Geometric Scaling Laws in Block Decompositions of the Dirichlet Eta Function

This paper presents a block-structured decomposition of the Dirichlet eta function $\eta(s) = \sum_{n=1}^{\infty} (-1)^{n-1} n^{-s}$, where $s = \sigma + it$, based on an exponential partitioning of the summation index into intervals of the form $N_k \sim 2e^{2k\pi/t}$. The series is reorganized into finite segments, which are analyzed as dynamically scaling components with approxipmately geometric decay behavior.

A ratio structure between successive block contributions is derived, leading to an approximate exponential scaling law of the form $R_k(s) \sim e^{-2\pi\sigma/t}$. This allows the eta function to be expressed as a coupled system of real and imaginary components, each defined over partitioned summation blocks.

Using this decomposition, a real–imaginary interaction structure is introduced, where intersection-type conditions between the real and imaginary parts are studied through a determinant-based formulation. The resulting system suggests a structured relationship between block interactions and phase behavior in the complex plane.

The framework provides a new perspective on the analytic structure of $\eta(s)$through interval scaling, phase coupling, and approximate geometric recursion. In particular, the model highlights a special role of the parameter $\sigma = 1/2$within the interaction structure, emerging from symmetry considerations in the decomposed representation.

This work is exploratory in nature and aims to develop a structured analytical model for studying oscillatory behavior in alternating Dirichlet series via block decomposition techniques.

$$roughly\ k = \left\lfloor \frac{t \ln 600000}{2\pi} \right\rfloor (excel\ limitation)$$

$$\eta(s) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^s} = \sum_{n=1}^{2\left\lfloor e^{\frac{2k}{t}} \right\rfloor} \frac{(-1)^{n-1}}{n^s} + \sum_{n=2\left\lfloor e^{\frac{2k\pi}{t}} \right\rfloor+1}^{2\left\lfloor e^{\frac{2(k+1)\pi}{t}} \right\rfloor} \frac{(-1)^{n-1}}{n^s} + \sum_{n=2\left\lfloor e^{\frac{2(k+1)\pi}{t}} \right\rfloor+1}^{2\left\lfloor e^{\frac{2(k+2)\pi}{t}} \right\rfloor} \frac{(-1)^{n-1}}{n^s} + \cdots$$

$$R_k(s) = \frac{\sum_{n=2\left\lfloor e^{\frac{2(k+1)\pi}{t}} \right\rfloor+1}^{2\left\lfloor e^{\frac{2(k+2)\pi}{t}} \right\rfloor} \frac{(-1)^{n-1}}{n^s}}{\sum_{n=2\left\lfloor e^{\frac{2k\pi}{t}} \right\rfloor+1}^{2\left\lfloor e^{\frac{2(k+1)\pi}{t}} \right\rfloor} \frac{(-1)^{n-1}}{n^s}}$$

$$s = \sigma + it$$

$$S_k = \sum_{n=N_k+1}^{N_{k+1}} \frac{(-1)^{n-1}}{n^s}$$

$$N_k = 2e^{2k\ \ /t}$$

$$S_k \approx \int_{N_k}^{N_{k+1}} x^{-s}\, e^{i\pi x}\, dx$$

$$S_k \sim N_k^{-s}$$

$$S_{k+1} \sim N_{k+1}^{-s}$$

$$\frac{S_{k+1}}{S_k} \sim \left(\frac{N_{k+1}}{N_k}\right)^{-s}$$

$$\frac{N_{k+1}}{N_k} \sim e^{2\pi/t}$$

$$R_k(s) \sim e^{-2\pi s/t}$$

$$R_k(s) \sim e^{-2\pi(\sigma+it)/t}$$

$$R_k(s) \sim e^{-2\pi\sigma/t} e^{-2\pi i}$$

$$e^{-2\pi i} = 1$$

$$\boxed{R_k(s) \sim e^{-2\pi\sigma/t}}$$

$$\boxed{\frac{\sum_{n=2\left\lfloor e^{\frac{2(\mathrm{k}+1)\pi}{t}}\right\rfloor+1}^{2\left\lfloor e^{\frac{2(\mathrm{k}+2)\pi}{t}}\right\rfloor} \frac{(-1)^{n-1}}{n^s}}{\sum_{n=2\left\lfloor e^{\frac{2\mathrm{k}\pi}{t}}\right\rfloor+1}^{2\left\lfloor e^{\frac{2(\mathrm{k}+1)\pi}{t}}\right\rfloor} \frac{(-1)^{n-1}}{n^s}} \approx e^{-2\pi\sigma/t}}$$

$$\sum_{n=1}^{2\left\lfloor e^{\frac{2\mathrm{k}\pi}{t}}\right\rfloor} \frac{(-1)^{n-1}}{n^s} = \aleph_k(\mathit{first\ abnormal\ series})$$

$$\sum_{n=2\left\lfloor e^{\frac{2k\pi}{t}}\right\rfloor+1}^{2\left\lfloor e^{\frac{2(k+1)\pi}{t}}\right\rfloor} \frac{(-1)^{n-1}}{n^s} = \aleph_{k+1}(almost\ geometric\ series)$$

$$\eta(s) = \aleph_s = \aleph_k + \aleph_{k+1} + \aleph_{k+2} + \aleph_{k+3} + \aleph_{k+4} + \cdots$$

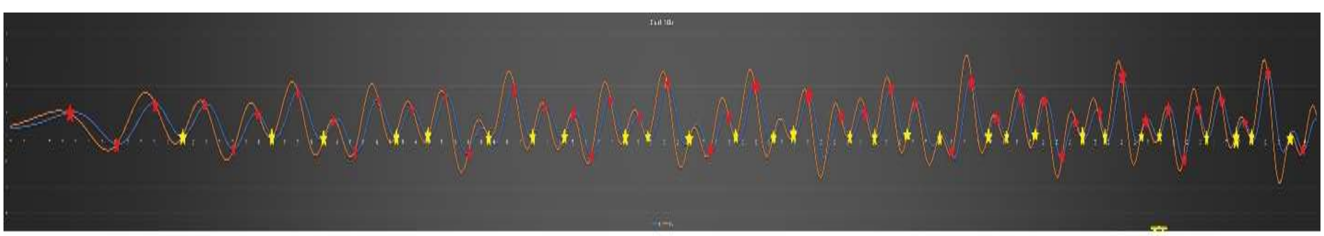
$$\aleph_s = \aleph_k + \aleph_{k+1} + \frac{\aleph_{k+1}}{e^{\frac{2\pi\sigma}{t}}} + \frac{\aleph_{k+2}}{e^{\frac{2\pi}{t}}} + \frac{\aleph_{k+3}}{e^{\frac{2\pi}{t}}} + \cdots = \aleph_k + \aleph_{k+1} + \frac{\aleph_{k+1}}{e^{\frac{2\pi\sigma}{t}}} + \frac{\aleph_{k+1}}{\left[e^{\frac{2\pi}{t}}\right]^2} + \frac{\aleph_{k+1}}{\left[e^{\frac{2\pi\sigma}{t}}\right]^2} + \cdots$$

$$\aleph_s = \aleph_k + \frac{\aleph_{k+1}}{1 - e^{-\frac{2\pi\sigma}{t}}}$$

$$\aleph_s = \sum_{n=1}^{2\left\lfloor e^{\frac{2k}{t}}\right\rfloor} \frac{(-1)^{n-1}}{n^{\sigma+it}} + \frac{\sum_{n=2\left\lfloor e^{\frac{2k\pi}{t}}\right\rfloor+1}^{2\left\lfloor e^{\frac{2(k+1)\pi}{t}}\right\rfloor} \frac{(-1)^{n-1}}{n^{\sigma+it}}}{1 - e^{-\frac{\sigma 2\pi}{t}}}$$

$$\aleph_{s,Re} = \sum_{n=1}^{2\left\lfloor e^{\frac{2k\pi}{t}}\right\rfloor} (-1)^{n-1} n^{-\sigma} \cos[t \ln n] + \frac{\sum_{n=2\left\lfloor e^{\frac{2k\pi}{t}}\right\rfloor+1}^{2\left\lfloor e^{\frac{2(k+1)\pi}{t}}\right\rfloor} (-1)^{n-1} n^{-\sigma} \cos[t \ln n]}{1 - e^{-\frac{\sigma 2\pi}{t}}}$$

$$\aleph_{s,Img} = -\sum_{n=1}^{2\left\lfloor e^{\frac{2k}{t}}\right\rfloor} (-1)^{n-1} n^{-\sigma} \sin[t \ln n] - \frac{\sum_{n=2\left\lfloor e^{\frac{2k}{t}}\right\rfloor+1}^{2\left\lfloor e^{\frac{2(k+1)\pi}{t}}\right\rfloor} (-1)^{n-1} n^{-\sigma} \sin[t \ln n]}{1 - e^{-\frac{\sigma 2\pi}{t}}}$$



Theorem

All intersection points of $\aleph_{s,Re}\sim$ and $\aleph_{s,Img}\sim$ functions are not Zeros.However all Zeros are intersection points of $\aleph_{s,Re}$ and $\aleph_{s,Img}$functions.

$$Z \subseteq P_{ic}(P_{ic} = \text{intersection points}\ , \text{Z} = \text{Zeros})$$

$$\aleph_{s,Re} = \aleph_{s,Img}(interaction\ property)$$

$$\sum_{n=1}^{2\left\lfloor e^{\frac{2k\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\cos[t\ln n] + \frac{\sum_{n=2\left\lfloor e^{\frac{2k\pi}{t}}\right\rfloor+1}^{2\left\lfloor e^{\frac{2(k+1)\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\cos[t\ln n]}{1-e^{-\frac{\sigma 2\pi}{t}}} = -\sum_{n=1}^{2\left\lfloor e^{\frac{2k\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\sin[t\ln n] - \frac{\sum_{n=2\left\lfloor e^{\frac{2k}{t}}\right\rfloor+1}^{2\left\lfloor e^{\frac{2(k+1)\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\sin[t\ln n]}{1-e^{-\frac{\sigma 2\pi}{t}}}$$

$$\frac{\sum_{n=2\left\lfloor e^{\frac{2k\pi}{t}}\right\rfloor+1}^{2\left\lfloor e^{\frac{2(k+1)\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\cos[t\ln n]}{1-e^{-\frac{\sigma 2\pi}{t}}} + \frac{\sum_{n=2\left\lfloor e^{\frac{2k}{t}}\right\rfloor+1}^{2\left\lfloor e^{\frac{2(k+1)\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\sin[t\ln n]}{1-e^{-\frac{\sigma 2\pi}{t}}} = -\sum_{n=1}^{2\left\lfloor e^{\frac{2k}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\cos[t\ln n] - \sum_{n=1}^{2\left\lfloor e^{\frac{2k\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\sin[t\ln n]$$

$$\frac{\sum_{n=2\left\lfloor e^{\frac{2k}{t}}\right\rfloor+1}^{2\left\lfloor e^{\frac{2(k+1)\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\cos[t\ln n] + \sum_{n=2\left\lfloor e^{\frac{2k\pi}{t}}\right\rfloor+1}^{2\left\lfloor e^{\frac{2(k+1)\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\sin[t\ln n]}{1-e^{-\frac{\sigma 2\pi}{t}}} = -\sum_{n=1}^{2\left\lfloor e^{\frac{2k\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\cos[t\ln n] - \sum_{n=1}^{2\left\lfloor e^{\frac{2k}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\sin[t\ln n]$$

$$1-e^{-\frac{\sigma 2\pi}{t}} = \frac{\sum_{n=2\left\lfloor e^{\frac{2k}{t}}\right\rfloor+1}^{2\left\lfloor e^{\frac{2(k+1)\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\cos[t\ln n] + \sum_{n=2\left\lfloor e^{\frac{2k\pi}{t}}\right\rfloor+1}^{2\left\lfloor e^{\frac{2(k+1)\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\sin[t\ln n]}{-\sum_{n=1}^{2\left\lfloor e^{\frac{2k\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\cos[t\ln n] - \sum_{n=1}^{2\left\lfloor e^{\frac{2k}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\sin[t\ln n]}(intersection\ or\ contact\ properties)$$

$$\aleph_{s,Re} = \sum_{n=1}^{2\left\lfloor e^{\frac{2k}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\cos[t\ln n] + \frac{\sum_{n=2\left\lfloor e^{\frac{2k\pi}{t}}\right\rfloor+1}^{2\left\lfloor e^{\frac{2(k+1)\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\cos[t\ln n]}{1-e^{-\frac{\sigma 2\pi}{t}}}$$

$$\aleph_{s,Re} = \sum_{n=1}^{2\left\lfloor e^{\frac{2k}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\cos[t\ln n] + \frac{\sum_{n=2\left\lfloor e^{\frac{2k}{t}}\right\rfloor+1}^{2\left\lfloor e^{\frac{2(k+1)\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\cos[t\ln n]\left[-\sum_{n=1}^{2\left\lfloor e^{\frac{2k}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\cos[t\ln n] - \sum_{n=1}^{2\left\lfloor e^{\frac{2k\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\sin[t\ln n]\right]}{\sum_{n=2\left\lfloor e^{\frac{2k\pi}{t}}\right\rfloor+1}^{2\left\lfloor e^{\frac{2(k+1)\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\cos[t\ln n] + \sum_{n=2\left\lfloor e^{\frac{2k\pi}{t}}\right\rfloor+1}^{2\left\lfloor e^{\frac{2(k+1)\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\sin[t\ln n]}$$

$$\aleph_{s,Re} = 0(when\ \aleph_{s,Re} = 0, \aleph_{s,Img} = 0\ due\ to\ intersection\ property)$$

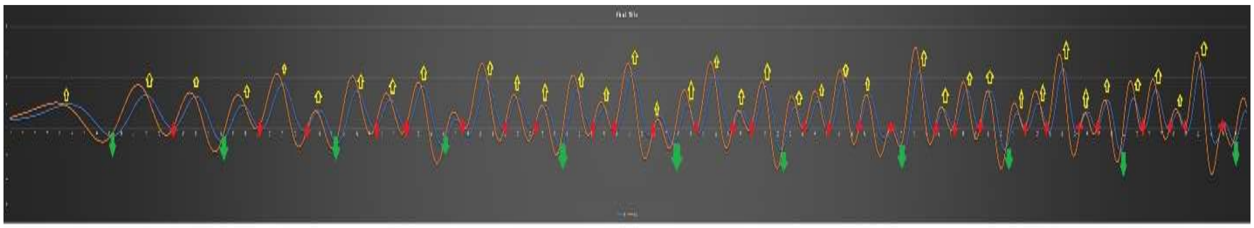
$$\sum_{n=1}^{2\left\lfloor e^{\frac{2k\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\cos[t\ln n] - \frac{\sum_{n=2\left\lfloor e^{\frac{2k\pi}{t}}\right\rfloor+1}^{2\left\lfloor e^{\frac{2(k+1)\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\cos[t\ln n]\left[\sum_{n=1}^{2\left\lfloor e^{\frac{2k}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\cos[t\ln n] + \sum_{n=1}^{2\left\lfloor e^{\frac{2k\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\sin[t\ln n]\right]}{\sum_{n=2\left\lfloor e^{\frac{2k\pi}{t}}\right\rfloor+1}^{2\left\lfloor e^{\frac{2(k+1)\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\cos[t\ln n] + \sum_{n=2\left\lfloor e^{\frac{2k}{t}}\right\rfloor+1}^{2\left\lfloor e^{\frac{2(k+1)\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\sin[t\ln n]} = 0$$

$$\sum_{n=1}^{2\left\lfloor e^{\frac{2k}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\cos[t\ln n]\left[\sum_{n=2\left\lfloor e^{\frac{2k}{t}}\right\rfloor+1}^{2\left\lfloor e^{\frac{2(k+1)\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\cos[t\ln n] + \sum_{n=2\left\lfloor e^{\frac{2k}{t}}\right\rfloor+1}^{2\left\lfloor e^{\frac{2(k+1)\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\sin[t\ln n]\right] = \sum_{n=2\left\lfloor e^{\frac{2k\pi}{t}}\right\rfloor+1}^{2\left\lfloor e^{\frac{2(k+1)\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\cos[t\ln n]\left[\sum_{n=1}^{2\left\lfloor e^{\frac{2k}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\cos[t\ln n] + \sum_{n=1}^{2\left\lfloor e^{\frac{2k\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\sin[t\ln n]\right]$$

$$\vartheta_Z = \left[\sum_{n=1}^{2\left\lfloor e^{\frac{2k\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\cos[t\ln n] \times \sum_{n=2\left\lfloor e^{\frac{2k}{t}}\right\rfloor+1}^{2\left\lfloor e^{\frac{2(k+1)\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\sin[t\ln n]\right] - \left[\sum_{n=2\left\lfloor e^{\frac{2k\pi}{t}}\right\rfloor+1}^{2\left\lfloor e^{\frac{2(k+1)\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\cos[t\ln n] \times \sum_{n=1}^{2\left\lfloor e^{\frac{2k\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\sin[t\ln n]\right]$$

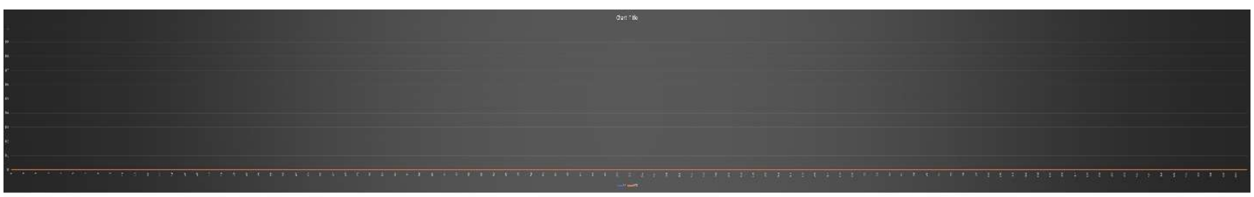
$$\vartheta_Z = \begin{bmatrix} \sum_{n=1}^{2\left\lfloor e^{\frac{2k}{t}} \right\rfloor} (-1)^{n-1} n^{-\sigma} \cos[t \ln n] & \sum_{n=1}^{2\left\lfloor e^{\frac{2k\pi}{t}} \right\rfloor} (-1)^{n-1} n^{-\sigma} \sin[t \ln n] \\ \sum_{n=2\left\lfloor e^{\frac{2k\pi}{t}} \right\rfloor+1}^{2\left\lfloor e^{\frac{2(k+1)\pi}{t}} \right\rfloor} (-1)^{n-1} n^{-\sigma} \cos[t \ln n] & \sum_{n=2\left\lfloor e^{\frac{2k\pi}{t}} \right\rfloor+1}^{2\left\lfloor e^{\frac{2(k+1)\pi}{t}} \right\rfloor} (-1)^{n-1} n^{-\sigma} \sin[t \ln n] \end{bmatrix}$$

$$determinant\ \vartheta_Z = 0 = zeta\ Zeros$$

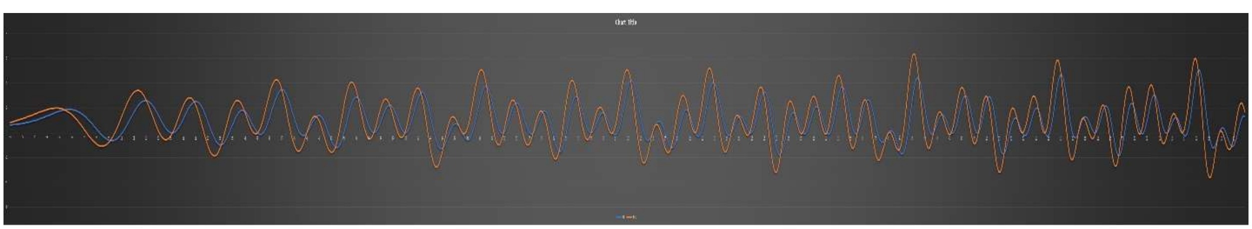


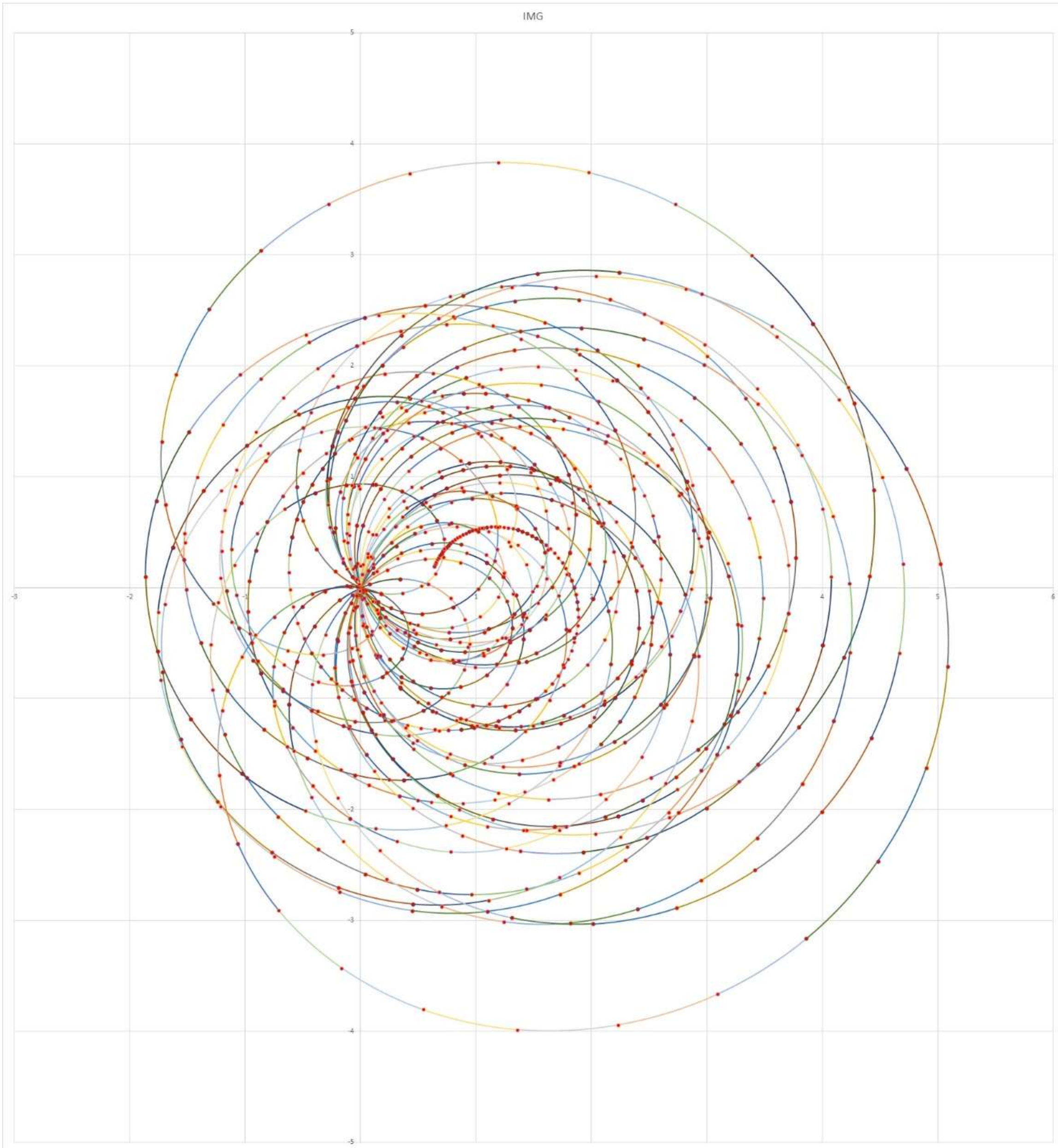
Classification of σ

There are three types of intersections such as upper,middle and lower intersections.

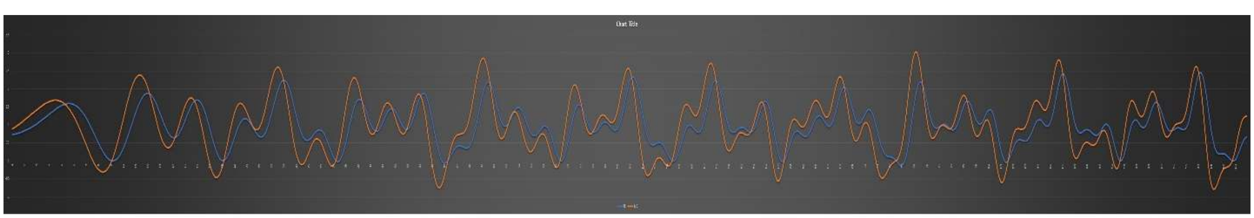


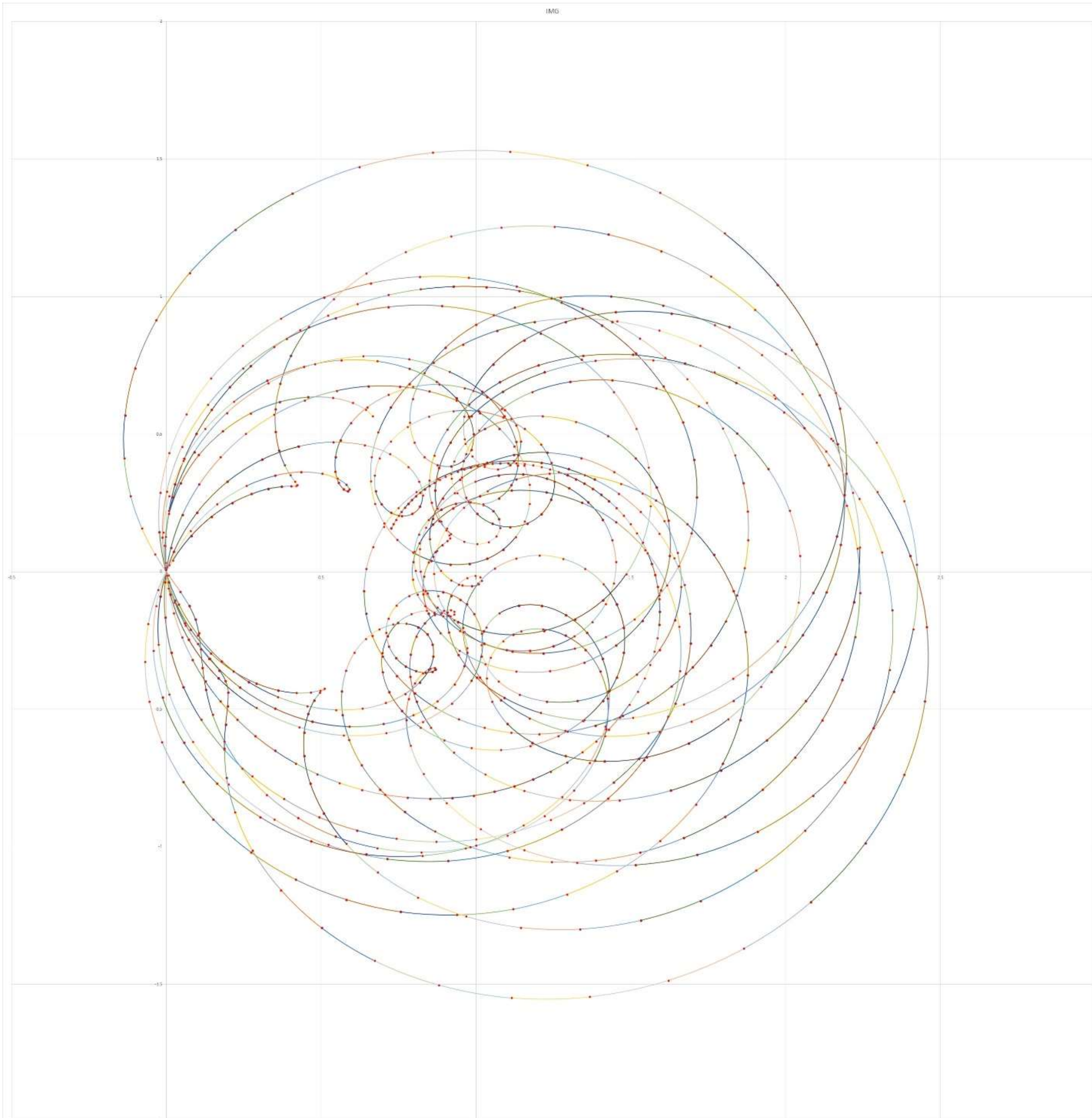
$$when\ \sigma = 0, undefine\ due\ to\ denomator = 0$$





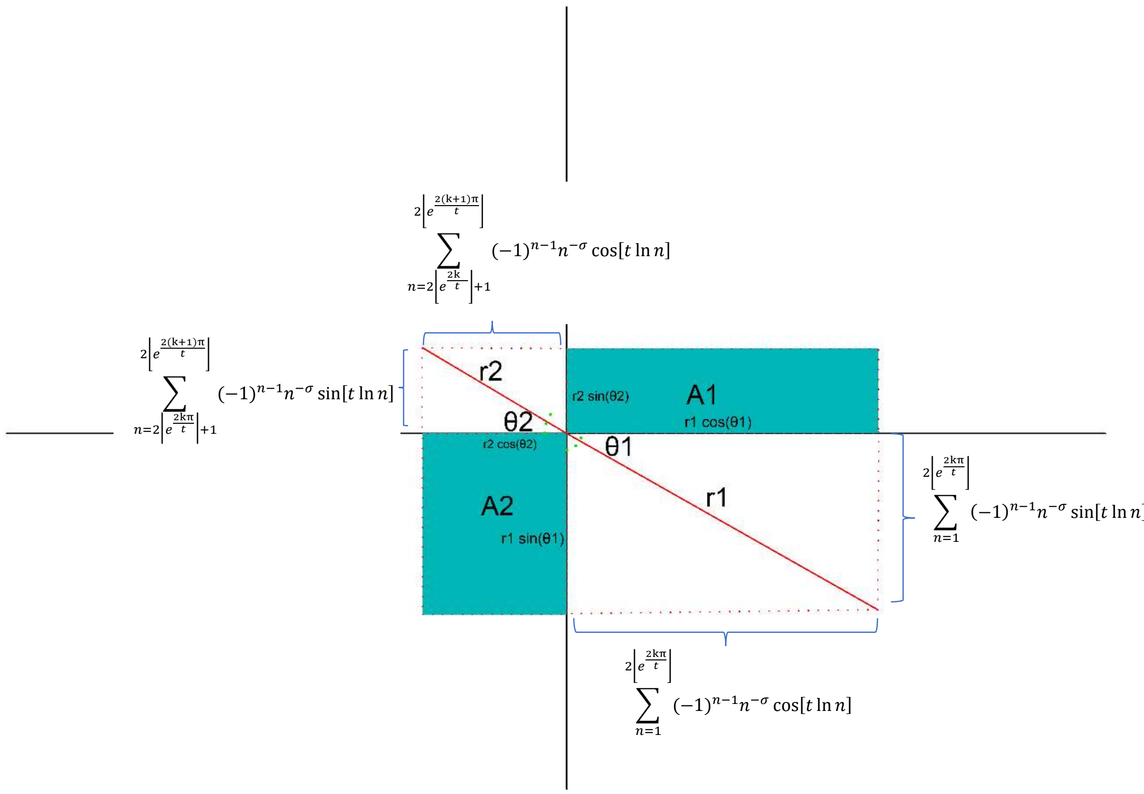
$$when\ \sigma = \frac{1}{2}, middle\ intersections\ are\ zeros$$





$$when\ \sigma = 1, lower\ intersections\ are\ zeros(but\ not\ Zeros\ due\ to\ 1 - 2^{1-s})$$

$$so\ \sigma = \frac{1}{2}\ is\ only\ satisfied\ zeta\ zeros$$



$$\vartheta_Z=\left[\sum_{n=1}^{2\left\lfloor e^{\frac{2k\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\cos[t\ln n]\times\sum_{n=2\left\lfloor e^{\frac{2k\pi}{t}}\right\rfloor+1}^{2\left\lfloor e^{\frac{2(k+1)\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\sin[t\ln n]\right]-\left[\sum_{n=2\left\lfloor e^{\frac{2k\pi}{t}}\right\rfloor+1}^{2\left\lfloor e^{\frac{2(k+1)\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\cos[t\ln n]\times\sum_{n=1}^{2\left\lfloor e^{\frac{2k\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\sin[t\ln n]\right]$$

$$\vartheta_Z=0$$

$$\left[\sum_{n=1}^{2\left\lfloor e^{\frac{2k\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\cos[t\ln n]\times\sum_{n=2\left\lfloor e^{\frac{2k\pi}{t}}\right\rfloor+1}^{2\left\lfloor e^{\frac{2(k+1)\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\sin[t\ln n]\right]-\left[\sum_{n=2\left\lfloor e^{\frac{2k\pi}{t}}\right\rfloor+1}^{2\left\lfloor e^{\frac{2(k+1)\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\cos[t\ln n]\times\sum_{n=1}^{2\left\lfloor e^{\frac{2k\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\sin[t\ln n]\right]=0$$

$$\left[\sum_{n=1}^{2\left\lfloor e^{\frac{2k\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\cos[t\ln n]\times\sum_{n=2\left\lfloor e^{\frac{2k\pi}{t}}\right\rfloor+1}^{2\left\lfloor e^{\frac{2(k+1)\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\sin[t\ln n]\right]=\left[\sum_{n=2\left\lfloor e^{\frac{2k\pi}{t}}\right\rfloor+1}^{2\left\lfloor e^{\frac{2(k+1)\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\cos[t\ln n]\times\sum_{n=1}^{2\left\lfloor e^{\frac{2k\pi}{t}}\right\rfloor}(-1)^{n-1}n^{-\sigma}\sin[t\ln n]\right]$$

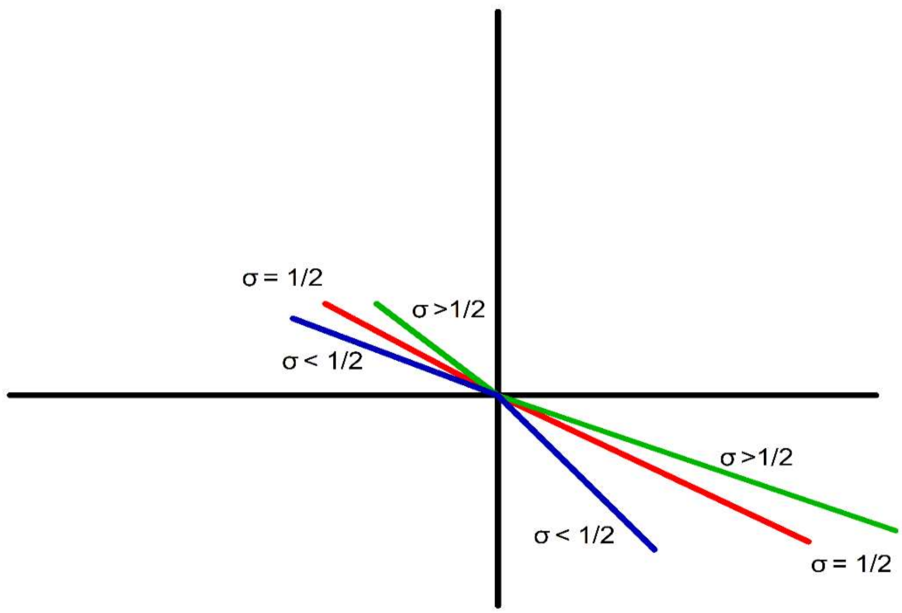
$$A_1=A_2$$

$$r_2\cos\theta_2\times r_1\sin\theta_1=r_1\cos\theta_1\times r_2\sin\theta_2$$

$$\frac{\sin\theta_1}{\cos\theta_1}=\frac{\sin\theta_2}{\cos\theta_2}$$

$$\tan\theta_1 = \tan\theta_2$$

$$\theta_1 = \theta_2$$



$$only\ \sigma = \frac{1}{2}, \theta_1 = \theta_2$$

$$when\ \sigma < \frac{1}{2}, \theta_1 < \theta_2$$

$$when\ \sigma > \frac{1}{2}, \theta_1 > \theta_2$$

$$so\ \sigma = \frac{1}{2}\ is\ only\ satisfied\ zeta\ zeros$$

So Riemann Hypothesis is true.

Zeros finder

https://docs.google.com/spreadsheets/d/1UnMW4XQVQFoJppUQX8tuDLwa4V7TY3pL/export?format=xlsx