_0 &= 2.27 \times 10^{-18} \text{ m s}^{-1} \text{ m}^{-1}, \\ m_0 &= 1.78 \times 10^{53} \text{ kg}. \end{aligned}$$

Note that Eq. (153) has previously been arrived at via other lines of reasoning [32][33]. For example, the relation between mass density ρ_0 and H_0 —which emerges from the Friedmann model of an expanding universe [34]—reduces as follows:

$$\begin{aligned} G\rho_0 &\simeq H_0^2, \\ m_0 &\sim \rho_0 r_0^3, \\ r_0 &= \frac{c}{H_0}, \\ m_0 &\sim \frac{H_0^2}{G} \frac{c^3}{H_0^3}, \\ m_0 &\sim \frac{c^3}{GH_0}. \end{aligned}$$

With Eq. (153) and mass–energy equivalence, the energy of the universe E_0 is thus:

$$\begin{aligned} E_0 &= m_0 c^2, \\ E_0 &= \frac{c^5 t_0}{G} = \frac{c^5}{GH_0}. \end{aligned} \quad (154)$$

Taking the derivative of E_0 with respect to time gives this:

$$\frac{dE_0}{dt} = \frac{c^5}{G} \frac{dt_0}{dt}, \quad (155)$$

$$P_0 = \frac{c^5}{G}. \quad (156)$$

Eq. (156), which gives the rate at which energy is added to the universe, is recognizable as the Planck power—theoretically, the upper power limit in the universe. Interestingly, there are no parameters in this equation—only constants.

In brief, Time-Driven Cosmology is the consequence of the universe running at two limits: the speed of light, which governs the rate of expansion, and Planck power, which caps the rate at which the universe gains energy. Further, it's posited that these two rates are directly coupled—one doesn't occur independently of the other. As space expands, energy is borrowed from the vacuum, and borrowing energy from the vacuum causes space to expand.

It's imagined that matter manifests via vacuum fluc-

tuations. It's gathered that these were more energetic and dense in the early universe as Planck power was distributed across a relatively small volume.

XV. EXPANSION PARAMETERS IN THE HUBBLE FRAME

From Sections X and XI: A frame moving in a parent frame sees an expansion parameter H_f that varies linearly with that frame's speed in the parent frame. For a given H_f , frame size r_f also varies linearly with speed. With this in mind, the following equations are written:

$$H_f = \frac{v_x}{r_0}, \quad (157)$$

$$v_x = r_f H_0, \quad (158)$$

$$r_0 H_f = r_f H_0. \quad (159)$$

Referring back to Fig. 8 and Eqs. (102) and (103), frame two—moving at speed v_2 relative to a parent frame of size r_f —sees an expansion parameter of H_1 . Recall from Section X that frame size r_f is an abstract parameter. Frame one moving at the lower speed v_1 relative to the same frame sees a smaller expansion parameter of H_2 . As described by these equations, for a given frame size r_f , the expansion parameter varies linearly in proportion to the frame speed.

It's thus imagined that there is a speed v_x relative to a parent frame of size r_0 where the moving frame sees an expansion parameter H_f proportional to that speed. This is embodied by Eq. (157). At $v_x = c$, Eq. (157) is equivalent to Eq. (112)—where a frame moving at c sees an expansion parameter of H_0 . As v_x goes to zero, H_f goes to zero. Here, r_0 is both the frame size *and* the size of the universe—as described in Section XI.

For a given H_f in Eq. (157), there is a speed v_f and frame size r_f that satisfy Eq. (107). It's postulated that v_f is also *some* function of v_x with the following stipulations: As v_x goes to zero, v_f goes to zero, and as v_x goes to c , v_f goes to c . Here, unless v_f is equivalent to v_x —which is assumed not to be the case—it's impossible for these two speeds to be linearly related, as that would violate one of the above stipulations.

Because H_f varies according to v_x —as described by Eq. (157), for Eq. (107) to hold—where v_f is a *non-linear function* of v_x , r_f must also vary as a function of v_x . With this in mind, Eq. (158) is written according to the following reasoning: In congruence with Eq. (108), r_f is a linear function of v_x . As v_x goes to zero, r_f goes to zero; and as v_x goes to c , r_f goes to r_0 —the two limits in terms of distance. Yet, for r_f to go to r_0 , the expansion parameter in Eq. (158) *must* be H_0 .

Combining Eqs. (157) and (158) gives Eq. (159), which is a relation between expansion parameters and frame sizes. Referring back to Section VI, notice the symmetry between Eqs. (62)–(63) and Eq. (159). In Eq. (62), for example, recall that r is the distance between two masses;

H_f is the expansion parameter of the two-mass frame; H_2 is the expansion parameter at mass two; and r_1 is the distance between the barycenter of the two-mass system to mass one. In comparison, r and H_f in Eq. (62) correspond to r_0 and H_0 , respectively, in Eq. (159)—where these parameters define the parent frame; and r_1 and H_2 correspond to r_f and H_f —which, in their respective equations, define a child frame.

Eqs. (157) and (158) have interesting implications. As a consequence of traveling at c —where v_x goes to c , light sees the two extents of the universe: its expansion parameter H_0 and its size r_0 . At the other extreme, where v_x goes to zero, both the expansion parameter H_f and frame size r_f go to zero in concert.

A relation similar to Eq. (159) is derived by equating Eq. (112) to Eq. (158):

$$\begin{aligned} H_0 &= \frac{c}{r_0} , \\ H_0 &= \frac{v_x}{r_f} , \\ cr_f &= v_x r_0 . \end{aligned} \quad (160)$$

Notice that Eq. (160) is reproduced by multiplying Eq. (159) by Eq. (157):

$$\begin{aligned} \frac{r_f}{H_f} &= \frac{r_0}{H_0} , \\ \frac{r_f}{H_f} \frac{H_f}{v_x} &= \frac{r_0}{H_0} \frac{1}{r_0} , \\ \frac{r_f}{v_x} &= \frac{r_0}{c} . \end{aligned} \quad (161)$$

Eq. (161) reduces as follows:

$$\begin{aligned} \frac{r_f}{v_x} &= \frac{r_0}{H_0} \frac{1}{r_0} , \\ \frac{r_f}{v_x} &= \frac{1}{H_0} = t_0 . \end{aligned}$$

As noted, r_f is an abstract parameter that doesn't reflect a measurable quantity. The same is true for v_x , which is also an abstract parameter that isn't directly observable. These are the only two abstract parameters used in the equations of this section. Interestingly, their quotient yields a non-abstract parameter: the inverse of H_0 , which is the Hubble time t_0 .

Plugging the relations for r_0 and r_f —Eqs. (112) and

(107), respectively—into Eq. (159), yields the following:

$$\begin{aligned} r_0 H_f &= r_f H_0 , \\ r_0 &= \frac{c}{H_0} , \\ r_f &= \frac{v_f}{H_f} , \\ \frac{c}{H_0} H_f &= \frac{v_f}{H_f} H_0 , \\ c H_f^2 &= v_f H_0^2 , \end{aligned} \quad (162)$$

$$H_f = \sqrt{\frac{v_f}{c}} H_0 . \quad (163)$$

Eq. (163) relates the expansion parameter H_f —seen by a moving frame with a speed of v_f —to H_0 . Notice that as v_f goes to c , H_f goes to H_0 ; but this leaves the question: Relative to what is v_f measured in this case?

The answer to this question can be divined from Eq. (162):

$$\begin{aligned} c &= r_0 H_0 , \\ c H_f^2 &= v_f H_0^2 , \\ r_0 H_f^2 &= v_f H_0^2 , \end{aligned} \quad (164)$$

$$\begin{aligned} r_0 H_f &= v_x , \\ v_x H_f &= v_f H_0 . \end{aligned} \quad (165)$$

Eq. (112) is plugged into Eq. (162), which gives Eq. (164). Plugging Eq. (157) into Eq. (164), reduces to Eq. (165).

Recalling Eq. (134) from Section XIII, both sides of Eq. (165) appear to be decelerations. $v_x H_f$ is a deceleration relative to the moving frame, and $v_f H_0$ is a deceleration relative to the Hubble frame—where v_f is a peculiar speed in the Hubble frame.

Plugging Eqs. (112) and (157) into Eq. (165) yields a direct relation between v_f and v_x :

$$\begin{aligned} v_x \frac{v_x}{r_0} &= v_f H_0 , \\ \frac{v_x^2}{r_0 H_0} &= v_f , \\ v_f &= \frac{v_x^2}{c} . \end{aligned} \quad (166)$$

Notice that Eq. (166) conforms to the above requirements: As v_f goes to zero, v_x goes to zero, and as v_f goes to c , v_x goes to c . As anticipated, v_f is *not* a linear function of v_x . Similarly, looking at Eq. (165), as v_f goes to zero, $v_x H_f$ goes to zero; and as v_f goes to c , $v_x H_f$ goes to $c H_0$ —the deceleration as seen by light.

There's an interesting parallel between Eqs. (166) and (18):

$$\begin{aligned} v_x^2 &= c v_f , \\ \Delta t_p^2 &= \Delta t_f \Delta t_u . \end{aligned}$$

Recall, in Eq. (18), that Δt_f is distance over c .

All parameters in Eq. (163) are non-abstract, measur-

able quantities. An observer moving at peculiar speed v_f would measure a dipole in the CMB. The Hubble parameter H_0 (and its inverse, the Hubble time t_0) are measurable. The expansion parameter H_f of a frame moving at v_f is also measurable. As v_f goes to c , H_f goes to H_0 —which is the expansion parameter as seen by light. This was surmised in Section XI where it was reasoned that, at subluminal speeds, moving frames would see expansion parameters less than H_0 .

In Section XIII, it was reasoned that a frame with a peculiar speed relative to the Hubble frame is decelerating in that frame. Building on this concept, in Section XIV, it was shown that the redshift of light could be imagined in two contexts—both having the same result: one where the wavelength of a photon is expanded over time according to H_0 and the other where a photon is decelerated according to cH_0 .

The following section will investigate the case where the peculiar speed v_f of the frame is less than c , which—akin to light—incurs measurable physical effects. As described by Eq. (163), a frame moving with a peculiar speed of v_f will have an expansion parameter of H_f . This, in turn, will result in an increase in the acceleration between particles in the system contained by the frame—as given by Eq. (70). Interestingly, the physical effect that manifests from a frame that is decelerating in the Hubble frame is an increase in acceleration within that decelerating frame.

XVI. GALACTIC-ROTATION CURVES

In this section, Eqs. (163) and (70), restated below, are applied to a selection of nearby galaxies from the SPARC (Spitzer Photometry & Accurate Rotation Curves) database to derive their rotation curves, which are then checked against observations:

$$H_f = \sqrt{\frac{v_f}{c}} H_0 ,$$

$$a_n = \frac{m_r G}{r} \left(\frac{1}{r} + \frac{2\sqrt{cH_f}}{\sqrt{m_t G}} \right) .$$

The SPARC database comprises a large catalog of galaxies that vary in mass, gas fraction, effective radii, and surface brightness [35]. As a cohesive dataset spanning a broad range of morphologies, the SPARC data provide an ideal test of modified-gravity as well as dark-matter halo models, the latter being the primary focus of the cited paper. The complete SPARC catalog contains too many galaxies to analyze here—where the intent is a quick sanity check of the above equations. A subset of the catalog was selected to be examined based on proximity to the Milky Way—with the cutoff set to 5 Mpc. These galaxies are listed in Tables II and III of Appendix XXIV. This appendix also includes the corresponding rotation curve for each galaxy as generated via Eq. (70). Distance estimates, listed in Table II, were sourced from the

NASA/IPAC Extragalactic Database [36].

Appendix XXIII is a synopsis of the procedure used to generate the rotation curves of Appendix XXIV. In short, discs are modeled as zero-thickness plates of varying density with respect to radius. Bulges (one galaxy in the selection has a bulge—NGC 6946) are modeled as spheres where density varies according to radius as well. Discs and bulges are broken into finite elements, and Eq. (70) is solved for each element, where r is the distance from the center of mass of that element to an offset from the center of rotation. The net acceleration a_n —toward the center of rotation—at a given offset is the vector sum of the accelerations with respect to each disc and bulge element. Net acceleration a_n is solved across a range of offsets for a given galaxy from the inner to the outermost data points—thereby establishing a rotation curve. The outermost data point, for each galaxy, is listed under the “ r_{max} ” column in Table II.

Looking at Eq. (70), when applied to galaxies, there are two free parameters: m_t , which is the total galactic mass, and H_f , the expansion parameter of the moving frame containing the galaxy. The total mass m_t is constrained by estimates of the matter comprising the stellar populations, dust, and gas. In practice, H_f is adjusted to fit a given galaxy’s rotation curve, but the resulting value should comport with Eq. (163). This equation has two free parameters: v_f and H_0 , which are only free to a certain extent. Even considering the Hubble tension, estimates of H_0 fall within a relatively narrow range. The same holds for the peculiar speed v_f of a body, which can be gauged via the CMB dipole and relative motions.

Each galaxy in the SPARC database includes curve fits for 12 different dark-matter halo models. To fit a given model, SPARC researchers adjusted the following free parameters: bulge mass-to-light ratio (Υ_b), stellar-disk mass-to-light ratio (Υ_d), galaxy distance, and disk inclination. This highlights another advantage of the SPARC catalog—as fitting the sundry dark-matter models required adjustments to these parameters, yielding a variety of stellar-mass models per galaxy from which to source data.

Looking at the two rotation curves for NGC 2976 in Appendix XXIV, for example, the curve with an $\Upsilon_d = 0.48$ was generated via the Burkert-Flat model (referred to simply as the “Burkert model” going forward), whereas the curve with an $\Upsilon_d = 0.60$ was generated via the DC14-LCDM model. The stellar-disk Newtonian curves in these figures—drawn as dashed blue lines—come directly from the SPARC catalog. The curves show the Keplerian decline that would be expected with only the stellar-disk mass as the source of centripetal acceleration—where that acceleration is purely Newtonian gravity. Notice that the stellar-disk curve from the Burkert model, with a smaller Υ_d , looks like a scaled-down version—vertically—of the DC14-LCDM model’s curve.

In addition to the stellar disk, Newtonian curves for the gas disk and the bulge (where appropriate) are published

in the SPARC catalog for each dark-matter model of each galaxy. Table II of Appendix XXIV relates a given galaxy to the dark-matter model used as the source of Newtonian curves for the components of that galaxy. These curves were translated to density profiles and fed into the finite-element analysis of Appendix XXIII—generating the net rotation curve for the galaxy in question. All source Newtonian curves from the SPARC catalog were reproduced in the figures of Appendix XXIV. The Newtonian contribution from the gas disk is drawn as a dashed magenta line (with the larger dashes), and the bulge contribution is shown as a dotted green line. The net rotation curve is shown as a solid cyan line.

Variance in the free parameters of distance and inclination can result in the error bars of one model looking off-set or scaled (or both) relative to those of another model with respect to the axes of distance and speed. This can be seen when comparing the Burkert and DC14- Λ CDM models of NGC 2976—where the error bars of the Burkert model peak at roughly 88 km s^{-1} at 2.1 kpc vs. a maximum speed of 81 km s^{-1} at 2.2 kpc for the error bars of the DC14- Λ CDM model.

The gas curves of these two models both peak at roughly 30 km s^{-1} but at different distances—with the Burkert curve peaking at 1.75 kpc vs. 1.8 kpc for the DC14- Λ CDM curve. The gas curves for a given galaxy are more consistent between models than the stellar curves, which are subject to being modified via the additional free parameter of mass-to-light ratio.

The rotation curves of the galaxies in Appendix XXIV were generated via the dark-matter model offering the best fit—as referenced in Table II. With density profiles for the stellar and gas disks (and bulge when relevant)—derived from the given Newtonian curves for these components, the expansion parameter H_f for a given galaxy is calibrated to fit the net rotation curve to the error bars associated with the source model. Referring, again, to NGC 2976, notice that the Burkert model (with a lower $\Upsilon_d = 0.48$) prefers the higher H_f of 1.9 than the DC14- Λ CDM model (with an $\Upsilon_d = 0.60$)—where fitting the curve favors an H_f of .20. It's concluded that, for NGC 2976, the expansion parameter H_f lies somewhere between .20 and $1.9 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

Table III shows the expansion parameter H_f for each galaxy in Table II used to generate that galaxy's rotation curve. Table III also includes the galactic mass m_t as well as the frame constant k_f —defined by Eq. (109)—for each galaxy, which is computed via H_f and m_t . The galactic mass m_t is calculated via the density curves of the associated dark-matter model. Because m_t is derived from data gleaned from the SPARC catalog, m_t is only a free parameter to the extent that there are 12 models per galaxy from which to source density curves. Out of the 29 galaxies listed in Table II, 12 used density profiles derived from the Burkert model, 5 used density profiles derived from the coreNFW Λ CDM model, and the rest used a combination of the remaining models.

Solving Eq. (163) for v_f for the cases where $H_f = .20$

and 1.9 gives the following:

$$v_f = c \left(\frac{H_f}{H_0} \right)^2,$$

$$v_f(.2) = 299\,792 \text{ km s}^{-1} \left(\frac{.2}{70} \right)^2 = 0.3 \text{ km s}^{-1},$$

$$v_f(1.9) = 299\,792 \text{ km s}^{-1} \left(\frac{1.9}{70} \right)^2 = 221 \text{ km s}^{-1}.$$

This predicts a peculiar speed v_f for NGC 2976 between .3 and 221 km s^{-1} .

As touched upon earlier, an observer with a peculiar speed will see a dipole in the CMB. In the case of the Milky Way, the amplitude of this dipole suggests that our galaxy has a speed of 565 km s^{-1} relative to the CMB [4].

Similarly, if the motion of a galaxy relative to the Milky Way is known, the peculiar speed of that galaxy relative to the CMB can be deduced. However, while the recessional component of a galaxy's relative velocity can be resolved by measuring its redshift—while accounting for cosmic redshift, determining the transverse component of the galaxy's velocity is more challenging [37].

If, for example, a galaxy were found along the axis of the Milky Way's dipole moving away from the blue-shifted side of the dipole with a peculiar speed of 565 km s^{-1} , it could be deduced that the galaxy was at rest relative to the CMB frame—but only in the radial direction. If the galaxy had a transverse velocity, it would have a dipole perpendicular to that of the Milky Way.

Conversely, if a galaxy were found perpendicular to the axis of the Milky Way's dipole with zero recessional peculiar speed, it could be concluded to be moving in parallel with the Milky Way—with a peculiar speed of 565 km s^{-1} —but only if its transverse velocity were verified to be zero.

It's surmised, however, that the speeds of galaxies with respect to the CMB that are derived purely from redshift data—without resolution of those galaxies' transverse velocities relative to us—are still useful data. Given a large enough data set, if Eq. (163) is valid, it seems reasonable that data points should concentrate along a trend line that comports with this relation.

Peculiar speeds relative to the CMB—based on redshift data—of the galaxies in Appendix XXIV are listed in Table II under the column, “CMB redshift”. These speeds were sourced from the *NASA/IPAC Extragalactic Database*. Notice that the CMB redshift for NGC 2976 is 90 km s^{-1} , which is nearly in the middle of the above estimated range: $0.3\text{--}221 \text{ km s}^{-1}$.

Fig. 10 is a plot of peculiar speed v_f vs. expansion parameter H_f for the galaxies in Appendix XXIV. The data points come from Tables II and III—with the value for v_f approximated by the CMB-redshift estimate published for a given galaxy and where the value for H_f is that which gives the best fit for that galaxy's rotation curve. Circles in the figure indicate fair to excellent curve

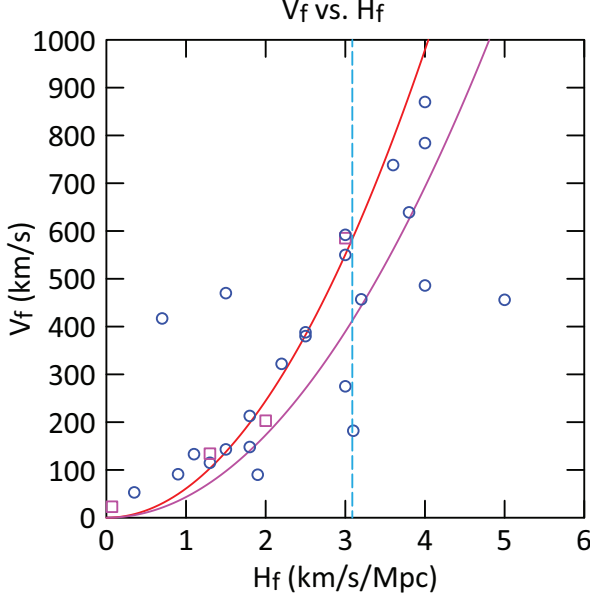


FIG. 10.

fits, while squares denote poor curve fits. Four galaxies have poor fits, which are highlighted in Table III. This table also notes the outlying galaxies relative to the data trend in the plot.

The left curve in the figure, drawn in red, traces Eq. (163). Galaxies plotted in Fig. 10 proximate to the left curve—assuming accurate values for H_f —have CMB redshifts that are nearly equal to their true peculiar velocities. A second curve to the right, drawn in magenta, accounts for uncertainty in the data—including that which stems from transverse velocities being unresolved. It's reckoned that the latter would tend to leave peculiar velocities underestimated. The right curve relates the true peculiar speed v_f to H_f as follows—where v_2 is the published CMB redshift:

$$v_f^2 = v_2^2 + v_2^2, \\ v_f = \frac{1}{\sqrt{2}}.$$

A galaxy plotted near the right curve thus has a true peculiar speed v_f that is approximately $\sqrt{2}$ times greater than the published CMB redshift. This scalar is the ratio of the magnitude of a vector—comprised of two orthogonal components of equal length—to the length of either component. This scalar seems to be congruent with variation in the present dataset—as about 83% of the galaxies plotted in Fig. 10 are near or contained within the left and right curves. In general, the data trend seems to comport with Eq. (163).

Looking at the rotation curves in Appendix XXIV, a similar percentage (about 86%) of the galaxies listed in Table III have good fits. The quality of fit for these galaxies was determined visually—no statistical analysis was performed. Again, the intent here was to present a

thesis with basic sanity checks made along the way.

NGC 2915 is an example of a galaxy having both a poor fit and, further, one that is significantly off trend in Fig. 10. With an H_f of 12, the (H_f, v_f) coordinate for NGC 2915 is outside the bounds of the figure. It's not clear if this galaxy has a high peculiar speed, which would translate to a high H_f , or has more mass than estimated in the SPARC data, or both.

Poor curve fits don't necessarily indicate a discrepancy in galactic mass, as the distribution thereof could also be at issue. Referring again to NGC 2976, the stellar disks in the Burkert and DC14-LCDM models ($\Upsilon_d = .48$ and $\Upsilon_d = .60$, respectively) have different masses, but both yield rotation curves with good fits. This example also highlights the sensitivity that H_f has with respect to mass—where just a 25% increase in Υ_d translates to a nearly 90% reduction in H_f .

The rotation curves of two galaxies were fit using disk mass-to-light ratios that were not specifically cited in one of the 12 dark-matter models. For NGC 0247, an Υ_d of 1.0 was used, which is about midway between the DC14 flat and coreNFW LCDM models: 0.50 and 1.52, respectively. The error bars for NGC 0247 came from the coreNFW LCDM model.

For NGC 6789, an Υ_d of 1.0 was used as well, which is about the midpoint of that of the Lucky13 LCDM and DC14 LCDM models: 0.69 and 1.20, respectively. The error bars for NGC 6789 were sourced from the DC14 LCDM model.

As r goes to infinity in Eq. (70), the reference mass m_r approaches the total mass m_t —as the galaxy looks increasingly like a point mass to an orbiting particle. Thus, at large distances, Eq. (70) reduces to Eq. (37) as follows:

$$a_n = \frac{m_r G}{r} \left(\frac{1}{r} + \frac{2\sqrt{cH_f}}{\sqrt{m_t G}} \right), \\ m_r \rightarrow m_t, \\ a_n = \frac{m_t G}{r^2} + \frac{2\sqrt{cH_f m_t G}}{r}, \\ a_n = \frac{2\sqrt{cH_f m_t G}}{r}, \\ \frac{v_{flat}^2}{\chi} = \frac{2\sqrt{cH_f m_t G}}{\chi}, \\ v_{flat} = \sqrt{2}(cH_f m_t G)^{\frac{1}{4}}. \quad (167)$$

Note that as m_t approaches a point mass, the local expansion parameter H_l —as seen by a particle of negligible mass in the field—approaches the frame's expansion parameter H_f . Further, in this regime, the Newtonian component of Eq. (37) loses significance in relation to the expansion component and thus, at a certain distance, this can be ignored. Here, Eq. (37) reduces to Eq. (36).

Assuming circular motion, Eq. (167) gives the asymptotic speed v_{flat} that is approached by particles as the expansion component of centripetal acceleration becomes dominant. Table III includes v_{flat} calculations for each

galaxy via this equation. As mentioned above, a sample of isolated galaxies from the KiDS survey has been found with rotation curves that remain flat for hundreds of kpc [17]. In Section XVIII, the question of how far a flat rotation curve can extend will be explored.

1. The Milky Way

Fitting the Milky Way’s rotation curve is left as a future exercise, but with an estimate of its peculiar speed v_f and (baryonic) mass m_t [38], the following parameters are easily calculated: H_f , k_f , and v_{flat} .

$$\begin{aligned} m_t &= 8.43 \times 10^{10} M_\odot = 1.68 \times 10^{41} \text{ kg} , \\ v_f &= 565 \text{ km s}^{-1} , \\ H_f &= 3.04 \text{ km s}^{-1} \text{ Mpc}^{-1} , \\ k_f &= 5.88 \times 10^{-61} \text{ kg}^{-1} \text{ s}^{-1} , \\ v_{flat} &= 191 \text{ km s}^{-1} . \end{aligned}$$

The parameter v_{flat} predicts that the Milky Way’s rotation curve flattens to 191 km s^{-1} , which comports with recent observations that extend out to 25 kpc [39][40]. Yoshiaki Sofue has compiled a “grand” rotation curve for the Milky Way that extends out to 400 kpc [41]. From 40 to 100 kpc, v_{flat} aligns well with this curve. Beyond 100 kpc, uncertainty in the data increases, and the error bars encompass a wide range of rotational velocities—from roughly 100 to 200 km s^{-1} .

The frame constant k_f , defined as the expansion parameter per unit mass, applies to all bodies in the system, which, in the case of the Milky Way, would include the Solar System. With the definition of k_f given by Eq. (109), Eq. (70) can be simplified as follows:

$$k_f = \frac{H_i}{m_i} = \frac{H_f}{m_t} , \quad (168)$$

$$a_n = \frac{m_r G}{r} \left(\frac{1}{r} + 2\sqrt{\frac{c}{G} k_f} \right) , \quad (169)$$

$$a_n = \frac{m_r G}{r^2} + \frac{2}{r} \sqrt{\frac{c}{G} k_f} , \quad (170)$$

$$k_e = \frac{2}{\chi} \sqrt{\frac{c}{G} k_f} \left(\frac{r^{\frac{1}{\chi}}}{m_r G} \right) , \quad (171)$$

Plugging Eq. (109) into Eq. (70) gives Eq. (169), which defines a_n as the sum of the two components of acceleration—the Newtonian and the expansion components. k_e , in Eq. (171), is the ratio of these components, which is an indication of their relative strength. Notice that k_e , which is a function of r , shows that the transition from the Newtonian to the non-Newtonian regime is linear with respect to distance.

Note that Section XVIII, as suggested, will explore the

expansion component of acceleration at large distances—far beyond the extent of the typical rotation curve. Here, it will be shown that curves do not stay flat indefinitely. However, from the front end of the curve to the portion where it flattens out—which is typically on the order of Mpc, Eqs. (70) and (169) are sufficiently accurate.

When applied to the Solar System, Eq. (171) shows that the acceleration regime is entirely Newtonian:

$$\begin{aligned} m_r &= 1 M_\odot = 1.98847 \times 10^{30} \text{ kg} , \\ r &= 1 \text{ AU} = 149\,597\,870.7 \text{ km} , \\ k_e &= 3.66 \times 10^{-30} . \end{aligned}$$

At the average radius of Earth’s orbit, the Newtonian component is 30 orders of magnitude larger than the non-Newtonian component.

At 4.25 ly—the distance to Proxima Centauri, the expansion component is still negligible at 25 orders of magnitude less than the Newtonian component:

$$\begin{aligned} r &= 4.2465 \text{ ly} = 4.017 \times 10^{13} \text{ km} , \\ k_e &= 9.84 \times 10^{-25} . \end{aligned}$$

Clearly, the effects of modified gravity are not detectable in or near the Solar System, where motions—within the precision of our measurements—remain Newtonian.

2. MOND

Well before Proxima Centauri—at around .111 ly (7300 AU), the strength of the Sun’s gravitational field drops below the MOND constant a_0 of $1.2\text{E}-10 \text{ m s}^{-2}$. According to MOND, as discussed, this constant marks the transition to the non-Newtonian acceleration regime, but, as just demonstrated, the Sun’s gravity remains Newtonian well past Proxima Centauri! In principle, MOND *does* account for such scenarios with a concept called the “external-field effect” (EFE)—where modified gravity is muted in systems that are in the presence of relatively strong external fields.

In contrast, under the present thesis, non-Newtonian effects in a galaxy are quantified by the frame constant k_f —which applies to all bodies and systems of bodies that are gravitationally bound to and part of the host galaxy. Looking at the Milky Way, at small scales—the scale of our Solar System and even out to nearby stars, k_f is too small for non-Newtonian effects to be observed. Non-Newtonian motion is only observed at galactic scales—where distances are such that the non-Newtonian acceleration component is dominant.

The vertical dashed cyan line in Fig. 10 is drawn at an H_f of $3.09 \text{ km s}^{-1} \text{ Mpc}^{-1}$, which is the value of H_f that corresponds to the MOND constant a_0 of $1.2\text{E}-10 \text{ m s}^{-2}$. Equating the deep-MOND-regime relation, Eq. (38),

with Eq. (36), distills as follows:

$$\begin{aligned} \frac{\sqrt{a_0 \overline{m_t G}}}{\chi} &= \frac{2\sqrt{cH_M \overline{m_t G}}}{\chi}, \\ a_0 &= 4cH_M, \\ H_M &= \frac{1}{4c}a_0, \\ H_M &= 3.09 \text{ km s}^{-1} \text{ Mpc}^{-1}. \end{aligned}$$

The expansion parameter corresponding to the MOND constant a_0 is designated as H_M . With Eq. (163), H_M equates to a frame speed v_M of 583 km s^{-1} , which is where the dashed blue line intersects the left red curve in Fig. 10. The blue line intersects the right magenta curve at 412 km s^{-1} . Interestingly, the Milky Way’s peculiar speed of 565 km s^{-1} is near v_M , and correspondingly, its expansion parameter of $3.04 \text{ km s}^{-1} \text{ Mpc}^{-1}$ is close to H_M . The MOND constant a_0 was arrived at empirically and is the number that yields the best curve fits—on average—for the many galaxies tested against MOND. Ultimately, is a_0 simply a gauge of the average peculiar speed of galaxies in the cosmos?

XVII. THE LOCAL GROUP

The salient question in Section IX was: “To what extent does external mass affect internal systems”? And from this, the immediate follow-on question is: What defines a system?

Here, a system is defined as a hierarchy of bodies and materials in stable orbits. These bodies move in unison such that the barycenter travels inertially. As such, all of the particles in a system are interconnected. Furthermore, it is reasoned that **all of the elements of a system share the same frame constant k_f** .

Is the top node in the hierarchy of systems typically a galaxy? a cluster of galaxies? To dig into this question, the main galaxies in the Local Group—Andromeda, Triangulum, and the Milky Way—are investigated to see where the evidence points. Eq. (69) shows that as mass is added to a system, the expansion parameter of a reference mass in that system decreases in concert. In general, this relation can be used as a starting point to explore the effects of combining galaxies into a system.

For example, if the Milky Way and Andromeda comprised a system, the Milky Way’s expansion parameter H_f , estimated above at $3.04 \text{ km s}^{-1} \text{ Mpc}^{-1}$, would be diluted by the system mass m_t , which would now include Andromeda’s mass:

$$\begin{aligned} H_r &= \frac{m_r}{m_t} H_f, \\ v_{flat} &= \sqrt{2m_r} \left(\frac{cH_r G}{m_t} \right)^{\frac{1}{4}}. \end{aligned} \quad (172)$$

Also, note that Eq. (167), the relation for v_{flat} , takes a different form when the reference mass is only a portion

of the system mass:

Assuming that the Milky Way and Andromeda are of roughly equal mass [42], which is fair for a quick illustrative calculation, m_r , which corresponds to the Milky Way here, and m_t are as follows:

$$\begin{aligned} m_r &= 1.68 \times 10^{41} \text{ kg}, \\ m_t &= 3.36 \times 10^{41} \text{ kg}. \end{aligned}$$

For a rough calculation, the peculiar speed v_f of the two-galaxy frame is assumed to be that of the Milky Way, which, in turn, leaves the frame constant H_f for the system at $3.04 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

$$\begin{aligned} v_f &= 565 \text{ km s}^{-1}, \\ H_f &= 3.04 \text{ km s}^{-1} \text{ Mpc}^{-1}. \end{aligned}$$

With these new parameters, H_r , k_f , and v_{flat} for the Milky Way recalculate as follows:

$$\begin{aligned} H_r &= 1.52 \text{ km s}^{-1} \text{ Mpc}^{-1}, \\ k_f &= 2.94 \times 10^{-61} \text{ kg}^{-1} \text{ s}^{-1}, \\ v_{flat} &= 135 \text{ km s}^{-1}. \end{aligned}$$

The expansion parameter of the Milky Way H_r is now half its previous value. The frame constant k_f , which now applies to the system as a whole—with double the mass—also gets reduced by one half. In concert, Eq. (172) gives a significantly lower value for v_{flat} — 135 km s^{-1} than the above value of 191 km s^{-1} . Interestingly, looking at v_{flat} , this two-galaxy system has a significantly lower energy state—both in terms of kinetic and potential energies—than the case where the galaxies are independent. This is with all else being equal—including mass and peculiar speed. The potential large disparity in energy states is an additional distinction in terms of what defines a system.

Based on Yoshiaki Sofue’s work, it’s possible that the Milky Way’s rotation curve does flatten to 135 km s^{-1} . Recall that past 100 kpc, the error bars of Sofue’s grand rotation curve allow for rotational velocities from 100 to 200 km s^{-1} [41]. With neither value of v_{flat} — 135 or 191 km s^{-1} —ruled out, the question as to whether the Milky Way and Andromeda comprise a system remains unresolved. Clearly, an analysis of Andromeda’s and the Milky Way’s rotation curves would offer further evidence in a particular direction, but a look at Triangulum (M33) has the potential to be more informative.

The mass of M33 is an order of magnitude less than Andromeda (M31) and the Milky Way [43][44]. The implication is that if M33 is part of a system comprising one or both of its larger neighbors, its expansion parameter would be reduced by an order of magnitude in comparison to its non-system value. Further, the distance between M33 and M31 is estimated at 200-240 kpc [45], which opens the possibility that M33 could be a satellite of M31. An analysis of M33’s rotation curve should provide a clear indication if this is indeed the case.

Fig. 11 shows M33’s rotation curve, which was generated using the same methodology—outlined in Ap-

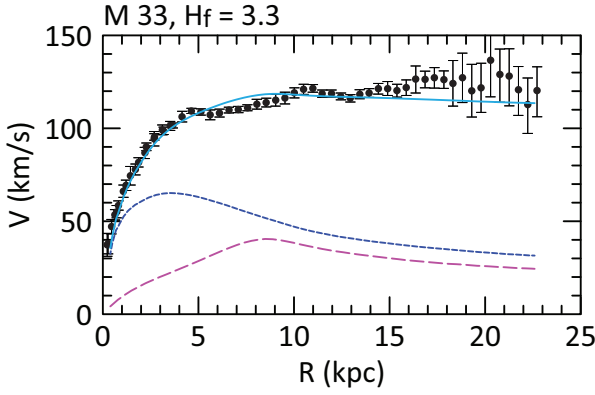


FIG. 11. The net rotation curve is drawn as a solid cyan line, and the Newtonian curves for the stellar and gas disks are drawn with dashed blue and magenta lines, respectively, with the magenta line having the larger dashes. Data are from Corbelli et al. 2014 [44].

pendix XXIII—that was employed for the rotation curves of Appendix XXIV. Density profiles for the stellar disk and gas components were derived from Newtonian curves rendered by Corbelli et al. 2014 [44].

The net rotation curve was fit with the following values for m_t and H_f :

$$\begin{aligned} m_t &= 7.51 \times 10^9 M_\odot = 1.49 \times 10^{40} \text{ kg}, \\ H_f &= 3.3 \text{ km s}^{-1} \text{ Mpc}^{-1}, \\ v_f &= 666 \text{ km s}^{-1}, \\ k_f &= 7.17 \times 10^{-60} \text{ kg}^{-1} \text{ s}^{-1}, \\ v_{flat} &= 106 \text{ km s}^{-1}. \end{aligned}$$

With these values for m_t and H_f , v_{flat} calculates to 106 km s^{-1} , which aligns with the rotation curve in the figure.

The total mass m_t , gleaned via these density profiles, is in line with estimates from Corbelli’s 2014 paper and a previous paper—Corbelli et al. 2003, which placed M33’s mass at $5.4 - 8.4 \text{ E}9 M_\odot$ [43].

The expansion parameter H_f of $3.3 \text{ km s}^{-1} \text{ Mpc}^{-1}$ translates to a peculiar frame speed v_f of 666 km s^{-1} . This is not too far from the estimated peculiar speed of the Milky Way—at 565 km s^{-1} —and that of the Local Group— 620 km s^{-1} . Two SPARC galaxies from Appendix XXIV that are also considered part of the Local Group—NGC 3109 and UGCA 444—have similar values for H_f : 3.6 and $3.2 \text{ km s}^{-1} \text{ Mpc}^{-1}$, respectively.

The frame constant k_f for M33 is an order of magnitude larger than that of the Milky Way. Given that M31 has a similar mass and peculiar speed to that of the Milky Way, M31’s frame constant should also be an order of magnitude smaller than that of M33. As such, while M33 is certainly gravitationally influenced by M31, the two galaxies do not constitute a system—based on the stipulation that all particles in a system share the same frame constant.

It’s thus unlikely that M31 and the Milky Way com-

prise a system. Further, based on their expansion parameters—which range between 3.04 and $3.6 \text{ km s}^{-1} \text{ Mpc}^{-1}$, each of the aforementioned galaxies in the Local Group (The Milky Way, M31, M33, NGC 3109, and UGCA 444) is seemingly independent of the others and does not likely constitute an element of a larger system.

Further investigation into the Local Group with the above questions in mind will be instructive. This should include an analysis of the rotation curves of M31 and the Milky Way, and the smaller pressure-supported galaxies, including the Globular Clusters on the outskirts of M31 and the Milky Way.

XVIII. RELATIVE EXPANSION PARAMETER

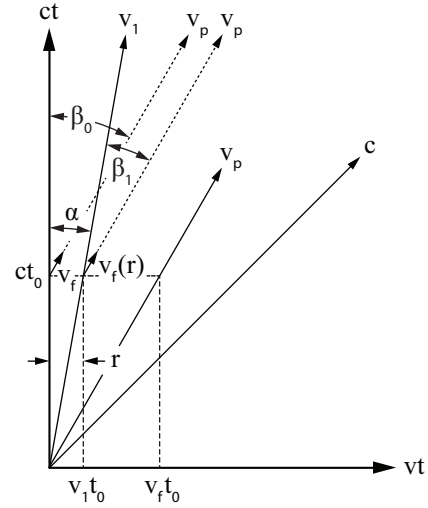


FIG. 12.

If rotation curves can remain flat for hundreds of kpc, can they remain flat for hundreds of Mpc? Can they remain flat indefinitely? To address this question, building on the framework of Section XIII, the concept of a “relative expansion parameter” $H_f(r)$ —as an extension of the “frame expansion parameter” H_f —will be investigated with distance taken into account.

Fig. 12, similar to Fig. 9, is a spacetime diagram with a frame moving at speed v_p in the rest frame of a Hubble coordinate. Recall from Section XIII that deceleration—as given by Eq. (163)—is calculated in the reference frame of the coincident coordinate—which is to say in the Hubble frame. In contrast to Fig. 9, the rest frame of Fig. 12 is that of the coincident coordinate—which is always the coordinate crossed by the moving frame at t_0 (the Hubble time). Notice that the moving frame is crossing the coordinate of the rest frame at t_0 . At this moment, v_f is equivalent to v_p .

Recall that while v_f is measured relative to the coincident Hubble coordinate, v_p is measured relative to a fixed coordinate, or alternatively said, relative to an ob-

server that is static in the Hubble frame. Thus, while v_p remains constant over time relative to this observer, v_f is always decreasing. Looking at the diagram, v_f decreases with distance as well. Notice, at distance r in the rest frame, peculiar speed $v_f(r)$, which is measured relative to coincident coordinate v_1 , is less than v_f —as shown by angle β_1 —which is less than β_0 . Here, peculiar speed $v_f(r)$ is defined as the peculiar speed at offset r in the rest frame.

As distance r in the rest frame increases, peculiar speed $v_f(r)$ decreases. At distance $v_f t_0$ from the ct axis, the slope of the coincident Hubble coordinate is equivalent to that of v_p . Across this distance—where the angle between v_p and the coincident coordinate is zero—a first observer in the frame at the ct axis would gauge that a second observer has a peculiar speed $v_f(r)$ of zero. Further, with the frame expansion parameter H_f being a function of v_f —as described by Eq. (163), this first observer would also gauge that the second observer sees the frame as having an expansion parameter H_f of zero.

This raises a question: In previous sections, the expansion parameter of the parent frame H_f was deemed to be invariant with respect to motion and position—a value that was agreed upon by all particles in the system. Is this still the case? The answer is yes. From the perspective of the first observer, the frame has an expansion parameter of H_f . Importantly, the second observer also agrees with this value and sees the frame as having the same expansion parameter of H_f . However, the first observer sees the second observer as existing in a frame with a smaller, relative expansion parameter of $H_f(r)$; and in congruence, the second observer sees the first as existing in the same relative frame—where each observer agrees on the value of other's relative expansion parameter $H_f(r)$.

Recognize that in the spacetime diagram of Fig. 12, there is no preferred observer in the moving frame. From the perspective of a first observer, a second observer at distance $v_f t_0$ has a peculiar speed $v_f(r)$ of zero. And likewise, from the perspective of the second observer, the first observer has a peculiar speed $v_f(r)$ of zero. Neither observer is preferred, and both agree that the other has a peculiar speed of zero. With Eq. (163), which relates peculiar speed to expansion parameter, each observer would see the other as existing in a frame with an expansion parameter of zero.

Further, because there is no preferred direction in the frame, two observers orthogonal to the first two—also at a distance of $v_f t_0$ —would agree that the frame has an expansion parameter of zero as well, but only with respect to each other! If these four observers formed a square, a pair of neighboring observers would be closer than $v_f t_0$, in which case, each observer in this pair would see the other as having a non-zero expansion parameter.

As a practical matter, at the scale of the typical rotation curve, $H_f(r)$ does not deviate with any significance from H_f . However, at large scales—typically on the order of Mpc, $H_f(r)$ can be substantially lower than H_f .

To explore the relation between expansion parameter and distance, refer again to Fig. 12. Angles β_0 and β_1 in the diagram are related to speed as follows:

$$\tan(\beta_0) = \frac{v_f}{c}, \quad (173)$$

$$\tan(\beta_1) = \frac{v_f(r)}{c}. \quad (174)$$

Note that v_f in Eq. (173) is relative to the coincident Hubble coordinate of the rest frame; and $v_f(r)$, which is the peculiar speed at offset r , is relative to coincident coordinate v_1 .

Similarly, a relation between distance r in the rest frame and angle α is as follows:

$$\tan(\alpha) = \frac{r}{ct_0}. \quad (175)$$

Expanding upon the equations above gives this:

$$\beta_1 = \beta_0 - \alpha, \quad (176)$$

$$\tan(\beta_1) = \tan(\beta_0 - \alpha), \quad (177)$$

$$\frac{v_f(r)}{c} = \tan\left(\arctan\left(\frac{v_f}{c}\right) - \arctan\left(\frac{r}{ct_0}\right)\right), \quad (178)$$

$$\frac{v_f(r)}{c} = \left(\frac{v_f}{c} - \frac{r}{ct_0}\right) \left(1 + \left(\frac{v_f}{c}\right) \left(\frac{r}{ct_0}\right)\right)^{-1}, \quad (179)$$

$$v_f(r) = (v_f - rH_0) \left(1 + \frac{v_f r H_0}{c^2}\right)^{-1}. \quad (180)$$

Eq. (176) is derived with simple geometry. Imagine v_1 rotating counter-clockwise α degrees about the origin in the diagram. This would bring v_1 in line with the ct axis—where β_1 would be equivalent to β_0 .

Plugging Eqs. (173), (174), and (175) into Eq. (178), reduces to Eq. (179) via trigonometry and simplification. Plugging Eq. (113) into Eq. (179) gives Eq. (180).

Notice that, as r goes to zero in Eq. (180), $v_f(r)$ goes to v_f . This is as expected, as both speeds in this case would be relative to the same coincident coordinate. Thus, $v_f(0) = v_f$. As r goes to $v_f t_0$ (or alternatively, v_f/H_0), v_f goes to zero, which begs the question: What happens when r goes beyond $v_f t_0$? Can v_f go negative?

While velocity, which has direction, can be negative, it doesn't make physical sense for speed—which is the magnitude of velocity (which has no direction)—to be negative. This comports with Eq. (134)—where a non-negative peculiar speed v_f translates to deceleration in the Hubble frame. Looking at the spacetime diagram, all peculiar motion decelerates in the Hubble frame—including light, which asymptotically approaches the edge of the light cone.

Conversely, a negative v_f would result in positive acceleration in the Hubble frame, which, according to the diagram, could only happen with time running in reverse! Further, would a body perpetually accelerating in the Hubble frame—which is to say the universe—be guaranteed to stay within the universe? Is deceleration the

universe's way of ensuring that nothing can escape?

Additionally, looking at the relation for H_f given by Eq. (163), a negative v_f would result in H_f being imaginary and thus ill-defined. For this and the above reasons, it's posited that v_f only has physical meaning as a non-negative value.

A relation for the expansion parameter $H_f(r)$ —as seen by observers separated by distance r —is given by Eq. (163), where the peculiar speed in this equation is the value agreed upon by the observers— $v_f(r)$:

$$H_f(r) = \sqrt{\frac{v_f(r)}{c}} H_0, \quad (181)$$

$$H_f(r)^2 = \frac{v_f(r)}{c} H_0^2. \quad (182)$$

Plugging Eqs. (182) and (163) into Eq. (179) gives the following:

$$\frac{H_f(r)^2}{H_0^2} = \left(\frac{H_f^2}{H_0^2} - \frac{r}{ct_0} \right) \left(1 + \left(\frac{H_f^2}{H_0^2} \right) \left(\frac{r}{ct_0} \right) \right)^{-1} \quad (183)$$

$$H_f(r)^2 = \frac{(H_f^2 - \frac{rH_0}{c} H_0^2) H_0^2}{(H_0^2 + \frac{rH_0}{c} H_f^2)}, \quad (184)$$

$$H_f(r) = H_0 \sqrt{\frac{H_f^2 - \frac{rH_0}{c} H_0^2}{H_0^2 + \frac{rH_0}{c} H_f^2}}. \quad (185)$$

Hence, in a frame with peculiar speed v_f and expansion parameter H_f , Eq. (185) gives the relative expansion parameter $H_f(r)$ as a function of distance r . Notice that, similar to Eq. (180), as r goes to zero, $H_f(r)$ goes to the frame expansion parameter H_f . Thus, $H_f(0) = H_f$.

1. Frame Extent x_0

Here, a new parameter x_0 , termed the “frame extent”, is introduced that is defined as the distance where $H_f(r)$ goes to zero. The frame extent x_0 is derived via Eq. (185) as follows:

$$\begin{aligned} H_f^2 - \frac{x_0 H_0}{c} H_0^2 &= 0, \\ x_0 &= \frac{c H_f^2}{H_0^3}. \end{aligned} \quad (186)$$

Notice, for light, which sees an expansion parameter of H_0 , Eq. (186) reduces to Eq. (112)—where x_0 goes to r_0 :

$$\begin{aligned} x_0 &= \frac{c H_0^2}{H_0^3}, \\ x_0 &= r_0 = \frac{c}{H_0}. \end{aligned}$$

Thus, the upper limit of the frame extent x_0 is the size of the universe r_0 . The lower limit of x_0 is zero, which is seen with a frame expansion parameter H_f of zero.

Note that there is a difference between the frame extent

x_0 and the frame size r_f : x_0 is an observable metric, whereas r_f is conceptual. With Eqs. (159) and (112), restated below, r_f relates to x_0 as follows:

$$\begin{aligned} r_f &= r_0 \frac{H_f}{H_0}, \\ r_0 &= \frac{c}{H_0}, \\ r_f &= \frac{c H_f}{H_0^2}, \\ \frac{H_f}{H_0} &= \frac{c H_f^2}{H_0^3} = x_0, \\ r_f H_f &= x_0 H_0. \end{aligned} \quad (187)$$

Looking at Eq. (187), with H_f being less than or equal to H_0 , the frame extent x_0 must be less than or equal to the frame size r_f . As H_f goes to H_0 , r_f goes to x_0 ; and as H_f goes to H_0 , both r_f and x_0 go to r_0 . As H_f goes to zero—looking at Eqs. (159) and (187)—both r_f and x_0 go to zero.

With Eq. (107), Eq. (187) further equates to:

$$r_f H_f = x_0 H_0 = v_f. \quad (188)$$

Thus, in relation to r_f , x_0 is the distance in the Hubble frame where the rate of expansion v_f is equivalent to the rate of expansion of the moving frame at r_f . Keep in mind that v_f is also the peculiar speed of the moving frame in the Hubble frame.

2. Relative Frame Constant $k_f(r)$

If measuring the expansion parameter of the parent frame across a distance can result in a relative value—described by $H_f(r)$, which deviates from H_f , where does this leave frame constant k_f ?

Recall from Section X that the positions and motions of the particles comprising a system have no bearing on the expansion parameter of the frame containing the system. The expansion parameter is a property of the frame alone.

This is still the case in the present situation. Further, it is still true that all particles in the system have the same frame constant k_f , which is defined by Eq. (109) as the expansion parameter per unit mass. Thus k_f can be calculated by dividing the frame expansion parameter H_f by the total system mass m_t . This relation still holds as well.

The frame expansion parameter H_f remains in effect and is distributed across all particles in the system—as described by Eq. (109). In relation to the peculiar speed of the frame, any point in the frame could be considered as the instantaneous “origin”—where $v_f(r) = v_f(0) = v_f$. Thus, any single point would see the frame having an expansion parameter of $H_f(r) = H_f(0) = H_f$. Because every particle in the system agrees on the frame's expansion parameter, it can be imagined that they all have the

same state—which is to say the same frame constant k_f .

However, while particles *in isolation* have the same frame constant k_f , akin to $H_f(r)$, a first particle will see a second particle in the system—at a distance r —as having a smaller, relative frame constant $k_f(r)$. Reciprocally, the second particle will see the first particle as having this same frame constant— $k_f(r)$. Building on Eq. (109), a relation for $k_f(r)$ is as follows:

$$k_f(r) = \frac{H_i(r)}{m_i} = \frac{H_f(r)}{m_t}. \quad (189)$$

If the relative expansion parameter $H_f(r)$ varies with distance—with one particle seeing another as existing in a relative frame that varies with distance—is it possible that $H_f(r)$ also varies with time and speed?

Looking at Eq. (163), H_f is already a function of time and speed— H_0 and v_f . Recall that H_0 is defined as the inverse of the Hubble time t_0 and thus, by definition, changes with time—asymptotically approaching zero. v_f is the peculiar speed relative to the Hubble frame and asymptotically approaches zero as well. Recall that inertial frames with non-zero peculiar speeds are always decelerating in the Hubble frame.

Thus, the expansion parameter H_f of a moving frame decreases over time—asymptotically approaching zero. It's not unreasonable, then, to imagine that the expansion parameter would decrease over distance as well—as described by relative expansion parameter $H_f(r)$. **Importantly, like H_f , $H_f(r)$ is a property of the frame and not its contents.**

Again, particles in isolation will all still measure the parent frame as having an expansion parameter of H_f —regardless of their positions and velocities relative to the frame. It's only when two particles, separated by a distance, measure the frame in unison that they see a relative value $H_f(r)$, which differs from the frame parameter H_f . Therefore, the frame's relative expansion parameter weakens with time *and* distance. Hence, measuring the frame at different times and across varying distances will give disparate results.

Note that while H_f will diminish over time, the rate of change of H_0 and v_f in Eq. (163) is negligible—as evidenced by $\dot{H}_0$ in Eq. (115), which calculates to an extremely small rate. To put this number in perspective: In another 13.8 billion years—the estimated age of the universe—the Hubble parameter will have not quite dropped from 70 to 69 $\text{km s}^{-1} \text{Mpc}^{-1}$.

3. A 10-Mpc Rotation Curve

Fig. 13 shows the rotation curve of a simulated galaxy that extends out to 10 Mpc. The curve was generated identically to the other curves herein—via Eq. (70)—but with H_f in this equation having been replaced by $H_f(r)$ —as given by Eq. (185).

The cyan line in Fig. 13 plots the curve for a galaxy

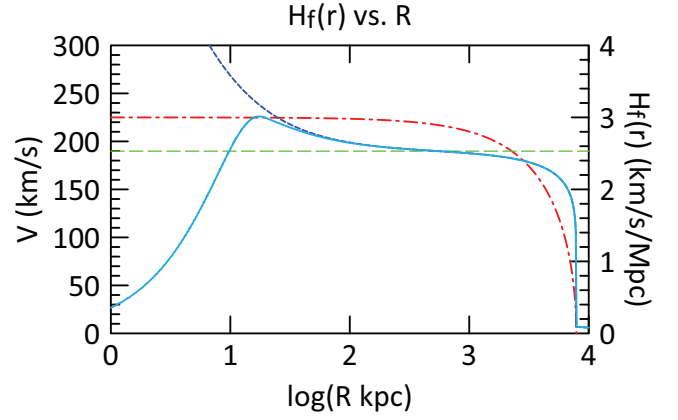


FIG. 13. A simulated galaxy with only a bulge. The net rotation curve is drawn as a solid cyan line. The blue line with small dashes is the rotation curve with the galaxy mass concentrated at a point. The green dashed line plots v_{flat} , and the red dash-dot line plots $H_f(r)$ vs. distance.

with only a bulge—no stellar and gas disks. The blue line with small dashes plots the curve for the case where the equivalent galaxy mass is concentrated at a point. The parameters for this galaxy are nearly identical to those of the Milky Way:

$$\begin{aligned} m_t &= 1.67 \times 10^{41} \text{ kg}, \\ H_f &= 3.0 \text{ km s}^{-1} \text{Mpc}^{-1}, \\ v_{flat} &= 190 \text{ km s}^{-1}, \\ x_0 &= 7.87 \text{ Mpc}. \end{aligned}$$

The red dash-dot line plots $H_f(r)$ vs. distance. Notice that $H_f(r)$ remains substantially flat out to 1 Mpc—where it has only dropped from 3.0 to 2.8 $\text{km s}^{-1} \text{Mpc}^{-1}$. From 1 to 6 Mpc, $H_f(r)$ declines nearly linearly to 1.5 $\text{km s}^{-1} \text{Mpc}^{-1}$. Beyond 6 Mpc, there is a rapid decline to zero at 7.87 Mpc, which is the value of frame extent x_0 for the parent frame—as calculated via Eq. (185). At this distance and beyond, $H_f(r)$ is zero, and the gravitational acceleration between particles in the frame is purely Newtonian.

The green dashed line plots v_{flat} for the galaxy, which calculates to 190 km s^{-1} . The rotation curve is nearly flat from 100 kpc to 2.8 Mpc. At 100 kpc, the rotational velocity is 5% above v_{flat} (at around 200 km/s). By 2.8 Mpc, the velocity has fallen to 5% below v_{flat} (at roughly 180 km/s), which constitutes a slope of $-7.4 \text{ km s}^{-1} \text{Mpc}^{-1}$.

Referring back to the isolated galaxies with rotation curves that remain substantially flat out to 1 Mpc—presented by Mistele et al. [17], it's predicted that, if the technology existed to look beyond 1 Mpc, these curves would eventually show declines similar to the curve of Fig. 13. Further, it's predicted that there's a distance—the frame extent x_0 —for each curve where the rotational velocity effectively drops to zero.

Notably, with x_0 being a function of the frame's expansion parameter H_f , galaxies with small expansion param-

TABLE I. A Sample of Galaxies With Relative Expansion Parameters at r_{max} .

Galaxy	H_f (km s ⁻¹ Mpc ⁻¹)	v_{flat} (km s ⁻¹)	r_{max} (kpc)	$H_f(r)$ (km s ⁻¹ Mpc ⁻¹)	v'_{flat} (km s ⁻¹)	x_0 (kpc)
Cam B	.07	10.7	1.68	.055	10.0	4.28
NGC 7793	.35	54.7	7.97	.337	54.1	107
NGC 2366	1.3	47.3	6.02	1.297	47.2	1480
NGC 0055	1.3	74.2	13.36	1.29	74.1	1480
NGC 6789	3.0	35.5	.73	3.0	35.5	7870
UGC 07524	3.0	82.9	10.50	3.0	82.9	7870

eters will have smaller frame extents and rotation curves that reflect this limit. This leaves the question: What is the consequence of not having corrected for distance (via Eq. (185)) on the rotation curves of Appendix XXIV—especially those with small expansion parameters?

Table I shows a sample of galaxies with the relative parameter $H_f(r)$ calculated at r_{max} —with r_{max} marking the extent of the rotation curve. Also included in the table is v'_{flat} , which is calculated via Eq. (167) using $H_f(r)$ as opposed to H_f . Note that v'_{flat} is purely illustrative. It's only meant to indicate any potential error in the rotation curve as a consequence of using a constant expansion parameter to generate the curve. As shown in Fig. 13, v_{flat} —as implied by the name—is flat with respect to distance by definition.

Looking at the galaxy with the smallest frame expansion parameter H_f in Appendix XXIV—with an H_f of .07 km s⁻¹ Mpc⁻¹—Cam B has an $H_f(r)$ that is 21% lower at .055 km s⁻¹ Mpc⁻¹. This, however, only translates to a 6% drop in rotational velocity at r_{max} —as rotational velocity is a function of H_f raised to the power of 1/4—as given by Eq. (167).

Hence, in Table I, with Cam B having a v'_{flat} of 10.0 vs. 10.7 for v_{flat} , it's surmised that the curve—as drawn in Appendix XXIV with a constant expansion parameter—is within the margin of error—especially given that this particular curve is a poor fit in the first place.

NGC 7793 has the next lowest H_f in the sample at .35 km s⁻¹ Mpc⁻¹. Its v'_{flat} is only 1% lower than v_{flat} —indicating that, as is the case with Cam B, the curve in Appendix XXIV is drawn with acceptable accuracy. The situation improves—the margin between v'_{flat} and v_{flat} decreases—for the remaining galaxies in the table with higher values of H_f . This could also be deduced by comparing r_{max} for these galaxies with x_0 . Qualitatively, looking at the table, when x_0 is an order of magnitude larger than r_{max} , it's reasonable to use a constant expansion parameter across the span of the rotation curve.

In summary, the expansion component of acceleration does not extend indefinitely but has a specific reach, and only within this range does it make a meaningful contribution to the motions of bodies. In galactic subsystems such as our Solar System, its effect is negligible and does not measurably contribute to the motions of bodies. Even out to 10 light-years, the expansion component is so small relative to the Newtonian component—orders of

magnitude less—that it can safely be ignored. However, in the range of kpc to Mpc, the expansion component becomes dominant, but as frame extent x_0 is approached, this component rapidly drops to zero.

Upon consideration, the expansion component of acceleration *must* have a limited range. If rotation curves remained flat indefinitely, then the accelerating fields underpinning those curves would have to extend indefinitely as well. Were this the case, the pull between bodies across vast distances would be far too large for the universe to have the structure and dynamics we observe, where galaxies, on average, are moving apart at speeds that increase with distance.

XIX. DYNAMIC MASS

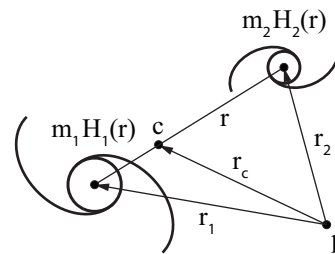


FIG. 14.

While the range of the expansion component is finite, it can still extend far enough to affect nearby galaxies. For example, the simulated galaxy of Section XVIII with a frame expansion parameter H_f of 3 km s⁻¹ Mpc⁻¹ has a frame extent x_0 of 7.87 Mpc, which encompasses most if not all of the galaxies in the Local Group. It's not unreasonable to imagine that 3 km s⁻¹ Mpc⁻¹ would be near the average expansion parameter for galaxies. Recall that the Milky Way has an estimated H_f of 3.04 km s⁻¹ Mpc⁻¹; and as shown in Fig. 10, an H_f of 3.09 km s⁻¹ Mpc⁻¹ corresponds to the MOND constant a_0 , which has been checked against a large catalog of galaxies.

In Section XVII, it was reasoned that the main galaxies of the Local Group—the Milky Way, M31, and M33, taken in any combination, do not comprise a system as defined in the section. Yet, they are gravitationally bound—with each having a frame extent of roughly 8

Mpc—corresponding to a frame expansion parameter on the order of $3 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

To explore how these galaxies influence each other, refer to Fig. 14, which shows two galaxies of different mass, expansion parameter, and frame constant moving in the Hubble frame. Note that there is no frame expansion parameter H_f for the galaxy pair because the pair does not comprise a system! Recall from Section XVII that all particles in a system, by definition, must have the same frame constant k_f . The frame constant of galaxy one in the figure is H_1/m_1 , whereas the frame constant of galaxy two is H_2/m_2 .

The acceleration with respect to galaxy one at distance r is given by Eqs. (185) and (70):

$$H_1(r) = H_0 \sqrt{\frac{H_1^2 - \frac{rH_0}{c} H_0^2}{H_0^2 + \frac{rH_0}{c} H_1^2}}, \quad (190)$$

$$a_1 = \frac{m_1 G}{r} \left(\frac{1}{r} + \frac{2\sqrt{cH_1(r)}}{\sqrt{m_1 G}} \right). \quad (191)$$

Here, it's assumed that galaxies one and two are sufficiently far apart that they can be considered as point masses.

The acceleration with respect to galaxy two at distance r is thus:

$$H_2(r) = H_0 \sqrt{\frac{H_2^2 - \frac{rH_0}{c} H_0^2}{H_0^2 + \frac{rH_0}{c} H_2^2}}, \quad (192)$$

$$a_2 = \frac{m_2 G}{r} \left(\frac{1}{r} + \frac{2\sqrt{cH_2(r)}}{\sqrt{m_2 G}} \right). \quad (193)$$

Looking at Eqs. (191) and (193), the following inequality is apparent:

$$m_2 a_1 \neq m_1 a_2.$$

Here, it's presumed that particles external to a system see the same acceleration as particles within the system. The particles of galaxy two experience an acceleration a_1 toward galaxy one and likewise, the particles of galaxy one experience an acceleration a_2 toward galaxy two.

Thus, referring to Fig. 14, an inertial observer at point p would not see point c —the barycenter of galaxies one and two—maintaining constant velocity over time:

$$\frac{d}{dt}(\vec{r}_c) \neq 0.$$

There is, however, a “dynamic mass” m_1' and dynamic mass m_2' such that the following equation is satisfied:

$$m_2' a_1 = m_1' a_2. \quad (194)$$

With Eq. (194), relations for m_1' and m_2' are as fol-

lows:

$$\begin{aligned} \frac{m_1'}{m_2'} &= \frac{a_1}{a_2}, \\ \frac{m_1'}{m_2'} &= \frac{\frac{m_1 G}{r} \left(\frac{1}{r} + \frac{2\sqrt{cH_1(r)}}{\sqrt{m_1 G}} \right)}{\frac{m_2 G}{r} \left(\frac{1}{r} + \frac{2\sqrt{cH_2(r)}}{\sqrt{m_2 G}} \right)}, \\ \frac{m_1'}{m_2'} &= \frac{m_1 \left(1 + \frac{2r\sqrt{cH_1(r)}}{\sqrt{m_1 G}} \right)}{m_2 \left(1 + \frac{2r\sqrt{cH_2(r)}}{\sqrt{m_2 G}} \right)}, \\ m_1' &= m_1 \left(1 + \frac{2r\sqrt{cH_1(r)}}{\sqrt{m_1 G}} \right), \end{aligned} \quad (195)$$

$$m_2' = m_2 \left(1 + \frac{2r\sqrt{cH_2(r)}}{\sqrt{m_2 G}} \right). \quad (196)$$

With Eq. (185), restated below, and Eqs. (195) and (196), a general relation for dynamic mass m' —as a function of baryonic mass m , frame expansion parameter H_f , and distance r —is as follows:

$$\begin{aligned} H_f(r) &= H_0 \sqrt{\frac{H_f^2 - \frac{rH_0}{c} H_0^2}{H_0^2 + \frac{rH_0}{c} H_f^2}}, \\ m' &= m \left(1 + \frac{2r\sqrt{cH_f(r)}}{\sqrt{mG}} \right). \end{aligned} \quad (197)$$

Looking at Eq. (197), the dynamic mass m' is comprised of two components—the baryonic term m and the dynamic term $m(2r\sqrt{cH_f(r)}/\sqrt{mG})$. Recognize that the dynamic term is only apparent with respect to a second mass at a distance r —and thus is only meaningful in the context of Eq. (194). The implication of this equation is that the momentum of two interacting particles—in the general case where those particles may or may not belong to the same system—is only conserved in the case where those masses are considered as dynamic—as described by Eqs. (185) and (197). As will be shown below, the dynamic terms in Eq. (194) cancel out when the two masses are part of the same system.

The logic behind dynamic mass is thus: A galaxy or, generally, a system of bodies, with a frame expansion parameter H_f of zero, has a dynamic mass m' that is equivalent to its baryonic mass m . All of the motions in this system are purely Newtonian. A system, however, with a non-zero H_f , has internal motions where bodies have elevated speeds that don't comport with the observable mass. Such a system could be construed as having missing mass or “dark mass”—where the larger the expansion parameter H_f , the greater the mass discrepancy. Thus, the expansion parameter of a given system can be thought of as amplifying the system's mass and increasing its inertia.

The equations for dynamic mass are applicable *within* systems as well as between systems. For example,

Eq. (194)—as applied to two bodies within and part of a system—resolves as follows:

$$\begin{aligned}
m_2' a_1 &= m_1' a_2, \\
m_2 \left(1 + \frac{2r\sqrt{cH_2(r)}}{\sqrt{m_2 G}} \right) a_1 &= m_1 \left(1 + \frac{2r\sqrt{cH_1(r)}}{\sqrt{m_1 G}} \right) a_2, \\
k_f(r) &= \frac{H_1(r)}{m_1} = \frac{H_2(r)}{m_2}, \\
m_2 \left(1 + \frac{2r\sqrt{ck_f(r)}}{\sqrt{G}} \right) a_1 &= m_1 \left(1 + \frac{2r\sqrt{ck_f(r)}}{\sqrt{G}} \right) a_2, \\
m_2 a_1 &= m_1 a_2.
\end{aligned}$$

Bodies within a system all see the same frame expansion parameter H_f and have the same frame constant k_f . Two bodies separated by distance r , in unison, see the frame as having a relative expansion parameter $H_f(r)$, and each body sees the other as having a relative frame constant $k_f(r)$, and both agree on its value. Thus, the dynamic terms of m_1' and m_2' cancel, and Eq. (194) reduces to the familiar Third Law of Motion—with non-dynamic mass terms.

If, however, m_1' and m_2' represent bodies or particles in separate systems with different relative frame constants $k_1(r)$ and $k_2(r)$, the dynamic terms of these masses do not cancel:

$$\begin{aligned}
m_2' a_1 &= m_1' a_2, \\
m_2 \left(1 + \frac{2r\sqrt{cH_2(r)}}{\sqrt{m_2 G}} \right) a_1 &= m_1 \left(1 + \frac{2r\sqrt{cH_1(r)}}{\sqrt{m_1 G}} \right) a_2, \\
\left(k_1(r) = \frac{H_1(r)}{m_1} \right) &\neq \left(k_2(r) = \frac{H_2(r)}{m_2} \right), \\
m_2 \left(1 + \frac{2r\sqrt{ck_2(r)}}{\sqrt{G}} \right) a_1 &= m_1 \left(1 + \frac{2r\sqrt{ck_1(r)}}{\sqrt{G}} \right) a_2.
\end{aligned}$$

Including the a_1 and a_2 terms, gives this:

$$\begin{aligned}
\frac{m_1 m_2 G}{r^2} \left(1 + \frac{2r\sqrt{cH_2(r)}}{\sqrt{m_2 G}} \right) \left(1 + \frac{2r\sqrt{cH_1(r)}}{\sqrt{m_1 G}} \right) &= \\
\frac{m_1 m_2 G}{r^2} \left(1 + \frac{2r\sqrt{cH_1(r)}}{\sqrt{m_1 G}} \right) \left(1 + \frac{2r\sqrt{cH_2(r)}}{\sqrt{m_2 G}} \right).
\end{aligned}$$

Notice that, beyond the frame extents of m_1 and m_2 —where $H_1(r)$ and $H_2(r)$ go to zero, the above equation resolves to the familiar relation for Newtonian gravity:

$$\begin{aligned}
\frac{m_1 m_2 G}{r^2} \left(1 + \frac{2r\sqrt{cH_2(r)}}{\sqrt{m_2 G}} \right) \left(1 + \frac{2r\sqrt{cH_1(r)}}{\sqrt{m_1 G}} \right) &= \\
\frac{m_1 m_2 G}{r^2} \left(1 + \frac{2r\sqrt{cH_1(r)}}{\sqrt{m_1 G}} \right) \left(1 + \frac{2r\sqrt{cH_2(r)}}{\sqrt{m_2 G}} \right), \\
\frac{m_1 m_2 G}{r^2} &= \frac{m_1 m_2 G}{r^2}.
\end{aligned}$$

The center of two dynamic masses is termed the “dynamic barycenter”. Referring back to Fig. 14, if point c were the dynamic barycenter of galaxies one and two—which is to say the center of m_1' and m_2' , an observer at point p *would* see point c maintaining constant velocity over time. However, with m_1' and m_2' being dynamic, the ratio of distances from c to each galaxy would evolve over time.

If the galaxies of the Local Group are bound within the frame extents of the three main galaxies—at roughly 8 Mpc, it’s reasonable to imagine that this is the case for other galaxy groups and clusters. When thinking about larger structures, it’s not necessary that every galaxy reside within the frame extent of every other galaxy. In other words, galaxies or groups can be chained together, where only the frame extents of individual links (or series of links) overlap. Extrapolating further, is it possible that the large-scale structures we observe in the universe—such as filaments and walls—are the result of such chaining—a phenomenon that allows these structures to form and remain stable over billions of years?

1. Gravitational Lensing

It’s presumed that large dynamic-mass terms would bend light at larger deflection angles than what the baryonic mass would predict—which is to say that the expansion component of acceleration should also affect light. However, gauging the deflection angle for a given dynamic mass will require relativistic extensions of Eqs. (21) and (70)—with attention paid to spatial curvature—the latter giving double the deflection angle than that accounted for by time dilation alone. How dynamic mass relates to spatial curvature is an open question.

It’s also necessary that any relativistic extension reduces to general relativity (in its current form) in systems where H_f goes to zero—such as the solar system—where decades of experiments have verified relativistic effects such as gravitational time dilation and the deflection of light around the sun from distant stars.

Further, this extension must also be concordant with gravitational waves that propagate at c , which has been verified by experiment [46]. This constraint effectively eliminates TeVeS as a possible modified-gravity theory. TeVeS (Tensor-Vector-Scalar) is a relativistic formulation for MOND that predicted gravitational waves from GW170817 should have arrived approximately 400 days out of sync with the light signal from the merger of binary stars GW170817 and GRB 170817A, which was not the case [47]. The authors of the cited paper note that not all modified gravity models are necessarily falsified by the data.

XX. MASS DENSITY OF THE UNIVERSE

Continuing the discussion from Section XIV, the mass m_0 of the universe is given by Eq. (150):

$$m_0 = \frac{cH_0 r_0^2}{G},$$

where H_0 and r_0 are the expansion parameter and size of the universe, respectively, in the rest frame of an inertial observer.

As reasoned in Section XIV, all inertial observers should agree on the total mass of the universe m_0 . And, as deduced in Section XIII, any given inertial observer—regardless of his peculiar speed—should perpetually see himself as being located at the center of the universe. A peculiar speed of zero is synonymous with being at rest in the Hubble frame—which is to say, at rest in the frame of the coincident Hubble coordinate.

It's posited that, with this and the above equation in mind, the contained mass m_c at a given radius r in the rest frame of a Hubble coordinate—where r is a distance in the frame—is as follows:

$$m_c = \frac{cH_0 r^2}{G}. \quad (198)$$

Eq. (198) describes an idealized case, termed the “ground state”, where mass is evenly distributed, and the particles that comprise the contained mass are at rest in the Hubble frame. As discussed, particles at rest in the Hubble frame are moving in the Hubble flow. In the ground state, all particles move precisely in the Hubble flow—with peculiar speeds of zero. Thus, in the ground state, particles in the universe are electrically neutral, have zero kinetic energy relative to the Hubble frame, and zero vibrational energy. This, in turn, applies to Eq. (150), which gives the net rest mass of the universe in the ground state.

In reality, however, particles and systems of particles—under the influence of gravity—are generally found to be in constant motion relative to the Hubble frame. This is termed the “dynamic state”. However, as the scale at which the universe is observed increases, the universe appears increasingly homogeneous and isotropic. Bodies look to largely be moving in the Hubble flow—which is to say, at large scales, the structure of the universe approaches that described by the ground state. With this in mind, investigating the ground state and how this state evolves over time can inform an understanding of the dynamic state. In this section, it will be shown that the deconvolution of Eq. (198) can reveal important aspects of the ground state—mass density, how mass moves over time, and how mass is created in exchange for the expansion of space.

The speed v of a coordinate at a distance r in the rest frame of an inertial observer is given by Eq. (110). The

derivative of this equation with respect to r is as follows:

$$\begin{aligned} v &= rH_0, \\ \frac{dv}{dr} &= H_0 + r \frac{dH_0}{dr}, \end{aligned} \quad (199)$$

$$\frac{d}{dt} \frac{dr}{dr} = \frac{dr}{dt} \left(H_0 + r \frac{dH_0}{dr} \right), \quad (200)$$

$$\frac{dH_0}{dr} = -\frac{H_0}{r}. \quad (201)$$

Notice that the left side of Eq. (200) goes to zero with the employment of Eq. (123)—which holds that the speed v of a receding coordinate remains constant over time. Plugging Eq. (201) into Eq. (199) shows that v remains constant over distance r as well:

$$\begin{aligned} \frac{dv}{dr} &= H_0 + \chi \left(-\frac{H_0}{\chi} \right), \\ \frac{dv}{dr} &= 0. \end{aligned} \quad (202)$$

Taking the derivative of Eq. (198) with respect to r gives the following:

$$\begin{aligned} \frac{dm_c}{dr} &= \frac{c}{G} \left(\frac{dH_0}{dr} r^2 + 2H_0 r \right), \\ \frac{dm_c}{dr} &= \frac{c}{G} \left(-\frac{H_0}{\chi} r^2 + 2H_0 r \right), \end{aligned} \quad (203)$$

$$\frac{dm_c}{dr} = \frac{cH_0 r}{G}, \quad (204)$$

$$\frac{dm_c}{dr} = \frac{cv_c}{G}. \quad (205)$$

Plugging Eq. (201) into Eq. (203) gives Eq. (204). Plugging Eq. (110) into Eq. (204) resolves to Eq. (205)—the latter showing that dm_c/dr is invariant with respect to distance *and time* for a given Hubble coordinate v_c . The reference frame of m_c is a Hubble coordinate, and any given coordinate v_c at a distance r in this rest frame—that is receding according to Eq. (110)—is also a Hubble coordinate.

The area density ρ_A of a layer as a function of v_c derives from Eq. (205):

$$\begin{aligned} dm_c &= \frac{cv_c dr}{G}, \\ A_c &= 4\pi r^2, \\ dA_c &= 8\pi r dr, \\ \frac{dm_c}{dA_c} &= \frac{cv_c}{8\pi Gr}, \\ \rho_A &= \frac{cv_c}{8\pi Gr}, \end{aligned} \quad (206)$$

$$\rho_A = \frac{cv_c H_0}{8\pi Gr H_0} = \frac{cv_c H_0}{8\pi G}, \quad (207)$$

$$\rho_A = \frac{cH_0}{8\pi G}. \quad (208)$$

Eq. (206) shows that ρ_A is a function of v_c and r . Multiplying Eq. (206) by H_0/H_0 gives Eq. (207), which—via Eq. (110)—reduces to Eq. (208). Interestingly, recalling Eq. (113), Eq. (208) shows that the area density ρ_A in a given frame is purely a function of time in that frame—with area density decreasing over time. This aligns with the Hubble flow, where the mass of the universe at a large scale is observed to be spreading apart—decreasing in area density.

With Eq. (205) and the relation for differential volume dV , the volume density (or density) ρ is thus:

$$\begin{aligned} dV_c &= 4\pi r^2 dr, \\ \frac{dm_c}{dV_c} &= \frac{cv_c dr}{4\pi Gr^2 dr} = \frac{cH_0}{4\pi Gr} \frac{dr}{dr}, \\ \rho &= \frac{cH_0}{4\pi Gr} = \frac{cH_0^2}{4\pi GrH_0}, \end{aligned} \quad (209)$$

$$\rho = \frac{cH_0^2}{4\pi Gv_c}, \quad (210)$$

$$\rho = \frac{2H_0}{v_c} \rho_A. \quad (211)$$

Eq. (210) gives the volume density ρ at coordinate v_c as a function of v_c . Eq. (211) gives density ρ_c as a function of area density ρ_A . Depth, in these two equations, is embodied by the Hubble coordinate v_c , which is a more convenient measure of distance than r —as it remains constant over time. Compared with the equation for ρ_A —where area density is decreasing along with H_0 (as described by Eq. (115)), ρ decreases by the square of H_0 , which accounts for mass spreading out both tangentially and radially.

Comparing the average density contained by coordinate v_c to the density at v_c gives this:

$$\begin{aligned} \rho_{ave} &= \frac{m_c}{V_c} = \frac{cH_0^2}{G} \frac{3}{4\pi r}, \\ \rho_{ave} &= \frac{3cH_0^2}{4\pi Gv_c} = 3\rho. \end{aligned} \quad (212)$$

$$\rho_{uave} = \frac{3H_0^2}{4\pi G}. \quad (213)$$

The average density ρ_{ave} is simply the mass contained by a volume divided by that volume. Eq. (212) shows that the average density in the volume is three times the density at the Hubble coordinate defining the volume; and thus, in the ground state, there is a gradient in density with respect to distance. The average density of the universe ρ_{uave} —where the volume is contained by r_0 —where $v_c = c$ in Eq. (212)—is given Eq. (213).

Note the similarity between Eq. (213) and the Friedmann equation for critical density—the density required for the universe to remain spatially flat [48]:

$$\rho_c = \frac{3H_0^2}{8\pi G}.$$

The critical density ρ_c is exactly half the value given by ρ_{uave} , but the Friedmann equation assumes uniform mass-energy density, which is not the case in the present thesis. Further, the present thesis holds that the mass of the universe is created continuously as the universe expands vs. having been created in an instant in the Big Bang. These complications make a direct comparison of these densities difficult as they derive from different premises—which makes it interesting that the two relations only differ by a factor of two.

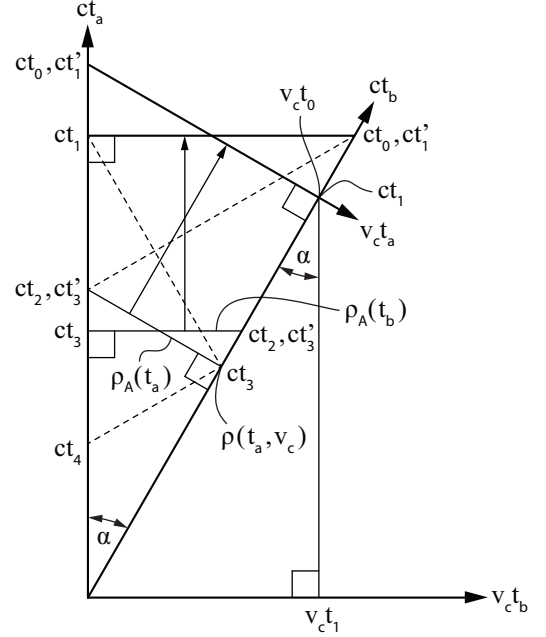


FIG. 15.

Fig. 15 is a spacetime diagram that relates time and distance in the rest frame of a Hubble coordinate to that of a moving frame. In this case, the moving frame b is a Hubble coordinate with speed v_c . The time and distance axes of the rest frame a are ct_a and $v_c t_a$, respectively; and the time and distance axes of the moving frame are ct_b and $v_c t_b$. The geometry of the diagram is such that two principles are satisfied: One, the speed between a fixed point in the rest frame and a fixed point in the moving frame is the same in both frames; and two, the speed of light calculates to c in both frames. Similar to the spacetime diagrams of Figs. 9 and 12, the world lines of Hubble coordinates a and b converge at the universe's initial event. As before, t_0 is the Hubble time in the rest frame.

From the perspective of an observer in the rest frame, the moving frame will have traveled a distance of $v_c t_0$ in the rest frame. Here, t_0 corresponds to t'_1 , which is timespan t_1 in the moving frame as observed in the rest frame. As taught by special relativity, a clock moving in the rest frame ticks more slowly than an identical clock that is static in the rest frame. This is known as “time dilation”. The speed of a clock is measured in tick events

per unit time. As the denominator in this fraction grows larger, the clock rate slows. Notice in the diagram that the span of time t'_1 —referred to as “coordinate time”—is longer than t_1 , which is “proper time” in the moving frame; and thus, *in the rest frame*, the moving clock is ticking slower.

An observer in the moving frame, counting off ticks on his clock, would calculate that he traveled a distance of $v_c t_1$ in the rest frame. An observer in the rest frame, *looking at his clock*—and with t_1 associated with an event that both observers agree upon—would calculate that the moving frame traveled a longer distance: $v_c t_0$. Thus, from the perspective of the moving observer, distances in the frame of travel, in the direction of travel, are “length contracted” relative to what an observer at rest in the frame of travel would measure.

In the diagram, distance and time are orthogonal. Specifically, distance in the rest frame—as measured by an observer in the moving frame—is orthogonal to time in the rest frame. In congruence, distance in the rest frame—as measured by an observer in the rest frame—is orthogonal to time in the moving frame. This geometry allows observers in both frames to agree on their relative speed and to agree on the speed of light.

Distance in the rest frame along the $v_c t_a$ axis increases at speed v_c as follows:

$$\sin(\alpha) = \frac{v_c \cancel{t_0}}{c \cancel{t_0}} = \frac{v_c}{c}.$$

In the moving frame, along the $v_c t_b$ axis, distance also increases at speed v_c :

$$\sin(\alpha) = \frac{v_c \cancel{t_1}}{c \cancel{t_1}} = \frac{v_c}{c}.$$

Thus, speed v_c is consistent in the two frames.

Light (represented by dashed lines) emitted by an observer in the rest frame at t_2 travels through time a span of $c(t_1 - t_2)$ to reach an observer in the moving frame. The distance in the rest frame that this beam of light will have traveled is represented by the line drawn between ct_2 on the ct_a axis and ct_3 on the ct_b axis. Notice that the lengths of these lines are equivalent, which is to say that light travels equal lengths through space and time.

In the moving frame, this beam will have traveled through time a span of $c(t_0 - t_3)$ and a distance in space depicted by the line between ct_1 on the ct_a axis and ct_0 on the ct_b axis. Again, these distances are equivalent; thus, both observers—in the rest frame and the moving frame—measure light as traveling at c .

Distance in the rest frame can be measured by bouncing light off a mirror. This is shown in the diagram by the dashed line from ct_4 on the ct_a axis to ct_3 on the ct_b axis, which reflects back to ct_1 on the ct_a axis. Dividing the total travel time by two, an observer would conclude that the beam of light hit the mirror at time t_2 and covered a distance in space (and time) of $c(t_2 - t_4)$.

Imagine, further, two lights in the rest frame offset by

a given distance. In the diagram, one light is at the midpoint of the solid line connecting ct_2 on the ct_a axis and ct_3 on the ct_b axis, and the second light is at the endpoint of this line. Both lights are timed to flash simultaneously at time t_2 in the rest frame. One path of light would trace the dashed line from ct_3 on the ct_b axis to ct_1 on the ct_a axis. The other path (not drawn) from the midpoint would be parallel to this dashed line and would be detected by the observer exactly midway between t_2 and t_1 . The observer—accounting for distance—would conclude that both lights flashed simultaneously. Thus, it can be said that all events in parallel with the $v_c t_a$ axis are simultaneous in the rest frame; and by the same logic, all events parallel with the $v_c t_b$ axis are simultaneous in the moving frame.

Under the principle that the laws of physics should be consistent between frames a and b , identical clocks at rest in each frame should tick at the same rate. Thus, it's imagined that observers in both frames would agree on the current age and mass of the universe— t_0 and m_0 . Further, looking at Eq. (208)—which is a function of H_0 (and thus, t_0), both observers would see the universe—in its ground state—as having the same area density ρ_A in relation to their respective clocks.

At a given point in time on the ct_a axis, ρ_A , which is invariant with respect to distance, is constant along the $v_c t_a$ axis—the line on which events in frame a are simultaneous. This line moves forward in time—with ρ_A decreasing simultaneously at points along its length according to Eq. (208). Likewise, in frame b , ρ_A is constant along the $v_c t_b$ axis. In the diagram, both lines of constant density move forward at the same rate in their respective frames.

With the understanding that there's nothing special about frames a and b in the diagram—as these two frames could represent any two Hubble coordinates in the universe, it's imagined that an observer in the rest frame of *any* Hubble coordinate in the universe would see a uniform area density extending indefinitely in all directions. This is in line with the cosmological principle, which holds that, at a large enough scale, the universe appears homogeneous and isotropic.

The volume density ρ as given by Eq. (211), however, is a function of two parameters: time t_0 and speed v_c of a distant Hubble coordinate—where ρ is the density at coordinate v_c at t_0 in the rest frame. Looking at the spacetime diagram, an observer in frame a , at local time t_0 , would calculate the same density at the point where the $v_c t_a$ and ct_b axes intersect—as an observer in frame b would calculate at the intersection of $v_c t_b$ and ct_a —at t_0 in his frame. Both observers would attribute the same volume density to the other's coordinate.

Thus, in the ground state, density can only be calculated in the rest frame of a Hubble coordinate relative to a second distant Hubble coordinate. Moreover, observers in different rest frames at different speeds relative to a given coordinate would attribute different densities to that coordinate. This is analogous to length contraction,

where physical distances are not absolute. It's concluded, therefore, that there is no absolute volume density in the universe. Volume density is relative to the speed of the Hubble coordinate at which density is calculated.

XXI. GROUND STATE OF THE UNIVERSE

Taking the derivative of Eq. (198) with respect to time gives this:

$$\frac{dm_c}{dt} = \frac{c}{G} \left(\frac{dH_0}{dt} r^2 + 2H_0 r \frac{dr}{dt} \right), \quad (214)$$

$$\dot{m}_c = \frac{c}{G} \left(-H_0^2 r^2 + 2H_0 r \frac{dr}{dt} \right), \quad (215)$$

$$\dot{m}_c = \frac{c}{G} (-v_c^2 + 2v_c^2), \quad (216)$$

$$\dot{m}_c = \frac{cv_c^2}{G}. \quad (217)$$

Eq. (115) is plugged into Eq. (214), which gives Eq. (215); and Eq. (110) is plugged into Eq. (215)—allowing $\dot{m}_c$ to be described in terms of v_c . Notice that, similar to Eq. (156) (which gives the power of the universe), the rate of mass change contained by a radius r —or said alternatively, encapsulated within a given Hubble coordinate v_c —is constant with respect to time. In the case where the Hubble coordinate demarks the extent of the universe—where v_c goes to c , Eq. (217) reduces to Eq. (156).

Alternatively, $\dot{m}_c$ can be defined as a function of m_c as follows:

$$\begin{aligned} m_c &= \frac{cH_0 r^2}{G} \frac{H_0}{H_0}, \\ m_c &= \frac{cv_c^2}{G} \frac{1}{H_0}, \\ \dot{m}_c &= m_c H_0. \end{aligned} \quad (218)$$

The implication of Eqs. (217) and (218) is that the mass contained by any Hubble coordinate is increasing over time. It, thus, has to be concluded that the mass of the universe is continuously increasing everywhere.

This can be visualized in Fig. 16, which shows the mass m_c contained by a given Hubble coordinate v_c depicted as the area A_c of a rectangle with side lengths of $(c/G)v_c$ and r :

$$m_c = A_c = \frac{c}{G} v_c \cdot r. \quad (219)$$

Line dm_c/dr vs. v_c is simply a plot of Eq. (205). Eq. (219) is derived by plugging Eq. (110) into Eq. (198).

As shown in the figure, line dm_c/dr vs. v_c maintains the same slope over time. The only movement in Fig. 16 is along $r(t)$. At the birth of the universe—when r_0 and t_0 are zero—the area of the rectangle at c is zero—along with all other rectangles along the v_c axis. As time progresses and the universe expands, these rectangles in-

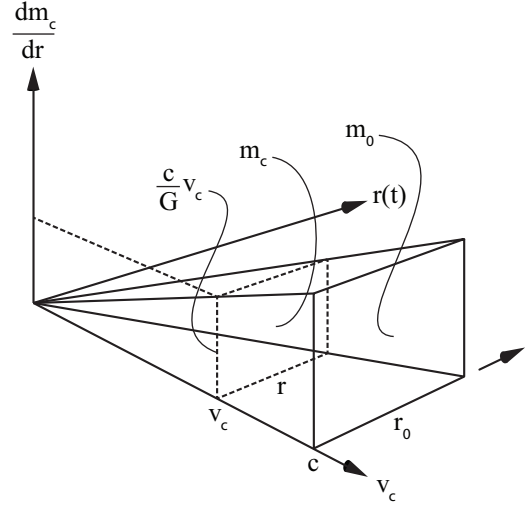


FIG. 16.

crease in area—representing the increase in mass across the span of Hubble coordinates from 0 to c .

Taking the partial derivative of Eq. (198) with respect to r —holding time constant—gives this:

$$\frac{\partial m_c}{\partial r} = \frac{2cH_0 r}{G}. \quad (220)$$

With H_0 defined as the inverse of t_0 , constant time translates to a constant expansion parameter H_0 .

To visualize how $\partial m_c/\partial r$ relates to dm_c/dr , refer to Fig. 17, which shows the $r(t)$ - dm_c/dr plane of Fig. 16 for a given Hubble coordinate v_c .

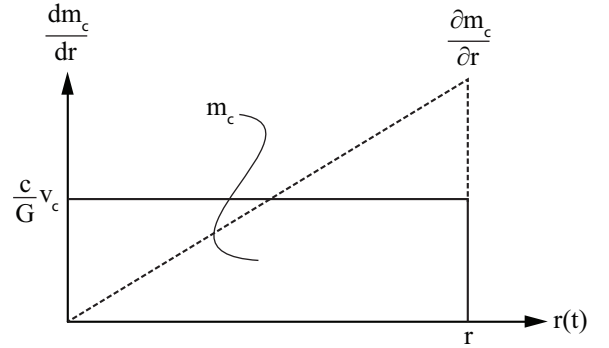


FIG. 17.

While the full derivative of m_c with respect to r , dm_c/dr —where r represents a moving coordinate—is constant over distance *and* time, the partial derivative $\partial m_c/\partial r$ increases linearly with r . Notice that in both cases, the area under the curve gives the contained mass m_c .

Alternatively, the partial derivative of Eq. (198) with

respect to time—at a constant r —is thus:

$$\begin{aligned} m_c &= \frac{cH_0 r^2}{G}, \\ \frac{\partial m_c}{\partial t} &= \frac{c}{G} \frac{dH_0}{dt} r^2, \\ \frac{\partial m_c}{\partial t} &= \frac{-cH_0^2 r^2}{G}, \end{aligned} \quad (221)$$

$$\frac{\partial m_c}{\partial t} = \frac{-cv_c^2}{G}, \quad (222)$$

$$\frac{\partial m_c}{\partial t} = -m_c H_0. \quad (223)$$

Note the negative sign in Eqs. (221) and (222)—indicating that a volume fixed at r actually loses mass, which implies that mass is moving across r . Interestingly, Eq. (223) shows that the rate of loss in this control volume is equivalent to the rate of mass-gain in an expanding volume with a moving r —the latter given by Eq. (218).

A fractional mass Δm_c moving across a small distance Δr at speed v_c has a mass-flow rate of $\dot{m}_c$:

$$\dot{m}_c = \frac{\Delta m_c v_c}{\Delta r}. \quad (224)$$

In differential form, Eq. (224) reduces as follows:

$$\begin{aligned} \dot{m}_c &= \frac{dm_c}{dr} v_c, \\ \dot{m}_c &= \frac{cv_c^2}{G}. \end{aligned}$$

Note the equivalence between the above mass-flow equation and Eq. (222), which indicates that the differential mass at a given Hubble coordinate is moving in concert with the coordinate at speed v_c . This movement applies to all Hubble coordinates, and thus, it's reasoned that all of the mass of the universe is moving in sync with these coordinates—which is to say, moving in the Hubble flow.

This mass flow describes the ground state—previously defined as the case where all particles comprising the mass of the universe are precisely in the Hubble flow with no particle having a non-zero peculiar speed—and where the mass at a given coordinate is distributed evenly across a surface defined by that coordinate.

However, Eq. (222) only accounts for the flow of existing mass across a boundary, but it doesn't account for the addition of mass, which is hypothetically increasing everywhere—as described by Eq. (217) and Fig. 16. Where and how is mass created?

1. Creation of Space and Mass

With Eq. (205), the fractional mass at two Hubble coordinates v_1 and v_2 —where v_2 is slightly greater than

v_1 —is thus:

$$\Delta m_1 = \frac{cv_1}{G} \Delta r, \quad (225)$$

$$\Delta m_2 = \frac{cv_2}{G} \Delta r. \quad (226)$$

Eqs. (225) and (226) confirm that the fractional mass across a minute distance Δr remains constant over time and distance—assuming that Δr remains constant, which is asserted here. Over time, as the distance between coordinates v_1 and v_2 increases, at a certain point, this distance will be greater than Δr —at which point, it's imagined that there is now space for a new coordinate.

The presumption here is that space does not expand continuously but in quanta—with each created quantum of space constituting a new coordinate. As reasoned in previous sections, mass is created as a consequence of this expansion. As space expands in quanta, mass is created in quanta; or alternatively said: Because the particles that make up matter could only be created in quanta—if they are manifested from expanding space, space must expand by quanta.

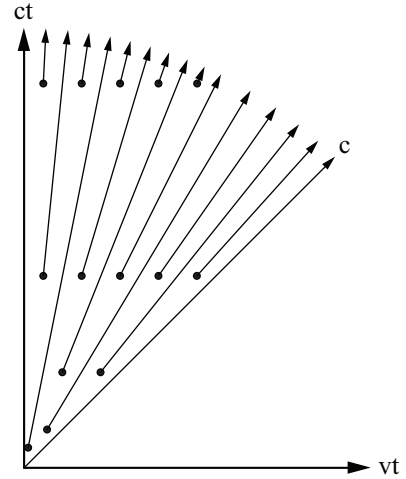


FIG. 18.

This mechanism of mass creation can be visualized in Fig. 18, which shows an array of Hubble coordinates fanning out. Black dots in the figure represent particles that are created in the spaces between these coordinates. The arrows emanating from these dots are the world lines of these particles and new Hubble coordinates created in concert. Notice that the world lines of coordinates created after the initial event all look to emanate from the initial event.

In Fig. 16, the increase in contained mass that is visualized as the growth of area A_c can be imagined as occurring as a result of the coordinate density increasing along the v_c axis. Expansion in r creates room for additional coordinates, which, in turn, increases the mass contained by r .

Imagined another way, consider Δm_1 and Δm_2 as layers of an expanding onion. As a void grows between these

layers—which constitutes expanding space, mass is created in unison with that expansion. Referring back to Eq. (206), the area density ρ_A at a given Hubble coordinate v_c decreases with r —which would be expected with a fixed mass being distributed over an increasing area. Importantly, new mass does not fill the voids within this expanding layer. The layer corresponds to an existing Hubble coordinate, and mass is only created upon the manifestation of a new coordinate—when a new layer is added to the onion.

If a particle is created in conjunction with a Hubble coordinate, there's no reason, upon creation, for this particle—at least, initially—to be moving relative to that coordinate. It's imagined that this is why there is a tendency for the matter of the universe to be moving in the Hubble flow. Mass is created in conjunction with Hubble coordinates and is initially at rest relative to these coordinates—which is to say at rest in the Hubble frame. These coordinates, however, are receding from each other, and thus, newly created mass will be moving in concert, which is to say, in the Hubble flow. But this leaves the following questions: Why would this motion persist? Why, under the influence of gravity, wouldn't the mass of the universe eventually reverse course and collapse in on itself?

2. Hubble Flow

To recap, in the ground state, all particles created in the Hubble flow perpetually remain in the Hubble flow as they expand out in the rest frame of a Hubble coordinate at constant velocities. In the rest frame of a Hubble coordinate, at a given moment in time, the area density ρ_A is invariant with respect to distance. This was shown by Eq. (208) and Fig. 15.

Further, if the universe is finite, which is deemed to be the case, an observer in the rest frame of a Hubble coordinate—moreover, *any* Hubble coordinate—will, thus, see himself as being located at the center of the universe. In the ground state, with time running reverse, not only will all particles in the universe converge on the observer's location, they will all converge at the same time—where $t_0 = 0$. Moreover, this will be the case for *any* observer in the rest frame of *any* Hubble coordinate. Thus, in the ground state, every particle sees itself at the center of the universe, and because mass, in the ground state, is homogeneous and isotropic, every particle is at the barycenter of all other particles.

At the barycenter, the gravitational fields of all surrounding particles cancel. Thus, when the universe is in the ground state, in the rest frame of *any* given particle, the net gravitational acceleration acting upon that particle is zero. Because this is the case for every particle in the universe, in the reference frame of a given particle, all other particles will continue to move indefinitely in the Hubble flow. Almost tautologically, because each particle sees all other particles moving in the Hubble flow, all

particles continue to move in the Hubble flow.

Importantly, in the ground state, a given particle does **not** recede from another particle because the space between those particles is expanding. Particles recede because they were created at rest in a dynamic frame—where any two particles separated by a distance will naturally be in motion relative to each other. A second particle will continue to move at constant velocity relative to the first because there are no net gravitational fields acting upon either. As these particles continue to drift apart, coincidentally, space expands in parallel at the rate of recession.

Further, because the recession rate of a given particle is constant over time, as the distance to this particle increases, for space to expand at the same rate, mathematically, the Hubble parameter H_0 must decrease in concert. This is described by Eq. (115), which shows H_0 decreasing at the necessary rate (as given by Eq. (123)) purely as a function of time—which *happens* to satisfy this requirement.

With that said, at the current age of the universe—13.8 billion years—in another 13.8 billion years, the Hubble parameter will only have decreased by roughly 1%. So, at present, the Hubble parameter can effectively be considered as constant. If the universe were in the ground state—*imagining the case where measurements could be made in this universe*, the expansion of the universe would look uniform and isotropic, and measurements of the Hubble parameter would be consistent across all regions of the cosmos.

3. The Dynamic State

The ground state, however, doesn't describe the universe we observe—which is highly dynamic. Even if the ground state existed for a moment—tantamount to a pencil balanced on its tip—it would quickly devolve to the dynamic state. Any perturbation would cause nearby particles to coalesce, and this slight concentration of mass would result in the net gravitational field in the vicinity of these particles increasing in strength above the baseline of zero. This strengthening field would, in turn, accelerate additional local particles toward the gathering mass—further amplifying the field in a feedback loop. As gathering particles increase in speed—with potential energy converting to kinetic energy—they would eventually collide and radiate heat. Clearly, the dynamic state is a higher-energy state than the ground state. Where did this energy come from?

It's posited that all energy in the universe is ultimately borrowed from the vacuum. This includes the creation of mass *and* the potential energy that induces the local movement of that mass. Energy flow is accounted for by changes in the size and shape of the vacuum. Energy is borrowed to create mass, which is reflected by the expansion (or creation) of space; and the concentration of mass is reflected by local curvature in spacetime.

In the ground state, all particles move in the Hubble flow at constant velocities with respect to each other because there is no net accelerating field acting upon these particles. It's thus concluded that the spacetime of the ground state is flat—as if the universe in the ground state has zero mass. As particles coalesce—transitioning from the ground state to the dynamic state, the spacetime around these particles becomes locally curved. Potential energy is converted to kinetic energy.

However, the net potential energy of the universe in the ground state is zero. Particles spread apart at constant velocities—indeinitely. Thus, as particles transition from the ground state to the dynamic state, the net potential energy of the universe decreases *from a baseline of zero*. In effect, energy is borrowed from the vacuum to accelerate these particles—with spacetime curving, increasingly, in concert.

If, at a moment in time immediately after the transition from the ground state to the dynamic state, all particles in motion—via force—were restored to their former positions in the ground state, spacetime would decurve. The work being done on these particles would increase the potential energy of the universe (which was briefly negative) back to zero as spacetime is pulled flat.

The ground state also describes a universe with zero entropy. Every particle is perfectly ordered—at rest in the Hubble frame. There is zero kinetic energy (relative to the Hubble frame) and zero vibrational energy in this state. Thus, as measured by an observer at rest in the Hubble frame, the temperature of the universe in the ground state is absolute zero.

As particles begin to coalesce in different regions, entropy increases in these regions, and energy flows from the vacuum to these particles as they gather and gain kinetic energy—raising the average temperature of the universe. As generally observed, energy flows in the direction that increases the total entropy of the universe.

In this vein, it can be imagined that increasing entropy is what makes the universe dynamic—putting its mass in motion. Both entropy and the curvature of spacetime immediately increase from baselines of zero as particles transition from the ground state to the dynamic state—implying that the two phenomena are related. Further, if the expansion of space is a function of time, is curvature a function of entropy? Interestingly, both time and the total entropy of the universe are only ever seen to be moving in one direction—increasing.

In the dynamic state—as is the case in the ground state, the recessional motion of two distant bodies is **not** a consequence of space expanding in the void—at least not directly. Once created, the motions of particles are *only* a function of their locations in the Hubble flow at the time of creation and the fields acting upon them. Particles aggregate into bodies, systems of bodies, and galaxies. The chaining of galaxies and mass flows gives rise to large-scale structures such as filaments and walls, and immense voids.

Yet—in measurements of the Hubble parameter based

on the recessional speeds of distant bodies in disparate regions of the universe—the Hubble parameter, when defined as the inverse of Hubble time, falls squarely in the range of measured values. Thus, the peculiar speeds of galaxies, at least at this point in time, have not overwhelmed their recessional speeds at large scales. Across large distances, bodies are largely receding according to Hubble's law, which is to say moving in the Hubble flow. Again, with the ground state in mind, it is imagined that bodies are largely found to be moving in the Hubble flow because the proximal particles that formed those bodies were created at rest in the Hubble frame.

4. Early Universe

Λ CDM holds that all of the matter of the universe was created in the Big Bang. BBN (also referred to as primordial nucleosynthesis) describes how light nuclei—predominantly ^2H , ^3He , ^4He , and ^7Li —were forged from a hot, dense plasma of photons and elementary particles as the early universe expanded and cooled. By the time BBN ends—20 minutes after the Big Bang—the theory predicts that the mass of the universe is about 75% hydrogen, 25% ^4He , and trace amounts of other light elements. These mass fractions are in good agreement with observations—factoring in the consequence of stellar activity having altered these fractions over the billions of years since this period [49]. However, there remains a persistent discrepancy between the predicted and observed primordial abundance of lithium [50].

Coasting cosmologies have different early-universe thermal histories than Λ CDM, which can result in mass fractions that don't comport with observations. Lewis et al. find that the linearly coasting model, as a result of a slow and prolonged cooling period, either produces a minuscule amount of helium or the correct mass fraction when the baryon-to-photon ratio is increased by an order of magnitude. In the latter case, however, lighter elements (above H and He in atomic weight) are fused into isotopes of Oxygen [51].

Well beyond the scope of this paper, BBN is an extremely complicated topic that combines cosmology, thermodynamics, and nuclear physics. With that said, the present thesis departs from other coasting models with the continuous creation of mass at the Planck power and density that varies with distance. While it's imagined that this mass comprises real particles, the nature of these particles—such as the ratio of photons to particles of mass and their average energy levels—is unknown. It's also unclear if and how particle composition would change over time.

As the present model describes an early universe where the Planck power is applied to a relatively small volume of space, in principle, conditions could be dense and hot enough to support some level of nucleosynthesis. However, given that most of the mass of the universe would be created *after* this period, nucleosynthesis would have

much less of an impact on the mass fractions of light nuclei in the late universe as compared to the case with Λ CDM.

A given cosmological model must allow for structure formation where small density perturbations allow matter to gather and eventually form galaxies on a timeline that agrees with observations of high-redshift galaxies from the early universe. Qualitatively, the additional acceleration term in Eq. (70) would tend to accelerate structure formation—especially considering that H_f , in this equation—being a linear function of H_0 , would be relatively large in a young universe. Whether structures would form, under this regime, in a way that resembles the universe we see today would require sophisticated simulations.

The last early-universe phenomenon to be addressed is the CMB. The interpretation of the CMB as the remnants of early-universe black-body radiation—the photons of which, having been stretched over billions of years such that they are currently seen as microwaves—is well accepted. Plasma in the early universe was opaque to photons as a result of Thomson scattering. As the universe expanded and cooled, this scattering was, in effect, switched off during recombination—the formation of neutral hydrogen from free electrons and protons—allowing the universe to be transparent to light. Attempts to explain the CMB in terms outside of the accepted scenario have largely failed [52].

As noted, the present model is not incompatible with a hot, dense early universe, which in principle could have undergone recombination. Because mass is created continuously in this model, there are open questions as to the timing of recombination and the photon density before and after the event.

A large aim of the present paper was focused on presenting a plausible explanation for the flat rotation curves of galaxies, which would obviate the need for dark matter in the universe. Dark matter, however, is an instrumental element in the current explanation for the peaks and valleys of the CMB spectra [53]. If dark matter doesn't exist, an alternative explanation for the shape of this curve will be needed—also beyond the scope of the current work. Further, philosophically, if dark matter doesn't exist, then it would be logical to presume that dark energy doesn't exist either.

5. Late Universe

A core building block of Λ CDM—comprising 70% of the energy of the universe—is dark energy, a spatially-smooth component with negative pressure that aims to explain the accelerating expansion rate of the universe [29]. Alexander et al. argue that when our location in an extensive void—with lower density than the spatial average over the cosmological horizon—is accounted for, an accelerating expansion rate is not needed to reproduce the observed luminosity-to-redshift relation [54]. Nielson

et al. argue that when statistical tests are performed against a broader database of supernovae—when certain corrections are made to their absolute magnitudes—the data are consistent with a constant rate of expansion [55].

The evidence for accelerating expansion is likely not settled. However, the present thesis allows for galaxies and clusters to have significant peculiar speeds relative to the Hubble flow. Further, these bodies may be entrained in large mass flows—possibly linked together in long chains. However, given a large enough catalog, on average, the average recessional speed should show a largely constant rate of expansion.

XXII. SUMMARY

The core concept of the paper is embodied by Eq. (21), which relates acceleration $\vec{a}$ in a field to the speed of information in the field c and time uncertainty Δt_u —the latter a measure of how efficiently that information can be transmitted to a particle in the field:

$$\vec{a} = -\frac{2\vec{c}}{\Delta t_u}.$$

Note that $\vec{a}$ —which is opposite the direction of information flow c —in Eq. (21) is a measure of the field strength that only translates to the kinematic acceleration of a particle in the field—in the Newtonian limit.

Eq. (21) was derived via the uncertainty principle as applied to Newtonian gravity. The relation was also used to recover the Coulomb force between two electrons. With this in mind, it's imagined that Δt_u may refer to the lifespan of a virtual particle, which would imply that gravity is a force carried by virtual gravitons. As such, curved spacetime would be an emergent phenomenon from quantum principles.

Eq. (21) was also used to calculate the mass of the universe—as given by Eq. (153):

$$m_0 = \frac{c^3}{GH_0} = \frac{c^3 t_0}{G}.$$

Note that this estimate only accounts for baryonic matter—disregarding dark matter and dark energy, which, under Λ CDM, comprise 95% of the mass of the universe [29]. However, the present thesis holds that baryonic matter is the only matter in the universe; there is no dark matter *nor* dark energy. At the present Hubble time, the mass of the universe m_0 calculates to 1.78E53 kg, which is reasonably close to the current estimate of 1.5E53 kg [3].

In Eq. (153), the Hubble time t_0 —which is mathematically the inverse of the Hubble parameter H_0 —is presumed to also represent the *actual* age of the universe. The inverse of the estimated age of the universe—13.787 billion years [4]—is 70.92 km s⁻¹ Mpc⁻¹, which falls squarely in the range of measurements for H_0 [19]. This correlation seems too improbable to be explainable as a

coincidence.

Similarly, Eq. (153) is arrived at under the premise that all observers see the universe as expanding at c in all directions—where r_0 is the most distant coordinate of the universe. This is described by Eq. (112):

$$c = r_0 H_0 .$$

Mathematically, for this relation to hold over time, H_0 must decrease as r_0 increases. Specifically, H_0 must be defined as the inverse of t_0 , and Hubble time t_0 must also refer to proper time in the frame in which r_0 is measured. Thus, the premise that the universe always expands linearly at c requires $H_0 = 1/t_0$, and vice versa.

Eq. (115) gives the decrease of H_0 with respect to time:

$$\dot{H}_0 = -H_0^2 .$$

The concept of a linearly expanding universe—expanding at c —is known as “coasting cosmology”. Coasting cosmologies solve the flatness and horizon problems in Λ CDM [30], but typically fail to align with early-universe thermal evolution—as currently understood. The latter is addressed in Section XXI.

The present coasting model is termed “Time-Driven Cosmology”—with the observation that the following metrics of the universe are all functions of time t_0 : the expansion parameter H_0 , the size r_0 , and mass m_0 . The time derivative of the relation for m_0 gives the Planck power, which suggests that mass is continuously created (at the Planck power) as the universe expands—and that the two phenomena are connected.

In Section XIII, it is shown that a frame with a non-zero peculiar speed relative to the Hubble frame is decelerating in the Hubble frame. This deceleration is proportional to peculiar speed v_f —as given by Eq. (134):

$$a_f = -v_f H_0 .$$

It’s presumed that this deceleration has physical effects. As v_f goes to zero, deceleration a_f goes to zero—where these physical effects disappear.

For light, deceleration a_f is maximized—as described by Eq. (135):

$$a_f = -c H_0 .$$

The observed effect on light—as it decelerates—is redshift. Interestingly, redshift can be explained in two contexts: the wavelength of a photon being stretched over time or a photon losing energy as it travels in a decelerating field. Combining Eqs. (135) and Eq. (21) ultimately reduces to Eq. (153)—the relation for the mass of universe m_0 .

Because any point in space can be the origin of expansion, it’s imagined that any frame moving in the Hubble frame can have its own origin of expansion—with the moving frame having an expansion parameter of H_f . A photon, for example, can be considered to exist in a frame

moving at a peculiar speed of c —with that frame having an expansion parameter of H_0 . A general relation for the expansion parameter H_f of a frame with a peculiar speed of v_f is given by Eq. (163):

$$H_f = \sqrt{\frac{v_f}{c}} H_0 .$$

Notice that, as v_f goes zero, H_f goes to zero and as v_f goes to c , H_f goes to H_0 . Thus, a frame traveling at c sees the full expansion parameter of the parent frame, which is to say the universe— H_0 .

Expanding space has no direct effect on matter. It does, however, affect fields—with the most salient example being the stretching of a photon’s wavelength as it travels through expanding space. It’s imagined that expanding space can also affect gravitational fields—magnifying those fields such that in large systems such as galaxies there are notable effects. Counterintuitively, expanding space increases the probability of bodies gathering into larger structures as opposed to drifting apart.

It’s posited that gravitational fields are also subject to being redshifted. It’s imagined that a redshifted gravitational field has an additional accelerating component termed the “expansion component” on top of the Newtonian component, which is embodied by Eq. (70). The net acceleration a_n relative to a reference mass m_r at distance r —in the Newtonian limit—is thus:

$$a_n = \frac{m_r G}{r} \left(\frac{1}{r} + \frac{2\sqrt{cH_f}}{\sqrt{m_t G}} \right) .$$

Here, m_t is the mass of a system with an expansion parameter H_f —the latter a function of the system’s peculiar speed v_f as described by Eq. (163). When applied to galaxies, this equation produces flat rotation curves in agreement with observations. Unlike MOND, these equations further predict that there will be cases where the rotation curve of a given galaxy has some level of Keplerian decline—and where a galaxy with a peculiar speed of zero would have entirely Newtonian orbits!

The mechanism for the expansion component is thus: While the redshift of light progresses over time, the strength of a gravitational field, as a function of the source mass, remains invariant—assuming a constant mass. Thus, as a gravitational field is redshifted, it is continuously re-informed by the source mass. The tension between redshift and re-information can be measured by time uncertainty Δt_u , which—via Eq. (21)—yields the additional acceleration term in Eq. (70). As before, Δt_u implies the existence of a virtual graviton—where this tension manifests as gravitons that amplify the existing field.

The universe can be described by two states—the “ground state” and the “dynamic state”. The ground state is an idealized case where all of the particles of the universe move precisely in the Hubble flow—where no particle has a peculiar speed. Every particle in the

universe, in the ground state, is perpetually at rest in the frame of a Hubble coordinate, and in this frame, all other particles are in recession—moving exactly according to Hubble’s law from zero to c . In the ground state, the contained mass m_c is given by Eq. (198):

$$m_c = \frac{cH_0 r^2}{G}.$$

From this equation, Eq. (208) is derived, which is a relation for area density ρ_A in the rest frame of any Hubble coordinate:

$$\rho_A = \frac{cH_0}{8\pi G}.$$

Notice that ρ_A is invariant with respect to distance and only changes with time—with H_0 being the inverse of Hubble time t_0 . Thus, in the ground state, all observers in their own rest frames will measure the same area density at a given proper time in their frames.

This, however, is not the case for volume density, which, in the rest frame of a Hubble coordinate—as given by Eq. (209)—decreases with distance:

$$\rho = \frac{cH_0}{4\pi Gr}.$$

Taking the time derivative of Eq. (198) yields interesting insights. For example, the mass at a given Hubble coordinate v_c remains invariant over time as the coordinate recedes. Another realization is that new mass is created between layers of moving mass as those layers separate—as a consequence of space expanding (or being created) between those layers.

In the ground state, as a consequence of the universe expanding at c in all frames of reference and mass being created in concert with space—as described by Eq. (198) and the derivatives thereof—each particle in the universe—in its rest frame—is at the barycenter of all other particles. As such, in its rest frame, there is no *net* accelerating field acting upon any individual particle. Thus, once created, particles move at constant velocities with respect to each other—each coincident with the Hubble coordinate that was created in unison. Particles naturally spread apart in the Hubble flow because there is no net gravitational field that would affect this motion.

Said another way, upon creation—in the ground state—particles are at rest in the Hubble frame. Because this frame is dynamic, two particles at rest in the Hubble frame, separated by a distance, will initially be receding from each other—with the recessional speed given by Hubble’s law. As long as there are no accelerating fields acting upon these particles, they will continue to move at constant velocity with respect to each other—drifting apart at the same speed in perpetuity. Coincidentally, space will always be expanding between these particles at the same rate. This is because the Hubble parameter H_0 decreases over time at precisely the rate where the re-

cessional speed given by Hubble’s law remains invariant.

The ground state, however, is purely hypothetical. In the real universe, particles are free to gather into dynamic structures. Even if the ground state were to exist for a moment, it would quickly devolve to the dynamic state. But concepts pertinent to the ground state can inform an understanding of the dynamic state. For example, it’s imagined that it would be unlikely for a body in the dynamic state to exist precisely at the barycenter of all other bodies. Even a slight offset from the barycenter would elicit some level of acceleration relative to those bodies. As the scale at which the universe is observed increases, however, it seems to look increasingly like that described by the ground state—homogeneous and isotropic, with bodies largely moving in the Hubble flow.

XXIII. THE GALACTIC MODEL

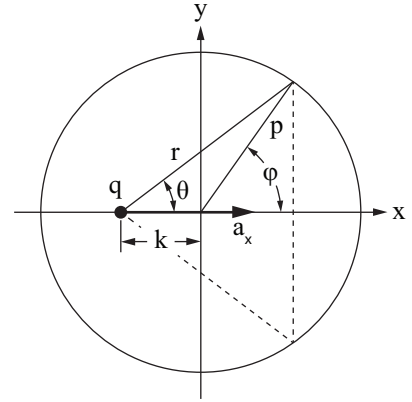


FIG. 19.

The galactic-rotation curves presented in Appendix XXIV were generated according to the following method: The bulge is modeled as a sphere with a density that varies according to radius p , and the disc is modeled as a zero-thickness plate that varies in surface density according to radius p . Density, at a given radius, is constant with respect to angle, and thus, local features in the disc such as spiral arms and bars are ignored.

Both models are described by Fig. 19, where, for the bulge, the circle represents a layer of the sphere and the vertical dashed line represents the ring that is formed by the intersection of that layer at radius p and a plane perpendicular to axis x . For the disc, the circle marks a radial offset p in a top-down view of the disc. In both models, $\vec{r}$ is a vector between point q at offset k from the y axis and a mass-element at p .

For the bulge, the mass m_r of a ring at radius p is thus:

$$V_r = \frac{2\pi}{3}(\cos \phi_1 - \cos \phi_2)(p_2^3 - p_1^3), \quad (227)$$

$$m_r = \rho(p)V_r. \quad (228)$$

The volume V_r of the ring bound by radii p_1 and p_2 , and angles ϕ_{i1} and ϕ_{i2} is given by Eq. (227). The mass is computed via Eq. (228), where the average density between p_1 and p_2 is multiplied by the volume. Notice that when $\phi_1 = 0$, $\phi_2 = \pi$, and $p_1 = 0$, V_r is the volume of a sphere of radius p_2 .

Relative to point q at offset k , the distance r to a differential element of ring mass is constant with respect to the revolved angle about the x axis, and thus, the acceleration along $\vec{r}$ from the gravitational field of a ring element is equal in magnitude to that of all elements of equal size across this angle. With a vector pointing to each element, the sum of these vectors yields the net acceleration a_x at point q with respect to a ring of mass m_r . The radial components of the vector sum all cancel, leaving a_x pointing along the x axis as follows:

$$p = \frac{p_1 + p_2}{2}, \quad (229)$$

$$\phi = \frac{\phi_1 + \phi_2}{2}, \quad (230)$$

$$r = \sqrt{p^2 + 2p \cos \phi k + k^2}, \quad (231)$$

$$\cos \theta = \frac{p \cos \phi + k}{r}, \quad (232)$$

$$a_x = \frac{m_r G}{r} \left(\frac{1}{r} + \frac{2\sqrt{cH_f}}{\sqrt{m_t G}} \right) \cos \theta, \quad (233)$$

$$\frac{v^2}{r} = a_x, \quad (234)$$

$$v = \sqrt{a_x r}. \quad (235)$$

The acceleration at point q along $\vec{r}$ from a mass element of the ring is given by Eq. (70), and the sum of these acceleration vectors is given by Eq. (233). Notice that when point q is centered in the ring—where $\theta = \pi/2$ —the net acceleration is zero, as expected. To compute only the Newtonian component of a_x , H_f in Eq. (233), is set to zero. Eq. (234) is the familiar relation between tangential speed v —where the motion is circular—and centripetal acceleration, which, in this case, is a_x . The rotation curves were drawn with the assumption of circular motion—with the rotational velocity v given by Eq. (235).

To calculate the net acceleration a_x at a given offset k from the gravitational field of the full bulge, the bulge is divided into a series of thin layers—with each layer having a certain density $\rho(p)$. Each layer is divided into a series of thin rings orthogonal to the x axis. Eqs. (227) and (228) are used to calculate the mass of a ring defined by radii p_1 and p_2 , and angles ϕ_1 and ϕ_2 . Eqs. (229)–(233) are then used to calculate the acceleration a_x towards the ring. Running this calculation for small increments of ϕ from 0 to π at a given p_1 and p_2 —which defines a shell—and summing a_x for each increment, gives the acceleration towards the shell. Running the shell calculation for small increments of p from 0 to the bulge radius, and summing the shell acceleration for each increment, gives the net acceleration a_b towards the bulge. As a test

of the code, offset k can be placed inside and outside of the bulge and a_b can be tested to see if Newton's shell theorem is confirmed—with H_f in Eq. (233) set to zero.

The algorithm is nearly identical for the disc calculation. Instead of calculating a ring mass, however, the mass of a pair of arc elements—where one element is the mirror of the other about the x axis—is calculated as follows:

$$A_r = (\phi_2 - \phi_1)(p_2^2 - p_1^2), \quad (236)$$

$$m_r = \rho(p)A_r. \quad (237)$$

Looking down at the disc—the view depicted in Fig. 19, a finite area element A_r of the disc is defined by radii p_1 and p_2 , and angles ϕ_1 and ϕ_2 . The mass of this element m_r is given by Eq. (237), where this area is multiplied by the surface density $\rho(p)$ at radius p . Notice that in the relation for A_r —Eq. (236), the area element is double the size that would be contained within the arc between ϕ_1 and ϕ_2 . The area A_r for the arc from 0 to π yields, with p_1 set to zero, the area of a circle of radius p_2 and not a half circle. A_r , and by extension, m_r , refer to an element–pair mirrored about the x axis.

Looking at Fig. 19, $\vec{r}$ points to a disc element, and the mirror of this vector, drawn as a dashed line, points to the reflected element in the pair. Because of the symmetry about the x axis, the radial components of the acceleration vectors from point q cancel, leaving a_x aligned along the x axis—as was the case for the bulge. Thus, m_r for a disc-element pair—as calculated by Eq. (237)—can be fed directly into Eqs. (229)–(233) to compute the gravitational acceleration a_x towards the pair. This calculation is run for small increments of ϕ from 0 to π at a given p_1 and p_2 , which gives the acceleration towards a ring of the disc. Integrating across the disk from zero to the disc radius by small increments of p and summing a_x for each ring gives the net acceleration a_d towards the disc.

XXIV. GALACTIC ROTATION CURVES

TABLE II. A Selection of Nearby SPARC Galaxies (mass models).

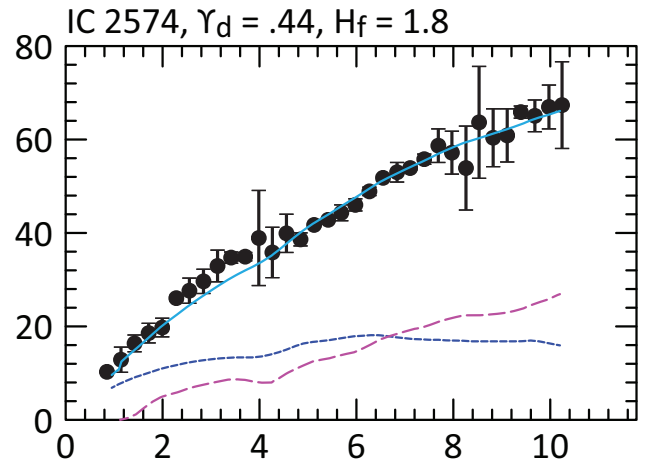
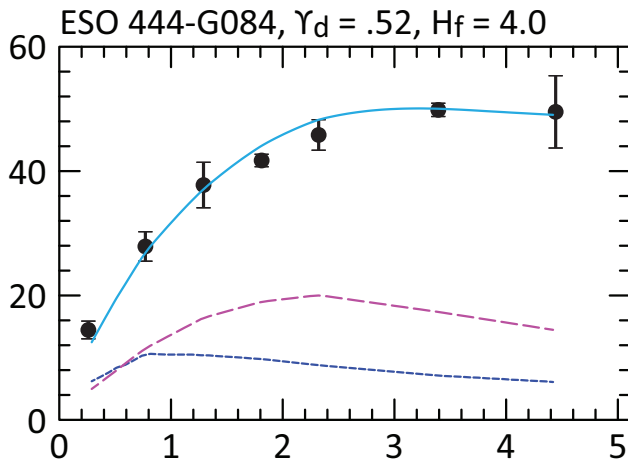
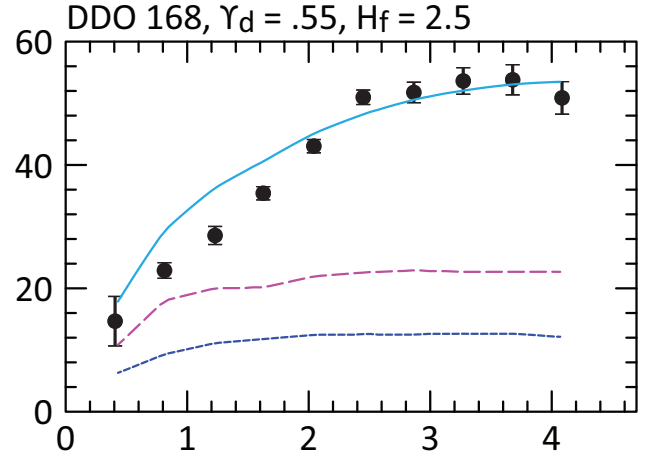
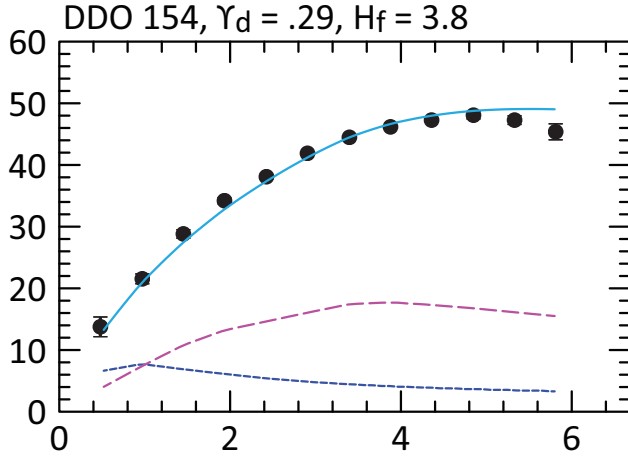
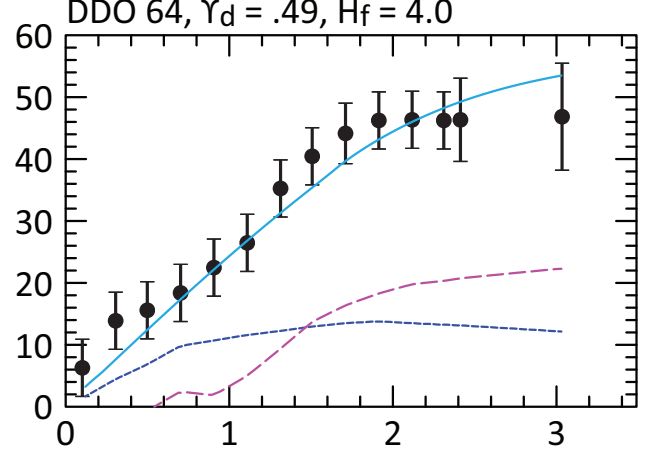
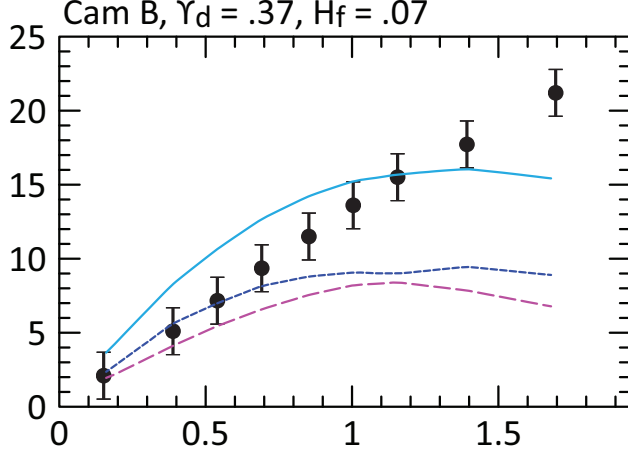
Galaxy	Dark-matter model	Υ_b	Υ_d	Distance (Mpc)	CMB redshift (km s ⁻¹)	r_{max} (kpc)
Cam B	NFW flat	0	.37	3.5	23	1.68
DDO 064	Burkert	0	.49	3.800-7.110	784	3.03
DDO 154	coreNFW flat	0	.29	4.04	639	5.97
DDO 168	Lucky13 LCDM	0	.55	4.25	380	4.07
ESO 444-G084	NFW LCDM	0	.54	4.53	870	4.42
IC 2574	DC14 flat	0	.44	3.890-3.940	148	10.20
NGC 0055	Einasto flat	0	.49	1.850-2.340	115	13.36
NGC 0247	coreNFW LCDM ^a	0	1.0	3.270-3.670	143	14.50
NGC 0300	Lucky13 LCDM	0	.73	1.790-2.170	91	12.04
NGC 2366	Burkert	0	.38	3.200-3.340	134	6.02
NGC 2403	Burkert	0	.90	3.010-3.930	182	19.60
NGC 2915	Burkert	0	.42	3.580-4.290	588	9.90
NGC 2976	Burkert	0	.48	3.520-3.630	90	2.25
NGC 3109	NFW LCDM	0	.51	1.333	738	7.16
NGC 3741	Burkert	0	.77	3.150-3.230	456	7.13
NGC 4068	coreNFW LCDM	0	.45	4.3	784	2.28
NGC 4214	NFW LCDM	0	.54	2.700-2.930	550	5.63
NGC 6789	DC14 LCDM ^a	0	1.0	3.6	275	0.73
NGC 6946	Burkert	.63	.58	4.51	133	18.90
NGC 7793	Burkert	0	.56	3.390-3.840	53	7.97
UGC 04305	Burkert	0	.50	3.24	203	5.45
UGC 04483	Burkert	0	.50	3.2	213	1.21
UGC 07232	Burkert	0	.50	2.95	486	.82
UGC 07524	DC14 flat	0	.43	4.410-4.610	585	10.50
UGC 07559	coreNFW LCDM	0	.43	4.97	470	2.50
UGC 07577	coreNFW LCDM	0	.37	2.6	417	1.50
UGC 07866	coreNFW LCDM	0	.48	4.57	592	2.29
UGC 08490	DC14 LCDM	0	.96	4.790-5.550	322	12.50
UGCA 444	Burkert	0	.49	0.93	457	2.60

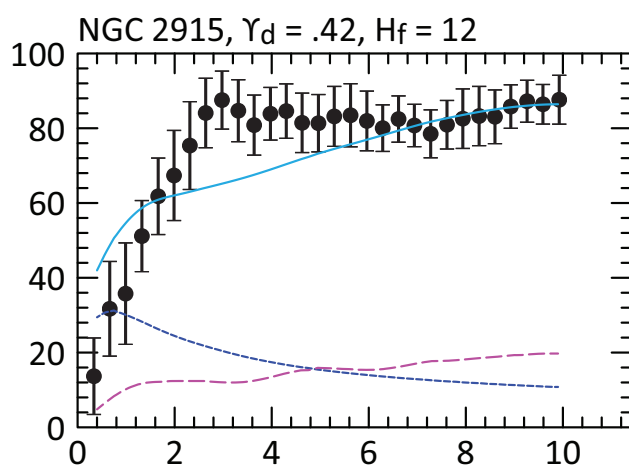
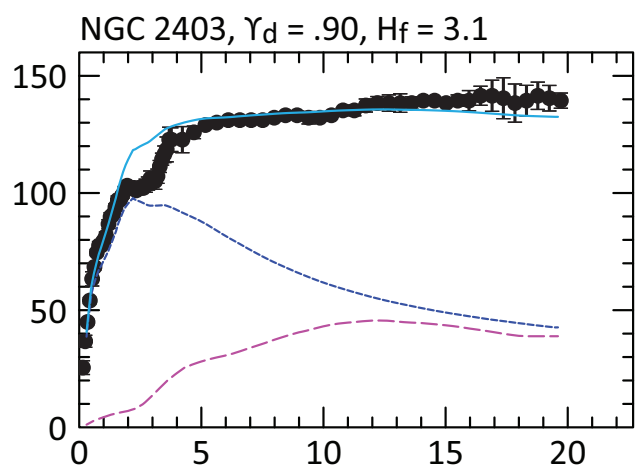
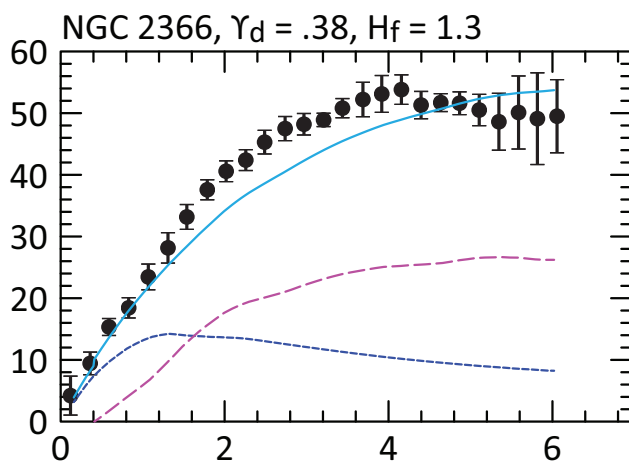
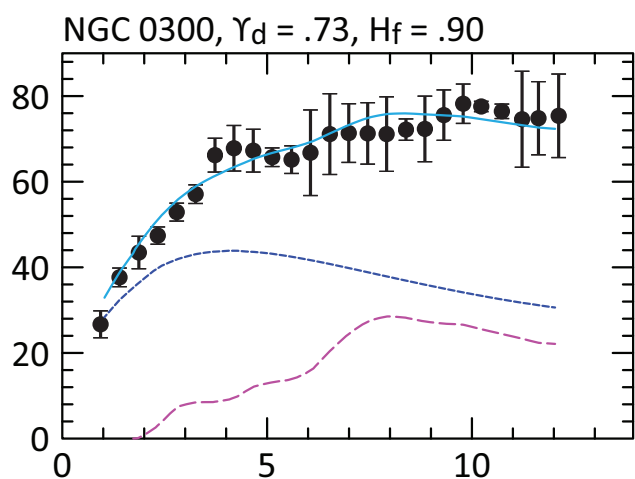
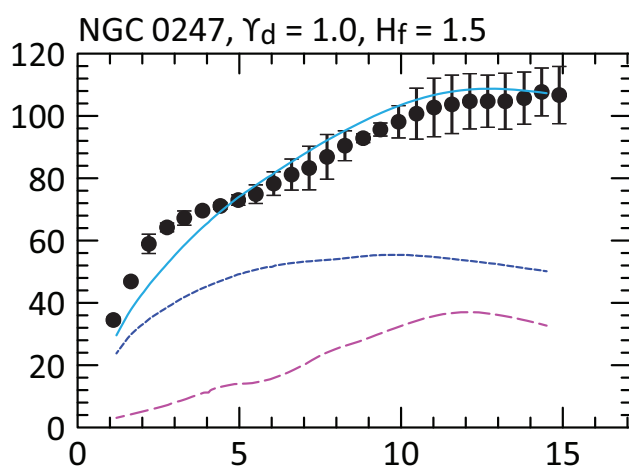
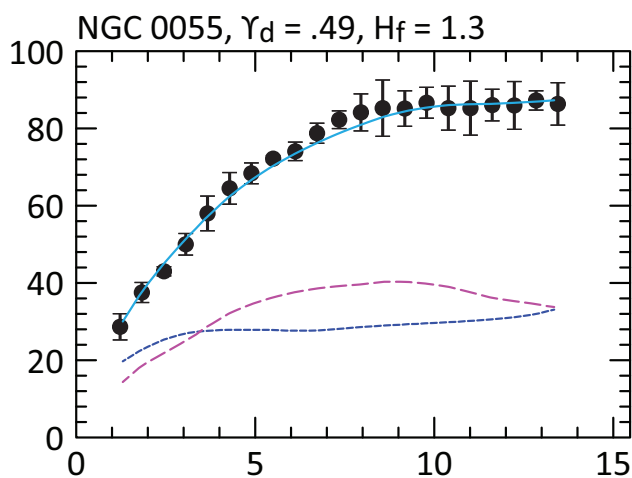
^a Error bars are from this dark-matter model, but the mass-to-light ratio is less than that given by the model.^b Mass-to-light ratios were sourced from the SPARC database [35].^c Distance and redshift data were sourced from the *NASA/IPAC Extragalactic Database* [36].

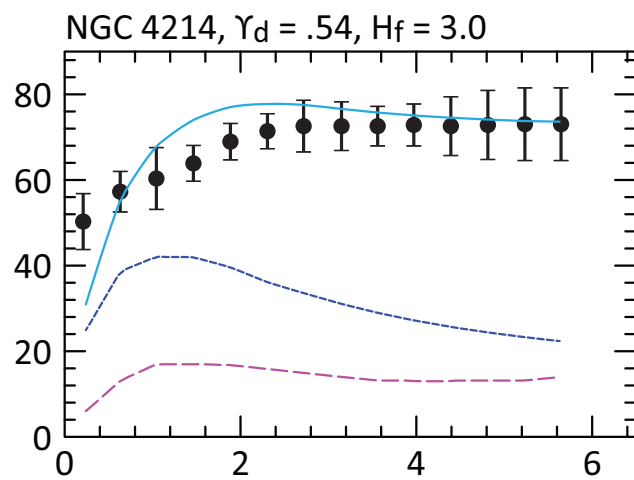
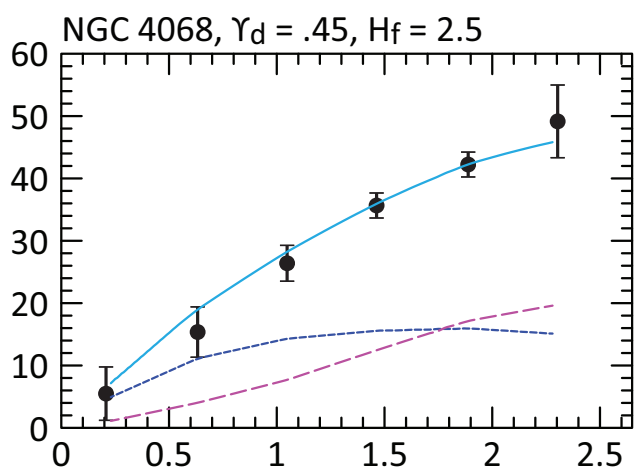
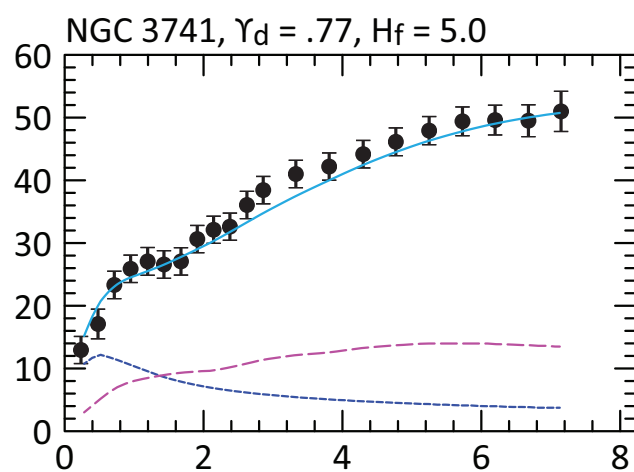
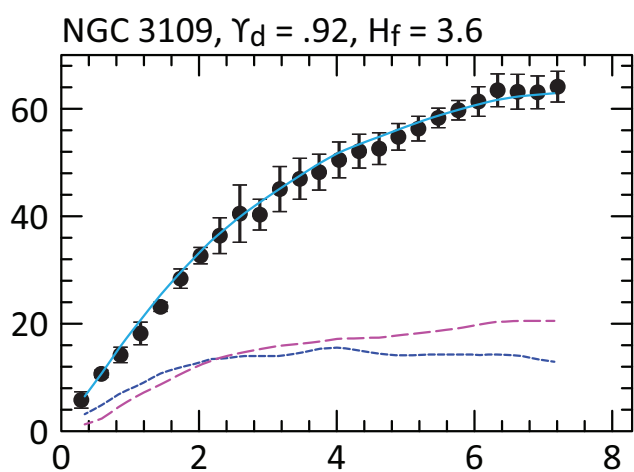
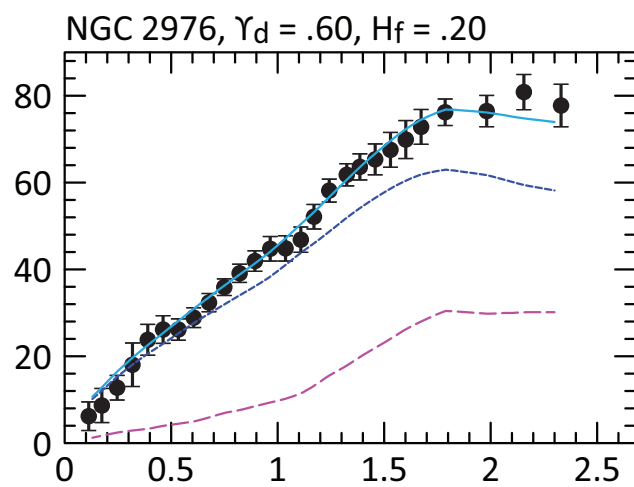
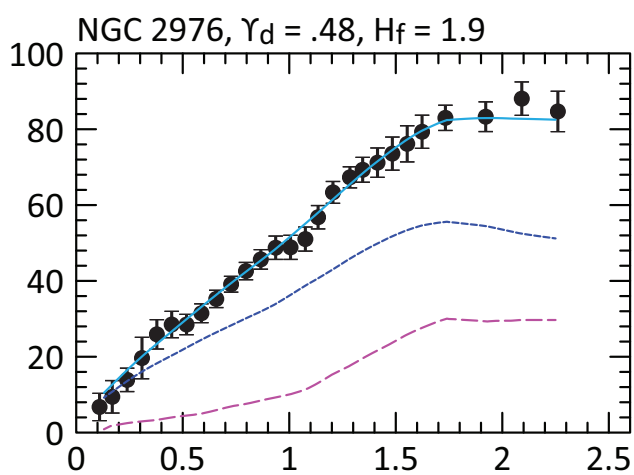
TABLE III. A Selection of Nearby SPARC Galaxies (H_f and k_f).

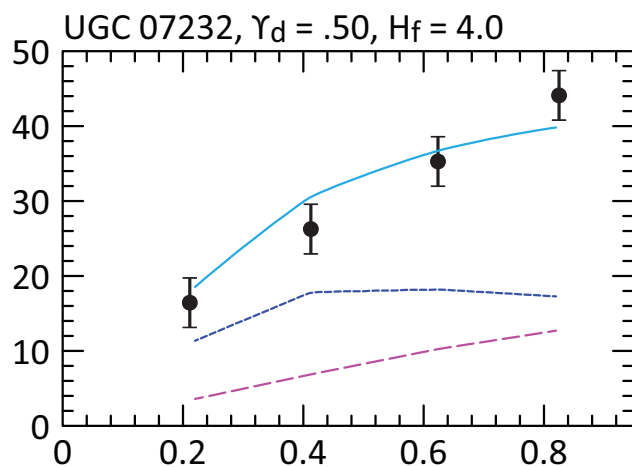
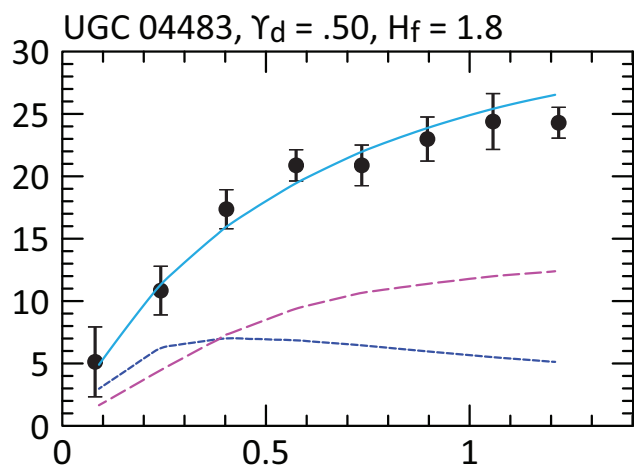
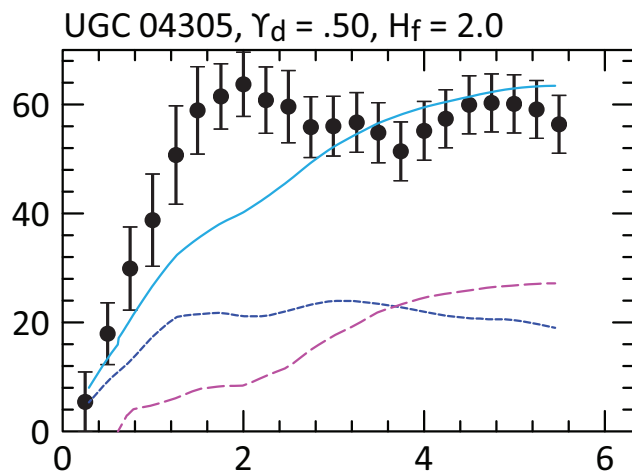
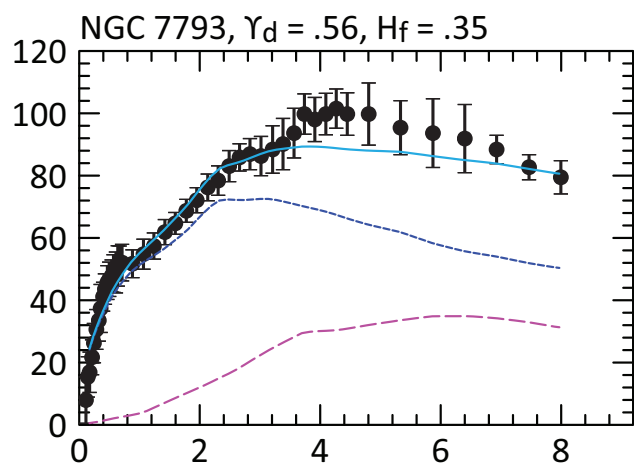
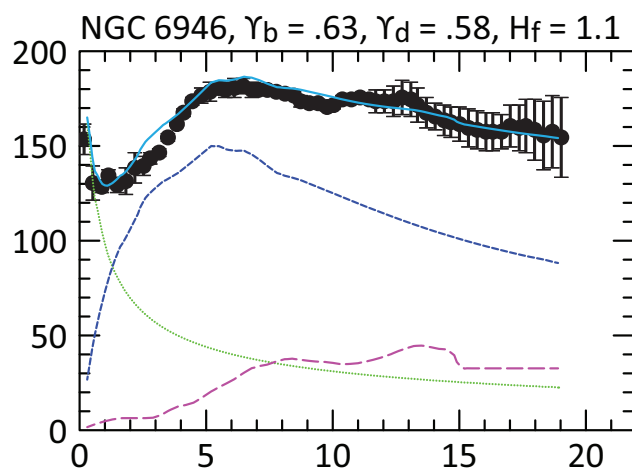
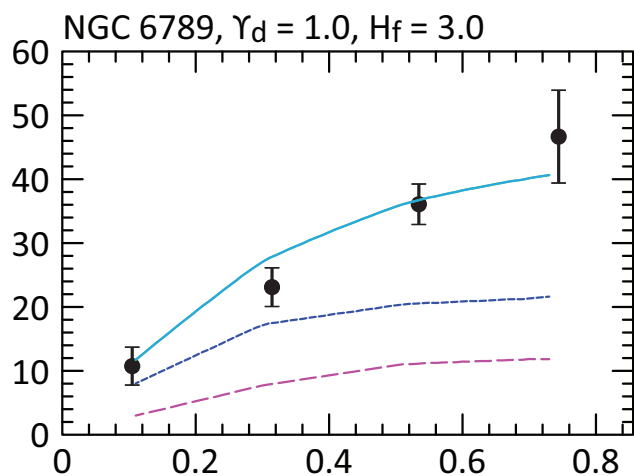
Galaxy	Mass (kg)	H_f (km s ⁻¹ Mpc ⁻¹)	k_f (kg ⁻¹ s ⁻¹)	v_{flat} (km s ⁻¹)	Poor fit	Outlier
Cam B	7.12E37	.07	3.19E-59	11	x	
DDO 064	6.90E38	4.0	1.88E-58	52		
DDO 154	4.75E38	3.8	2.59E-58	47		
DDO 168	7.69E38	2.5	1.05E-58	47		
ESO 444-G084	4.49E38	4.0	2.89E-58	46		
IC 2574	3.24E39	1.8	1.80E-59	62		
NGC 0055	8.97E39	1.3	4.70E-60	74		
NGC 0247	1.62E40	1.5	3.00E-60	89		
NGC 0300	6.21E39	.90	4.70E-60	62		
NGC 2366	1.48E39	1.3	2.85E-59	47		
NGC 2403	2.52E40	3.1	3.99E-60	119		x
NGC 2915	1.56E39	12	2.49E-58	83	x	x
NGC 2976	2.30E39	1.9	2.68E-59	58		
NGC 3109	1.35E39	3.6	8.64E-59	60		
NGC 3741	4.99E38	5.0	3.25E-58	50		
NGC 4068	4.84E38	2.5	1.67E-58	42		
NGC 4214	1.44E39	3.0	6.75E-59	58		
NGC 6789	2.05E38	3.0	4.74E-58	36		
NGC 6946	7.23E40	1.1	4.93E-61	120		
NGC 7793	9.83E39	.35	1.15E-60	55		
UGC 04305	1.70E39	2.0	3.81E-59	54	x	
UGC 04483	8.18E37	1.8	7.13E-58	25		
UGC 07232	1.74E38	4.0	7.45E-58	37		x
UGC 07524	6.06E39	3.0	1.60E-59	83	x	
UGC 07559	3.10E38	1.5	1.57E-58	33		x
UGC 07577	7.55E37	.70	3.00E-58	19		x
UGC 07866	2.23E38	3.0	4.36E-58	36		
UGC 08490	4.95E39	2.2	1.44E-59	73		
UGCA 444	1.88E38	3.2	5.52E-58	35		

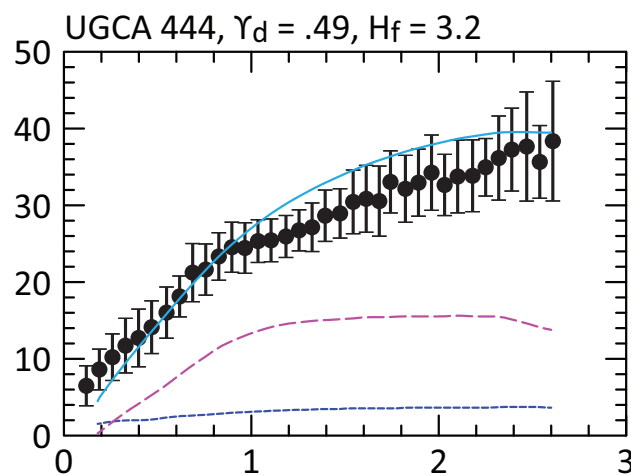
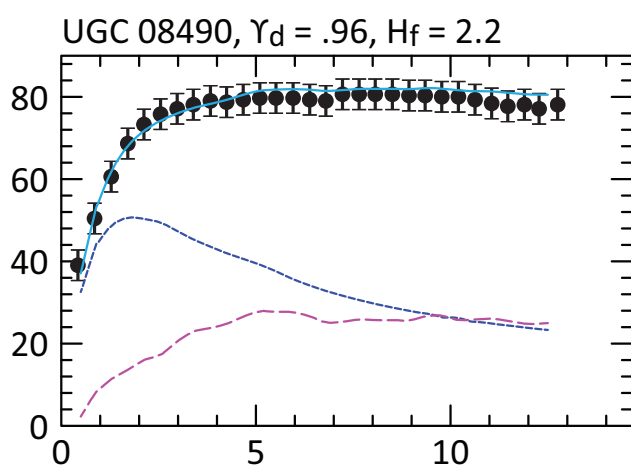
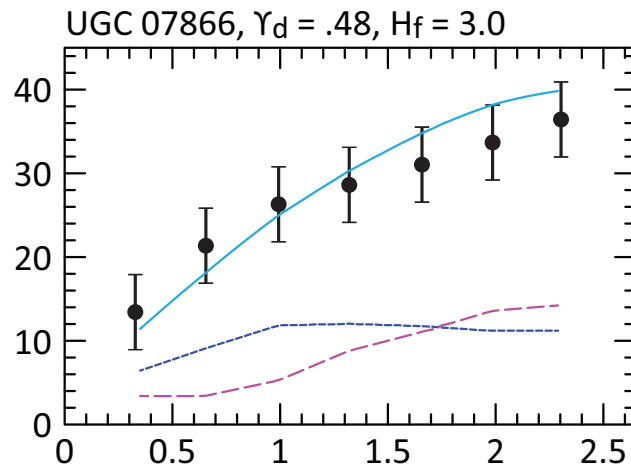
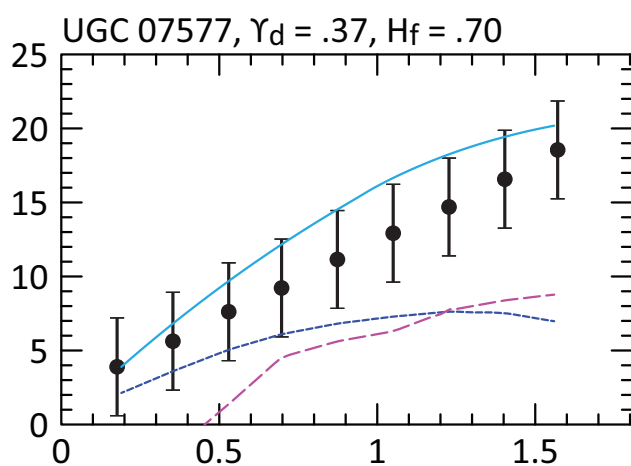
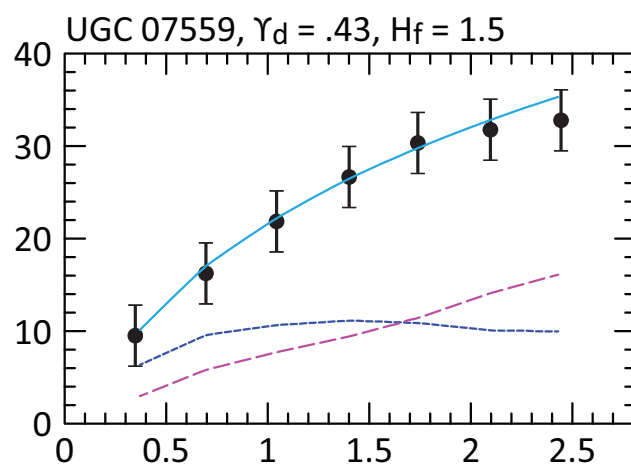
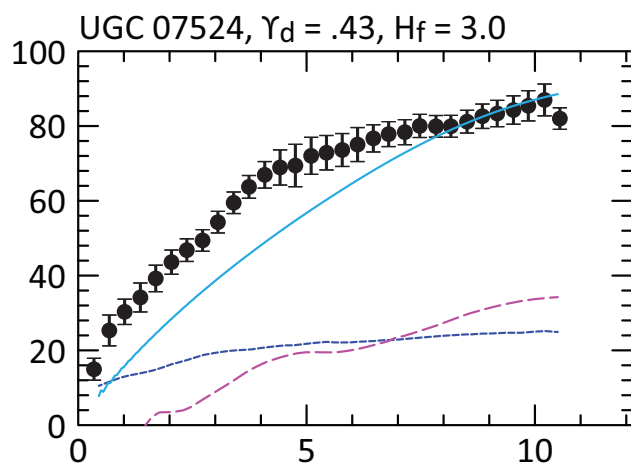
In the following graphs, the units for the vertical axes are km s^{-1} , and the units for the horizontal axes are kpc. The Newtonian curve for the stellar disk is drawn with a dashed blue line. The Newtonian contribution from the gas disk is drawn with a dashed magenta line with larger dashes, and the bulge contribution is drawn as a dotted green line. The net rotation curve, generated via Eq. (70), is drawn as a solid cyan line.











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