THREE THEOREMS FROM WHICH FOLLOW TWO NEW LAWS OF GRAVITATION AND A NEW LAW OF UNIVERSAL GRAVITATION.

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Abstract. *New theorems on the flux of the gravitational field strength vector are presented, free from the limitations inherent in Gauss's theorem. Gauss's theorem is not sufficient for a complete description of gravity. It is shown that the relationship between the gravitational field flux and the mass of sources is not the only method for representing the gravitational field flux. Unlike Gauss's theorem, the new theorems do not include the mass of sources. The first theorem establishes a relationship between the flux of the gravitational field strength vector and the parameters of the body's orbit. The second theorem establishes a relationship between the flux of the gravitational field strength vector and the parameters of the universe. The third theorem concerns the total gravitational force that actually acts on the body and is caused by the gravitational interaction of the test body with all bodies in the universe. From these theorems, two new laws of gravitation and a new law of universal gravitation are derived, which for the first time describes gravity taking into account the action of all bodies in the universe. Unlike Newton's law of gravity, the new law of universal gravitation does not include the mass of sources and the gravitational constant G. The new theorems relate to the gravitational field, which obeys the inverse-square law, and to the gravitational field of the universe, where the inverse-square law does not apply. The new theorems and new laws of gravity provide a complete description of the gravitational interactions of all N-body bodies in the universe. The new theorems demonstrate the existence of a workaround instead of searching for a direct solution to the gravitational N-body problem (for $N \geq 3$).*

Keywords: *Gauss's theorem, gravitational field strength vector, cosmological force law, Hooke-Kepler's law of gravitation, law of universal gravitation, universal gravitation theorem, local gravitation theorem.*

1. Introduction

In mathematical physics, there are a number of fundamental theorems that prove the properties of gravity and the behavior of matter. These include Newton's spherical shell theorem, the Maclaurin-Laplace theorem on the attraction of ellipsoids of revolution, the Penrose-Hawking singularity theorems, Birkhoff's theorem on the Schwarzschild metric, and Gauss's theorem on the gravitational flux through a closed surface.

Gauss's theorem relates the flux of the gravitational field strength vector to the mass of the sources. Newton's law of gravitation follows from Gauss's theorem. Both Newton's law of gravitation and Gauss's theorem include gravitational mass as a parameter.

Here we show that gravitational mass is neither the only nor the best parameter for determining the gravitational field strength vector flux and the law of gravity. Below, we demonstrate that the flux of the gravitational field strength vector through a closed surface is directly related to the orbital parameters and integral parameters of the Universe, and we present new theorems for gravity. We also demonstrate that Newton's law of gravitation is not the only physical law representing the gravitational force. Based on these new theorems, we derive new laws of gravity that are more accurate and perfect than Newton's law of universal gravitation.

2. Gauss's Theorem of Gravitation.

The Gauss Theorem of Gravitation is a fundamental law establishing the relationship between the flux of the gravitational field strength vector *g* through a closed surface and the mass within the closed surface. Gauss's Theorem states that the flux of the gravitational field strength vector *g* through a closed surface is proportional to the mass of the sources:

$$\Phi_g = \oint_S g \cdot dS = -4\pi GM \tag{1}$$

Where: Φ_g is the gravitational field flux, g is the gravitational field strength vector, dS is the surface area element, G is the gravitational constant, M is the total mass inside the surface.

Gauss's theorem is valid for a gravitational field that obeys an inverse-square law. The minus sign indicates that gravitational forces are always attractive (the lines of force enter the surface), unlike electrostatics, where the flux can be positive or negative. The mathematical formulation of Gauss's theorem for forces depending on the inverse square of the distance was first systematically presented by Gauss in 1839 [1]. The very idea of the flux of the gravitational field strength vector through a closed surface began to take shape in the works of Lagrange in 1773. In his work on the attraction of ellipsoids, Lagrange first mathematically derived the relationship between the flux of the gravitational field strength vector and mass [2]. Lagrange found the "seed" of the idea in gravity, and Gauss transformed it into a universal law of nature and a powerful mathematical tool. Gauss showed that for any field obeying an inverse-square law (whether gravity or electrostatics), the flux through a closed surface is determined exclusively by the sources within that surface.

From Gauss's theorem, Newton's law of two-body gravity, which also obeys an inverse-square law, is derived. However, Gauss's theorem cannot be used to derive the law of gravitation, which describes the gravity of all N bodies in the universe where the inverse-square law does not apply. auss's theorem is not sufficient to solve this problem. This is the limit of applicability of Gauss's theorem for gravity. Furthermore, Gauss's theorem for gravity uses the mass of the sources "M" as a parameter, which is not a primary empirical fact but is calculated from observed orbits. Gauss's theorem for gravity is applicable on small scales. On the scale of the universe, Gauss's theorem is not applicable.

In electromagnetism, the flux of the electric field strength vector is associated only with electric charge. It is proportional to the total electric charge located inside the surface. In gravity, the situation is different. Gauss's theorem implies that the flux of the gravitational field strength vector is related to the mass of the sources. It is known that the central mass determines the parameters of the body's orbit. Therefore, the flux of the gravitational field strength vector can be expressed in fundamentally different ways: through the field source (the mass inside) and through the kinematic parameters of the test body's orbit. The first method is represented by Gauss's theorem. There is no theorem for the second method. One must formulate it. This is a theorem relating the flux of the gravitational field strength vector to the kinematic parameters of the orbit. The value of such a theorem is that the orbit is a primary empirical fact. We consider the orbital parameters (observed data) as the primary basis for determining the magnitude of the gravitational flux.

All of the above suggests that Newton's law of gravitation is not the only law of gravitational interaction. Since Newton's law, which includes the mass "M", is derived from Gauss's theorem, a new law of gravity will follow from the new theorem, which will not include the mass "M". Therefore, it is of interest to consider how the flow of the gravitational field strength vector is related to the orbital parameters.

3. Theorem on the relationship between the flux of the gravitational field strength vector and orbital parameters.

THEOREM 1. (Theorem on the relationship between the flux of the gravitational field strength vector and orbital parameters): "*The flux of the gravitational field strength vector g through any closed surface S is expressed in terms of the parameters of the body's elliptical orbit and is proportional to the ratio of the cube of the semimajor axis R^3 to the square of the body's orbital period T^2.*"

Theorem 1 is represented by the mathematical formula in Figure 1:

$$\Phi_1 = \oint_S g \cdot dS = -4\pi R^3 / T^2$$

Fig. 1. Mathematical formula for the relationship between the flux of the gravitational field strength vector and the orbital parameters. Where: Φ_1 is the gravitational field flux, g is the gravitational field strength vector, dS is the surface area element, and R and T are the orbital parameters.

Here, we use the number of revolutions per unit time (1/T) rather than radians per unit time (2π/T) as units for measuring the frequency. This simplifies the formulas, eliminates dimensionless coefficients, and makes them coherent.

The proof of Theorem 1 is based on combining the geometric properties of the field (Gauss's law) and the dynamics of the body's motion in the gravitational field of the source (Kepler's law).

The ratio of R^3 to the square of the orbital period T^2 is an invariant for the test mass in the gravitational field of the source. It can be used to express the magnitude of the flux of the gravitational field strength vector without knowing the mass "M" itself directly.

Theorem 1 applies to a gravitational field if its intensity decreases inversely with the square of the distance from the source. This is its similarity to Gauss's theorem. It differs from Gauss's theorem in that Theorem 1 operates not with the mass "M," but with the kinematic parameters of the orbit R and T, which are the primary empirical fact.

4. Deriving a new law of gravity from Theorem 1.

Let's consider which law of gravity follows from Theorem 1. For simplicity, consider a Gaussian sphere of radius r. The field strength vector g at any point on this sphere is directed toward the center, and the magnitude of g is the same at all points on the sphere's surface. The flux of the field strength vector g through the sphere's surface is equal to the product of the field strength and the sphere's area:

$$\Phi_g = \oint_S g \cdot dS = g \cdot (4\pi r^2) \qquad (2)$$

According to Theorem 1, this flow is equal to:

$$\Phi_g = -4\pi R^3 / T^2 \qquad (3)$$

From here:

$$\Phi_g = -4\pi R^3 / T^2 = g \cdot (4\pi r^2) \qquad (4)$$

Field strength g:

$$g = -R^3 / r^2 T^2 \qquad (5)$$

Since the field strength g is the force acting on a unit mass (g = F/m), we obtain a new law of gravity based only on the field geometry and orbital parameters:

$$F = mR^3 / r^2 T^2 \qquad (6)$$

The new law of gravity shows the magnitude of the force for the local gravitational interaction of two bodies. In [3], we called this law of gravity the Hooke-Kepler law (Fig. 2).

$$F_{H-K} = \frac{mR^3}{T^2 r^2}$$

Fig. 2. The Hooke-Kepler law of gravitation. Where **m** is the mass of the body, **R** and **T** are the orbital parameters, and **r** is the distance.

This is a more refined and accurate law of gravitation than Newton's law. It reflects the local force of gravitational interaction between two bodies. Instead of mass, it includes orbital parameters, since the orbit is the primary empirical fact. An approximate verbal formula for this law of gravitation, with an emphasis on the Keplerian orbit, was first formulated by Robert Hooke in 1679 [4]. However, Newton preferred mass over orbital parameters. As a result, a more refined and accurate law of gravitation between two bodies remained undiscovered.

5. Theorem on the relationship between the flux of the gravitational field strength vector and the parameters of the Universe.

The relationship between the flux of the gravitational field strength vector and the parameters of the Universe is established by Theorem 2.

THEOREM 2. (Theorem on the relationship between the flux of the gravitational field strength vector and the parameters of the Universe). "*The flux of the gravitational field strength vector* g_U *through a closed surface* S_U *of the Universe is equal to the product* $4\pi c^2 R_U^2 \sqrt{\Lambda}$."

Theorem 2 is represented by the mathematical formula in Figure 3:

$$\Phi_{Cos} = \oint_S g_U \cdot dS_U = -4\pi c^2 R_U^2 \sqrt{\Lambda}$$

Fig. 3. Theorem on the relationship between the flux of the gravitational field strength vector and the parameters of the Universe. Where: Φ_{Cos} is the flux of the gravitational field of the Universe, g_U is the gravitational field strength vector of the Universe, dS_U is the surface area element, c is the speed of light, R_U is the radius of the Universe, and Λ is the cosmological constant.

The resulting gravitational field of the Universe is the vector sum of the gravitational fields generated by each object in the Universe. If the gravitational field is generated by several sources, then the flux of the resulting field vector through a closed surface is equal to the sum of the fluxes of the field vectors generated by each source individually:

$$\Phi_{Cos} = \oint_S g_1 \cdot dS + \oint_S g_2 \cdot dS + \ldots + \oint_S g_n \cdot dS \quad (7)$$

We consider the Universe as a whole, so the gravitational flux through a closed surface is directly related to the parameters of the Universe. Using the parameters of the Universe eliminates the problem associated with the unknown value of "n" in equation (7). Direct summation of the forces from an infinite number of stars leads to a false infinite result (Seeliger's gravitational paradox). Einstein was the first to realize that Gauss's theorem leads to a contradiction: If the Universe is infinite and uniformly filled with mass, then the gravitational flux through any closed surface must tend to infinity. Physics does not operate with infinities. In a finite Universe, the mass of the entire Universe can be considered a separate source of the gravitational field. This mass creates a flux Φ_{Cos} of the field strength vector g_U.

The total gravitational field strength from the entire Universe is defined as:

$$g_U = \sum_{i=1}^{n} g_i = c^2 \sqrt{\Lambda} \quad (8)$$

Where: c is the speed of light, Λ is the cosmological constant.

The area of the Gaussian sphere for the Universe:

$$S_U = 4\pi R_U^2 \quad (9)$$

We substitute the value of g_U and the value of S_U into the definition of the flow:

$$\Phi_{Cos} = \oint_S g_U \cdot dS_U = -g_U \cdot 4\pi R_U^2 = -4\pi R_U^2 c^2 \sqrt{\Lambda} \quad (10)$$

Theorem 2 is valid for the gravitational field created by all bodies in the Universe where the inverse square law does not apply.

6. Deriving a New Law of Gravity from Theorem 2.

Let's consider which physical law of gravity would derive from Theorem 2. For simplicity, we'll consider the universe as a Gaussian sphere of radius R_U. The field strength vector g_U at any point on this sphere is directed toward the center, and the magnitude of g_U is the same at all points on the sphere's surface.

The flux of the field strength vector g_U through the surface of the sphere is equal to the product of the field strength and the sphere's area:

$$\Phi_{Cos} = \oint_S g_U \cdot dS_U = -g_U \cdot (4\pi R_U^2) \quad (11)$$

According to Theorem 2, this flow is equal to:

$$\Phi_{Cos} = -4\pi c^2 R_U^2 \sqrt{\Lambda} \quad (12)$$

From here:

$$\Phi_{Cos} = -4\pi c^2 R_U^2 \sqrt{\Lambda} = -g_U \cdot (4\pi R_U^2) \quad (13)$$

Field strength g_U:

$$g_U = c^2 \sqrt{\Lambda} \quad (14)$$

Since the field strength gU is the force acting on a unit mass ($g_U = F/m$), we obtain a new law of gravity based only on the field geometry and the parameters of the Universe:

$$F = mc^2 \sqrt{\Lambda} \quad (15)$$

The new law of gravity shows the magnitude of the gravitational force of all bodies in the Universe. In [5, 6], we called this law of gravity the law of cosmological force:

$$F_{Cos} = mc^2 \sqrt{\Lambda}$$

Fig. 4. The law of cosmological force. Where m is the mass of the object, c is the speed of light, and Λ is the cosmological constant.

This new law of gravitation describes the attractive force exerted by the entire mass of the universe on a test mass. This is the part of the gravitational force that Newton's law "does not detect." Instead of the gravitational constant G, the law of cosmological force contains the cosmological constant Λ. The new law of gravitation shows that any body of mass m experiences a cosmological force proportional to its mass and the cosmological constant Λ. The cosmological force is linear in mass and does not obey the inverse-square law. On small scales, the additional cosmological force is significantly smaller than the Newtonian force. On the scale of the universe, the cosmological force is enormous. In the limit, it is exactly equal to the Planck force:

$$\lim_{m \to M_U} F_{Cos} = \lim_{m \to M_U} mc^2\sqrt{\Lambda} = \frac{c^4}{G} = 1.21027 \bullet 10^{44}\,N$$

Fig. 5. Limiting value of the cosmological force F_{Cos}.

The theoretical limit of the cosmological force at $m \to M_U$ reaches the enormous value c^4/G = 1.21027 x 10^{44} N. At large distances, the main part of the universal gravitational force is the cosmological force F_{Cos}.

7. The Theorem of Universal Gravitation

Here we present a new law of universal gravitation that takes into account the gravitational pull of all bodies in the universe. Hooke gave an approximate verbal formula for the law of universal gravitation as early as 1679, seven years before the publication of Newton's Principia [4, 6]. Newton rejected Hooke's proposal, solving the two-body problem. This idealized model of two-body gravitation proved inapplicable to the real universe. Through the efforts of popularizers, Newton's law was called the law of universal gravitation [4]. Newton never called his two-body gravitational law the law of universal gravitation. The real law of universal gravitation, which takes into account the gravitational pull of all bodies in the universe, remained undiscovered. Theorem 3 provides a solution to this problem, outlined in Hooke's verbal formula.

THEOREM 3. (Theorem of Universal Gravitation). "*The total gravitational force acting on a body is equal to the sum of the local gravitational force of the two bodies and the gravitational force of all bodies in the Universe (the cosmological force).*"

Theorem 3 is represented by the formula (Fig. 6):

$$F_U = \frac{mR^3}{T^2 r^2} + mc^2\sqrt{\Lambda}$$

Fig. 6. Law of universal gravitation. Where: **m** is the mass of the body, **R** and **T** are the orbital parameters, **r** is the distance, **c** is the speed of light in a vacuum, and **Λ** is the cosmological constant.

The proof of the law of universal gravitation follows from the superposition of the fluxes of the gravitational field strength vectors and the linearity of the integral. The resulting gravitational field at any point in space is the vector sum of the gravitational fields generated by each source individually. If the gravitational field is generated by multiple sources, then the flux of the total gravitational field strength vector through a closed surface is equal to the sum of the fluxes of the strength vectors.

$$\Phi_U = \Phi_1 + \Phi_{Cos} \qquad (16)$$

Figure 7 shows the formula for the total flux Φ_U, represented by the orbital parameters and the parameters of the universe.

$$\Phi_U = \oint_S g \cdot dS + \oint_S g_U \cdot dS_U = -4\pi(R^3/T^2 + c^2 R_U^2 \sqrt{\Lambda})$$

Fig. 7. Total Φ_U flux represented by orbital parameters and Universe parameters.

8. Conclusion.

Newton, in his mathematical formulation of the law of gravitation, and Gauss, in his mathematical formulation of the theorem on the flux of the gravitational field strength vector, used gravitational mass. Both Newton and Gauss ignored observable parameters, such as orbital parameters and the parameters of the universe. The price of this "inattention" is the undiscovered laws of gravity more perfect than Newton's law of gravity.

Rejecting gravitational mass as a parameter for determining the flux of the gravitational field strength vector, in favor of orbital parameters, leads to three new theorems, from which a new law of universal gravitation is derived without the gravitational constant G. This new law of universal gravitation, for the first time, yields a real gravitational force taking into account the gravitational attraction of all bodies in the universe. As a result, the low accuracy of the gravitational constant G ceases to be a problem in gravity theory. The N-body gravitational problem, which has no analytical solution, ceases to be a problem in gravity theory, since new theorems demonstrate the existence of a workaround for taking into account the gravitational attraction of all bodies in the Universe instead of searching for a solution to the N-body gravitational problem (for $N \geq 3$).

9. Final Remarks.

1. Gravitational mass is neither the only nor the best parameter for the law of universal gravitation and for determining the flux of the gravitational field strength vector.

2. The flux of the gravitational field strength vector through a closed surface can be expressed in three fundamentally different ways: through the source of the field (the mass within), through the kinematic parameters of the orbit of a test body, and through the parameters of the Universe. These three approaches demonstrate that gravitational flux is a fundamental quantity linking the local distribution of mass, the motion of objects, and the large-scale structure of the Universe.

3. Gauss's theorem is insufficient for a complete description of gravity. Gauss's theorem allows one to express the flux of the gravitational field strength vector through the field source (the mass within), but does not allow one to express it through the kinematic parameters of the orbit of a test body or through the parameters of the Universe.

4. Three new theorems are presented, from which two new laws of gravitation and a new law of universal gravitation follow. The new theorems apply both to gravitational fields that obey the inverse-square law and to the gravitational field of the Universe, where the inverse-square law does not apply.

5. Theorem 1 is a theorem on the relationship between the flux of the gravitational field strength vector and the orbital parameters. Theorem 1 applies to a gravitational field if its intensity decreases inversely with the square of the distance from the source. This is its similarity to Gauss's theorem. It differs from Gauss's theorem in that Theorem 1 operates not with the mass "M," but with the kinematic parameters of the orbit.

6. Theorem 2 is a theorem on the relationship between the flux of the gravitational field strength vector and the parameters of the universe. Theorem 2 is valid for the gravitational field created by all bodies in the universe, where the inverse-square law does not apply.

7. Theorem 3 is a theorem of universal gravitation. Theorem 3 states that the total gravitational force acting on a body is equal to the sum of the local gravitational force of the two bodies and the gravitational force of all bodies in the universe (the cosmological force).

8. Newton's law of universal gravitation is not the only physical law describing the gravitational force. New laws of gravity, more precise than Newton's law, were derived from new theorems.

9. Newton solved the two-body problem. He didn't call his law of gravity the law of universal gravitation. Thanks to the efforts of popularizers who were too far removed from understanding gravity, Newton's law was called the law of universal gravitation. The real law of universal gravitation, which takes into account the gravity of all bodies in the universe, remained undiscovered.

10. The new theorems and new laws of gravity provide a complete description of the gravitational interactions of all N-body bodies in the universe. The new theorems demonstrate the existence of a workaround instead of searching for a solution to the N-body gravitational problem (for $N \geq 3$).

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