Simple Proof of Fermat's Last Theorem for odd powers

Minho Baek

It was already proved right that x^n+y^n=z^n, (n>2) has no solutions in positive integers which we called Fermat's Last Theorem (FLT) by Andrew Wiles. But his proof would be impossible in the 17th century. I took the idea from Euler proof and proved in case of n=odd by simple method.

1. Introduction

Pierre de Fermat claimed he had proof that no three positive integer x, y and z satisfy the equation $x^n + y^n = z^n$ for n greater than 2 which we called Fermat's Last Theorem(FLT). For about 3 century, many people have tried to prove FLT. Finally, FLT was proved right by Andrew Wiles in 1995. However, the proof of Wiles is a modern math that is difficult to understand and complex. So, some people, who think Fermat prove FLT by himself, still believe elementary method exist. We know that FLT can be proved by proving n=odd primes because case of n=even proved by Fermat himself. I got an idea to solve for n=odd from Euler's proof. In this paper, I proved the case of n=odd by simple method.

2. Proof for n=3

$$x^3 + y^3 = z^3, (x < y < z)$$

Where x, y and z = positive integer, relatively prime

Assume that $x^3 + y^3 = z^3, (x < y < z)$ has a solution

This equation can be classified into three categories as follows.

Case 1. (x, y, z) = (even, odd, odd)

Case 2. (x, y, z) = (odd, even, odd)

Case 3. (x, y, z) = (odd, odd, even)

Case 1. (x, y, z) = (even, odd, odd)

Step 1

Let $y = (u - v), z = (u + v)$.

Assume that u and v are not relatively prime.

Let $u = fU, v = fV$.

$$y = f(U - V)$$

$$z = f(U + V)$$

y and z have common factor of f.

But this is a contradiction because y and z are relatively prime.

So, u and v are relatively prime.

Also u and v are opposite parity because y and z are odd.

$$x^3 = (u+v)^3 - (u-v)^3$$

$$x^3 = 2v(v^2 + 3u^2)$$

Assume that $u = \text{even}, v = \text{odd}$.

$v^2 + 3u^2$ is odd.

Assume that $u = \text{odd}, v = \text{even}$.

$v^2 + 3u^2$ is odd.

So, $v^2 + 3u^2$ is always odd.

Thus the greatest common factor of 2v and $v^2 + 3u^2$ is odd.

Assume that the common factor of 2v and $v^2 + 3u^2$ is odd except 1 and 3.

Let $v = fV$ and $v^2 + 3u^2 = fN$.

$$3u^2 = f(N - fV^2)$$

u and v have common factor of f.

But this is a contradiction because u and v are relatively prime.

So, the greatest common factor of 2v and $v^2 + 3u^2$ is either 1 or 3.

Assume that the greatest common factor of 2v and $\mathbf{v^2 + 3u^2}$ is 1.

It would be that $2v = p^3$ and $v^2 + 3u^2 = q^3$.

$$x^3 = p^3 q^3$$

Assume that the greatest common factor of 2v and $\mathbf{v^2 + 3u^2}$ is 3.

Let $v = 3r$.

u and r are also relatively prime because u and v are relatively prime.

$$x^3 = 6r(9r^2 + 3u^2)$$

$$x^3 = 18r(u^2 + 3r^2)$$

The greatest common factor of 18r and $u^2 + 3r^2$ would be 1.

It is possible that $18r = p'^3$ and $u^2 + 3r^2 = q'^3$.

$$x^3 = p'^3 q'^3$$

Step 2

Assume that $z = y + i$.

From $y = (u - v), z = (u + v)$,

$$z - y = (u + v) - (u - v) = 2v$$

$$2v = i$$

Assume that $y = k + i$.

Since y is odd and i is even, k is odd.

Assume that y and i are not relatively prime.

Let $y = fY, i = fI$.

$$z = y + i$$

$$z = f(Y + I)$$

y and z have common factor of f.

But this is a contradiction because y and z are relatively prime.

So, y and i are relatively prime.

Assume that k and y are not relatively primes.

Let $k = fK, y = fY$.

$$i = y - k$$

$$i = f(Y - K)$$

y and i have common factor of f.

But this is a contradiction because y and i are relatively prime.

So, k and y are relatively prime.

Let $k = (u' - v)$, $y = (u' + v)$.

Where $u' < u$

Assume that u' and v are not relatively prime.

Let $u' = fU', v = fV$.

$$k = f(U' - V)$$

$$y = f(U' + V)$$

k and y have common factor of f.

But this is a contradiction because k and y are relatively prime.

So, u' and v are relatively prime.

Also u' and v are opposite parity because k and y are odd.

$$y^3 - k^3 = (u' + v)^3 - (u' - v)^3$$

$$y^3 - k^3 = 2v(v^2 + 3u'^2)$$

Assume that $u' = \text{even}, v = \text{odd}$.

$v^2 + 3u'^2$ is odd.

Assume that $u' = \text{odd}, v = \text{even}$.

$v^2 + 3u'^2$ is odd.

So, $v^2 + 3u'^2$ is always odd.

Thus the greatest common factor of 2v and $v^2 + 3u'^2$ is odd.

Assume that the common factor of 2v and $v^2 + 3u'^2$ is odd number except 1 and 3.

Let $v = fV$ and $v^2 + 3u'^2 = fN'$.

$$3u'^2 = f(N' - fV^2)$$

u' and v have common factor of f.

But this is a contradiction because u' and v are relatively prime.

So, the greatest common factor of 2v and $v^2 + 3u'^2$ is either 1 or 3.

Assume that the greatest common factor of 2v and $\mathbf{v^2 + 3u'^2}$ is 1.

It is possible that $2v = p^3$ and $v^2 + 3u'^2 = \alpha$.

$$y^3 - k^3 = \alpha p^3$$

Assume that $\alpha = {s_1}^3$.

$$y^3 - k^3 = {s_1}^3 p^3$$

Where s_1 and p are relatively prime

Let $y^3 = {c_1}^3, k^3 = {b_1}^3, {s_1}^3 p^3 = {a_1}^3$

$${a_1}^3 + {b_1}^3 = {c_1}^3$$

But this equation is a contradiction by the method of infinite descent.

Assume that $\alpha \neq {s_1}^3$.

$$y^3 - k^3 = \alpha p^3$$

Where α and p are relatively prime

Assume that the greatest common factor of 2v and $\mathbf{v^2 + 3u'^2}$ is 3.

Let $v = 3r$.

u' and r are relatively prime because u' and v are relatively prime.

$$y^3 - k^3 = 6r(9r^2 + 3u'^2)$$

$$y^3 - k^3 = 18r(u'^2 + 3r^2)$$

The greatest common factor of 18r and $u'^2 + 3r^2$ would be 1.

It is possible that $18r = p'^3$ and $u'^2 + 3r^2 = \alpha'$.

$$y^3 - k^3 = \alpha' p'^3$$

Assume that $\alpha' = {s_1}'^3$.

$$y^3 - k^3 = {s_1}'^3 p'^3$$

Where ${s_1}'$ and p' are relatively prime

Let $y^3 = {c'_1}^3, k^3 = {b'_1}^3, {s_1}'^3 p'^3 = {a'_1}^3$

$${a'_1}^3 + {b'_1}^3 = {c'_1}^3$$

But this equation is a contradiction by the method of infinite descent.

Assume that $\alpha' \neq {s_1}^3$.

$$y^3 - k^3 = \alpha' p'^3$$

Where α' and p' are relatively prime

Step 3

Assume that $k = t + i$.

Since k is odd and i is even, t is odd.

Assume that k and i are not relatively prime.

Let $k = fK, i = fI$.

$$y = k + i$$

$$y = f(K + I)$$

k and y have common factor of f.

But this is a contradiction because k and y are relatively prime.

So, k and i are relatively prime.

Assume that t and k are not relatively prime.

Let $t = fT, k = fK$.

$$i = k - t$$

$$i = f(K - T)$$

k and i have common factor of f.

But this is a contradiction because k and i are relatively prime.

So, t and k are relatively prime.

Let $t = (u'' - v), k = (u'' + v)$.

Where $u'' < u'$

Assume that u'' and v are not relatively prime.

Let $u'' = fU'', v = fV$.

$$t = f(U'' - V)$$

$$k = f(U'' + V)$$

t and k have common factor of f.

But this is a contradiction because t and k are relatively prime.

So, u'' and v are relatively prime.

Also u'' and$v$ are opposite parity because t and k are odd.

$$k^3 - t^3 = (u'' + v)^3 - (u'' - v)^3$$

$$k^3 - t^3 = 2v(v^2 + 3u''^2)$$

Assume that $u'' = \text{even}, v = \text{odd}$.

$v^2 + 3u''^2$ is odd.

Assume that $u'' = \text{odd}, v = \text{even}$.

$v^2 + 3u''^2$ is odd.

So, $v^2 + 3u''^2$ is always odd.

Thus the greatest common factor of 2v and $v^2 + 3u''^2$ is odd.

Assume that the common factor of 2v and $v^2 + 3u''^2$ is odd number except 1 and 3.

Let $v = fV$ and $v^2 + 3u''^2 = fN''$.

$$3u''^2 = f(N'' - fV^2)$$

u'' and v have common factor of f.

But this is a contradiction because u'' and v are relatively prime.

So, the greatest common factor of 2v and $v^2 + 3u''^2$ is either 1 or 3.

Assume that the greatest common factor of 2v and $\mathbf{v^2 + 3u''^2}$ is 1,

It is possible that $2v = p^3$ and $v^2 + 3u''^2 = \beta$.

$$k^3 - t^3 = \beta p^3$$

Assume that $\beta = {s_2}^3$.

$$k^3 - t^3 = {s_2}^3 p^3$$

Where s_2 and p are relatively prime

Let $k^3 = {c_2}^3, t^3 = {b_2}^3, {s_2}^3 p^3 = {a_2}^3$

$${a_2}^3 + {b_2}^3 = {c_2}^3$$

But this equation is a contradiction by the method of infinite descent.

Assume that $\beta \neq {s_2}^3$.

$$k^3 - t^3 = \beta p^3$$

Where β and p are relatively prime, $\beta < \alpha$

This equation also leads to a contradiction by the method of infinite descent.

Assume that the greatest common factor of 2v and $\mathbf{v^2 + 3u''^2}$ is 3.

Let $v = 3r$.

u'' and r are relatively prime because u'' and v are relatively prime.

$$k^3 - t^3 = 6r(9r^2 + 3u''^2)$$

$$k^3 - t^3 = 18r(u''^2 + 3r^2)$$

The greatest common factor of 18r and $u''^2 + 3r^2$ would be 1.

It is possible that $18r = p'^3$ and $u''^2 + 3r'^2 = \beta'$.

$$k^3 - t^3 = \beta' p'^3$$

Assume that $\beta' = {s_2}'^3$.

$$k^3 - t^3 = {s_2}'^3 p'^3$$

Where ${s_2}'$ and p' are relatively prime

Let $k^3 = {c'_2}^3, t^3 = {b'_2}^3, {s_2}'^3 p'^3 = {a'_2}^3$

$${a'_2}^3 + {b'_2}^3 = {c'_2}^3$$

But this equation is a contradiction by the method of infinite descent.

Assume that $\beta' \neq {s_2}'^3$.

$$k^3 - t^3 = \beta' p'^3$$

Where β' and p' are relatively prime, $\beta' < \alpha'$

This equation also leads to a contradiction by the method of infinite descent.

Thus case 1 has no solution by the method of infinite descent.

Case 2. (x, y, z) = (odd, even, odd)

Case 2 has no solution because it can be proved by the same method as Case 1.

Case 3. (x, y, z) = (odd, odd, even)

Step 1

Let $x = (u - v), y = (u + v)$.

Assume that u and v are not relatively prime.

Let $u = fU, v = fV$.

$$x = f(U - V)$$

$$y = f(U + V)$$

But this is a contradiction because x and y are relatively prime.

So, u and v are relatively prime.

Also u and v are opposite parity because x and y are odd.

$$z^3 = (u + v)^3 + (u - v)^3$$

$$z^3 = 2u(u^2 + 3v^2)$$

Assume that $u = \text{even}, v = \text{odd}$.

$u^2 + 3v^2$ is odd.

Assume that $u = \text{odd}, v = \text{even}$.

$u^2 + 3v^2$ is odd.

So, $u^2 + 3v^2$ is always odd.

Thus the greatest common factor of 2u and $u^2 + 3v^2$ is odd.

Assume that the common factor of 2u and $u^2 + 3v^2$is odd except 1 and 3.

Let $u = fU$ and $u^2 + 3v^2 = fN$.

$$3v^2 = f(N - fU^2)$$

u and v have common factor of f.

But this is a contradiction because u and v are relatively prime.

So, the greatest common factor of 2u and $u^2 + 3v^2$ is either 1 or 3.

Assume that the greatest common factor of 2u and $\mathbf{u^2 + 3v^2}$ is 1.

It is possible that $2u = p^3$ and $u^2 + 3v^2 = q^3$.

Where p and q are relatively prime

$$z^3 = p^3q^3$$

Assume that the greatest common factor of 2u and $\mathbf{u^2 + 3v^2}$ is 3.

Let $u = 3r$.

v and r are relatively prime because v and u are relatively prime.

$$z^3 = 6r(9r^2 + 3v^2)$$

$$z^3 = 18r(v^2 + 3r^2)$$

The greatest common factor of 18r and $v^2 + 3r^2$ would be 1.

It is possible that $18r = p'^3$ and $v^2 + 3r^2 = q'^3$.

$$z^3 = p'^3q'^3$$

Step 2

Assume that $v = iv'$.

Where $i = \text{positive integer}$

Let $k = (u - v'), l = (u + v')$.

u and v' are relatively prime because u and v are relatively prime.

Assume that k and u are not relatively prime.

Let $k = fK, u = fU$.

$$k = u - v'$$

$$v' = f(U - K)$$

u and v' have common factor of f.

But this is a contradiction because u and v' are relatively prime.

So, k and u are relatively prime.

Assume that l and u are not relatively prime.

Let $l = fL, u = fU$.

$$l = u + v'$$

$$v' = f(L - U)$$

u and v' have common factor of f.

But this is a contradiction because u and v' are relatively prime.

So, l and u are relatively prime.

Assume that k and l are not relatively prime.

Let $k = fK, l = fL$.

$$2u = k + l$$

$$2u = f(K + L)$$

k and u have common factor of f.

But this is a contradiction because k and u are relatively prime.

So, k and l are relatively prime.

$$k^3 + l^3 = (u - v')^3 + (u + v')^3$$

$$k^3 + l^3 = 2u(u^2 + 3v'^2)$$

Where u and v' are opposite parity

Assume that $u = \text{even}, v' = \text{odd}$.

$u^2 + 3v'^2$ is odd.

Assume that $u = \text{odd}, v' = \text{even}$.

$u^2 + 3v'^2$ is odd.

So, $u^2 + 3v'^2$ is always odd.

Thus the greatest common factor of 2u and $u^2 + 3v'^2$ is odd.

Assume that the common factor of 2u and $u^2 + 3v'^2$ is odd number except 1 and 3.

Let $u = fU$ and $u^2 + 3v'^2 = fN'$.

$$3v'^2 = f(N' - fU^2)$$

u and v′ have common factor of f.

But this is a contradiction because u and v′ are relatively prime.

So, the greatest common factor of 2u and $u^2 + 3v'^2$ is either 1 or 3.

Assume that the greatest common factor of 2u and $\mathbf{u^2 + 3v'^2}$ is 1.

It is possible that $2u = p^3$ and $v^2 + 3u'^2 = \alpha$.

$$k^3 + l^3 = \alpha p^3$$

Assume that $\alpha = {s_1}^3$.

$$k^3 + l^3 = {s_1}^3 p^3$$

Where s_1 and p are relatively prime

Let $k^3 = {a_1}^3, l^3 = {b_1}^3, {s_1}^3 p^3 = {c_1}^3$

$${a_1}^3 + {b_1}^3 = {c_1}^3$$

But this equation is a contradiction by the method of infinite descent.

Assume that $\alpha \neq {s_1}^3$.

$$k^3 + l^3 = \alpha p^3$$

Where α and p are relatively prime

Assume that the greatest common factor of 2u and $\mathbf{u^2 + 3v'^2}$ is 3.

Let $u = 3r$.

v′ and r are relatively prime because v′ and u are relatively prime.

$$k^3 + l^3 = 6r(9r^2 + 3v'^2)$$

$$k^3 + l^3 = 18r(v'^2 + 3r^2)$$

The greatest common factor of 18r and $v'^2 + 3r^2$ would be 1.

It is possible that $18r = p'^3$ and $v'^2 + 3r^2 = \alpha'$.

$$k^3 + l^3 = \alpha' p'^3$$

Assume that $\alpha' = {s_1}'^3$.

$$k^3 + l^3 = {s_1}'^3 p'^3$$

Where ${s_1}'$ and p' are relatively prime

Let $k^3 = {a'_1}^3, l^3 = {b'_1}^3, {s_1}'^3 p'^3 = {c'_1}^3$

$${a'_1}^3 + {b'_1}^3 = {c'_1}^3$$

But this equation is a contradiction by the method of infinite descent.

Assume that $\alpha' \neq {s_1}'^3$.

$$k^3 + l^3 = \alpha' p'^3$$

Where α' and p' are relatively prime

Step 3

Assume that $v' = iv''$.

Where $i =$ positive integer

Let $m = (u - v''), t = (u + v'')$.

u and v'' are relatively prime because u and v' are relatively prime.

Assume that m and u are not relatively prime.

Let $m = fM, u = fU$.

$$m = u - v''$$

$$v'' = f(U - M)$$

u and v''have common factor of f.

But this is a contradiction because u and v'' are relatively prime.

So, m and u are relatively prime.

Assume that t and u are not relatively prime.

Let $t = fT, u = fU$.

$$t = u + v''$$

$$v'' = f(T - U)$$

u and v'' have common factor of f.

But this is a contradiction because u and v'' are relatively prime.

So, t and u are relatively prime.

Assume that m and t are not relatively prime.

Let $m = fM, t = fT$.

$$2u = m + t$$

$$2u = f(M + T)$$

m and u have common factor of f.

But this is a contradiction because m and u are relatively prime.

So, m and t are relatively prime.

$$m^3 + t^3 = (u - v'')^3 + (u + v'')^3$$

$$m^3 + t^3 = 2u(u^2 + 3v''^2)$$

Where u and v'' are opposite parity

Assume that $u = \text{even}, v'' = \text{odd}$.

$u^2 + 3v''^2$ is odd.

Assume that $u = \text{odd}, v'' = \text{even}$.

$u^2 + 3v''^2$ is odd.

So, $u^2 + 3v''^2$ is always odd.

Thus the greatest common factor of 2u and $u^2 + 3v''^2$ is odd.

Assume that the common factor of 2u and $u^2 + 3v''^2$ is odd number except 1 and 3.

Let $u = fU$ and $u^2 + 3v''^2 = fN''$.

$$3v''^2 = f(N'' - fU^2)$$

u and v'' have common factor of f.

But this is a contradiction because u and v'' are relatively prime.

So, the greatest common factor of 2u and $u^2 + 3v''^2$ is either 1 or 3.

Assume thatthe greatest common factor of 2u and $\mathbf{u^2 + 3v''^2}$ is 1,

It is possible that $2u = p^3$ and $u^2 + 3v''^2 = \beta$.

$$m^3 + t^3 = \beta p^3$$

Assume that$\beta = {s_2}^3$.

$$m^3 + t^3 = {s_2}^3 p^3$$

Where s_2 and p are relatively prime

Let $m^3 = {a_2}^3, t^3 = {b_2}^3, {s_2}^3 p^3 = {c_2}^3$

$${a_2}^3 + {b_2}^3 = {c_2}^3$$

But this equation is a contradiction by the method of infinite descent.

Assume that $\beta \neq {s_2}^3$.

$$m^3 + t^3 = \beta p^3$$

Where β and p are relatively prime, $\beta < \alpha$

This equation also leads to a contradiction by the method of infinite descent.

Assume thatthe greatest common factor of 2u and $\mathbf{u^2 + 3v''^2}$ is 3.

Let $u = 3r$.

v'' and r are relatively prime because v'' and u are relatively prime.

$$m^3 + t^3 = 6r(9r^2 + 3v''^2)$$

$$m^3 + t^3 = 18r(v''^2 + 3r^2)$$

The greatest common factor of 18r and $v''^2 + 3r^2$ would be 1.

It is possible that $18r = p'^3$ and $v''^2 + 3r^2 = \beta'$.

$$m^3 + t^3 = \beta' p'^3$$

Assume that $\beta' = {s_2}'^3$.

$$m^3 + t^3 = {s_2}'^3 p'^3$$

Where ${s_2}'$ and p' are relatively prime

Let $m^3 = {a'_2}^3, t^3 = {b'_2}^3, {s_2}'^3 p'^3 = {c'_2}^3$

$${a'_2}^3 + {b'_2}^3 = {c'_2}^3$$

But this equation is a contradiction by the method of infinite descent.

Assume that $\beta' \neq {s_2}'^3$.

$$m^3 + t^3 = \beta' p'^3$$

Where β' and p' are relatively prime, $\beta' < \alpha'$

This equation also leads to a contradiction by the method of infinite descent.

Thus case 3 has no solution by the method of infinite descent.

In conclusion, there are no positive integers in case of n=3 since all of Case 1, 2 and 3 are contradictions.

3. Proof for n=odd primes

$$x^n + y^n = z^n, (x < y < z)$$

Where x, y and z = positive integer, relatively prime.

Assume that $x^n + y^n = z^n, (x < y < z)$ has a solution

This equation can be classified into three categories as follows.

Case 1. (x, y, z) = (even, odd, odd)

Case 2. (x, y, z) = (odd, even, odd)

Case 3. (x, y, z) = (odd, odd, even)

Case 1. (x, y, z) = (even, odd, odd)

Step 1

Let $y = (u - v), z = (u + v)$.

Assume that u and v are not relatively prime.

Let $u = fU, v = fV$.

$$y = f(U - V)$$

$$z = f(U + V)$$

y and z have common factor of f.

But this is a contradiction because y and z are relatively prime.

So, u and v are relatively prime.

Also u and v are opposite parity because y and z are odd.

$$x^n = (u + v)^n - (u - v)^n$$

$$x^n = (u^n + C_1u^{n-1}v^1 + C_2u^{n-2}v^2 + \cdots + C_{n-2}u^2v^{n-2} + C_{n-1}u^1v^{n-1} + v^n)$$
$$-(u^n - C_1u^{n-1}v^1 + C_2u^{n-2}v^2 + \cdots - C_{n-2}u^2v^{n-2} + C_{n-1}u^1v^{n-1} - v^n)$$

Where $C = \{C_1, C_2, \cdots, C_{n-2}, C_{n-1}\} = \{\frac{n}{1}, \frac{n(n-1)}{1\cdot2}, \cdots, \frac{n(n-1)\cdots4\cdot3}{1\cdot2\cdots(n-3)(n-2)}, \frac{n(n-1)\cdots\cdot2\cdot1}{1\cdot2\cdots(n-2)(n-1)}\}$

$$x^n = 2v(v^{n-1} + n(u^{n-1} + C'_3u^{n-3}v^2 + \cdots + C'_{n-2}u^2v^{n-3}))$$

Where $C' = \{C'_1, C'_2, \cdots, C'_{n-2}, C'_{n-1}\} = \{\frac{1}{1}, \frac{(n-1)}{1\cdot2}, \cdots, \frac{(n-1)\cdots4\cdot3}{1\cdot2\cdots(n-3)(n-2)}, \frac{(n-1)\cdots\cdot2\cdot1}{1\cdot2\cdots(n-2)(n-1)}\}$

$$x^n = 2v(v^{n-1} + n(u^{n-1} + \gamma))$$

Where $\gamma = C'_3 u^{n-3} v^2 + C'_5 u^{n-5} v^4 + \cdots + C'_{n-4} u^4 v^{n-5} + C'_{n-2} u^2 v^{n-3}$

Assume that $u = \text{even}, v = \text{odd}$.

$v^{n-1} + n(u^{n-1} + \gamma)$ is odd.

Assume that $u = \text{odd}, v = \text{even}$.

$v^{n-1} + n(u^{n-1} + \gamma)$ is odd.

So, $v^{n-1} + n(u^{n-1} + \gamma)$ is always odd.

Thus the greatest common factor of 2v and $v^{n-1} + n(u^{n-1} + \gamma)$ is odd.

Assume that the common factor of 2v and $v^{n-1} + n(u^{n-1} + \gamma)$ is odd except 1 and n.

Let $v = fV$, $\gamma = f\Gamma$ and $v^{n-1} + n(u^{n-1} + \gamma) = fN$.

$$f^{n-1}V^{n-1} + n(u^{n-1} + f\Gamma) = fN$$

Where $f\Gamma = f(C'_3 u^{n-3} f^1 V^2 + C'_5 u^{n-5} f^3 V^4 + \cdots + C'_{n-4} u^4 f^{n-6} V^{n-5} + C'_{n-2} u^2 f^{n-4} V^{n-3})$

$$nu^{n-1} = f(N - f^{n-2}V^{n-1} - n\Gamma)$$

u and v have common factor of f.

But this is a contradiction because u and v are relatively prime.

So, the greatest common factor of 2v and $v^{n-1} + n(u^{n-1} + \gamma)$ is either 1 or n.

Assume that the greatest common factor of 2v and $\mathbf{v^{n-1} + n(u^{n-1} + \gamma)}$ is 1.

It is possible that $2v = p^n$ and $v^{n-1} + n(u^{n-1} + \gamma) = q^n$.

Where p and q are relatively prime

$$x^n = p^n q^n$$

Assume that the greatest common factor of 2v and $\mathbf{v^{n-1} + n(u^{n-1} + \gamma)}$ is n.

Let $v = nr$.

u and r are relatively prime because u and v are relatively prime.

$$x^n = 2nr(n^{n-1}r^{n-1} + n(u^{n-1} + \gamma))$$

$$x^n = 2n^2 r(n^{n-2}r^{n-1} + (u^{n-1} + \gamma))$$

The greatest common factor of $2n^2 r$ and $n^{n-2}r^{n-1} + (u^{n-1} + \gamma)$ would be 1.

It is possible that $2n^2 r = p'^n$ and $n^{n-2}r^{n-1} + (u^{n-1} + \gamma) = q'^n$.

$$x^n = p'^n q'^n$$

Step 2

Assume that $z = y + i$.

From $y = (u - v), z = (u + v)$,

$$z - y = (u + v) - (u - v) = 2v$$

$$2v = i$$

Assume that $y = k + i$.

Since y is odd and i is even, k is odd.

Assume that y and i are not relatively prime.

Let $y = fY, i = fI$.

$$z = y + i$$

$$z = f(Y + I)$$

y and z have common factor of f.

But this is a contradiction because y and z are relatively prime.

So, y and i are relatively prime.

Assume that k and y are not relatively prime.

Let $k = fK, y = fY$.

$$i = y - k$$

$$i = f(Y - K)$$

y and i have common factor of f.

But this is a contradiction because y and i are relatively prime.

So, k and y are relatively prime.

Let $k = (u' - v)$, $y = (u' + v)$.

Where $u' < u$

Assume that u' and v are not relatively prime.

Let $u' = fU', v = fV$.

$$k = f(U' - V)$$

$$y = f(U' + V)$$

k and y have common factor of f.

But this is a contradiction because k and y are relatively prime.

So, u' and v are relatively prime.

Also u' and v are opposite parity because k and y are odd.

$$y^n - k^n = (u' + v)^n - (u' - v)^n$$

$$y^n - k^n = 2v\left(v^{n-1} + n({u'}^{n-1} + \gamma')\right)$$

Where $\gamma' = C'_3 u'^{n-3} v^2 + C'_5 u'^{n-5} v^4 + \cdots + C'_{n-4} u'^4 v^{n-5} + C'_{n-2} u'^2 v^{n-3}$

Assume that $u' = \text{even}, v = \text{odd}$.

$v^{n-1} + n({u'}^{n-1} + \gamma')$ is odd.

Assume that $u' = \text{odd}, v = \text{even}$.

$v^{n-1} + n({u'}^{n-1} + \gamma')$ is odd.

So, $v^{n-1} + n({u'}^{n-1} + \gamma')$ is always odd.

Thus the greatest common factor of 2v and $v^{n-1} + n({u'}^{n-1} + \gamma')$ is odd.

Assume that the common factor of 2v and $v^{n-1} + n({u'}^{n-1} + \gamma')$ is odd number except 1 and n.

Let $v = fV$, $\gamma' = f\Gamma'$ and $v^{n-1} + n({u'}^{n-1} + \gamma') = fN'$.

$$f^{n-1}V^{n-1} + n(u'^{n-1} + f\Gamma') = fN'$$

Where $f\Gamma' = f(C'_3 {u'}^{n-3} f^1 V^2 + C'_5 {u'}^{n-5} f^3 V^4 + \cdots + C'_{n-4} u'^4 f^{n-6} V^{n-5} + C'_{n-2} u'^2 f^{n-4} V^{n-3})$

$$nu'^{n-1} = f(N' - f^{n-2}V^{n-1} - n\Gamma')$$

u' and v have common factor of f.

But this is a contradiction because u' and v are relatively prime.

So, the greatest common factor of 2v and $v^{n-1} + n({u'}^{n-1} + \gamma')$ is either 1 or n.

Assume that the greatest common factor of 2v and $\mathbf{v^{n-1} + n({u'}^{n-1} + \gamma')}$ is 1.

It is possible that $2v = p^n$ and $v^{n-1} + n({u'}^{n-1} + \gamma') = \alpha$.

$$y^n - k^n = \alpha p^n$$

Assume that $\alpha = {s_1}^3$.

$$y^n - k^n = {s_1}^n p^n$$

Where s_1 and p are relatively prime

Let $y^n = {c_1}^n, k^n = {b_1}^n, {s_1}^n p^n = {a_1}^n$

$${a_1}^n + {b_1}^n = {c_1}^n$$

But this equation is a contradiction by the method of infinite descent.

Assume that $\alpha \neq {s_1}^3$.

$$y^n - k^n = \alpha p^n$$

Where α and p are relatively prime

Assume that the greatest common factor of 2v and $\mathbf{v^{n-1} + n({u'}^{n-1} + m')}$ is n.

Let $v = nr$.

u' and r are relatively prime because u' and v are relatively prime.

$$y^n - k^n = 2nr\left(n^{n-1}r^{n-1} + n({u'}^{n-1} + \gamma')\right)$$

$$y^n - k^n = 2n^2r\left(n^{n-2}r^{n-1} + ({u'}^{n-1} + \gamma')\right)$$

The greatest common factor of $2n^2r$ and $n^{n-2}r^{n-1} + ({u'}^{n-1} + \gamma')$ would be 1.

It is possible that $2n^2r = p'^n$ and $n^{n-2}r^{n-1} + ({u'}^{n-1} + \gamma') = \alpha'$.

$$y^n - k^n = \alpha' p'^n$$

Assume that $\alpha' = {s_1}'^n$.

$$y^n - k^n = {s'_1}^n p'^n$$

Where s_1' and p' are relatively prime

Let $y^n = {c'_1}^n, k^n = {b'_1}^n, {s'_1}^n p'^n = {a'_1}^n$

$${a'_1}^n + {b'_1}^n = {c'_1}^n$$

But this equation is a contradiction by the method of infinite descent.

Assume that $\alpha \neq {s_1}^n$.

$$y^n - k^n = \alpha' p'^n$$

Where α' and p' are relatively prime

Step 3

Assume that $k = t + i$.

Since k is odd and i is even, t is odd.

Assume that k and i are not relatively prime.

Let $k = fK, i = fI$.

$$y = k + i$$

$$y = f(K + I)$$

k and y have common factor of f.

But this is a contradiction because k and y are relatively prime.

So, k and i are relatively prime.

Assume that t and k are not relatively prime.

Let $t = fT, k = fK$.

$$i = k - t$$

$$i = f(K - T)$$

k and i have common factor of f.

But this is a contradiction because k and i are relatively prime.

So, t and k are relatively prime.

Let $t = (u'' - v)$, $k = (u'' + v)$.

Where $u'' < u'$

Assume that u'' and v are not relatively prime.

Let $u'' = fU'', v = fV$.

$$t = f(U'' - V)$$

$$k = f(U'' + V)$$

t and k have common factor of f.

But this is a contradiction because t and k are relatively prime.

So, u'' and v are relatively prime.

Also u'' and v are opposite parity because t and k are odd.

$$k^n - t^n = (u'' + v)^n - (u'' - v)^n$$

$$k^n - t^n = 2v\left(v^{n-1} + n({u''}^{n-1} + \gamma'')\right)$$

Where $\gamma'' = C'_3 u''^{n-3} v^2 + C'_5 u''^{n-5} v^4 + \cdots + C'_{n-4} u''^4 v^{n-5} + C'_{n-2} u''^2 v^{n-3}$

Assume that $u'' = \text{even}, v = \text{odd}$.

$v^{n-1} + n(u''^{n-1} + m'')$ is odd.

Assume that $u'' = \text{odd}, v = \text{even}$.

$v^{n-1} + n(u''^{n-1} + m'')$ is odd.

So, $v^{n-1} + n(u''^{n-1} + m'')$ is always odd.

Thus the greatest common factor of 2v and $v^{n-1} + n(u''^{n-1} + \gamma'')$ is odd.

Assume that the common factor is odd number except 1 and n.

Let $v = fV$, $\gamma'' = f\Gamma''$ and $v^{n-1} + n(u''^{n-1} + \gamma'') = fN$.

$$f^{n-1}V^{n-1} + n(u''^{n-1} + f\Gamma'') = fN$$

Where $f\Gamma'' = f(C'_3 u''^{n-3} f^1 V^2 + C'_5 u''^{n-5} f^3 V^4 + \cdots + C'_{n-4} u''^4 f^{n-6} V^{n-5} + C'_{n-2} u''^2 f^{n-4} V^{n-3})$

$$nu''^{n-1} = f(N - f^{n-2}V^{n-1} - n\Gamma'')$$

u'' and v have common factor of f.

But this is a contradiction because u'' and v are relatively prime.

So, the greatest common factor of 2v and $v^{n-1} + n(u''^{n-1} + \gamma'')$ is either 1 or n.

Assume that the greatest common factor of 2v and $\mathbf{v^{n-1} + n(u''^{n-1} + \gamma'')}$ is 1.

It is possible that $2v = p^n$ and $v^{n-1} + n(u'^{n-1} + \gamma'') = \beta$.

$$k^n - t^n = \beta p^n$$

Assume that $\beta = {s_2}^3$.

$$k^n - t^n = {s_2}^n p^n$$

Where ${s_2}^n$ and p are relatively prime

Let $k^n = {c_2}^n, t^n = {b_2}^n, {s_2}^n p^n = {a_2}^n$

$${a_2}^n + {b_2}^n = {c_2}^n$$

But this equation is a contradiction by the method of infinite descent.

Assume that $\beta \neq {s_2}^n$.

$$k^n - t^n = \beta p^n$$

Where β and p are relatively prime, $\beta < \alpha$

This equation also leads to a contradiction by the method of infinite descent.

Assume that the greatest common factor of 2v and $\mathbf{v^{n-1} + n(u''^{n-1} + \gamma'')}$ is n.

Let $v = nr$.

u'' and r are relatively prime because u'' and v are relatively prime.

$$k^n - t^n = 2nr\left(v^{n-1} + n({u''}^{n-1} + \gamma'')\right)$$

$$k^n - t^n = 2n^2r\left(n^{n-2}r^{n-1} + ({u''}^{n-1} + \gamma'')\right)$$

The greatest common factor of $2n^2r$ and $n^{n-2}r^{n-1} + ({u''}^{n-1} + \gamma'')$ would be 1.

It is possible that $2n^2r = p'^n$ and $n^{n-2}r^{n-1} + ({u''}^{n-1} + \gamma'') = \beta'$.

$$k^n - t^n = \beta' p'^n$$

Assume that $\beta' = {s_2}'^n$.

$$k^n - t^n = {s_2}'^n p'^n$$

Where ${s_2}'^n$ and p' are relatively prime

Let $k^n = {c'_2}^n, t^n = {b'_2}^n, {s_2}'^n p'^n = {a'_2}^n$

$${a'_2}^n + {b'_2}^n = {c'_2}^n$$

But this equation is a contradiction by the method of infinite descent.

Assume that $\beta' \neq {s_2}'^n$.

$$k^n - t^n = \beta' p'^n$$

Where β' and p' are relatively prime, $\beta' < \alpha'$

This equation also leads to a contradiction by the method of infinite descent.

Thus case 1 has no solution by the method of infinite descent.

Case 2. (x, y, z) = (odd, even, odd)

Case 2 has no solution because it can be proved by the same method as Case 1.

Case 3. (x, y, z) = (odd, odd, even)

Step 1

Let $x = (u - v), y = (u + v)$.

Assume that u and v are not relatively prime.

Let $u = fU, v = fV$.

$$x = f(U - V)$$

$$y = f(U + V)$$

But this is a contradiction because x and y are relatively prime.

So, u and v are relatively prime.

Also u and v are opposite parity because x and y are odd.

$$z^n = (u - v)^n + (u + v)^n$$

$$z^n = (u^n - C_1 u^{n-1} v^1 + C_2 u^{n-2} v^2 + \cdots - C_{n-2} u^2 v^{n-2} + C_{n-1} u^1 v^{n-1} - v^n) \\ +(u^n + C_1 u^{n-1} v^1 + C_2 u^{n-2} v^2 + \cdots + C_{n-2} u^2 v^{n-2} + C_{n-1} u^1 v^{n-1} + v^n)$$

Where $C = \{C_1, C_2, \cdots, C_{n-2}, C_{n-1}\} = \{\frac{n}{1}, \frac{n(n-1)}{1\cdot 2}, \cdots, \frac{n(n-1)\cdots 4\cdot 3}{1\cdot 2\cdots(n-3)(n-2)}, \frac{n(n-1)\cdots 2\cdot 1}{1\cdot 2\cdots(n-2)(n-1)}\}$

$$z^n = 2u(u^{n-1} + n(v^{n-1} + C'_{n-3} v^{n-3} u^2 + \cdots + C'_2 v^2 u^{n-3}))$$

Where $C' = \{C'_1, C'_2, \cdots, C'_{n-2}, C'_{n-1}\} = \{\frac{1}{1}, \frac{(n-1)}{1\cdot 2}, \cdots, \frac{(n-1)\cdots 4\cdot 3}{1\cdot 2\cdots(n-3)(n-2)}, \frac{(n-1)\cdots 2\cdot 1}{1\cdot 2\cdots(n-2)(n-1)}\}$

$$z^n = 2u(u^{n-1} + n(v^{n-1} + \gamma))$$

Where $\gamma = C'_{n-3} v^{n-3} u^2 + C'_{n-5} v^{n-5} u^4 + \cdots + C'_4 v^4 u^{n-5} + C'_2 v^2 u^{n-3}$

Assume that $u = \text{even}, v = \text{odd}$.

$u^{n-1} + n(v^{n-1} + \gamma)$ is odd.

Assume that $uu = \text{odd}, v = \text{even}$.

$u^{n-1} + n(v^{n-1} + \gamma)$ is odd.

So, $u^{n-1} + n(v^{n-1} + \gamma)$ is always odd.

Thus the greatest common factor of 2u and $u^{n-1} + n(v^{n-1} + \gamma)$ is odd.

Assume that the common factor is odd except 1 and n.

Let $u = fU$ and $u^{n-1} + n(v^{n-1} + \gamma) = fN$.

$$f^{n-1} U^{n-1} + n(u^{n-1} + f\Gamma) = fN$$

Where $f\Gamma = f(C'_{n-3} v^{n-3} f^1 U^2 + C'_{n-5} v^{n-5} f^3 U^4 + \cdots + C'_4 v^4 f^{n-6} U^{n-5} + C'_2 v^2 f^{n-4} U^{n-3})$

$$nu^{n-1} = f(N - f^{n-2} U^{n-1} - n\Gamma)$$

u and v have common factor of f.

But this is a contradiction because u and v are relatively prime.

So, the greatest common factor of 2u and $u^{n-1} + n(v^{n-1} + \gamma)$ is either 1 or n.

Assume that the greatest common factor of 2u and $\mathbf{u^{n-1} + n(v^{n-1} + \gamma)}$ is 1.

It is possible that $2u = p^n$ and $u^{n-1} + n(v^{n-1} + \gamma) = q^n$.

Where p and q are relatively prime

$$z^n = p^n q^n$$

Assume thatthe greatest common factor of 2u and $\mathbf{u^{n-1} + n(v^{n-1} + \gamma)}$ is n.

Let $u = nr$.

v and r are relatively prime because v and u are relatively prime.

$$z^n = 2nr(u^{n-1} + n(v^{n-1} + \gamma))$$

$$z^n = 2n^2 r(n^{n-2} r^{n-1} + (v^{n-1} + \gamma))$$

The greatest common factor of $2n^2 r$ and $n^{n-2} r^{n-1} + (v^{n-1} + \gamma)$ would be 1.

It is possible that $2n^2 r = p'^n$ and $n^{n-2} r^{n-1} + (v^{n-1} + \gamma) = q'^n$.

$$z^n = p'^n q'^n$$

Step 2

Assume that $v = iv'$.

Where $i =$ positive integer

Let k=(u−v'), l=(u+v').

u and v' are relatively prime because u and v are relatively prime.

Assume that k and u are not relatively prime.

Let $k = fK, u = fU$.

$$k = u - v'$$

$$v' = f(U - K)$$

u and v' have common factor of f.

But this is a contradiction because u and v' are relatively prime.

So, k and u are relatively prime.

Assume that l and u are not relatively prime.

Let $l = fL, u = fU$.

$$l = u + v'$$

$$v' = f(L - U)$$

u and v' have common factor of f.

But this is a contradiction because u and v' are relatively prime.

So, l and u are relatively prime.

Assume that k and l are not relatively prime.

Let $k = fK, l = fL$.

$$2u = k + l$$

$$2u = f(K + L)$$

u and k have common factor of f.

But this is a contradiction because u and k are relatively prime.

So, k and l are relatively prime.

$$k^n + l^n = (u - v')^n + (u + v')^n$$

$$k^n + l^n = 2u\left(u^{n-1} + n({v'}^{n-1} + \gamma')\right)$$

Where $\gamma' = C'_{n-3}v'^{n-3}u^2 + C'_{n-5}v'^{n-5}u^4 + \cdots + C'_4v'^4u^{n-5} + C'_2v'^2u^{n-3}$

Assume that $u = \text{even}, v' = \text{odd}$.

$u^{n-1} + n(v'^{n-1} + \gamma')$ is odd.

Assume that $u = \text{odd}, v' = \text{even}$.

$u^{n-1} + n(v'^{n-1} + \gamma')$ is odd.

So, $u^{n-1} + n(v'^{n-1} + \gamma')$ is always odd.

Thus the greatest common factor of 2u and $u^{n-1} + n(v'^{n-1} + \gamma')$ is odd.

Assume that the common factor of 2u and $u^{n-1} + n(v'^{n-1} + \gamma')$is odd number except 1 and n.

Let $u = fU$, $\gamma' = f\Gamma'$ and $u^{n-1} + n({v'}^{n-1} + \gamma') = fN'$.

$$f^{n-1}U^{n-1} + n({v'}^{n-1} + f\Gamma') = fN'$$

Where $f\Gamma' = f(C'_{n-3}{v'}^{n-3}f^1U^2 + C'_{n-5}{v'}^{n-5}f^3U^4 + \cdots + C'_4{v'}^4f^{n-6}U^{n-5} + C'_2{v'}^2f^{n-4}U^{n-3})$

$$nv'^{n-1} = f(N - f^{n-2}U^{n-1} - n\Gamma')$$

u and v' have common factor of f.

But this is a contradiction because u and v' are relatively prime.

So, the greatest common factor of 2u and $u^{n-1} + n({v'}^{n-1} + \gamma')$ is either 1 or n.

Assume that the greatest common factor of 2u and $\mathbf{u^{n-1} + n(v'^{n-1} + \gamma')}$ **is 1.**

It is possible that $2u = p^n$ and $u^{n-1} + n(v'^{n-1} + \gamma') = \alpha$.

$$k^n + l^n = \alpha p^n$$

Assume that $\alpha = {s_1}^n$.

$$k^n + l^n = {s_1}^n p^n$$

Where s_1 and p are relatively prime

Let $k^n = {c_1}^n, l^n = {b_1}^n, {s_1}^n p^n = {a_1}^n$

$${a_1}^n + {b_1}^n = {c_1}^n$$

But this equation is a contradiction by the method of infinite descent.

Assume that $\alpha \neq {s_1}^n$.

$$k^n + l^n = \alpha p^n$$

Where α and p are relatively prime

Assume that the greatest common factor of 2u and $\mathbf{u^{n-1} + n(v'^{n-1} + \gamma')}$ **is n.**

Let $u = nr$.

v' and r are relatively prime because v' and u are relatively prime.

$$k^n + l^n = 2nr(u^{n-1} + n(v'^{n-1} + \gamma'))$$

$$k^n + l^n = 2n^2r(n^{n-2}r^{n-1} + (v'^{n-1} + \gamma'))$$

The greatest common factor of $2n^2r$ and $n^{n-2}r^{n-1} + (v'^{n-1} + \gamma')$ would be 1.

It is possible that $2n^2r = p'^n$ and $n^{n-2}r^{n-1} + (v'^{n-1} + \gamma') = \alpha'$.

$$k^n + l^n = \alpha' p'^n$$

Assume that $\alpha' = {s_1}'^n$.

$$k^n + l^n = {s_1}'^n p'^n$$

Where ${s_1}'$ and p' are relatively prime

Let $k^n = {a_1}'^n, l^n = {b_1}'^n, {s_1}'^n p'^n = {c_1}'^n$

$${a_1}'^n + {b_1}'^n = {c_1}'^n$$

But this equation is a contradiction by the method of infinite descent.

Assume that $\alpha' \neq {s_1}'^n$.

$$k^n + l^n = \alpha' p'^n$$

Where α' and p' are relatively prime

Step 3

Assume that $v' = iv''$.

Where $i =$ positive integer

Let $m = (u - v''), t = (u + v'')$.

u and v'' are relatively prime because u and v' are relatively prime.

Assume that m and u are not relatively prime.

Let $m = fM, u = fU$.

$$m = u - v''$$

$$v'' = f(U - M)$$

u and v'' have common factor of f.

But this is a contradiction because u and v'' are relatively prime.

So, m and u are relatively prime.

Assume that t and u are not relatively prime.

Let $t = fT, u = fU$.

$$t = u + v''$$

$$v'' = f(T - U)$$

u and v'' have common factor of f.

But this is a contradiction because u and v'' are relatively prime.

So, t and u are relatively prime.

Assume that m and t are not relatively prime.

Let $m = fM, t = fT$.

$$2u = m + t$$

$$2u = f(M + T)$$

m and u have common factor of f.

But this is a contradiction because m and u are relatively prime.

So, m and t are relatively prime.

$$m^n + t^n = (u - v'')^n + (u + v'')^n$$

$$m^n + t^n = 2u(u^{n-1} + n(v''^{n-1} + \gamma''))$$

Where $\gamma'' = C'_{n-3}v''^{n-3}u^2 + C'_{n-5}v''^{n-5}u^4 + \cdots + C'_4 v''^4 u^{n-5} + C'_2 v''^2 u^{n-3}$

Assume that $u = \text{even}, v'' = \text{odd}$.

$u^{n-1} + n(v''^{n-1} + \gamma'')$ is odd.

Assume that $u = \text{odd}, v'' = \text{even}$.

$u^{n-1} + n(v''^{n-1} + \gamma'')$ is odd.

So, $u^{n-1} + n(v''^{n-1} + \gamma'')$ is always odd.

Thus the greatest common factor of 2u and $u^{n-1} + n(v''^{n-1} + \gamma'')$ is odd.

Assume that the common factor of 2u and $u^{n-1} + n(v''^{n-1} + \gamma'')$ is odd number except 1 and n.

Let $u = fU$, $\gamma'' = f\Gamma''$ and $u^{n-1} + n(v''^{n-1} + \gamma'') = fN''$.

$$f^{n-1}U^{n-1} + n(v''^{n-1} + f\Gamma'') = fN''$$

Where $f\Gamma'' = f(C'_{n-3}v''^{n-3}f^1U^2 + C'_{n-5}v''^{n-5}f^3U^4 + \cdots + C'_4 v''^4 f^{n-6}U^{n-5} + C'_2 v''^2 f^{n-4}U^{n-3})$

$$nv''^{n-1} = f(N - f^{n-2}U^{n-1} - n\Gamma'')$$

u and v'' have common factor of f.

But this is a contradiction because u and v'' are relatively prime.

So, the greatest common factor of 2u and $u^{n-1} + n(v''^{n-1} + \gamma'')$ is either 1 or n.

Assume that the greatest common factor of 2u and $\mathbf{u^{n-1} + n(v''^{n-1} + \gamma'')}$ is 1.

It is possible that $2u = p^n$ and $u^{n-1} + n(v''^{n-1} + \gamma'') = \beta$.

$$m^n + t^n = \beta p^n$$

Assume that $\beta = s_2{}^n$.

$$m^n + t^n = s_2{}^n p^n$$

Where s_2 and p are relatively prime

Let $m^n = a_2{}^n, t^n = b_2{}^n, s_2{}^n p^n = c_2{}^n$

$$a_2{}^n + b_2{}^n = c_2{}^n$$

But this equation is a contradiction by the method of infinite descent.

Assume that $\beta \neq s_2{}^n$.

$$m^n + t^n = \beta p^n$$

Where β and p are relatively prime, $\beta < \alpha$

This equation also leads to a contradiction by the method of infinite descent.

Assume that the greatest common factor of 2u and $\mathbf{u^{n-1} + n(v''^{n-1} + \gamma'')}$ is n.

Let $u = nr$.

v'' and r are relatively prime because v'' and u are relatively prime.

$$m^n + t^n = 2nr\left(u^{n-1} + n(v''^{n-1} + \gamma'')\right)$$

$$m^n + t^n = 2n^2r\left(n^{n-2}r^{n-1} + (v''^{n-1} + \gamma'')\right)$$

The greatest common factor of $2n^2r$ and $n^{n-2}r^{n-1} + (v''^{n-1} + \gamma'')$ would be 1.

It is possible that $2n^2r = p'^n$ and $n^{n-2}r^{n-1} + (v''^{n-1} + \gamma'') = \beta'$.

$$m^n + t^n = \beta' p'^n$$

Assume that $\beta' = {s_2}'^n$.

$$m^n + t^n = {s_2}'^n p'^n$$

Where ${s_2}'$ and p' are relatively prime

Let $m^n = {a_2}'^n, t^n = {b_2}'^n, {s_2}'^n p'^n = {c_1}'^n$

$${a_2}'^n + {b_2}'^n = {c_2}'^n$$

But this equation is a contradiction by the method of infinite descent.

Assume that $\beta' \neq {s_2}'^n$.

$$k^n + l^n = \beta' p'^n$$

Where β' and p' are relatively prime, $\beta' < \alpha'$

This equation also leads to a contradiction by the method of infinite descent.

Thus case 3 has no solution by the method of infinite descent.

In conclusion, there are no positive integers in case of n=odd number since all of Case 1, 2 and 3 are contradictions.

5. REFERENCES

[1] A. Wilds. Modular elliptic curves and Fermat's last theorem. Ann. of Math., 141:443-551, 1995.
[2] Dickson, LE (2005) [1920], History of the theory of numbers. Vol. II: Diophantine analysis, New York: Dover Publications, ISBN 978-0-486-44233-4, MR 0245500