Analysis of solution of equations for magnetic field of rotating ball using polynomials

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Abstract: The exact form of the solution for the vector potential and magnetic field of a rotating uniformly charged ball is explicitly found. Expressions for specific vector spherical polynomials associated with the corresponding components of the potential are used to represent the solution. Inside and outside a charged ball uniformly rotating around its axis, the components of the potential and magnetic field are determined up to terms containing the sixth power of the speed of light in the denominator. To do this, it was necessary to use spherical coordinates and eight polynomials of each type. In addition, the solutions inside and outside the ball were equated to each other on the surface of the ball, taking into account the symmetry of the ball. The accuracy of the approach used can be increased, since it is determined only by the number of polynomials used, whose contribution to solutions decreases rapidly as the degree of the polynomials increases. To calculate the vector potential and magnetic field of a rotating ball within the framework of special relativity, it is sufficient to substitute the coordinates of the observation point, the invariant volumetric charge density, the angular velocity of rotation and the radius of the ball into the formulas.

Keywords: polynomial expansion; vector potential; magnetic field; rotating ball.

1. Introduction

There are various methods for determining the magnetic field of moving charge configurations. Thus, in [1], rotating charges create currents that are considered as currents in coils with many turns. In [2-3], the Biot-Savart law is used to determine the magnetic field. These approaches allow only first-order accuracy when determining terms containing the square of the speed of light in the denominator. Calculations using retarded potentials allow improving the accuracy [4], but the complexity of the calculations quickly increases as the accuracy increases.

There are known works in which the electromagnetic field inside and outside a rotating uniformly charged or conducting body is calculated in various ways [5-8]. In the case when there are arbitrarily directed currents in a spherical configuration, the solution of the Laplace equation contains spherical harmonic functions [9]. In order to simplify the solution for the magnetic field of rotating charge configurations, in [10-11] the magnetic field was calculated using the electric potential and electric field strength. In [12], cylindrical and spherical coordinates, as well as spherical polynomials, were used to determine the magnetic field of a spherical system of chaotically moving particles.

One of the widely used methods for approximately solving differential equations in modern engineering is the Finite Element Method (FEM). Some examples of using this method to determine magnetic fields can be found in [13-15]. A comparison of the FEM with the harmonic method is presented in [16], where it is indicated that in some situations, the harmonic method provides a faster convergence rate for the vector potential and magnetic field components than the finite element method.

In magnetostatics, a method of determining the magnetic field in the form of a gradient from a scalar magnetic potential is often used. Thus, in [17] one can find a solution for the magnetic field of a uniformly magnetized sphere, which is expressed through Legendre polynomials. This method is suitable outside the body, as well as in the body's substance in the case where there are no currents in this substance. Beginning with the classical work of Gauss [18], the Earth's magnetic field is modeled in a similar way, and about 150 spherical harmonics were used to study the small-scale magnetic field of the lithosphere in [19].

A more general approach, suitable for electric currents, involves the use of a vector magnetic potential. This was used, for example, in [20], where formulas are presented that relate the scalar magnetic potential to the vector magnetic potential.

The use of polynomials allows one to find a solution quite simply also in the case of rotating charged bodies. In this case, in the solution of the Laplace equation for the vector magnetic potential outside the rotating body generating a stationary magnetic field, as a rule, the associated Legendre polynomials appear.

The purpose of this work is detailed derivation of specific formulas describing vector magnetic potential and magnetic field inside and outside a uniformly charged rotating ball. In the spherical coordinates r,θ,ϕ, the radial component A_r of the vector potential $\mathbf{A}=\left(A_r,A_\theta,A_\phi\right)$ turns out to depend on polynomials that are the product of $\sin\theta$ by the associated Legendre polynomials of the first order in the form $P_n^1\left(\cos\theta\right)$. As for the component A_θ, it turns out to depend on the products of $\cos\theta$ by $P_n^1\left(\cos\theta\right)$. However, solving the inhomogeneous Laplace equation inside the body leads to an unexpected peculiarity, which lies in the fact that the component A_ϕ contains the polynomial $Z_0(\cos\theta)$ that does not coincide with the polynomial $P_0^1(\cos\theta)$. The difference arises as a consequence of the fact that the polynomial $Z_0(\cos\theta)$ is not only one of the solutions of the Laplace equation for the component A_ϕ, but also exactly agrees with the Lorentz factor of rotating charges, which must be taken into account in the special theory of relativity.

Another purpose is not just to present the solution in a general form with undetermined coefficients, as is the case in most papers, but also to describe a specific procedure for determining such coefficients. For rotating ball, using a sufficient number of polynomials, it is possible to obtain the values of vector potential and magnetic field at any given point with the required accuracy. In this case, to determine the components of the field at an arbitrary point with specified coordinates, it is sufficient to know only the invariant volume charge density, the angular velocity of rotation, and the radius of the rotating ball. An essential part of the method is the use of specific spherical polynomials containing associated Legendre polynomials and harmonic functions as multipliers.

The results obtained may be relevant in those models of generation of the Earth's and planets' magnetic fields, which assume spatial radial separation of charges in matter at high temperatures and high pressure. The rotation of the positive and negative charges separated from each other, together with the matter of the rotating planet, can serve as a source of the observed magnetic field [21-26]. An additional effect occurs if tidal forces from the Sun or the moons of the planets are taken into account [27].

Of particular interest is calculation of the magnetic field of those neutron stars that have an electric charge like a proton [28-31], and at the same time rotate so fast that the velocity of the surface of the stars becomes comparable to the speed of light. In this case, even those terms in the formulas that contain c^4 and c^6 in the denominator cease to be small and require their own accounting. From the relationship between the charge density, angular velocity, and radius of the star with the observed magnetic field, one can obtain an estimate of the charge that such stars could have. In addition, it becomes possible to calculate the energy and angular momentum of the electromagnetic field of a neutron star with increased accuracy.

The formulas obtained for the magnetic field of the ball, given their simple appearance and sufficiently high accuracy, can be used for calibration of magnetic field standards and sensors in engineering and applied research. If necessary, these formulas can easily take into account the magnetic permeability of the substance of the ball itself and the magnetic permeability of the medium surrounding the ball.

2. Method

Let us assume that there is charged matter continuously distributed in a certain volume in the Cartesian reference frame. Also we will assume that this matter is in stationary motion in the planes parallel to the plane XOY, but it does not move in the direction of the axis OZ. This is possible, for example, when the charged matter rotates about the axis OZ. Another example is a body that is motionless as a whole, in which electric currents distributed over the volume carry the charge in the planes parallel to the plane XOY.

For the sake of certainty, let us consider the first case, which corresponds to the rotation of a spherical body uniformly charged throughout its entire volume. With a uniform rotation of such a ball around its axis, the vector potential $\mathbf{A}$ is independent of time and satisfies the following equation [32-33], which follows from Maxwell's equations:

$$\Delta \mathbf{A} = -\mu \mu_0 \mathbf{j} = -\frac{\mu \gamma \rho_{0q}}{\varepsilon_0 c^2} \mathbf{v} . \tag{1}$$

where $\Delta \mathbf{A}$ is the vector Laplace operator acting on the vector potential $\mathbf{A}$; $\mu_0 = \frac{1}{\varepsilon_0 c^2}$ is the vacuum magnetic permeability; μ is the magnetic permeability; ε_0 is the vacuum permittivity; c is the speed of light; $\mathbf{j} = \gamma \rho_{0q} \mathbf{v}$ is the current density; $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ is the Lorentz factor of motion of a charged element of matter at the observation point; ρ_{0q} is the invariant charge density of the element of matter; $\mathbf{v}$ is the velocity of motion of the element of matter. We will further assume that $\mu = 1$ and both in the substance of the ball and in the space outside the ball there are no magnetic moments of particles that could lead to the magnetization of matter and to a change in the vector potential and magnetic field.

If equation (1) is considered in Cartesian coordinates, then the component A_z of the vector potential directed toward the axis OZ, according to [34], is equal to zero, $A_z = 0$. This is due to the condition that the velocity component v_z in (1) is equal to zero, since the elements of charged matter move in planes perpendicular to the axis OZ.

Based on the symmetry of the ball, the components of the vector potential should be expressed in spherical three-dimensional coordinates r,θ,ϕ as $\mathbf{A}=\left(A_r,A_\theta,A_\phi\right)$. Among all possible solutions, we will limit ourselves to those in which the components A_r,A_θ,A_ϕ of the vector potential are independent of the angle ϕ of the observation point. In this case, all solutions will be symmetrical with respect to the axis of the ball, which is associated with the axis OZ of the Cartesian coordinate system and has its origin at the center of the ball.

Taking this into account, we can write the following for the vector potential:

$$\mathbf{A}=A_r\left(r,\theta\right)\mathbf{e}_r+A_\theta\left(r,\theta\right)\mathbf{e}_\theta+A_\phi\left(r,\theta\right)\mathbf{e}_\phi, \tag{2}$$

where $\mathbf{e}_r$, $\mathbf{e}_\theta$, $\mathbf{e}_\phi$ are unit vectors of the spherical coordinate system. In this case, the unit vector $\mathbf{e}_r=\frac{\mathbf{r}}{r}$ is directed along the radial coordinate r; the radius-vector $\mathbf{r}=\left(x,y,z\right)$ is directed from the center of the spherical reference frame to the point of observation; the unit vector $\mathbf{e}_\theta$ is directed parallel to the line of meridians (longitude line) in the direction opposite to the axis OZ, that is, towards the South pole; the unit vector $\mathbf{e}_\phi$ is directed parallel to the line of parallels (latitude line) and counterclockwise, when viewed from the side of the axis OZ. In (2), the components of the vector potential $A_r\left(r,\theta\right)$, $A_\theta\left(r,\theta\right)$ and $A_\phi\left(r,\theta\right)$ depend only on the radial coordinate r and on the angle θ.

In general, the vector Laplace operator for a vector $\mathbf{b}$ with spherical components $\mathbf{b}=\left(b_r,b_\theta,b_\phi\right)$ in spherical coordinates has the following form:

$$\begin{aligned}\Delta\mathbf{b}=&\left(\Delta b_r-\frac{2b_r}{r^2}-\frac{2}{r^2\sin\theta}\frac{\partial\left(b_\theta\sin\theta\right)}{\partial\theta}-\frac{2}{r^2\sin\theta}\frac{\partial b_\phi}{\partial\phi}\right)\mathbf{e}_r+\\&+\left(\Delta b_\theta-\frac{b_\theta}{r^2\sin^2\theta}+\frac{2}{r^2}\frac{\partial b_r}{\partial\theta}-\frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial b_\phi}{\partial\phi}\right)\mathbf{e}_\theta+\\&+\left(\Delta b_\phi-\frac{b_\phi}{r^2\sin^2\theta}+\frac{2}{r^2\sin\theta}\frac{\partial b_r}{\partial\phi}+\frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial b_\theta}{\partial\phi}\right)\mathbf{e}_\phi.\end{aligned} \tag{3}$$

Replacing the vector $\mathbf{b}=(b_r,b_\theta,b_\phi)$ in (3) with the vector $\mathbf{A}=(A_r,A_\theta,A_\phi)$ from (2), we rewrite equation (1) in spherical coordinates as follows:

$$\left(\Delta A_r-\frac{2A_r}{r^2}-\frac{2}{r^2\sin\theta}\frac{\partial\left(A_\theta\sin\theta\right)}{\partial\theta}\right)\mathbf{e}_r+\left(\Delta A_\theta-\frac{A_\theta}{r^2\sin^2\theta}+\frac{2}{r^2}\frac{\partial A_r}{\partial\theta}\right)\mathbf{e}_\theta+$$
$$+\left(\Delta A_\phi-\frac{A_\phi}{r^2\sin^2\theta}\right)\mathbf{e}_\phi=-\frac{\rho_{0q}\,\omega r\sin\theta}{\varepsilon_0c^2\sqrt{1-\frac{\omega^2r^2\sin^2\theta}{c^2}}}\mathbf{e}_\phi. \tag{4}$$

In (4), it was taken into account that the Lorentz factor on the right-hand side of (1) is expressed by the formula $\gamma=\frac{1}{\sqrt{1-\frac{\omega^2r^2\sin^2\theta}{c^2}}}$, and derivatives of the vector potential components with respect to the angle ϕ are zero. The velocity components of a charged element of the ball's matter in spherical coordinates have the form $\mathbf{v}=(v_r,v_\theta,v_\phi)=(0,0,\omega r\sin\theta)$, where $\omega=\frac{d\phi}{dt}$ is the angular velocity of the ball's rotation; r is the radial coordinate, specifying the distance from the origin at the center of the ball to the element of matter; θ is the angle between the axis OZ of the Cartesian coordinate system and the radius vector , specifying the position of the element of matter.

In spherical coordinates, for an arbitrary scalar function $f=f(r,\theta,\phi)$, the Laplacian of this function for the case $r\neq 0$ has the form:

$$\Delta f=\frac{1}{r}\frac{\partial^2}{\partial r^2}(rf)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}. \tag{5}$$

We will use (5) to represent the Laplacians ΔA_r, ΔA_θ and ΔA_ϕ in (4). The components of the magnetic induction vector can be calculated through the curl of the vector potential in spherical coordinates using the formula

$$\mathbf{B}=\nabla\times\mathbf{A}=\frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(A_\phi\sin\theta\right)-\frac{\partial A_\theta}{\partial\phi}\right]\mathbf{e}_r+\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi}-\frac{\partial}{\partial r}\left(rA_\phi\right)\right]\mathbf{e}_\theta+$$
$$+\frac{1}{r}\left[\frac{\partial}{\partial r}\left(rA_\theta\right)-\frac{\partial A_r}{\partial\theta}\right]\mathbf{e}_\phi=B_r\mathbf{e}_r+B_\theta\mathbf{e}_\theta+B_\phi\mathbf{e}_\phi. \qquad (6)$$

3. Results

3.1 External vector potential

Similarly to (2), for the components of the external vector potential we have:

$$\mathbf{A}_o=A_{or}\left(r,\theta\right)\mathbf{e}_r+A_{o\theta}\left(r,\theta\right)\mathbf{e}_\theta+A_{o\phi}\left(r,\theta\right)\mathbf{e}_\phi. \qquad (7)$$

There is no charged matter outside the ball, therefore, on the right-hand side of (1) we should assume $\rho_{0q}=0$, so that the equation for the vector potential takes the form $\Delta\mathbf{A}_o=0$. In spherical coordinates, taking into account the equality of the right-hand side of (4) to zero for $\rho_{0q}=0$, we obtain the following:

$$\left(\Delta A_{or}-\frac{2A_{or}}{r^2}-\frac{2}{r^2\sin\theta}\frac{\partial\left(A_{o\theta}\sin\theta\right)}{\partial\theta}\right)\mathbf{e}_r+\left(\Delta A_{o\theta}-\frac{A_{o\theta}}{r^2\sin^2\theta}+\frac{2}{r^2}\frac{\partial A_{or}}{\partial\theta}\right)\mathbf{e}_\theta+$$
$$+\left(\Delta A_{o\phi}-\frac{A_{o\phi}}{r^2\sin^2\theta}\right)\mathbf{e}_\phi=0. \qquad (8)$$

Let us now take into account the fact that according to (1) the component A_{oz} of the vector potential, directed towards the axis OZ and written in Cartesian coordinates, is equal to zero, $A_{oz}=0$. We will use the fact that the components of an arbitrary vector in Cartesian coordinates can be expressed in terms of components of the same vector in spherical coordinates. We proceed from the relations between the unit vectors in Cartesian and spherical coordinates:

$$\mathbf{e}_x=\sin\theta\cos\phi\,\mathbf{e}_r+\cos\theta\cos\phi\,\mathbf{e}_\theta-\sin\phi\,\mathbf{e}_\phi.$$

$$\mathbf{e}_y = \sin\theta\sin\phi\,\mathbf{e}_r + \cos\theta\sin\phi\,\mathbf{e}_\theta + \cos\phi\,\mathbf{e}_\phi\,.$$

$$\mathbf{e}_z = \cos\theta\,\mathbf{e}_r - \sin\theta\,\mathbf{e}_\theta\,. \qquad (9)$$

$$\mathbf{e}_r = \sin\theta\cos\phi\,\mathbf{e}_x + \sin\theta\sin\phi\,\mathbf{e}_y + \cos\theta\,\mathbf{e}_z\,.$$

$$\mathbf{e}_\theta = \cos\theta\cos\phi\,\mathbf{e}_x + \cos\theta\sin\phi\,\mathbf{e}_y - \sin\theta\,\mathbf{e}_z\,.$$

$$\mathbf{e}_\phi = -\sin\phi\,\mathbf{e}_x + \cos\phi\,\mathbf{e}_y\,. \qquad (10)$$

In Cartesian coordinates, the external vector potential is written as follows:

$$\mathbf{A}_o = A_{ox}\mathbf{e}_x + A_{oy}\mathbf{e}_y + A_{oz}\mathbf{e}_z\,. \qquad (11)$$

Let us equate $\mathbf{A}_o$ in (7) to $\mathbf{A}_o$ in (11) and use the expressions for the unit vectors $\mathbf{e}_r$, $\mathbf{e}_\theta$, $\mathbf{e}_\phi$ from (10). As a result, for the Cartesian components of the vector potential outside the rotating charged ball, the following relations are obtained:

$$A_{ox} = A_{or}\sin\theta\cos\phi + A_{o\theta}\cos\theta\cos\phi - A_{o\phi}\sin\phi\,.$$

$$A_{oy} = A_{or}\sin\theta\sin\phi + A_{o\theta}\cos\theta\sin\phi + A_{o\phi}\cos\phi\,.$$

$$A_{oz} = A_{or}\cos\theta - A_{o\theta}\sin\theta\,. \qquad (12)$$

Hence, from (12) with the condition $A_{oz} = 0$ we obtain the relation

$$A_{or}\cos\theta - A_{o\theta}\sin\theta = 0\,. \qquad (13)$$

In order for vector equation (8) to be satisfied, it is necessary that each parenthesis in (8) vanishes. In view of (13), this leads to three equations

$$\Delta A_{or} - \frac{2A_{or}}{r^2} - \frac{2}{r^2 \sin\theta} \frac{\partial\left(A_{or}\cos\theta\right)}{\partial\theta} = 0. \tag{14}$$

$$\Delta A_{o\theta} - \frac{A_{o\theta}}{r^2 \sin^2\theta} + \frac{2}{r^2} \frac{\partial}{\partial\theta}\left(\frac{A_{o\theta}\sin\theta}{\cos\theta}\right) = 0. \tag{15}$$

$$\Delta A_{o\phi} - \frac{A_{o\phi}}{r^2 \sin^2\theta} = 0. \tag{16}$$

Relation (13) has led to the fact that in each equation in (14-16) there is only one of the components of the external vector potential. Since the components A_{or}, $A_{o\theta}$ and $A_{o\phi}$ do not mix with each other in the equations, this makes the solution of equations (14-16) noticeably easier.

Let us substitute into (5) the components A_{or}. $A_{o\theta}$, $A_{o\phi}$ instead of f and find the Laplacians ΔA_{or}, $\Delta A_{o\theta}$, $\Delta A_{o\phi}$, respectively, given that the components A_{or}. $A_{o\theta}$, $A_{o\phi}$ do not depend on the angle ϕ. Then substituting ΔA_{or}, $\Delta A_{o\theta}$, $\Delta A_{o\phi}$ into (14-16), we arrive at three partial differential equations for the vector potential components:

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(rA_{or}) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial A_{or}}{\partial\theta}\right) - \frac{2A_{or}}{r^2} - \frac{2}{r^2\sin\theta}\frac{\partial\left(A_{or}\cos\theta\right)}{\partial\theta} = 0. \tag{17}$$

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(rA_{o\theta}) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial A_{o\theta}}{\partial\theta}\right) - \frac{A_{o\theta}}{r^2\sin^2\theta} + \frac{2}{r^2}\frac{\partial}{\partial\theta}\left(\frac{A_{o\theta}\sin\theta}{\cos\theta}\right) = 0. \tag{18}$$

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(rA_{o\phi}) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial A_{o\phi}}{\partial\theta}\right) - \frac{A_{o\phi}}{r^2\sin^2\theta} = 0. \tag{19}$$

3.2 Calculation of component A_{or}

In order to solve equation (17), we assume that a specific solution for the vector potential component A_{or} can be represented as follows:

$$\bar{A}_{or} = N(r)M(\theta). \tag{20}$$

After substituting (20) into (17) and separating variables, we have:

$$\frac{r}{N(r)}\frac{\partial^2}{\partial r^2}\left[rN(r)\right]-2=-\frac{1}{M(\theta)\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial M(\theta)}{\partial\theta}\right)+\frac{2}{M(\theta)\sin\theta}\frac{\partial\left(M(\theta)\cos\theta\right)}{\partial\theta}=\zeta. \tag{21}$$

The left-hand side (21) depends only on r, and the middle part of the equality depends only on θ. In this connection, both parts of the equality, according to the standard procedure for separating variables, are equated to some constant value ζ.

This yields two equations for the functions $N(r)$ and $M(\theta)$, expressed through the parameter ζ:

$$r\frac{\partial^2 N(r)}{\partial r^2}+2\frac{\partial N(r)}{\partial r}-\frac{\left(\zeta+2\right)N(r)}{r}=0. \tag{22}$$

$$\sin\theta\frac{\partial^2 M(\theta)}{\partial\theta^2}-\cos\theta\frac{\partial M(\theta)}{\partial\theta}+\left(\zeta+2\right)M(\theta)\sin\theta=0. \tag{23}$$

Substituting into (22) a trial solution of the form $N(r)\sim Cr^n$, where C is some constant, allows us to find two possible values of the exponent n by solving a quadratic equation. This yields a solution to equation (22), expressed in terms of ζ and two undetermined coefficients C_1 and C_2:

$$N(r)=C_1 r^{\frac{-1+\sqrt{1+4(\zeta+2)}}{2}}+C_2 r^{\frac{-1-\sqrt{1+4(\zeta+2)}}{2}}. \tag{24}$$

In order to find regular solutions for $\bar{A}_{or}$ in (20), it is necessary to obtain integer powers in (24). To do this, we should set

$$\zeta+2=n\left(n+1\right), \qquad \sqrt{1+4\left(\zeta+2\right)}=2n+1. \tag{25}$$

$$N_n(r) = C_{1,n} r^n + \frac{C_{2,n}}{r^{n+1}} \, , \tag{26}$$

where n is an integer, and $n \geq 0$; $C_{1,n}$ and $C_{2,n}$ are constant coefficients depending on n.

Equation (23), taking into account (25), becomes dependent on n as a discrete parameter:

$$\frac{\partial^2 M(\theta)}{\partial \theta^2} - \frac{\cos\theta}{\sin\theta} \frac{\partial M(\theta)}{\partial \theta} + n(n+1) M(\theta) = 0 \, . \tag{27}$$

Individual solutions of equation (27) can be represented as $M_n(\theta) = C_{3,n} V_n(\theta)$, where $V_n(\theta)$ depend on $\cos\theta$ and are vector spherical polynomials for A_{or}, $C_{3,n}$ are constant coefficients associated with the corresponding polynomials $V_n(\theta)$.

The first eight such vector spherical polynomials have the following form:

$$V_0(\theta) = \cos\theta \, , \qquad V_1(\theta) = \sin^2\theta \, , \qquad V_2(\theta) = \sin^2\theta \cos\theta \, .$$

$$V_3(\theta) = \sin^2\theta \left(5\cos^2\theta - 1\right), \qquad V_4(\theta) = \sin^2\theta \cos\theta \left(7\cos^2\theta - 3\right).$$

$$V_5(\theta) = \sin^2\theta \left(21\cos^4\theta - 14\cos^2\theta + 1\right).$$

$$V_6(\theta) = \sin^2\theta \cos\theta \left(33\cos^4\theta - 30\cos^2\theta + 5\right).$$

$$V_7(\theta) = \sin^2\theta \left(429\cos^6\theta - 495\cos^4\theta + 135\cos^2\theta - 5\right). \tag{28}$$

To obtain a polynomial $V_n(\theta)$, it is necessary to find a solution to equation (27) for a given n. We can assume that each such solution depends on $\cos\theta$. The polynomials in (28) are not normalized and are given up to an arbitrary constant coefficient. This is due to the fact that equation (27) is linear with respect to any constant coefficient multiplied by the polynomials in (28), so that each such coefficient is cancelled out as a result.

Assuming that the function $M(\theta)$ in (27) depends only on $\cos\theta$, we find

$$\frac{\partial M(\theta)}{\partial \theta} = \frac{\partial M(\theta)}{\partial \cos\theta}\frac{\partial \cos\theta}{\partial \theta} = -\sin\theta \frac{\partial M(\theta)}{\partial \cos\theta}.$$

$$\frac{\partial^2 M(\theta)}{\partial \theta^2} = -\cos\theta \frac{\partial M(\theta)}{\partial \cos\theta} + \sin^2\theta \frac{\partial^2 M(\theta)}{\partial \left(\cos\theta\right)^2}. \tag{29}$$

Substituting (29) into (27) yields the equation

$$\sin^2\theta \frac{\partial^2 M(\theta)}{\partial \left(\cos\theta\right)^2} + n\left(n+1\right)M(\theta) = 0. \tag{30}$$

Making the substitutions $\cos\theta = x$, $M(\theta) = M(\cos\theta) = M(x)$ in (30), we obtain:

$$\left(1-x^2\right)\frac{\partial^2 M(x)}{\partial x^2} + n\left(n+1\right)M(x) = 0. \tag{31}$$

In (31) we replace the function in the form $M(x) = \left(1-x^2\right)^k S\left(x\right)$, passing from $M(x)$ to the new function $S\left(x\right)$. For the derivatives of the function $M(x)$ and equation (31) as a whole, we find:

$$\frac{\partial M(x)}{\partial x} = -2kx\left(1-x^2\right)^{k-1} S\left(x\right) + \left(1-x^2\right)^k \frac{\partial S\left(x\right)}{\partial x}.$$

$$\frac{\partial^2 M(x)}{\partial x^2} = \left(1-x^2\right)^k \frac{\partial^2 S\left(x\right)}{\partial x^2} - 4kx\left(1-x^2\right)^{k-1}\frac{\partial S\left(x\right)}{\partial x} - 2k\left(1-x^2\right)^{k-1} S\left(x\right) +$$
$$+4k\left(k-1\right)x^2\left(1-x^2\right)^{k-2} S\left(x\right).$$

$$\left(1-x^2\right)\frac{\partial^2 S\left(x\right)}{\partial x^2} - 4kx\frac{\partial S\left(x\right)}{\partial x} + \left[n\left(n+1\right) - 2k + \frac{4k\left(k-1\right)x^2}{1-x^2}\right]S\left(x\right) = 0. \tag{32}$$

By definition, associated Legendre polynomials $P_n^m(x)$ of degree n and order m are expressed in terms of Legendre polynomials $P_n(x)$ using the formula [17]:

$$P_n^m(x) = (-1)^m (1-x^2)^{\frac{m}{2}} \frac{d^m}{dx^m} P_n(x). \tag{33}$$

The first eight Legendre polynomials are:

$$P_0 = 1, \quad P_1 = x, \quad P_2 = \frac{1}{2}(3x^2 - 1), \quad P_3 = \frac{1}{2}(5x^3 - 3x), \quad P_4 = \frac{1}{8}(35x^4 - 30x^2 + 3).$$

$$P_5 = \frac{1}{8}(63x^5 - 70x^3 + 15x), \qquad P_6 = \frac{1}{16}(231x^6 - 315x^4 + 105x^2 - 5).$$

$$P_7 = \frac{1}{16}(429x^7 - 693x^5 + 315x^3 - 35x). \tag{34}$$

Legendre polynomials satisfy the Legendre equation [35]

$$(1-x^2)\frac{d^2 P_n(x)}{dx^2} - 2x\frac{dP_n(x)}{dx} + n(n+1)P_n(x) = 0, \tag{35}$$

and are found according to Rodrigues' formula [36]

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2 - 1)^n. \tag{36}$$

For the associated Legendre polynomials $P_n^m(x)$, the differential equation and Rodrigues' formula for normalized solutions of this equation of the following form are valid [17]:

$$(1-x^2)\frac{d^2 P_n^m(x)}{dx^2} - 2x\frac{dP_n^m(x)}{dx} + \left[n(n+1) - \frac{m^2}{1-x^2}\right]P_n^m(x) = 0. \tag{37}$$

$$P_n^m(x) = \frac{(-1)^m}{2^n n!}\left(1-x^2\right)^{m/2}\frac{d^{n+m}}{dx^{n+m}}\left(x^2-1\right)^n . \qquad (38)$$

A comparison of equations (32) and (37) shows that for $k = \frac{1}{2}$ and for $m = 1$ the equations have the same form, and the solutions $S_n(x)$ in (32), up to the sign and constant coefficients, are equal to the associated Legendre polynomials $P_n^1(x)$. The first eight such associated Legendre polynomials have the following form:

$$P_0^1(x) = 0, \quad P_1^1(x) = -\sqrt{1-x^2}, \quad P_2^1(x) = -3x\sqrt{1-x^2} .$$

$$P_3^1(x) = -\frac{3}{2}\sqrt{1-x^2}\left(5x^2-1\right), \qquad P_4^1(x) = -\frac{5}{2}x\sqrt{1-x^2}\left(7x^2-3\right).$$

$$P_5^1(x) = -\frac{15}{8}\sqrt{1-x^2}\left(21x^4-14x^2+1\right), \quad P_6^1(x) = -\frac{21}{8}x\sqrt{1-x^2}\left(33x^4-30x^2+5\right).$$

$$P_7^1(x) = -\frac{7}{16}\sqrt{1-x^2}\left(429x^6-495x^4+135x^2-5\right). \qquad (39)$$

To calculate the polynomials $P_n^1(x)$ in (39), one can use formulas (33) taking into account (34), or use (38) when $m = 1$.

In (31-32) a substitution in the form $M(x) = \left(1-x^2\right)^k S(x)$ was used. Replacing $S(x)$ with polynomials $P_n^1(x)$ at $k = \frac{1}{2}$, we obtain up to constant coefficients $M_n(x) = \sqrt{1-x^2}P_n^1(x)$, where $P_n^1(x)$ are represented in (39). Making the substitution $x = \cos\theta$, we arrive at $M_n(\theta) = \sin\theta P_n^1(\cos\theta)$. Since $M_n(\theta) = C_{3,n}V_n(\theta)$, it turns out that the polynomials $V_n(\theta)$ in (28) up to the sign and constant coefficients are expressed through the product of $\sin\theta$ and the associated Legendre polynomials $P_n^m(x)$ at order $m = 1$ and at $x = \cos\theta$. The only exception is the polynomial $V_0(\theta)=\cos\theta$ in (28), for which the relation $M_0(\theta)=C_{3,0}V_0=\sin\theta P_0^1(\cos\theta)$ is not satisfied.

The general solution for the component A_{or} can be represented as a sum of partial solutions (20): $A_{or}=\sum_{n=0}^{\infty}M_n(\theta)N_n(r)$. Substituting $N_n(r)$ (26) and $M_n(\theta)=C_{3,n}V_n(\theta)$ into this expression and taking into account $V_n(\theta)$ (28), we find A_{or}:

$$A_{or}=\sum_{n=0}^{\infty}C_{3,n}V_n(\theta)\left(C_{1,n}r^n+\frac{C_{2,n}}{r^{n+1}}\right), \tag{40}$$

where the constant coefficients $C_{1,n}$, $C_{2,n}$ and $C_{3,n}$ in order to distinguish them from each other are given by two indices.

3.3 Calculation of component $A_{o\theta}$

A specific solution for the component $A_{o\theta}$ in (18) can be expressed through the product of two functions:

$$\bar{A}_{o\theta}=G(r)H(\theta). \tag{41}$$

We substitute (41) into (18) and separate the variables:

$$\frac{r}{G(r)}\frac{\partial^2}{\partial r^2}\left[rG(r)\right]=-\frac{1}{H(\theta)\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial H(\theta)}{\partial\theta}\right)+\frac{1}{\sin^2\theta}-\frac{2}{H(\theta)}\frac{\partial}{\partial\theta}\left[\frac{H(\theta)\sin\theta}{\cos\theta}\right]=\psi\,. \tag{42}$$

As a consequence of (42), two equations are obtained for the functions $G(r)$, $H(\theta)$, and the parameter ψ:

$$r\frac{\partial^2 G(r)}{\partial r^2}+2\frac{\partial G(r)}{\partial r}-\frac{\psi G(r)}{r}=0\,. \tag{43}$$

$$\sin^2\theta\cos^2\theta\frac{\partial^2 H(\theta)}{\partial\theta^2}+\sin\theta\cos\theta\left(1+\sin^2\theta\right)\frac{\partial H(\theta)}{\partial\theta}+$$
$$+\left(2\sin^2\theta-\cos^2\theta+\psi\sin^2\theta\cos^2\theta\right)H(\theta)=0. \tag{44}$$

Regular solutions of equation (43) similarly to (26) are obtained at $\psi=n\left(n+1\right)$ in the form:

$$G_n(r)=C_{4,n}r^n+\frac{C_{5,n}}{r^{n+1}}, \tag{45}$$

where n is an integer, and $n\geq 0$; $C_{4,n}$ and $C_{5,n}$ are constant coefficients depending on n.

Substitution $\psi=n\left(n+1\right)$ into (44) leads to an equation in which the function $H(\theta)$ will depend on both the angle θ and the parameter n:

$$\sin^2\theta\cos^2\theta\frac{\partial^2 H(\theta)}{\partial\theta^2}+\sin\theta\cos\theta\left(1+\sin^2\theta\right)\frac{\partial H(\theta)}{\partial\theta}+$$
$$+\left[2\sin^2\theta-\cos^2\theta+n\left(n+1\right)\sin^2\theta\cos^2\theta\right]H(\theta)=0. \tag{46}$$

Solutions (46) depending on n can be represented as $H_n(\theta)=C_{6,n}W_n(\theta)$, where $W_n(\theta)$ are vector spherical polynomials for $A_{o\theta}$, $C_{6,n}$ are constant coefficients associated with the corresponding polynomials. $W_n(\theta)$. The first eight such polynomials have the following form:

$$W_0(\theta)=\frac{\cos^2\theta+C_{70}\cos\theta}{\sin\theta},\qquad W_1(\theta)=\sin\theta\cos\theta,\qquad W_2(\theta)=\sin\theta\cos^2\theta.$$

$$W_3(\theta)=\sin\theta\cos\theta\left(5\cos^2\theta-1\right),\qquad W_4(\theta)=\sin\theta\cos^2\theta\left(7\cos^2\theta-3\right).$$

$$W_5(\theta)=\sin\theta\cos\theta\left(21\cos^4\theta-14\cos^2\theta+1\right).$$

$$W_6(\theta)=\sin\theta\cos^2\theta\left(33\cos^4\theta-30\cos^2\theta+5\right).$$

$$W_7(\theta) = \sin\theta\cos\theta\left(429\cos^6\theta - 495\cos^4\theta + 135\cos^2\theta - 5\right). \tag{47}$$

Assuming that the function $H(\theta)$ is a function of $\cos\theta$, the derivatives in equation (46) and the equation itself can be expressed as follows:

$$\frac{\partial H(\theta)}{\partial\theta} = \frac{\partial H(\theta)}{\partial\cos\theta}\frac{\partial\cos\theta}{\partial\theta} = -\sin\theta\frac{\partial H(\theta)}{\partial\cos\theta}.$$

$$\frac{\partial^2 H(\theta)}{\partial\theta^2} = -\cos\theta\frac{\partial H(\theta)}{\partial\cos\theta} + \sin^2\theta\frac{\partial^2 H(\theta)}{\partial\left(\cos\theta\right)^2}.$$

$$\begin{aligned}&\sin^4\theta\cos^2\theta\frac{\partial^2 H(\theta)}{\partial\left(\cos\theta\right)^2} - 2\sin^2\theta\cos\theta\frac{\partial H(\theta)}{\partial\cos\theta} + \\ &+\left[2\sin^2\theta - \cos^2\theta + n\left(n+1\right)\sin^2\theta\cos^2\theta\right]H(\theta) = 0.\end{aligned} \tag{48}$$

Let's make a change of variables in (48) in the form $\cos\theta = x$ and move from the function $H(\theta)$ to the function $H(x)$:

$$x^2\left(1-x^2\right)^2\frac{\partial^2 H(x)}{\partial x^2} - 2x\left(1-x^2\right)\frac{\partial H(x)}{\partial x} + \left[2 - 3x^2 + n\left(n+1\right)x^2\left(1-x^2\right)\right]H(x) = 0. \tag{49}$$

Let us represent the function $H(x)$ as $H(x) = x^p T(x)$. This gives the following in (49):

$$\frac{\partial H(x)}{\partial x} = px^{p-1}T(x) + x^p\frac{\partial T(x)}{\partial x}.$$

$$\frac{\partial^2 H(x)}{\partial x^2} = x^p\frac{\partial^2 T(x)}{\partial x^2} + 2px^{p-1}\frac{\partial T(x)}{\partial x} + p\left(p-1\right)x^{p-2}T(x).$$

$$\left(1-x^2\right)\frac{\partial^2 T(x)}{\partial x^2}+\frac{2\left(p-px^2-1\right)}{x}\frac{\partial T(x)}{\partial x}+$$
$$+\left[n(n+1)+\frac{p(p-1)\left(1-x^2\right)^2-2p\left(1-x^2\right)+2-3x^2}{x^2\left(1-x^2\right)}\right]T(x)=0. \tag{50}$$

Comparison of the last equation (50) and equation (37) for $p=1$ and for $m=1$ yields the coincidence of the equations and the equality of the functions $T_n(x)=P_n^1(x)$. Consequently, $H_n(x)=xP_n^1(x)$, where $P_n^1(x)$ are presented in (39). After substitution $x=\cos\theta$, we have $H_n(\cos\theta)=\cos\theta P_n^1(\cos\theta)$. On the other hand, in (46) we had $H_n(\theta)=C_{6,n}W_n(\theta)$. Therefore, up to the sign and constant coefficients, we obtain an expression for the polynomials (47) in the form $W_n(\theta)=\cos\theta P_n^1(\cos\theta)$, that is, through the associated Legendre polynomials $P_n^m(x)$ of order $m=1$, in which $x=\cos\theta$. However, the first polynomial $W_0(\theta)=\frac{\cos^2\theta+C_{70}\cos\theta}{\sin\theta}$ presented in (47) is not determined in this way.

The general solution for the component $A_{o\theta}$ can be represented as the sum of specific solutions (41): $A_{o\theta}=\sum_{n=0}^{\infty}H_n(\theta)G_n(r)$. Substituting $G_n(r)$ (45) and $H_n(\theta)=C_{6,n}W_n(\theta)$ into this expression, taking into account $W_n(\theta)$ (47), we find $A_{o\theta}$:

$$A_{o\theta}=\sum_{n=0}^{\infty}C_{6,n}W_n(\theta)\left(C_{4,n}r^n+\frac{C_{5,n}}{r^{n+1}}\right), \tag{51}$$

where $C_{4,n}$, $C_{5,n}$ and $C_{6,n}$ are constant coefficients.

3.4 Calculation of component $A_{o\phi}$

To solve equation (19) for the vector potential component $A_{o\phi}$, we assume that a specific solution for the component $A_{o\phi}$ can be represented as $\bar{A}_{o\phi}=R(r)\Theta(\theta)$. This leads to a separation of variables in (19) and to two equations for $R(r)$ and $\Theta(\theta)$:

$$\frac{r}{R(r)}\frac{\partial^2}{\partial r^2}\left[rR(r)\right]=-\frac{1}{\Theta(\theta)\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\frac{\partial\Theta(\theta)}{\partial\theta}\right]+\frac{1}{\sin^2\theta}=n\left(n+1\right).$$

$$r\frac{\partial^2 R(r)}{\partial r^2}+2\frac{\partial R(r)}{\partial r}-\frac{n\left(n+1\right)R(r)}{r}=0. \tag{52}$$

$$\frac{\partial^2\Theta(\theta)}{\partial\theta^2}+\frac{\cos\theta}{\sin\theta}\frac{\partial\Theta(\theta)}{\partial\theta}+\left[n\left(n+1\right)-\frac{1}{\sin^2\theta}\right]\Theta(\theta)=0. \tag{53}$$

Regular solutions of equations (52-53) depend on n. The solution of (52) is the expression

$$R_n(r)=C_{7,n}r^n+\frac{C_{8,n}}{r^{n+1}}, \tag{54}$$

where n is an integer, and $n\geq 0$; $C_{7,n}$ and $C_{8,n}$ are constant coefficients depending on n.

Denoting a specific solution of equation (53) as $\Theta_n(\theta)=C_{9,n}Z_n(\theta)$, where $C_{9,n}$ are constant coefficients associated with the corresponding polynomials $Z_n(\theta)$, for vector spherical polynomials $Z_n(\theta)$ we obtain the following:

$$Z_0(\theta)=\frac{1}{\sin\theta},\qquad Z_1(\theta)=\sin\theta,\qquad Z_2(\theta)=\sin\theta\cos\theta.$$

$$Z_3(\theta)=\sin\theta\left(5\cos^2\theta-1\right),\qquad Z_4(\theta)=\sin\theta\cos\theta\left(7\cos^2\theta-3\right).$$

$$Z_5(\theta)=\sin\theta\left(21\cos^4\theta-14\cos^2\theta+1\right),\quad Z_6(\theta)=\sin\theta\cos\theta\left(33\cos^4\theta-30\cos^2\theta+5\right).$$

$$Z_7(\theta)=\sin\theta\left(429\cos^6\theta-495\cos^4\theta+135\cos^2\theta-5\right). \tag{55}$$

In (55) the first eight vector polynomials $Z_n(\theta)$ for computing $A_{o\phi}$ are presented.

Assuming that the function $\Theta(\theta)$ is a function of $\cos\theta$ in the form $\Theta(\theta)=\Theta(\cos\theta)$, we calculate the derivatives in equation (53) and transform the equation itself:

$$\frac{\partial\Theta(\theta)}{\partial\theta}=\frac{\partial\Theta(\theta)}{\partial\cos\theta}\frac{\partial\cos\theta}{\partial\theta}=-\sin\theta\frac{\partial\Theta(\theta)}{\partial\cos\theta}.$$

$$\frac{\partial^2\Theta(\theta)}{\partial\theta^2}=-\cos\theta\frac{\partial\Theta(\theta)}{\partial\cos\theta}+\sin^2\theta\frac{\partial^2\Theta(\theta)}{\partial\left(\cos\theta\right)^2}.$$

$$\sin^2\theta\frac{\partial^2\Theta(\theta)}{\partial\left(\cos\theta\right)^2}-2\cos\theta\frac{\partial\Theta(\theta)}{\partial\cos\theta}+\left[n\left(n+1\right)-\frac{1}{\sin^2\theta}\right]\Theta(\theta)=0. \quad (56)$$

Denoting $\cos\theta=x$, we make a change of variables in (56), passing from $\Theta(\theta)$ to $\Theta(x)$:

$$\left(1-x^2\right)\frac{\partial^2\Theta(x)}{\partial x^2}-2x\frac{\partial\Theta(x)}{\partial x}+\left[n\left(n+1\right)-\frac{1}{1-x^2}\right]\Theta(x)=0. \quad (57)$$

A comparison of equation (57) and equation (37) shows that $\Theta_n(x)=P_n^1\left(x\right)$, where the polynomials $P_n^1\left(x\right)$ are given in (39). Therefore, for $x=\cos\theta$ we have $\Theta_n(\theta)=P_n^1\left(\cos\theta\right)$. Since $\Theta_n(\theta)=C_{9,n}Z_n(\theta)$, the polynomials Z_n in (55), up to a sign and constant coefficients, are the associated Legendre polynomials $P_n^m\left(\cos\theta\right)$, in which $m=1$.

In (55) at $\cos\theta=x$ it is clear that $Z_0\left(x\right)=\frac{1}{\sqrt{1-x^2}}$. In this case, substituting the polynomial $Z_0\left(x\right)=\frac{1}{\sqrt{1-x^2}}$ into equation (37) instead of $P_n^m\left(x\right)$ at $m=1$ and at $n=0$ satisfies this equation. It turns out that $Z_0\left(x\right)=\frac{1}{\sqrt{1-x^2}}$ is not equal to $P_0^1\left(x\right)=0$ in (39), although for the remaining polynomials at $n\geq 1$ up to the sign and constant coefficient the equality holds $Z_n\left(x\right)=P_n^1\left(x\right)$.

Summing up all possible specific solutions $\overline{A}_{o\phi}=\Theta(\theta)R(r)$, taking into account $R_n(r)$ (54) and the relation $\Theta_n(\theta)=C_{9,n}Z_n(\theta)$, where $Z_n(\theta)$ are presented in (55), we find $A_{o\phi}$:

$$A_{o\phi} = \sum_{n=0}^{\infty} \Theta_n(\theta) R_n(r) = \sum_{n=0}^{\infty} C_{9,n} Z_n(\theta) \left(C_{7,n} r^n + \frac{C_{8,n}}{r^{n+1}} \right), \tag{58}$$

where $C_{7,n}$, $C_{8,n}$, $C_{9,n}$ are constant coefficients.

3.5 Expressions for external vector potential and external magnetic field

Considering that in (6) the components of the vector potential do not depend on the angle ϕ, the following is obtained for the components of the external magnetic field:

$$\mathbf{B}_o = B_{or}(r,\theta)\mathbf{e}_r + B_{o\theta}(r,\theta)\mathbf{e}_\theta + B_{o\phi}(r,\theta)\mathbf{e}_\phi = \\ = \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(A_{o\phi}\sin\theta\right)\mathbf{e}_r - \frac{1}{r}\frac{\partial}{\partial r}\left(rA_{o\phi}\right)\mathbf{e}_\theta + \frac{1}{r}\left[\frac{\partial}{\partial r}\left(rA_{o\theta}\right) - \frac{\partial A_{or}}{\partial\theta}\right]\mathbf{e}_\phi. \tag{59}$$

Let us substitute the polynomials $V_n(\theta)$ (28) into the expression A_{or} (40), using the first eight polynomials:

$$A_{or} = \sum_{n=0}^{\infty} C_{3,n} V_n(\theta)\left(C_{1,n} r^n + \frac{C_{2,n}}{r^{n+1}} \right) \approx C_{3,0}\cos\theta\left(C_{1,0} + \frac{C_{2,0}}{r} \right) + C_{3,1}\sin^2\theta\left(C_{1,1} r + \frac{C_{2,1}}{r^2} \right) + \\ + C_{3,2}\sin^2\theta\cos\theta\left(C_{1,2} r^2 + \frac{C_{2,2}}{r^3} \right) + C_{3,3}\sin^2\theta\left(5\cos^2\theta - 1\right)\left(C_{1,3} r^3 + \frac{C_{2,3}}{r^4} \right) + \\ + C_{3,4}\sin^2\theta\cos\theta\left(7\cos^2\theta - 3\right)\left(C_{1,4} r^4 + \frac{C_{2,4}}{r^5} \right) + \\ + C_{3,5}\sin^2\theta\left(21\cos^4\theta - 14\cos^2\theta + 1\right)\left(C_{1,5} r^5 + \frac{C_{2,5}}{r^6} \right) + \\ + C_{3,6}\sin^2\theta\cos\theta\left(33\cos^4\theta - 30\cos^2\theta + 5\right)\left(C_{1,6} r^6 + \frac{C_{2,6}}{r^7} \right) + \\ + C_{3,7}\sin^2\theta\left(429\cos^6\theta - 495\cos^4\theta + 135\cos^2\theta - 5\right)\left(C_{1,7} r^7 + \frac{C_{2,7}}{r^8} \right). \tag{60}$$

Similarly, we substitute polynomials $W_n(\theta)$ (47) into the expression $A_{o\theta}$ (51), and also polynomials $Z_n(\theta)$ (55) into the expression $A_{o\phi}$ (58):

$$A_{o\theta}=\sum_{n=0}^{\infty}C_{6,n}W_n(\theta)\left(C_{4,n}r^n+\frac{C_{5,n}}{r^{n+1}}\right)\approx C_{6,0}\frac{\cos^2\theta+C_{70}\cos\theta}{\sin\theta}\left(C_{4,0}+\frac{C_{5,0}}{r}\right)+$$
$$+C_{6,1}\sin\theta\cos\theta\left(C_{4,1}r+\frac{C_{5,1}}{r^2}\right)+C_{6,2}\sin\theta\cos^2\theta\left(C_{4,2}r^2+\frac{C_{5,2}}{r^3}\right)+$$
$$+C_{6,3}\sin\theta\cos\theta\left(5\cos^2\theta-1\right)\left(C_{4,3}r^3+\frac{C_{5,3}}{r^4}\right)+C_{6,4}\sin\theta\cos^2\theta\left(7\cos^2\theta-3\right)\left(C_{4,4}r^4+\frac{C_{5,4}}{r^5}\right)+$$
$$+C_{6,5}\sin\theta\cos\theta\left(21\cos^4\theta-14\cos^2\theta+1\right)\left(C_{4,5}r^5+\frac{C_{5,5}}{r^6}\right)+$$
$$+C_{6,6}\sin\theta\cos^2\theta\left(33\cos^4\theta-30\cos^2\theta+5\right)\left(C_{4,6}r^6+\frac{C_{5,6}}{r^7}\right)+$$
$$+C_{6,7}\sin\theta\cos\theta\left(429\cos^6\theta-495\cos^4\theta+135\cos^2\theta-5\right)\left(C_{4,7}r^7+\frac{C_{5,7}}{r^8}\right).$$

(61)

$$A_{o\phi}=\sum_{n=0}^{\infty}C_{9,n}Z_n(\theta)\left(C_{7,n}r^n+\frac{C_{8,n}}{r^{n+1}}\right)\approx C_{9,0}\frac{1}{\sin\theta}\left(C_{7,0}+\frac{C_{8,0}}{r}\right)+C_{9,1}\sin\theta\left(C_{7,1}r+\frac{C_{8,1}}{r^2}\right)+$$
$$+C_{9,2}\sin\theta\cos\theta\left(C_{7,2}r^2+\frac{C_{8,2}}{r^3}\right)+C_{9,3}\sin\theta\left(5\cos^2\theta-1\right)\left(C_{7,3}r^3+\frac{C_{8,3}}{r^4}\right)+$$
$$+C_{9,4}\sin\theta\cos\theta\left(7\cos^2\theta-3\right)\left(C_{7,4}r^4+\frac{C_{8,4}}{r^5}\right)+$$
$$+C_{9,5}\sin\theta\left(21\cos^4\theta-14\cos^2\theta+1\right)\left(C_{7,5}r^5+\frac{C_{8,5}}{r^6}\right)+$$
$$+C_{9,6}\sin\theta\cos\theta\left(33\cos^4\theta-30\cos^2\theta+5\right)\left(C_{7,6}r^6+\frac{C_{8,6}}{r^7}\right)+$$
$$+C_{9,7}\sin\theta\left(429\cos^6\theta-495\cos^4\theta+135\cos^2\theta-5\right)\left(C_{7,7}r^7+\frac{C_{8,7}}{r^8}\right).$$

(62)

From the relationship between Cartesian and spherical coordinates we obtain the relation $\cos\theta=\frac{z}{r}$. Since the charged ball rotates about the axis OZ, it follows from symmetry that when z replaced by $-z$ the components of the vector potential in (60-62) must remain unchanged. Only $\cos^2\theta = \frac{z^2}{r^2}$ and other even powers of the form $\cos^{2k}\theta = \frac{z^{2k}}{r^{2k}}$ satisfy this condition. Therefore, in (60-62) we should choose the corresponding coefficients equal to zero in those terms that contain $\cos\theta$.

All this gives the following:

$$\begin{aligned} A_{or} \approx\ & C_{3,1}\sin^2\theta\left(C_{1,1}r + \frac{C_{2,1}}{r^2}\right) + C_{3,3}\sin^2\theta\left(5\cos^2\theta - 1\right)\left(C_{1,3}r^3 + \frac{C_{2,3}}{r^4}\right) + \\ & + C_{3,5}\sin^2\theta\left(21\cos^4\theta - 14\cos^2\theta + 1\right)\left(C_{1,5}r^5 + \frac{C_{2,5}}{r^6}\right) + \\ & + C_{3,7}\sin^2\theta\left(429\cos^6\theta - 495\cos^4\theta + 135\cos^2\theta - 5\right)\left(C_{1,7}r^7 + \frac{C_{2,7}}{r^8}\right). \end{aligned} \tag{63}$$

$$\begin{aligned} A_{o\theta} \approx\ & C_{6,0}\frac{\cos^2\theta}{\sin\theta}\left(C_{4,0} + \frac{C_{5,0}}{r}\right) + C_{6,2}\sin\theta\cos^2\theta\left(C_{4,2}r^2 + \frac{C_{5,2}}{r^3}\right) + \\ & + C_{6,4}\sin\theta\cos^2\theta\left(7\cos^2\theta - 3\right)\left(C_{4,4}r^4 + \frac{C_{5,4}}{r^5}\right) + \\ & + C_{6,6}\sin\theta\cos^2\theta\left(33\cos^4\theta - 30\cos^2\theta + 5\right)\left(C_{4,6}r^6 + \frac{C_{5,6}}{r^7}\right). \end{aligned} \tag{64}$$

$$\begin{aligned} A_{o\phi} \approx\ & C_{9,0}\frac{1}{\sin\theta}\left(C_{7,0} + \frac{C_{8,0}}{r}\right) + C_{9,1}\sin\theta\left(C_{7,1}r + \frac{C_{8,1}}{r^2}\right) + \\ & + C_{9,3}\sin\theta\left(5\cos^2\theta - 1\right)\left(C_{7,3}r^3 + \frac{C_{8,3}}{r^4}\right) + \\ & + C_{9,5}\sin\theta\left(21\cos^4\theta - 14\cos^2\theta + 1\right)\left(C_{7,5}r^5 + \frac{C_{8,5}}{r^6}\right) + \\ & + C_{9,7}\sin\theta\left(429\cos^6\theta - 495\cos^4\theta + 135\cos^2\theta - 5\right)\left(C_{7,7}r^7 + \frac{C_{8,7}}{r^8}\right). \end{aligned} \tag{65}$$

We substitute (63-65) into (59) and after differentiation with respect to spherical coordinates we determine components of external magnetic field B_{or}, $B_{o\theta}$ and $B_{o\phi}$:

$$B_{or} \approx 2C_{9,1}\cos\theta\left(C_{7,1}+\frac{C_{8,1}}{r^3}\right)+4C_{9,3}\cos\theta\left(5\cos^2\theta-3\right)\left(C_{7,3}r^2+\frac{C_{8,3}}{r^5}\right)+$$
$$+2C_{9,5}\cos\theta\left(63\cos^4\theta-70\cos^2\theta+15\right)\left(C_{7,5}r^4+\frac{C_{8,5}}{r^7}\right)+ \tag{66}$$
$$+8C_{9,7}\cos\theta\left(429\cos^6\theta-693\cos^4\theta+315\cos^2\theta-35\right)\left(C_{7,7}r^6+\frac{C_{8,7}}{r^9}\right).$$

$$B_{o\theta} \approx -C_{7,0}C_{9,0}\frac{1}{r\sin\theta}-C_{91}\sin\theta\left(2C_{7,1}-\frac{C_{8,1}}{r^3}\right)-$$
$$-C_{9,3}\sin\theta\left(5\cos^2\theta-1\right)\left(4C_{7,3}r^2-\frac{3C_{8,3}}{r^5}\right)-$$
$$-C_{9,5}\sin\theta\left(21\cos^4\theta-14\cos^2\theta+1\right)\left(6C_{7,5}r^4-\frac{5C_{8,5}}{r^7}\right)- \tag{67}$$
$$-C_{9,7}\sin\theta\left(429\cos^6\theta-495\cos^4\theta+135\cos^2\theta-5\right)\left(8C_{7,7}r^6-\frac{7C_{8,7}}{r^9}\right).$$

$$B_{o\phi} \approx C_{4,0}C_{6,0}\frac{\cos^2\theta}{r\sin\theta}-2C_{3,1}\sin\theta\cos\theta\left(C_{1,1}+\frac{C_{2,1}}{r^3}\right)+C_{6,2}\sin\theta\cos^2\theta\left(3C_{4,2}r-\frac{2C_{5,2}}{r^4}\right)-$$
$$-4C_{3,3}\sin\theta\cos\theta\left(5\cos^2\theta-3\right)\left(C_{1,3}r^2+\frac{C_{2,3}}{r^5}\right)+$$
$$+C_{6,4}\sin\theta\cos^2\theta\left(7\cos^2\theta-3\right)\left(5C_{4,4}r^3-\frac{4C_{5,4}}{r^6}\right)-$$
$$-2C_{3,5}\sin\theta\cos\theta\left(63\cos^4\theta-70\cos^2\theta+15\right)\left(C_{1,5}r^4+\frac{C_{2,5}}{r^7}\right)+$$
$$+C_{6,6}\sin\theta\cos^2\theta\left(33\cos^4\theta-30\cos^2\theta+5\right)\left(7C_{4,6}r^5-\frac{6C_{5,6}}{r^8}\right)-$$
$$-8C_{3,7}\sin\theta\cos\theta\left(429\cos^6\theta-693\cos^4\theta+315\cos^2\theta-35\right)\left(C_{1,7}r^6+\frac{C_{2,7}}{r^9}\right).$$
(68)

The expressions for the components of the vector potential in (63-65) and for the components of the magnetic field in (66-68) can be simplified further. Indeed, the vector potential and the magnetic field outside the ball cannot be directly proportional to the radial coordinate r to avoid infinite values at large values of r. The corresponding coefficients in the terms proportional to r must be zero. In addition, all terms in the components of the vector potential and magnetic field must depend on r, otherwise even at infinity there will be nonzero terms depending only on the angle θ . Consequently, the coefficients $C_{1,1}$, $C_{4,0}$, $C_{7,0}$ and $C_{7,1}$ for the external field must be zero. Taking this into account, we have:

$$
\begin{aligned}
A_{or} \approx\ & C_{2,1}C_{3,1}\frac{1}{r^2}\sin^2\theta + C_{2,3}C_{3,3}\frac{1}{r^4}\sin^2\theta\left(5\cos^2\theta - 1\right) + \\
& + C_{2,5}C_{3,5}\frac{1}{r^6}\sin^2\theta\left(21\cos^4\theta - 14\cos^2\theta + 1\right) + \\
& + C_{2,7}C_{3,7}\frac{1}{r^8}\sin^2\theta\left(429\cos^6\theta - 495\cos^4\theta + 135\cos^2\theta - 5\right).
\end{aligned} \tag{69}
$$

$$
\begin{aligned}
A_{o\theta} \approx\ & C_{5,0}C_{6,0}\frac{\cos^2\theta}{r\sin\theta} + C_{5,2}C_{6,2}\frac{1}{r^3}\sin\theta\cos^2\theta + C_{5,4}C_{6,4}\frac{1}{r^5}\sin\theta\cos^2\theta\left(7\cos^2\theta - 3\right) + \\
& + C_{5,6}C_{6,6}\frac{1}{r^7}\sin\theta\cos^2\theta\left(33\cos^4\theta - 30\cos^2\theta + 5\right).
\end{aligned} \tag{70}
$$

$$
\begin{aligned}
A_{o\phi} \approx\ & C_{8,0}C_{9,0}\frac{1}{r\sin\theta} + C_{8,1}C_{9,1}\frac{1}{r^2}\sin\theta + C_{8,3}C_{9,3}\frac{1}{r^4}\sin\theta\left(5\cos^2\theta - 1\right) + \\
& + C_{8,5}C_{9,5}\frac{1}{r^6}\sin\theta\left(21\cos^4\theta - 14\cos^2\theta + 1\right) + \\
& + C_{8,7}C_{9,7}\frac{1}{r^8}\sin\theta\left(429\cos^6\theta - 495\cos^4\theta + 135\cos^2\theta - 5\right).
\end{aligned} \tag{71}
$$

$$
\begin{aligned}
B_{or} \approx\ & 2C_{8,1}C_{9,1}\frac{1}{r^3}\cos\theta + 4C_{8,3}C_{9,3}\frac{1}{r^5}\cos\theta\left(5\cos^2\theta - 3\right) + \\
& + 2C_{8,5}C_{9,5}\frac{1}{r^7}\cos\theta\left(63\cos^4\theta - 70\cos^2\theta + 15\right) + \\
& + 8C_{8,7}C_{9,7}\frac{1}{r^9}\cos\theta\left(429\cos^6\theta - 693\cos^4\theta + 315\cos^2\theta - 35\right).
\end{aligned} \tag{72}
$$

$$
\begin{aligned}
B_{o\theta} \approx\ & C_{8,1}C_{9,1}\frac{1}{r^3}\sin\theta + 3C_{8,3}C_{9,3}\frac{1}{r^5}\sin\theta\left(5\cos^2\theta - 1\right) + \\
& + 5C_{8,5}C_{9,5}\frac{1}{r^7}\sin\theta\left(21\cos^4\theta - 14\cos^2\theta + 1\right) + \\
& + 7C_{8,7}C_{9,7}\frac{1}{r^9}\sin\theta\left(429\cos^6\theta - 495\cos^4\theta + 135\cos^2\theta - 5\right).
\end{aligned} \tag{73}
$$

$$B_{o\phi} \approx -2C_{2,1}C_{3,1}\frac{1}{r^3}\sin\theta\cos\theta - 2C_{5,2}C_{6,2}\frac{1}{r^4}\sin\theta\cos^2\theta -$$
$$-4C_{2,3}C_{3,3}\frac{1}{r^5}\sin\theta\cos\theta\left(5\cos^2\theta-3\right) - 4C_{5,4}C_{6,4}\frac{1}{r^6}\sin\theta\cos^2\theta\left(7\cos^2\theta-3\right)-$$
$$-2C_{2,5}C_{3,5}\frac{1}{r^7}\sin\theta\cos\theta\left(63\cos^4\theta-70\cos^2\theta+15\right)-$$
$$-6C_{5,6}C_{6,6}\frac{1}{r^8}\sin\theta\cos^2\theta\left(33\cos^4\theta-30\cos^2\theta+5\right)-$$
$$-8C_{2,7}C_{3,7}\frac{1}{r^9}\sin\theta\cos\theta\left(429\cos^6\theta-693\cos^4\theta+315\cos^2\theta-35\right).$$

(74)

3.6 Components of vector potential inside ball

Let's take a closer look at how the expression $\mathbf{v}=\left(0,0,\omega r\sin\theta\right)$ for the velocity of a charged element of matter is derived in spherical coordinates. The velocity $\mathbf{v}$ arises due to the rotation of the ball around the axis OZ. The Cartesian coordinates of the rotating element of matter are determined in terms of spherical coordinates by the expressions:

$$x = r\sin\theta\cos\phi\,, \qquad y = r\sin\theta\sin\phi\,, \qquad z = r\cos\theta\,. \tag{75}$$

When the ball rotates, the radial coordinate r and angle θ of an arbitrary particle of the ball remain constant. If we denote the angular velocity of rotation as $\omega=\frac{d\phi}{dt}$, then the components of the velocity, taking into account (75), will be equal to:

$$v_x = \frac{dx}{dt} = -r\sin\theta\sin\phi\frac{d\phi}{dt} = -\omega y\,, \qquad v_y = \frac{dy}{dt} = r\sin\theta\cos\phi\frac{d\phi}{dt} = \omega x\,, \qquad v_z = \frac{dz}{dt} = 0\,.$$

(76)

Using (76), we express the velocity through Cartesian unit vectors:

$$\mathbf{v} = \left(v_x, v_y, v_z\right) = -\omega r\sin\theta\sin\phi\,\mathbf{e}_x + \omega r\sin\theta\cos\phi\,\mathbf{e}_y\,. \tag{77}$$

Similarly, in spherical coordinates, velocity can be represented in terms of spherical unit vectors:

$$\mathbf{v}=\left(v_r,v_\theta,v_\phi\right)=v_r\mathbf{e}_r+v_\theta\mathbf{e}_\theta+v_\phi\mathbf{e}_\phi\,. \tag{78}$$

Replacing the Cartesian unit vectors in (77) with unit vectors (9) and comparing with (78), we find the velocity components in spherical coordinates:

$$\mathbf{v}=\left(0,0,v_\phi\right)=\omega r\sin\theta\,\mathbf{e}_\phi. \tag{79}$$

According to (79) the components of the velocity v_r and v_θ are equal to zero, and the non-zero component v_ϕ of the velocity at each point is directed along the unit vector $\mathbf{e}_\phi$, that is, along the parallels of the corresponding spherical surface.

The vector potential inside the ball is written similarly to (2):

$$\mathbf{A}_i=\left(A_{ir},A_{i\theta},A_{i\phi}\right)=A_{ir}\left(r,\theta\right)\mathbf{e}_r+A_{i\theta}\left(r,\theta\right)\mathbf{e}_\theta+A_{i\phi}\left(r,\theta\right)\mathbf{e}_\phi\,. \tag{80}$$

Due to symmetry, in the case under consideration, the components of the vector potential do not depend on the angle ϕ.

From (4) follows the equation for the components of the vector potential inside the ball:

$$\begin{aligned}\Delta\mathbf{A}_i=&\left(\Delta A_{ir}-\frac{2A_{ir}}{r^2}-\frac{2}{r^2\sin\theta}\frac{\partial\left(A_{i\theta}\sin\theta\right)}{\partial\theta}\right)\mathbf{e}_r+\left(\Delta A_{i\theta}-\frac{A_{i\theta}}{r^2\sin^2\theta}+\frac{2}{r^2}\frac{\partial A_{ir}}{\partial\theta}\right)\mathbf{e}_\theta+\\&+\left(\Delta A_{i\phi}-\frac{A_{i\phi}}{r^2\sin^2\theta}\right)\mathbf{e}_\phi=-\frac{\rho_{0q}\omega r\sin\theta}{\varepsilon_0c^2\sqrt{1-\dfrac{\omega^2r^2\sin^2\theta}{c^2}}}\mathbf{e}_\phi.\end{aligned} \tag{81}$$

According to (1), the component of the vector potential in Cartesian coordinates directed along the axis OZ must be equal to zero. This leads to relation (13), which will also be valid for the components of the vector potential inside the ball in the following form:

$$A_{ir}\cos\theta-A_{i\theta}\sin\theta=0\,. \tag{82}$$

Taking into account (82), three equations for the components of the vector potential follow from (81):

$$\Delta A_{ir}-\frac{2A_{ir}}{r^2}-\frac{2}{r^2\sin\theta}\frac{\partial\left(A_{ir}\cos\theta\right)}{\partial\theta}=0. \tag{83}$$

$$\Delta A_{i\theta}-\frac{A_{i\theta}}{r^2\sin^2\theta}+\frac{2}{r^2}\frac{\partial}{\partial\theta}\left(\frac{A_{i\theta}\sin\theta}{\cos\theta}\right)=0. \tag{84}$$

$$\Delta A_{i\phi}-\frac{A_{i\phi}}{r^2\sin^2\theta}=-\frac{\rho_{0q}\,\omega r\sin\theta}{\varepsilon_0 c^2\sqrt{1-\frac{\omega^2 r^2\sin^2\theta}{c^2}}}. \tag{85}$$

Equations (83-84), as well as equation (85) with the zero right-hand side, have the same form as equations (14-16). Therefore, for solutions of equations (83-84), as well as equation (85) with the zero right-hand side, we can use previously found solutions of equations (14-16).

The general solutions of equations (14-16) are presented in (63-65). We replace the coefficients $C_{k,n}$ in (63-65) with coefficients $D_{k,n}$ and write down the corresponding solutions of equations (83-84), as well as equation (85) with the zero right-hand side:

$$\begin{aligned}A_{ir}\approx{}& D_{3,1}\sin^2\theta\left(D_{1,1}r+\frac{D_{2,1}}{r^2}\right)+D_{3,3}\sin^2\theta\left(5\cos^2\theta-1\right)\left(D_{1,3}r^3+\frac{D_{2,3}}{r^4}\right)+\\&+D_{3,5}\sin^2\theta\left(21\cos^4\theta-14\cos^2\theta+1\right)\left(D_{1,5}r^5+\frac{D_{2,5}}{r^6}\right)+\\&+D_{3,7}\sin^2\theta\left(429\cos^6\theta-495\cos^4\theta+135\cos^2\theta-5\right)\left(D_{1,7}r^7+\frac{D_{2,7}}{r^8}\right).\end{aligned} \tag{86}$$

$$\begin{aligned}A_{i\theta}\approx{}& D_{6,0}\frac{\cos^2\theta}{\sin\theta}\left(D_{4,0}+\frac{D_{5,0}}{r}\right)+D_{6,2}\sin\theta\cos^2\theta\left(D_{4,2}r^2+\frac{D_{5,2}}{r^3}\right)+\\&+D_{6,4}\sin\theta\cos^2\theta\left(7\cos^2\theta-3\right)\left(D_{4,4}r^4+\frac{D_{5,4}}{r^5}\right)+\\&+D_{6,6}\sin\theta\cos^2\theta\left(33\cos^4\theta-30\cos^2\theta+5\right)\left(D_{4,6}r^6+\frac{D_{5,6}}{r^7}\right).\end{aligned} \tag{87}$$

$$A_{i\phi 1} \approx D_{9,0}\frac{1}{\sin\theta}\left(D_{7,0}+\frac{D_{8,0}}{r}\right)+D_{9,1}\sin\theta\left(D_{7,1}r+\frac{D_{8,1}}{r^2}\right)+$$
$$+D_{9,3}\sin\theta\left(5\cos^2\theta-1\right)\left(D_{7,3}r^3+\frac{D_{8,3}}{r^4}\right)+$$
$$+D_{9,5}\sin\theta\left(21\cos^4\theta-14\cos^2\theta+1\right)\left(D_{7,5}r^5+\frac{D_{8,5}}{r^6}\right)+ \quad (88)$$
$$+D_{9,7}\sin\theta\left(429\cos^6\theta-495\cos^4\theta+135\cos^2\theta-5\right)\left(D_{7,7}r^7+\frac{D_{8,7}}{r^8}\right).$$

In (88) the quantity $A_{i\phi 1}$ is the general solution of equation (85) with the zero right-hand side. Now we need to find a specific solution $A_{i\phi 2}$ of the equation with the non-zero right-hand side in (85).

To determine $\Delta A_{i\phi}$ in (85) in spherical coordinates, we use expression (5), in which $A_{i\phi 2}$ instead of f should be substituted taking into account that $A_{i\phi}$ and $A_{i\phi 2}$ do not depend on the angle ϕ and therefore $\frac{\partial^2 A_{i\phi 2}}{\partial \phi^2}=0$. For the quantity $A_{i\phi 2}$ according to (5) and (85), the following equation follows:

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}\left(rA_{i\phi 2}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial A_{i\phi 2}}{\partial\theta}\right)-\frac{A_{i\phi 2}}{r^2\sin^2\theta}=-\frac{\rho_{0q}\,\omega r\sin\theta}{\varepsilon_0 c^2\sqrt{1-\frac{\omega^2 r^2\sin^2\theta}{c^2}}}. \quad (89)$$

To find a specific solution to the inhomogeneous equation (89), we will assume that $A_{i\phi 2}$ is a function of the new variable $\rho = r\sin\theta$, that is $A_{i\phi 2}=A_{i\phi 2}\left(\rho\right)$. In this case, it will be

$$\frac{\partial A_{i\phi 2}}{\partial\theta}=\frac{\partial A_{i\phi 2}}{\partial\rho}\frac{\partial\rho}{\partial\theta}=r\cos\theta\frac{\partial A_{i\phi 2}}{\partial\rho}.$$

$$\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial A_{i\phi 2}}{\partial\theta}\right)=\frac{\partial}{\partial\theta}\left(r\sin\theta\cos\theta\frac{\partial A_{i\phi 2}}{\partial\rho}\right)=$$
$$=r\cos^2\theta\frac{\partial A_{i\phi 2}}{\partial\rho}-r\sin^2\theta\frac{\partial A_{i\phi 2}}{\partial\rho}+r\sin\theta\cos\theta\frac{\partial\rho}{\partial\theta}\frac{\partial}{\partial\rho}\left(\frac{\partial A_{i\phi 2}}{\partial\rho}\right)=$$
$$=r\cos^2\theta\frac{\partial A_{i\phi 2}}{\partial\rho}-r\sin^2\theta\frac{\partial A_{i\phi 2}}{\partial\rho}+r^2\sin\theta\cos^2\theta\frac{\partial^2 A_{i\phi 2}}{\partial\rho^2}.$$

$$\frac{\partial}{\partial r}\left(rA_{i\phi 2}\right)=A_{i\phi 2}+r\frac{\partial A_{i\phi 2}}{\partial r}=A_{i\phi 2}+r\frac{\partial A_{i\phi 2}}{\partial\rho}\frac{\partial\rho}{\partial r}=A_{i\phi 2}+r\sin\theta\frac{\partial A_{i\phi 2}}{\partial\rho}.$$

$$\frac{\partial^2}{\partial r^2}\left(rA_{i\phi 2}\right)=\frac{\partial}{\partial r}\left[\frac{\partial}{\partial r}\left(rA_{i\phi 2}\right)\right]=\frac{\partial}{\partial r}\left(A_{i\phi 2}+r\sin\theta\frac{\partial A_{i\phi 2}}{\partial\rho}\right)=$$
$$=\frac{\partial A_{i\phi 2}}{\partial\rho}\frac{\partial\rho}{\partial r}+\sin\theta\frac{\partial A_{i\phi 2}}{\partial\rho}+r\sin\theta\frac{\partial\rho}{\partial r}\frac{\partial}{\partial\rho}\left(\frac{\partial A_{i\phi 2}}{\partial\rho}\right)=$$
$$=2\sin\theta\frac{\partial A_{i\phi 2}}{\partial\rho}+r\sin^2\theta\frac{\partial^2 A_{i\phi 2}}{\partial\rho^2}.$$

(90)

Taking into account (90), relation (89) is transformed into an equation for the function $A_{i\phi 2}(\rho)$:

$$\frac{\partial^2 A_{i\phi 2}}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial A_{i\phi 2}}{\partial\rho}-\frac{A_{i\phi 2}}{\rho^2}=-\frac{\rho_{0q}\,\omega\,\rho}{\varepsilon_0 c^2\sqrt{1-\frac{\omega^2\rho^2}{c^2}}}. \qquad (91)$$

To find the solution to equation (91), it is convenient to proceed as follows. We replace $A_{i\phi 2}$ with the function $g=g(\rho)$ and first consider the homogeneous equation that is part of (91). This equation must correspond to some function $\overline{g}=\overline{g}(\rho)$:

$$\frac{\partial^2\overline{g}}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial\overline{g}}{\partial\rho}-\frac{\overline{g}}{\rho^2}=0. \qquad (92)$$

Let us substitute a trial solution in the form $\overline{g} = \rho^n$ into (92), assuming that $\overline{g}$ depends only on ρ. We obtain two admissible values for the exponent n, so $\overline{g}_1 = \rho$ and $\overline{g}_2 = \frac{1}{\rho}$ turn out to be two independent specific solutions of (92). Moreover, $\overline{g} = C_3 \overline{g}_1 + C_4 \overline{g}_2$, where C_3 and C_4 are arbitrary constant coefficients, will be the general solution of (92).

Next, we substitute $g = g(\rho)$ in (91) instead of $A_{i\phi 2}$, and since g depends only on ρ, we replace the partial derivatives with ordinary derivatives:

$$g'' + \frac{1}{\rho} g' - \frac{g}{\rho^2} = -\frac{\rho_{0q} \omega \rho}{\varepsilon_0 c^2 \sqrt{1 - \frac{\omega^2 \rho^2}{c^2}}} . \tag{93}$$

To find a specific solution to equation (93), we use the variation of parameters method. To do this, we replace the constant coefficients C_3 and C_4 with the functions $C_3(\rho)$ and $C_4(\rho)$, to be determined, in the form

$$g = C_3(\rho)\overline{g}_1 + C_4(\rho)\overline{g}_2 = \rho C_3(\rho) + \frac{1}{\rho} C_4(\rho). \tag{94}$$

The derivatives of the function g are equal to

$$g' = C_3'(\rho)\overline{g}_1 + C_3(\rho)\overline{g}_1' + C_4'(\rho)\overline{g}_2 + C_4(\rho)\overline{g}_2' .$$

$$g'' = \frac{d}{d\rho}\left[C_3'(\rho)\overline{g}_1 + C_4'(\rho)\overline{g}_2\right] + C_3'(\rho)\overline{g}_1' + C_3(\rho)\overline{g}_1'' + C_4'(\rho)\overline{g}_2' + C_4(\rho)\overline{g}_2'' . \tag{95}$$

In (95), notations of the form $C_3'(\rho) = \frac{dC_3(\rho)}{d\rho}$, $\overline{g}_1' = \frac{d\overline{g}_1}{d\rho}$, $\overline{g}_1'' = \frac{d\overline{g}_1'}{d\rho} = \frac{d^2\overline{g}_1}{d\rho^2}$ are used.

Substituting (94-95) into (93) yields:

$$\frac{d}{d\rho}\left[C_3'(\rho)\overline{g}_1 + C_4'(\rho)\overline{g}_2\right] + \frac{1}{\rho}\left[C_3'(\rho)\overline{g}_1 + C_4'(\rho)\overline{g}_2\right] +$$
$$+ C_3(\rho)\left[\overline{g}_1'' + \frac{1}{\rho}\overline{g}_1' - \frac{\overline{g}_1}{\rho^2}\right] + C_4(\rho)\left[\overline{g}_2'' + \frac{1}{\rho}\overline{g}_2' - \frac{\overline{g}_2}{\rho^2}\right] + \tag{96}$$
$$+ C_3'(\rho)\overline{g}_1' + C_4'(\rho)\overline{g}_2' = -\frac{\rho_{0q}\,\omega\,\rho}{\varepsilon_0 c^2\sqrt{1-\frac{\omega^2\rho^2}{c^2}}}.$$

In (96) we assume that

$$\overline{g}_1'' + \frac{1}{\rho}\overline{g}_1' - \frac{\overline{g}_1}{\rho^2} = 0, \qquad \overline{g}_2'' + \frac{1}{\rho}\overline{g}_2' - \frac{\overline{g}_2}{\rho^2} = 0. \tag{97}$$

$$C_3'(\rho)\overline{g}_1 + C_4'(\rho)\overline{g}_2 = 0. \tag{98}$$

$$C_3'(\rho)\overline{g}_1' + C_4'(\rho)\overline{g}_2' = -\frac{\rho_{0q}\,\omega\,\rho}{\varepsilon_0 c^2\sqrt{1-\frac{\omega^2\rho^2}{c^2}}}. \tag{99}$$

Relations (97) are satisfied because $\overline{g}_1 = \rho$ and $\overline{g}_2 = \frac{1}{\rho}$ are two independent specific solutions of the homogeneous equation (92). Substituting $\overline{g}_1 = \rho$, $\overline{g}_1' = \frac{d\overline{g}_1}{d\rho} = 1$, $\overline{g}_2 = \frac{1}{\rho}$, $\overline{g}_2' = \frac{d\overline{g}_2}{d\rho} = -\frac{1}{\rho^2}$ into (98-99) yields:

$$\rho\, C_3'(\rho) + \frac{1}{\rho} C_4'(\rho) = 0. \tag{100}$$

$$C_3'(\rho) - \frac{1}{\rho^2} C_4'(\rho) = -\frac{\rho_{0q}\,\omega\,\rho}{\varepsilon_0 c^2\sqrt{1-\frac{\omega^2\rho^2}{c^2}}}. \tag{101}$$

Expressing $C_4'(\rho)$ from (100) and substituting into (101), we arrive at a first-order differential equation for determining $C_3(\rho)$ and solution of this equation:

$$C_3'(\rho)=\frac{dC_3(\rho)}{d\rho}=-\frac{\rho_{0q}\,\omega\,\rho}{2\varepsilon_0 c^2\sqrt{1-\frac{\omega^2\rho^2}{c^2}}}. \tag{102}$$

$$C_3(\rho)=\frac{\rho_{0q}}{2\varepsilon_0\,\omega}\sqrt{1-\frac{\omega^2\rho^2}{c^2}}+C_5. \tag{103}$$

From (100) and (102) it follows:

$$C_4'(\rho)=\frac{dC_4(\rho)}{d\rho}=\frac{\rho_{0q}\,\omega\,\rho^3}{2\varepsilon_0 c^2\sqrt{1-\frac{\omega^2\rho^2}{c^2}}}.$$

$$C_4(\rho)=-\frac{c^2\rho_{0q}\left(1+\frac{\omega^2\rho^2}{2c^2}\right)}{3\varepsilon_0\,\omega^3}\sqrt{1-\frac{\omega^2\rho^2}{c^2}}+C_6. \tag{104}$$

From (94), (103-104) we find a specific solution to equation (93):

$$g=\frac{\rho_{0q}\,\rho}{2\varepsilon_0\,\omega}\sqrt{1-\frac{\omega^2\rho^2}{c^2}}+C_5\,\rho-\frac{c^2\rho_{0q}\left(1+\frac{\omega^2\rho^2}{2c^2}\right)}{3\varepsilon_0\,\omega^3\rho}\sqrt{1-\frac{\omega^2\rho^2}{c^2}}+\frac{C_6}{\rho}. \tag{105}$$

Let's transform the bracket in the third term on the right-hand side into (105):

$$1+\frac{\omega^2\rho^2}{2c^2}=\frac{3\omega^2\rho^2}{2c^2}+1-\frac{\omega^2\rho^2}{c^2}.$$

This yields in (105) the following:

$$g = -\frac{c^2 \rho_{0q}\left(1-\frac{\omega^2 \rho^2}{c^2}\right)^{3/2}}{3\varepsilon_0 \omega^3 \rho} + C_5 \rho + \frac{C_6}{\rho}. \tag{106}$$

Taking into account the correspondence $A_{i\phi 2}$ and g in (91) and (93), and making the substitution $\rho = r\sin\theta$ in (106), for a specific solution of the inhomogeneous equation (89) we find:

$$A_{i\phi 2} = -\frac{c^2 \rho_{0q}\left(1-\frac{\omega^2 r^2 \sin^2\theta}{c^2}\right)^{3/2}}{3\varepsilon_0 \omega^3 r\sin\theta} + C_5 r\sin\theta + \frac{C_6}{r\sin\theta}. \tag{107}$$

The complete solution of equation (85) will then be equal to the sum $A_{i\phi} = A_{i\phi 1} + A_{i\phi 2}$. Taking into account (88) and (107), we find:

$$\begin{aligned} A_{i\phi} = A_{i\phi 1} + A_{i\phi 2} \approx D_{9,0}\frac{1}{\sin\theta}\left(D_{7,0} + \frac{D_{8,0}}{r}\right) - \frac{c^2 \rho_{0q}\left(1-\frac{\omega^2 r^2 \sin^2\theta}{c^2}\right)^{3/2}}{3\varepsilon_0 \omega^3 r\sin\theta} + \frac{C_6}{r\sin\theta} + \\ + D_{9,1}\sin\theta\left(D_{7,1} r + \frac{D_{8,1}}{r^2}\right) + C_5 r\sin\theta + D_{9,3}\sin\theta\left(5\cos^2\theta - 1\right)\left(D_{7,3} r^3 + \frac{D_{8,3}}{r^4}\right) + \\ + D_{9,5}\sin\theta\left(21\cos^4\theta - 14\cos^2\theta + 1\right)\left(D_{7,5} r^5 + \frac{D_{8,5}}{r^6}\right) + \\ + D_{9,7}\sin\theta\left(429\cos^6\theta - 495\cos^4\theta + 135\cos^2\theta - 5\right)\left(D_{7,7} r^7 + \frac{D_{8,7}}{r^8}\right). \end{aligned} \tag{108}$$

In the expressions for the components of the vector potential (86-87) and (108), the coefficients of those terms that contain r^2, r^3, r^4 etc. in the denominator should be equated to zero. This is necessary so that the components of the vector potential do not turn to infinity inside the body at $r = 0$. Taking this into account, we have:

$$
\begin{aligned}
A_{ir} &\approx D_{1,1}D_{3,1}\, r\sin^2\theta + D_{1,3}D_{3,3}\, r^3\sin^2\theta\left(5\cos^2\theta-1\right)+ \\
&+D_{1,5}D_{3,5}\, r^5\sin^2\theta\left(21\cos^4\theta-14\cos^2\theta+1\right)+ \\
&+D_{1,7}D_{3,7}\, r^7\sin^2\theta\left(429\cos^6\theta-495\cos^4\theta+135\cos^2\theta-5\right).
\end{aligned} \tag{109}
$$

$$
\begin{aligned}
A_{i\theta} &\approx D_{6,0}\frac{\cos^2\theta}{\sin\theta}\left(D_{4,0}+\frac{D_{5,0}}{r}\right)+D_{4,2}D_{6,2}\, r^2\sin\theta\cos^2\theta+ \\
&+D_{4,4}D_{6,4}\, r^4\sin\theta\cos^2\theta\left(7\cos^2\theta-3\right)+ \\
&+D_{4,6}D_{6,6}\, r^6\sin\theta\cos^2\theta\left(33\cos^4\theta-30\cos^2\theta+5\right).
\end{aligned} \tag{110}
$$

$$
\begin{aligned}
A_{i\phi} &\approx D_{9,0}\frac{1}{\sin\theta}\left(D_{7,0}+\frac{D_{8,0}}{r}\right)-\frac{c^2\rho_{0q}\left(1-\frac{\omega^2 r^2\sin^2\theta}{c^2}\right)^{3/2}}{3\varepsilon_0\omega^3 r\sin\theta}+\frac{C_6}{r\sin\theta}+ \\
&+D_{7,1}D_{9,1}\, r\sin\theta+C_5 r\sin\theta+D_{7,3}D_{9,3}\, r^3\sin\theta\left(5\cos^2\theta-1\right)+ \\
&+D_{7,5}D_{9,5}\, r^5\sin\theta\left(21\cos^4\theta-14\cos^2\theta+1\right)+ \\
&+D_{7,7}D_{9,7}\, r^7\sin\theta\left(429\cos^6\theta-495\cos^4\theta+135\cos^2\theta-5\right).
\end{aligned} \tag{111}
$$

In (110-111) the terms containing $D_{4,0}$ and $D_{7,0}$ were retained, which depend on the angle θ but do not depend on r. Similar terms $C_{4,0}$ and $C_{7,0}$ in (64-65) in the components of the vector potential outside the ball were equated to zero based on the fact that at infinity the vector potential in any case vanishes. But inside the ball, due to the limited size of its dimensions, the radial coordinate cannot be infinite. In this case, the terms containing $D_{4,0}$ and $D_{7,0}$ are admissible.

3.7 Components of internal magnetic field induction vector

Based on formula (6) for the magnetic field induction in spherical coordinates, for the components of the magnetic field inside a rotating ball, similar to (59), we have:

$$
\begin{aligned}
\mathbf{B}_i &= B_{ir}(r,\theta)\mathbf{e}_r + B_{i\theta}(r,\theta)\mathbf{e}_\theta + B_{i\phi}(r,\theta)\mathbf{e}_\phi = \\
&=\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(A_{i\phi}\sin\theta\right)\mathbf{e}_r-\frac{1}{r}\frac{\partial}{\partial r}\left(rA_{i\phi}\right)\mathbf{e}_\theta+\frac{1}{r}\left[\frac{\partial}{\partial r}\left(rA_{i\theta}\right)-\frac{\partial A_{ir}}{\partial\theta}\right]\mathbf{e}_\phi.
\end{aligned} \tag{112}
$$

Substituting the components of the vector potential inside the ball (109-111) into (112) we find the components of the internal magnetic field:

$$B_{ir} \approx \frac{\rho_{0q} \cos\theta \sqrt{1-\frac{\omega^2 r^2 \sin^2\theta}{c^2}}}{\varepsilon_0 \omega} + 2D_{7,1}D_{9,1}\cos\theta + 2C_5 \cos\theta +$$
$$+4D_{7,3}D_{9,3}\, r^2 \cos\theta\left(5\cos^2\theta - 3\right) + 2D_{7,5}D_{9,5}\, r^4 \cos\theta\left(63\cos^4\theta - 70\cos^2\theta + 15\right) +$$
$$+8D_{7,7}D_{9,7}\, r^6 \cos\theta\left(429\cos^6\theta - 693\cos^4\theta + 315\cos^2\theta - 35\right).$$

(113)

$$B_{i\theta} \approx -D_{7,0}D_{9,0}\frac{1}{r\sin\theta} - \frac{\rho_{0q} \sin\theta \sqrt{1-\frac{\omega^2 r^2 \sin^2\theta}{c^2}}}{\varepsilon_0 \omega} - 2D_{7,1}D_{9,1}\sin\theta - 2C_5 \sin\theta -$$
$$-4D_{7,3}D_{9,3}\, r^2 \sin\theta\left(5\cos^2\theta - 1\right) - 6D_{7,5}D_{9,5}\, r^4 \sin\theta\left(21\cos^4\theta - 14\cos^2\theta + 1\right) -$$
$$-8D_{7,7}D_{9,7}\, r^6 \sin\theta\left(429\cos^6\theta - 495\cos^4\theta + 135\cos^2\theta - 5\right).$$

(114)

$$B_{i\phi} \approx D_{4,0}D_{6,0}\frac{\cos^2\theta}{r\sin\theta} - 2D_{1,1}D_{3,1}\sin\theta\cos\theta + 3D_{4,2}D_{6,2}\, r\sin\theta\cos^2\theta -$$
$$-4D_{1,3}D_{3,3}\, r^2 \sin\theta\cos\theta\left(5\cos^2\theta - 3\right) + 5D_{4,4}D_{6,4}\, r^3 \sin\theta\cos^2\theta\left(7\cos^2\theta - 3\right) -$$
$$-2D_{1,5}D_{3,5}\, r^4 \sin\theta\cos\theta\left(63\cos^4\theta - 70\cos^2\theta + 15\right) +$$
$$+7D_{4,6}D_{6,6}\, r^5 \sin\theta\cos^2\theta\left(33\cos^4\theta - 30\cos^2\theta + 5\right) -$$
$$-8D_{1,7}D_{3,7}\, r^6 \sin\theta\cos\theta\left(429\cos^6\theta - 693\cos^4\theta + 315\cos^2\theta - 35\right).$$

(115)

3.8 Clarification of components of vector potential and magnetic field induction vector

We transform $A_{i\phi}$ in (111), using the expansion of the following function in the form

$$\left(1-\frac{\omega^2 r^2 \sin^2\theta}{c^2}\right)^{3/2} \approx 1 - \frac{3\omega^2 r^2 \sin^2\theta}{2c^2} + \frac{3\omega^4 r^4 \sin^4\theta}{8c^4} + \frac{\omega^6 r^6 \sin^6\theta}{16c^6} + \frac{3\omega^8 r^8 \sin^8\theta}{128c^8},$$

(116)

and then expressing $A_{i\phi}$ only through $\sin\theta$:

$$A_{i\phi} \approx D_{7,0} D_{9,0} \frac{1}{\sin\theta} + D_{8,0} D_{9,0} \frac{1}{r\sin\theta} - \frac{c^2 \rho_{0q}}{3\varepsilon_0 \omega^3 r\sin\theta} + \frac{C_6}{r\sin\theta} + \frac{\rho_{0q} r\sin\theta}{2\varepsilon_0 \omega} +$$
$$+D_{7,1} D_{9,1} r\sin\theta + C_5 r\sin\theta + 4D_{7,3} D_{9,3} r^3 \sin\theta + 8D_{7,5} D_{9,5} r^5 \sin\theta + 64 D_{7,7} D_{9,7} r^7 \sin\theta -$$
$$-\frac{\rho_{0q} \omega r^3 \sin^3\theta}{8c^2 \varepsilon_0} - 5D_{7,3} D_{9,3} r^3 \sin^3\theta - 28 D_{7,5} D_{9,5} r^5 \sin^3\theta - 432 D_{7,7} D_{9,7} r^7 \sin^3\theta -$$
$$-\frac{\rho_{0q} \omega^3 r^5 \sin^5\theta}{48 c^4 \varepsilon_0} + 21 D_{7,5} D_{9,5} r^5 \sin^5\theta + 792 D_{7,7} D_{9,7} r^7 \sin^5\theta -$$
$$-\frac{\rho_{0q} \omega^5 r^7 \sin^7\theta}{128 c^6 \varepsilon_0} - 429 D_{7,7} D_{9,7} r^7 \sin^7\theta. \tag{117}$$

Similarly, we express $A_{o\phi}$ in (71) only through $\sin\theta$:

$$A_{o\phi} \approx C_{8,0} C_{9,0} \frac{1}{r\sin\theta} + C_{8,1} C_{9,1} \frac{1}{r^2} \sin\theta + 4C_{8,3} C_{9,3} \frac{1}{r^4} \sin\theta + 8C_{8,5} C_{9,5} \frac{1}{r^6} \sin\theta +$$
$$+64 C_{8,7} C_{9,7} \frac{1}{r^8} \sin\theta - 5C_{8,3} C_{9,3} \frac{1}{r^4} \sin^3\theta - 28 C_{8,5} C_{9,5} \frac{1}{r^6} \sin^3\theta - 432 C_{8,7} C_{9,7} \frac{1}{r^8} \sin^3\theta +$$
$$+21 C_{8,5} C_{9,5} \frac{1}{r^6} \sin^5\theta + 792 C_{8,7} C_{9,7} \frac{1}{r^8} \sin^5\theta - 429 C_{8,7} C_{9,7} \frac{1}{r^8} \sin^7\theta. \tag{118}$$

On the surface of a rotating charged ball of radius a, the external and internal vector potentials must coincide and transform into each other. In this connection, at an arbitrarily chosen angle θ and at $r = a$, we equate the components of the external vector potential (69-70) to the components of the internal vector potential (109-110), and also equate (117) and (118). Considering terms with identical dependencies on $\sin\theta$ and $\cos\theta$, we obtain the following equalities:

$$C_{2,1} C_{3,1} = D_{1,1} D_{3,1} a^3, \qquad C_{2,3} C_{3,3} = D_{1,3} D_{3,3} a^7, \qquad C_{2,5} C_{3,5} = D_{1,5} D_{3,5} a^{11}, \qquad D_{4,0} = 0.$$

$$C_{2,7} C_{3,7} = D_{1,7} D_{3,7} a^{15}, \qquad C_{5,0} C_{6,0} = D_{5,0} D_{6,0}, \qquad C_{5,2} C_{6,2} = D_{4,2} D_{6,2} a^5, \qquad D_{7,0} = 0.$$

$$C_{5,4}C_{6,4} = D_{4,4}D_{6,4}\,a^9, \qquad C_{5,6}C_{6,6} = D_{4,6}D_{6,6}\,a^{13}, \qquad C_{8,0}C_{9,0} = D_{8,0}D_{9,0} - \frac{c^2\rho_{0q}}{3\varepsilon_0\omega^3} + C_6\,. \tag{119}$$

$$C_{8,1}C_{9,1}\frac{1}{a^2} + 4C_{8,3}C_{9,3}\frac{1}{a^4} + 8C_{8,5}C_{9,5}\frac{1}{a^6} + 64C_{8,7}C_{9,7}\frac{1}{a^8} =$$
$$= \frac{\rho_{0q}\,a}{2\varepsilon_0\,\omega} + D_{7,1}D_{9,1}\,a + C_5\,a + 4D_{7,3}D_{9,3}\,a^3 + 8D_{7,5}D_{9,5}\,a^5 + 64D_{7,7}D_{9,7}\,a^7. \tag{120}$$

$$5C_{8,3}C_{9,3}\frac{1}{a^4} + 28C_{8,5}C_{9,5}\frac{1}{a^6} + 432C_{8,7}C_{9,7}\frac{1}{a^8} =$$
$$= \frac{\rho_{0q}\,\omega\,a^3}{8c^2\varepsilon_0} + 5D_{7,3}D_{9,3}\,a^3 + 28D_{7,5}D_{9,5}\,a^5 + 432D_{7,7}D_{9,7}\,a^7. \tag{121}$$

$$21C_{8,5}C_{9,5}\frac{1}{a^6} + 792C_{8,7}C_{9,7}\frac{1}{a^8} = -\frac{\rho_{0q}\,\omega^3\,a^5}{48c^4\varepsilon_0} + 21D_{7,5}D_{9,5}\,a^5 + 792D_{7,7}D_{9,7}\,a^7\,. \tag{122}$$

$$429C_{8,7}C_{9,7} = \frac{\rho_{0q}\,\omega^5\,a^{15}}{128c^6\varepsilon_0} + 429D_{7,7}D_{9,7}\,a^{15}\,. \tag{123}$$

From (120-123) we find the following:

$$C_{8,7}C_{9,7} = \frac{\rho_{0q}\,\omega^5\,a^{15}}{54912c^6\varepsilon_0} + D_{7,7}D_{9,7}\,a^{15}, \qquad C_{8,5}C_{9,5} = D_{7,5}D_{9,5}\,a^{11} - \frac{\rho_{0q}\,\omega^3\,a^{11}}{1008c^4\varepsilon_0} - \frac{\rho_{0q}\,\omega^5\,a^{13}}{1456c^6\varepsilon_0}\,.$$

$$C_{8,3}C_{9,3} = D_{7,3}D_{9,3}\,a^7 + \frac{\rho_{0q}\,\omega\,a^7}{40c^2\varepsilon_0} + \frac{\rho_{0q}\,\omega^3\,a^9}{180c^4\varepsilon_0} + \frac{\rho_{0q}\,\omega^5\,a^{11}}{440c^6\varepsilon_0}\,.$$

$$C_{8,1}C_{9,1} = D_{7,1}D_{9,1}\,a^3 + C_5\,a^3 + \frac{\rho_{0q}\,a^3}{2\varepsilon_0\,\omega} - \frac{\rho_{0q}\,\omega\,a^5}{10c^2\varepsilon_0} - \frac{\rho_{0q}\,\omega^3\,a^7}{70c^4\varepsilon_0} - \frac{\rho_{0q}\,\omega^5\,a^9}{210c^6\varepsilon_0}\,. \tag{124}$$

Thus, the constant coefficients in the components of the external vector potential are expressed through the coefficients in the components of the internal vector potential.

Let us now compare the components of magnetic field on the surface of the ball at $r = a$. Analogously to (116), we expand $\sqrt{1-\frac{\omega^2 r^2 \sin^2\theta}{c^2}}$ in the form

$$\sqrt{1-\frac{\omega^2 r^2 \sin^2\theta}{c^2}} \approx 1-\frac{\omega^2 r^2 \sin^2\theta}{2c^2}-\frac{\omega^4 r^4 \sin^4\theta}{8c^4}-\frac{\omega^6 r^6 \sin^6\theta}{16c^6} \approx$$
$$\approx 1-\frac{\omega^2 r^2}{2c^2}-\frac{\omega^4 r^4}{8c^4}-\frac{\omega^6 r^6}{16c^6}+\frac{\omega^2 r^2 \cos^2\theta}{2c^2}+\frac{\omega^4 r^4 \cos^2\theta}{4c^4}+\frac{3\omega^6 r^6 \cos^2\theta}{16c^6}-$$
$$-\frac{\omega^4 r^4 \cos^4\theta}{8c^4}-\frac{3\omega^6 r^6 \cos^4\theta}{16c^6}+\frac{\omega^6 r^6 \cos^6\theta}{16c^6}.$$

(125)

and then we substitute (125) into (113) and into (114). In this case, we express B_{ir} in (113) only through $\cos\theta$, and $B_{i\theta}$ in (114) only through $\sin\theta$:

$$B_{ir} \approx 2D_{7,1}D_{9,1}\cos\theta+2C_5\cos\theta+\frac{\rho_{0q}\cos\theta}{\varepsilon_0\omega}\left(1-\frac{\omega^2 r^2}{2c^2}-\frac{\omega^4 r^4}{8c^4}-\frac{\omega^6 r^6}{16c^6}\right)-$$
$$-12D_{7,3}D_{9,3}\,r^2\cos\theta+30D_{7,5}D_{9,5}\,r^4\cos\theta-280D_{7,7}D_{9,7}\,r^6\cos\theta+$$
$$+20D_{7,3}D_{9,3}\,r^2\cos^3\theta+\frac{\rho_{0q}\omega^2 r^2\cos^3\theta}{2c^2\varepsilon_0\omega}\left(1+\frac{\omega^2 r^2}{2c^2}+\frac{3\omega^4 r^4}{8c^4}\right)-140D_{7,5}D_{9,5}\,r^4\cos^3\theta+$$
$$+2520D_{7,7}D_{9,7}\,r^6\cos^3\theta+126D_{7,5}D_{9,5}\,r^4\cos^5\theta-\frac{\rho_{0q}\omega^4 r^4\cos^5\theta}{8c^4\varepsilon_0\omega}\left(1+\frac{3\omega^2 r^2}{2c^2}\right)-$$
$$-5544D_{7,7}D_{9,7}\,r^6\cos^5\theta+3432D_{7,7}D_{9,7}\,r^6\cos^7\theta+\frac{\rho_{0q}\omega^5 r^6\cos^7\theta}{16c^6\varepsilon_0}.$$

(126)

$$B_{i\theta} \approx -D_{7,0}D_{9,0}\frac{1}{r\sin\theta} - 2D_{7,1}D_{9,1}\sin\theta - 2C_5\sin\theta - \frac{\rho_{0q}\sin\theta}{\varepsilon_0\omega} - 16D_{7,3}D_{9,3}\,r^2\sin\theta -$$
$$-48D_{7,5}D_{9,5}\,r^4\sin\theta - 512D_{7,7}D_{9,7}\,r^6\sin\theta + 20D_{7,3}D_{9,3}\,r^2\sin^3\theta +$$
$$+\frac{\rho_{0q}\omega r^2\sin^3\theta}{2c^2\varepsilon_0} + 168D_{7,5}D_{9,5}\,r^4\sin^3\theta + 3456D_{7,7}D_{9,7}\,r^6\sin^3\theta - 126D_{7,5}D_{9,5}\,r^4\sin^5\theta +$$
$$+\frac{\rho_{0q}\omega^3 r^4\sin^5\theta}{8c^4\varepsilon_0} - 6336D_{7,7}D_{9,7}\,r^6\sin^5\theta + 3432D_{7,7}D_{9,7}\,r^6\sin^7\theta + \frac{\rho_{0q}\omega^5 r^6\sin^7\theta}{16c^6\varepsilon_0}. \qquad (127)$$

Similarly, we express B_{or} in (72) through powers of $\cos\theta$, and also $B_{o\theta}$ in (73) through powers of $\sin\theta$:

$$B_{or} \approx 2C_{8,1}C_{9,1}\frac{1}{r^3}\cos\theta - 12C_{8,3}C_{9,3}\frac{1}{r^5}\cos\theta + 30C_{8,5}C_{9,5}\frac{1}{r^7}\cos\theta - 280C_{8,7}C_{9,7}\frac{1}{r^9}\cos\theta +$$
$$+20C_{8,3}C_{9,3}\frac{1}{r^5}\cos^3\theta - 140C_{8,5}C_{9,5}\frac{1}{r^7}\cos^3\theta + 2520C_{8,7}C_{9,7}\frac{1}{r^9}\cos^3\theta +$$
$$+126C_{8,5}C_{9,5}\frac{1}{r^7}\cos^5\theta - 5544C_{8,7}C_{9,7}\frac{1}{r^9}\cos^5\theta + 3432C_{8,7}C_{9,7}\frac{1}{r^9}\cos^7\theta. \qquad (128)$$

$$B_{o\theta} \approx C_{8,1}C_{9,1}\frac{1}{r^3}\sin\theta + 12C_{8,3}C_{9,3}\frac{1}{r^5}\sin\theta + 40C_{8,5}C_{9,5}\frac{1}{r^7}\sin\theta + 448C_{8,7}C_{9,7}\frac{1}{r^9}\sin\theta -$$
$$-15C_{8,3}C_{9,3}\frac{1}{r^5}\sin^3\theta - 140C_{8,5}C_{9,5}\frac{1}{r^7}\sin^3\theta - 3024C_{8,7}C_{9,7}\frac{1}{r^9}\sin^3\theta +$$
$$+105C_{8,5}C_{9,5}\frac{1}{r^7}\sin^5\theta + 5544C_{8,7}C_{9,7}\frac{1}{r^9}\sin^5\theta - 3003C_{8,7}C_{9,7}\frac{1}{r^9}\sin^7\theta. \qquad (129)$$

Comparison of (126) and (128), (127) and (129), (74) and (115) at $r=a$ gives the relationships between the coefficients:

$$C_{8,1}C_{9,1}\frac{1}{a^3} - 6C_{8,3}C_{9,3}\frac{1}{a^5} + 15C_{8,5}C_{9,5}\frac{1}{a^7} - 140C_{8,7}C_{9,7}\frac{1}{a^9} = D_{7,1}D_{9,1} + C_5 +$$
$$+\frac{\rho_{0q}}{2\varepsilon_0\omega}\left(1 - \frac{\omega^2 a^2}{2c^2} - \frac{\omega^4 a^4}{8c^4} - \frac{\omega^6 a^6}{16c^6}\right) - 6D_{7,3}D_{9,3}\,a^2 + 15D_{7,5}D_{9,5}\,a^4 - 140D_{7,7}D_{9,7}\,a^6.$$

$$C_{8,3}C_{9,3}\,\frac{1}{a^5}-7C_{8,5}C_{9,5}\,\frac{1}{a^7}+126C_{8,7}C_{9,7}\,\frac{1}{a^9}=$$
$$=D_{7,3}D_{9,3}\,a^2+\frac{\rho_{0q}\,\omega\,a^2}{40c^2\varepsilon_0}\left(1+\frac{\omega^2\,a^2}{2c^2}+\frac{3\omega^4\,a^4}{8c^4}\right)-7D_{7,5}D_{9,5}\,a^4+126D_{7,7}D_{9,7}\,a^6.$$

$$C_{8,5}C_{9,5}-44C_{8,7}C_{9,7}\,\frac{1}{a^2}=D_{7,5}D_{9,5}\,a^{11}-\frac{\rho_{0q}\,\omega^3\,a^{11}}{1008c^4\varepsilon_0}\left(1+\frac{3\omega^2\,a^2}{2c^2}\right)-44D_{7,7}D_{9,7}\,a^{13}.$$

$$C_{8,7}C_{9,7}=D_{7,7}D_{9,7}\,a^{15}+\frac{\rho_{0q}\,\omega^5\,a^{15}}{54912c^6\varepsilon_0}.$$

$$C_{8,1}C_{91}\,\frac{1}{a^3}+12C_{8,3}C_{9,3}\,\frac{1}{a^5}+40C_{8,5}C_{9,5}\,\frac{1}{a^7}+448C_{8,7}C_{9,7}\,\frac{1}{a^9}=$$
$$=-2D_{7,1}D_{9,1}-2C_5-\frac{\rho_{0q}}{\varepsilon_0\,\omega}-16D_{7,3}D_{9,3}\,a^2-48D_{7,5}D_{9,5}\,a^4-512D_{7,7}D_{9,7}\,a^6.$$

$$15C_{8,3}C_{9,3}\,\frac{1}{a^5}+140C_{8,5}C_{9,5}\,\frac{1}{a^7}+3024C_{8,7}C_{9,7}\,\frac{1}{a^9}=$$
$$=-20D_{7,3}D_{9,3}\,a^2-\frac{\rho_{0q}\,\omega\,a^2}{2c^2\varepsilon_0}-168D_{7,5}D_{9,5}\,a^4-3456D_{7,7}D_{9,7}\,a^6.$$

$$105C_{8,5}C_{9,5}\,\frac{1}{a^7}+5544C_{8,7}C_{9,7}\,\frac{1}{a^9}=-126D_{7,5}D_{9,5}\,a^4+\frac{\rho_{0q}\,\omega^3\,a^4}{8c^4\varepsilon_0}-6336D_{7,7}D_{9,7}\,a^6.$$

$$91C_{8,7}C_{9,7}=-104D_{7,7}D_{9,7}\,a^{15}-\frac{\rho_{0q}\,\omega^5\,a^{15}}{528c^6\varepsilon_0}. \qquad (130)$$

$$D_{4,0}=0, \qquad D_{7,0}=0, \qquad C_{2,1}C_{3,1}=D_{1,1}D_{3,1}a^3, \qquad C_{5,2}C_{6,2}=-\frac{3}{2}D_{4,2}D_{6,2}\,a^5.$$

$$C_{2,3}C_{3,3}=D_{1,3}D_{3,3}\,a^7, \qquad C_{5,4}C_{6,4}=-\frac{5}{4}D_{4,4}D_{6,4}\,a^9, \qquad C_{2,5}C_{3,5}=D_{1,5}D_{3,5}\,a^{11}.$$

$$C_{5,6}C_{6,6} = -\frac{7}{6}D_{4,6}D_{6,6}\,a^{13}, \qquad C_{2,7}C_{3,7} = D_{1,7}D_{3,7}a^{15}. \tag{131}$$

In (130) the coefficients can be separated from each other:

$$D_{7,1}D_{9,1} + C_5 = -\frac{\rho_{0q}}{2\varepsilon_0\,\omega}\left(1 - \frac{\omega^2 a^2}{3c^2} - \frac{\omega^4 a^4}{15c^4} - \frac{\omega^6 a^6}{35c^6}\right).$$

$$D_{7,3}D_{9,3} = -\frac{\rho_{0q}\,\omega}{40c^2\varepsilon_0}\left(1 + \frac{2\omega^2 a^2}{7c^2} + \frac{\omega^4 a^4}{7c^4}\right), \qquad D_{7,5}D_{9,5} = \frac{\rho_{0q}\,\omega^3}{1008c^4\varepsilon_0}\left(1 + \frac{9\omega^2 a^2}{11c^2}\right).$$

$$D_{7,7}D_{9,7} = -\frac{\rho_{0q}\,\omega^5}{54912c^6\varepsilon_0}. \tag{132}$$

$$C_{8,1}C_{9,1} = \frac{\rho_{0q}\,\omega a^5}{15c^2\varepsilon_0}\left(1 + \frac{2\omega^2 a^2}{7c^2} + \frac{\omega^4 a^4}{7c^4}\right), \qquad C_{8,3}C_{9,3} = -\frac{\rho_{0q}\,\omega^3 a^9}{630c^4\varepsilon_0}\left(1 + \frac{9\omega^2 a^2}{11c^2}\right).$$

$$C_{8,5}C_{9,5} = \frac{\rho_{0q}\,\omega^5 a^{13}}{8008c^6\varepsilon_0}, \qquad C_{8,7} = 0. \tag{133}$$

By substituting the coefficients (132-133) into the relations (124), we can verify that these relations are satisfied.

In order for the values of the coefficients in (119) to coincide with the values of the coefficients found in (131), it is necessary to assume that

$$D_{4,2} = 0, \quad C_{5,2} = 0, \quad D_{4,4} = 0, \quad C_{5,4} = 0, \quad D_{4,6} = 0, \quad C_{5,6} = 0. \tag{134}$$

Taking into account (119) and (132-134), we can clarify the vector potential components (109-111) and the magnetic field components (113-115) inside the rotating charged ball:

$$A_{ir} \approx D_{1,1}D_{3,1}\, r\sin^2\theta + D_{1,3}D_{3,3}\, r^3\sin^2\theta\left(5\cos^2\theta - 1\right) +$$
$$+ D_{1,5}D_{3,5}\, r^5\sin^2\theta\left(21\cos^4\theta - 14\cos^2\theta + 1\right) + \tag{135}$$
$$+ D_{1,7}D_{3,7}\, r^7\sin^2\theta\left(429\cos^6\theta - 495\cos^4\theta + 135\cos^2\theta - 5\right).$$

$$A_{i\theta} \approx D_{5,0}D_{6,0}\,\frac{\cos^2\theta}{r\sin\theta}. \tag{136}$$

$$A_{i\phi} \approx D_{8,0}D_{9,0}\,\frac{1}{r\sin\theta} - \frac{c^2\rho_{0q}\left(1 - \dfrac{\omega^2 r^2\sin^2\theta}{c^2}\right)^{3/2}}{3\varepsilon_0\omega^3 r\sin\theta} + \frac{C_6}{r\sin\theta} -$$
$$- \frac{\rho_{0q}\, r\sin\theta}{2\varepsilon_0\omega}\left(1 - \frac{\omega^2 a^2}{3c^2} - \frac{\omega^4 a^4}{15c^4} - \frac{\omega^6 a^6}{35c^6}\right) -$$
$$- \frac{\rho_{0q}\,\omega r^3\sin\theta\left(5\cos^2\theta - 1\right)}{40c^2\varepsilon_0}\left(1 + \frac{2\omega^2 a^2}{7c^2} + \frac{\omega^4 a^4}{7c^4}\right) + \tag{137}$$
$$+ \frac{\rho_{0q}\,\omega^3 r^5\sin\theta\left(21\cos^4\theta - 14\cos^2\theta + 1\right)}{1008c^4\varepsilon_0}\left(1 + \frac{9\omega^2 a^2}{11c^2}\right) -$$
$$- \frac{\rho_{0q}\,\omega^5 r^7\sin\theta\left(429\cos^6\theta - 495\cos^4\theta + 135\cos^2\theta - 5\right)}{54912c^6\varepsilon_0}.$$

$$B_{ir} \approx \frac{\rho_{0q}\cos\theta\sqrt{1 - \dfrac{\omega^2 r^2\sin^2\theta}{c^2}}}{\varepsilon_0\omega} - \frac{\rho_{0q}\cos\theta}{\varepsilon_0\omega}\left(1 - \frac{\omega^2 a^2}{3c^2} - \frac{\omega^4 a^4}{15c^4} - \frac{\omega^6 a^6}{35c^6}\right) -$$
$$- \frac{\rho_{0q}\,\omega r^2\cos\theta\left(5\cos^2\theta - 3\right)}{10c^2\varepsilon_0}\left(1 + \frac{2\omega^2 a^2}{7c^2} + \frac{\omega^4 a^4}{7c^4}\right) + \tag{138}$$
$$+ \frac{\rho_{0q}\,\omega^3 r^4\cos\theta\left(63\cos^4\theta - 70\cos^2\theta + 15\right)}{504c^4\varepsilon_0}\left(1 + \frac{9\omega^2 a^2}{11c^2}\right) -$$
$$- \frac{\rho_{0q}\,\omega^5 r^6\cos\theta\left(429\cos^6\theta - 693\cos^4\theta + 315\cos^2\theta - 35\right)}{6864c^6\varepsilon_0}.$$

$$B_{i\theta} \approx -\frac{\rho_{0q} \sin\theta \sqrt{1-\frac{\omega^2 r^2 \sin^2\theta}{c^2}}}{\varepsilon_0 \omega} + \frac{\rho_{0q} \sin\theta}{\varepsilon_0 \omega}\left(1-\frac{\omega^2 a^2}{3c^2}-\frac{\omega^4 a^4}{15c^4}-\frac{\omega^6 a^6}{35c^6}\right)+$$
$$+\frac{\rho_{0q}\omega r^2 \sin\theta\left(5\cos^2\theta-1\right)}{10c^2\varepsilon_0}\left(1+\frac{2\omega^2 a^2}{7c^2}+\frac{\omega^4 a^4}{7c^4}\right)-$$
$$-\frac{\rho_{0q}\omega^3 r^4 \sin\theta\left(21\cos^4\theta-14\cos^2\theta+1\right)}{168c^4\varepsilon_0}\left(1+\frac{9\omega^2 a^2}{11c^2}\right)+ \tag{139}$$
$$+\frac{\rho_{0q}\omega^5 r^6 \sin\theta\left(429\cos^6\theta-495\cos^4\theta+135\cos^2\theta-5\right)}{6864c^6\varepsilon_0}.$$

$$B_{i\phi} \approx -2D_{1,1}D_{3,1}\sin\theta\cos\theta-4D_{1,3}D_{3,3}\,r^2\sin\theta\cos\theta\left(5\cos^2\theta-3\right)-$$
$$-2D_{1,5}D_{3,5}\,r^4\sin\theta\cos\theta\left(63\cos^4\theta-70\cos^2\theta+15\right)- \tag{140}$$
$$-8D_{1,7}D_{3,7}\,r^6\sin\theta\cos\theta\left(429\cos^6\theta-693\cos^4\theta+315\cos^2\theta-35\right).$$

In the same way, taking into account (119) and (132-134), we can clarify the vector potential components (69-71) and the magnetic field components (72-74) outside the ball:

$$A_{or} \approx D_{1,1}D_{3,1}\frac{a^3}{r^2}\sin^2\theta+D_{1,3}D_{3,3}\frac{a^7}{r^4}\sin^2\theta\left(5\cos^2\theta-1\right)+$$
$$+D_{1,5}D_{3,5}\frac{a^{11}}{r^6}\sin^2\theta\left(21\cos^4\theta-14\cos^2\theta+1\right)+ \tag{141}$$
$$+D_{1,7}D_{3,7}\frac{a^{15}}{r^8}\sin^2\theta\left(429\cos^6\theta-495\cos^4\theta+135\cos^2\theta-5\right).$$

$$A_{o\theta} \approx D_{5,0}D_{6,0}\frac{\cos^2\theta}{r\sin\theta}. \tag{142}$$

$$A_{o\phi} \approx \frac{1}{r\sin\theta}\left(D_{8,0}D_{9,0}-\frac{c^2\rho_{0q}}{3\varepsilon_0\omega^3}+C_6\right)+\frac{\rho_{0q}\omega a^5}{15c^2\varepsilon_0}\frac{\sin\theta}{r^2}\left(1+\frac{2\omega^2 a^2}{7c^2}+\frac{\omega^4 a^4}{7c^4}\right)-$$
$$-\frac{\rho_{0q}\omega^3 a^9}{630c^4\varepsilon_0}\frac{\sin\theta\left(5\cos^2\theta-1\right)}{r^4}\left(1+\frac{9\omega^2 a^2}{11c^2}\right)+ \tag{143}$$
$$+\frac{\rho_{0q}\omega^5 a^{13}}{8008c^6\varepsilon_0}\frac{\sin\theta}{r^6}\left(21\cos^4\theta-14\cos^2\theta+1\right).$$

$$B_{or} \approx \frac{2\rho_{0q}\omega a^5}{15c^2\varepsilon_0}\frac{1}{r^3}\cos\theta\left(1+\frac{2\omega^2a^2}{7c^2}+\frac{\omega^4a^4}{7c^4}\right)-$$
$$-\frac{2\rho_{0q}\omega^3a^9}{315c^4\varepsilon_0}\frac{1}{r^5}\cos\theta\left(5\cos^2\theta-3\right)\left(1+\frac{9\omega^2a^2}{11c^2}\right)+ \quad (144)$$
$$+\frac{\rho_{0q}\omega^5a^{13}}{4004c^6\varepsilon_0}\frac{1}{r^7}\cos\theta\left(63\cos^4\theta-70\cos^2\theta+15\right).$$

$$B_{o\theta} \approx \frac{\rho_{0q}\omega a^5}{15c^2\varepsilon_0}\frac{1}{r^3}\sin\theta\left(1+\frac{2\omega^2a^2}{7c^2}+\frac{\omega^4a^4}{7c^4}\right)-$$
$$-\frac{\rho_{0q}\omega^3a^9}{210c^4\varepsilon_0}\frac{1}{r^5}\sin\theta\left(5\cos^2\theta-1\right)\left(1+\frac{9\omega^2a^2}{11c^2}\right)+ \quad (145)$$
$$+\frac{5\rho_{0q}\omega^5a^{13}}{8008c^6\varepsilon_0}\frac{1}{r^7}\sin\theta\left(21\cos^4\theta-14\cos^2\theta+1\right).$$

$$B_{o\phi} \approx -2D_{1,1}D_{3,1}\frac{a^3}{r^3}\sin\theta\cos\theta-4D_{1,3}D_{3,3}\frac{a^7}{r^5}\sin\theta\cos\theta\left(5\cos^2\theta-3\right)-$$
$$-2D_{1,5}D_{3,5}\frac{a^{11}}{r^7}\sin\theta\cos\theta\left(63\cos^4\theta-70\cos^2\theta+15\right)- \quad (146)$$
$$-8D_{1,7}D_{3,7}\frac{a^{15}}{r^9}\sin\theta\cos\theta\left(429\cos^6\theta-693\cos^4\theta+315\cos^2\theta-35\right).$$

Note that in (143-145) there are terms with round brackets containing quantities of the type $\frac{2\omega^2a^2}{7c^2}$, $\frac{\omega^4a^4}{7c^4}$ etc. These quantities appeared as a result of taking into account the Lorentz factor γ in the inhomogeneous Laplace equation (81) and applying the procedure of comparing the external and internal field components on the surface of the ball. As a result, the components of the external magnetic field in (144-145) acquire an additional and nonlinear dependence on the radius a of the ball and on the angular velocity ω of rotation. Quantities of the type $\frac{\omega^2a^2}{c^2}$ also appeared in [4] when calculating the field components using retarded potentials, but for a different reason. related to integration over the volume of the ball.

According to (136) and (142), the expressions for the component of the internal vector potential $A_{i\theta}$ and for the component of the external vector potential $A_{o\theta}$ are identical in form.

However, the component $A_{i\theta}$ in (136) at $r=0$ and at an angle $\theta=0$ tends to infinity, which should not be the case for a ball. Therefore, in (119), it is necessary to choose $D_{5,0}=0$, $C_{5,0}=0$. In this case, instead of (136) and (142), the following will be obtained:

$$A_{i\theta}=0, \qquad A_{o\theta}=0. \tag{147}$$

If we take into account (116) in (137), we get the following:

$$\begin{aligned}
A_{i\phi} \approx\; & D_{8,0}D_{9,0}\frac{1}{r\sin\theta}-\frac{c^2\rho_{0q}}{3\varepsilon_0\omega^3 r\sin\theta}+\frac{C_6}{r\sin\theta}+\frac{\rho_{0q}r\sin\theta}{2\varepsilon_0\omega}- \\
& -\frac{\rho_{0q}r\sin\theta}{2\varepsilon_0\omega}\left(1-\frac{\omega^2a^2}{3c^2}-\frac{\omega^4a^4}{15c^4}-\frac{\omega^6a^6}{35c^6}\right)-\frac{\rho_{0q}\omega r^3\sin^3\theta}{8c^2\varepsilon_0}- \\
& -\frac{\rho_{0q}\omega^3r^5\sin^5\theta}{48c^4\varepsilon_0}-\frac{\rho_{0q}\omega^5r^7\sin^7\theta}{128c^6\varepsilon_0}-\frac{\rho_{0q}\omega r^3\sin\theta\left(5\cos^2\theta-1\right)}{40c^2\varepsilon_0}\left(1+\frac{2\omega^2a^2}{7c^2}+\frac{\omega^4a^4}{7c^4}\right)+ \\
& +\frac{\rho_{0q}\omega^3r^5\sin\theta\left(21\cos^4\theta-14\cos^2\theta+1\right)}{1008c^4\varepsilon_0}\left(1+\frac{9\omega^2a^2}{11c^2}\right)- \\
& -\frac{\rho_{0q}\omega^5r^7\sin\theta\left(429\cos^6\theta-495\cos^4\theta+135\cos^2\theta-5\right)}{54912c^6\varepsilon_0}.
\end{aligned} \tag{148}$$

In (148) the component $A_{i\phi}$ at $r=0$ and at the angle $\theta=0$ turns to infinity, which is not permissible for a ball. Consequently, the coefficients must satisfy the condition

$$D_{8,0}D_{9,0}-\frac{c^2\rho_{0q}}{3\varepsilon_0\omega^3}+C_6=0. \tag{149}$$

Substituting (149) into (137) and into (143) gives the following:

$$A_{i\phi} \approx \frac{c^2 \rho_{0q}}{3\varepsilon_0 \omega^3 r \sin\theta} - \frac{c^2 \rho_{0q} \left(1 - \frac{\omega^2 r^2 \sin^2\theta}{c^2}\right)^{3/2}}{3\varepsilon_0 \omega^3 r \sin\theta} -$$
$$- \frac{\rho_{0q} r \sin\theta}{2\varepsilon_0 \omega} \left(1 - \frac{\omega^2 a^2}{3c^2} - \frac{\omega^4 a^4}{15c^4} - \frac{\omega^6 a^6}{35c^6}\right) -$$
$$- \frac{\rho_{0q} \omega r^3 \sin\theta \left(5\cos^2\theta - 1\right)}{40c^2 \varepsilon_0} \left(1 + \frac{2\omega^2 a^2}{7c^2} + \frac{\omega^4 a^4}{7c^4}\right) + \tag{150}$$
$$+ \frac{\rho_{0q} \omega^3 r^5 \sin\theta \left(21\cos^4\theta - 14\cos^2\theta + 1\right)}{1008c^4 \varepsilon_0} \left(1 + \frac{9\omega^2 a^2}{11c^2}\right) -$$
$$- \frac{\rho_{0q} \omega^5 r^7 \sin\theta \left(429\cos^6\theta - 495\cos^4\theta + 135\cos^2\theta - 5\right)}{54912c^6 \varepsilon_0}.$$

$$A_{o\phi} \approx \frac{\rho_{0q} \omega a^5}{15c^2 \varepsilon_0} \frac{\sin\theta}{r^2} \left(1 + \frac{2\omega^2 a^2}{7c^2} + \frac{\omega^4 a^4}{7c^4}\right) -$$
$$- \frac{\rho_{0q} \omega^3 a^9}{630c^4 \varepsilon_0} \frac{\sin\theta \left(5\cos^2\theta - 1\right)}{r^4} \left(1 + \frac{9\omega^2 a^2}{11c^2}\right) + \tag{151}$$
$$+ \frac{\rho_{0q} \omega^5 a^{13}}{8008c^6 \varepsilon_0} \frac{1}{r^6} \sin\theta \left(21\cos^4\theta - 14\cos^2\theta + 1\right).$$

Now note that all terms $B_{i\phi}$ in (140) contain $\cos\theta$ as a factor. It is known that $\cos(\pi - \theta) = -\cos\theta$. It turns out that if on the globe we lay off an angle θ from the north pole towards the equator and it turns out $B_{i\phi}$ to be positive, then if we lay off the same angle θ from the south pole towards the equator, getting an angle $\pi - \theta$, the value $B_{i\phi}$ will be negative.

A contradiction arises: when a charged ball rotates, currents arise from the moving charges of the ball, which are all directed in one direction, determined by the angular velocity of rotation. In this case, it should be expected that the component $B_{i\phi}$ directed along the parallels of the ball should also be symmetrical relative to the plane of the equator and have the same sign everywhere. But according to (140), the signs of the component $B_{i\phi}$ in the northern and southern hemispheres are opposite. From the point of view of magnetic lines, the following picture is obtained: in one of the hemispheres, the magnetic lines pass inside the ball, go outside and then go to the equator, twisting relative to the axis OZ of rotation of the ball. After the magnetic field lines pass the equator, they begin to twist in the opposite direction, returning to their original state inside the ball.

To avoid a contradiction, we must assume that $B_{i\phi}=0$. This requires that the coefficients $D_{1,1}$, $D_{1,3}$, $D_{1,5}$ and $D_{1,7}$. be equal to zero. This leads to changes in (135), (140-141), (146) and gives the following:

$$A_{ir}=0, \qquad B_{i\phi}=0, \qquad A_{or}=0, \qquad B_{o\phi}=0. \tag{152}$$

From (138-152) the following is obtained:

$$A_{ir}=0, \qquad A_{i\theta}=0.$$

$$\begin{aligned}A_{i\phi}\approx{}&\frac{c^2\rho_{0q}}{3\varepsilon_0\omega^3 r\sin\theta}-\frac{c^2\rho_{0q}\left(1-\frac{\omega^2 r^2\sin^2\theta}{c^2}\right)^{3/2}}{3\varepsilon_0\omega^3 r\sin\theta}-\frac{\rho_{0q}r\sin\theta}{2\varepsilon_0\omega}\left(1-\frac{\omega^2 a^2}{3c^2}-\frac{\omega^4 a^4}{15c^4}-\frac{\omega^6 a^6}{35c^6}\right)-\\&-\frac{\rho_{0q}\omega r^3\sin\theta\left(5\cos^2\theta-1\right)}{40c^2\varepsilon_0}\left(1+\frac{2\omega^2 a^2}{7c^2}+\frac{\omega^4 a^4}{7c^4}\right)+\\&+\frac{\rho_{0q}\omega^3 r^5\sin\theta\left(21\cos^4\theta-14\cos^2\theta+1\right)}{1008c^4\varepsilon_0}\left(1+\frac{9\omega^2 a^2}{11c^2}\right)-\\&-\frac{\rho_{0q}\omega^5 r^7\sin\theta\left(429\cos^6\theta-495\cos^4\theta+135\cos^2\theta-5\right)}{54912c^6\varepsilon_0}.\end{aligned} \tag{153}$$

$$\begin{aligned}B_{ir}\approx{}&\frac{\rho_{0q}\cos\theta\sqrt{1-\frac{\omega^2 r^2\sin^2\theta}{c^2}}}{\varepsilon_0\omega}-\frac{\rho_{0q}\cos\theta}{\varepsilon_0\omega}\left(1-\frac{\omega^2 a^2}{3c^2}-\frac{\omega^4 a^4}{15c^4}-\frac{\omega^6 a^6}{35c^6}\right)-\\&-\frac{\rho_{0q}\omega r^2\cos\theta\left(5\cos^2\theta-3\right)}{10c^2\varepsilon_0}\left(1+\frac{2\omega^2 a^2}{7c^2}+\frac{\omega^4 a^4}{7c^4}\right)+\\&+\frac{\rho_{0q}\omega^3 r^4\cos\theta\left(63\cos^4\theta-70\cos^2\theta+15\right)}{504c^4\varepsilon_0}\left(1+\frac{9\omega^2 a^2}{11c^2}\right)-\\&-\frac{\rho_{0q}\omega^5 r^6\cos\theta\left(429\cos^6\theta-693\cos^4\theta+315\cos^2\theta-35\right)}{6864c^6\varepsilon_0}.\end{aligned}$$

$$B_{i\theta} \approx -\frac{\rho_{0q}\sin\theta\sqrt{1-\frac{\omega^2 r^2\sin^2\theta}{c^2}}}{\varepsilon_0\omega}+\frac{\rho_{0q}\sin\theta}{\varepsilon_0\omega}\left(1-\frac{\omega^2 a^2}{3c^2}-\frac{\omega^4 a^4}{15c^4}-\frac{\omega^6 a^6}{35c^6}\right)+$$
$$+\frac{\rho_{0q}\omega r^2\sin\theta\left(5\cos^2\theta-1\right)}{10c^2\varepsilon_0}\left(1+\frac{2\omega^2 a^2}{7c^2}+\frac{\omega^4 a^4}{7c^4}\right)-$$
$$-\frac{\rho_{0q}\omega^3 r^4\sin\theta\left(21\cos^4\theta-14\cos^2\theta+1\right)}{168c^4\varepsilon_0}\left(1+\frac{9\omega^2 a^2}{11c^2}\right)+$$
$$+\frac{\rho_{0q}\omega^5 r^6\sin\theta\left(429\cos^6\theta-495\cos^4\theta+135\cos^2\theta-5\right)}{6864c^6\varepsilon_0}.$$

$$B_{i\phi}=0. \tag{154}$$

$$A_{or}=0, \qquad A_{o\theta}=0.$$

$$A_{o\phi} \approx \frac{\rho_{0q}\omega a^5}{15c^2\varepsilon_0}\frac{\sin\theta}{r^2}\left(1+\frac{2\omega^2 a^2}{7c^2}+\frac{\omega^4 a^4}{7c^4}\right)-$$
$$-\frac{\rho_{0q}\omega^3 a^9}{630c^4\varepsilon_0}\frac{\sin\theta\left(5\cos^2\theta-1\right)}{r^4}\left(1+\frac{9\omega^2 a^2}{11c^2}\right)+ \tag{155}$$
$$+\frac{\rho_{0q}\omega^5 a^{13}}{8008c^6\varepsilon_0}\frac{1}{r^6}\sin\theta\left(21\cos^4\theta-14\cos^2\theta+1\right).$$

$$B_{or} \approx \frac{2\rho_{0q}\omega a^5}{15c^2\varepsilon_0}\frac{1}{r^3}\cos\theta\left(1+\frac{2\omega^2 a^2}{7c^2}+\frac{\omega^4 a^4}{7c^4}\right)-$$
$$-\frac{2\rho_{0q}\omega^3 a^9}{315c^4\varepsilon_0}\frac{1}{r^5}\cos\theta\left(5\cos^2\theta-3\right)\left(1+\frac{9\omega^2 a^2}{11c^2}\right)+$$
$$+\frac{\rho_{0q}\omega^5 a^{13}}{4004c^6\varepsilon_0}\frac{1}{r^7}\cos\theta\left(63\cos^4\theta-70\cos^2\theta+15\right).$$

$$B_{o\theta} \approx \frac{\rho_{0q}\omega a^5}{15c^2\varepsilon_0}\frac{1}{r^3}\sin\theta\left(1+\frac{2\omega^2 a^2}{7c^2}+\frac{\omega^4 a^4}{7c^4}\right)-$$
$$-\frac{\rho_{0q}\omega^3 a^9}{210c^4\varepsilon_0}\frac{1}{r^5}\sin\theta\left(5\cos^2\theta-1\right)\left(1+\frac{9\omega^2 a^2}{11c^2}\right)+$$
$$+\frac{5\rho_{0q}\omega^5 a^{13}}{8008c^6\varepsilon_0}\frac{1}{r^7}\sin\theta\left(21\cos^4\theta-14\cos^2\theta+1\right).$$

$$B_{o\phi}=0. \qquad (156)$$

The solutions (153-154) for the vector potential and the magnetic field inside a rotating charged ball are smooth and do not contain discontinuities and infinities. These solutions contain functions such as $\sin\theta$, $\cos\theta$, and associated Legendre polynomials $P_n^1(x)$, which have continuous derivatives of any order. The same can be said about the solutions (155-156) for the vector potential and the magnetic field outside the rotating charged ball.

4. Discussion

Our results can be compared with the results in [37], where the magnetic field of axisymmetric current configurations was studied and cases where circular currents flowed in a certain layer inside a stationary body were considered. In this case, it was found, similarly to (153) and (155), that the main component of the vector potential is expressed in terms of the sum of terms in the form $A_\phi=\sum_n F_n(r)P_n^1(\cos\theta)$ where the functions $F_n(r)$ depend on the radial coordinate r and on the degree n of the associated Legendre polynomials $P_n^1(\cos\theta)$.

In [2], the following was found inside a rotating uniformly charged ball:

$$B_{ir}=\frac{\mu_0\rho_{0q}\omega\cos\theta}{3}\left(a^2-\frac{3r^2}{5}\right), \qquad B_{i\theta}=\frac{\mu_0\rho_{0q}\omega\sin\theta}{3}\left(\frac{6r^2}{5}-a^2\right). \qquad (157)$$

$$A_{i\phi}=\frac{\mu_0\rho_{0q}\omega\sin\theta}{6}\left(ra^2-\frac{3r^3}{5}\right). \qquad (158)$$

If we decompose the square root in (154) by the formula (125), then it becomes clear that the expressions for B_{ir} and $B_{i\theta}$ coincide with expressions (157) in the first approximation, given that in (157) $\mu_0 = \frac{1}{c^2 \varepsilon_0}$ is the vacuum permeability. Similarly, if we decompose the expression $\left(1 - \frac{\omega^2 r^2 \sin^2\theta}{c^2}\right)^{3/2}$ in (153) by the formula (116), then in (153) $A_{i\phi}$ will coincide with (158) in the first approximation. However, expressions (154) and (153) are much more accurate, since they contain additional terms containing c^6 in the denominator. In addition, in [2], the Lorentz factor $\gamma = \frac{1}{\sqrt{1 - \frac{\omega^2 r^2 \sin^2\theta}{c^2}}}$ of moving charges was not taken into account when calculating the field.

In order to verify the obtained solutions for the magnetic field induction $\mathbf{B}$ of a rotating ball, we substitute these solutions into Maxwell's equation with a source in the form of a current density $\mathbf{j}$:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j}. \tag{159}$$

In spherical coordinates, equation (159), taking into account the relations $\mu_0 = \frac{1}{\varepsilon_0 c^2}$, $\mathbf{j} = \gamma \rho_{0q} \mathbf{v}$, $\gamma = \frac{1}{\sqrt{1 - \frac{\omega^2 r^2 \sin^2\theta}{c^2}}}$, $\mathbf{v} = (v_r, v_\theta, v_\phi) = (0, 0, \omega r \sin\theta)$, can be written as follows:

$$\begin{gathered}\nabla \times \mathbf{B} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(B_\phi \sin\theta) - \frac{\partial B_\theta}{\partial\phi}\right]\mathbf{e}_r + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial B_r}{\partial\phi} - \frac{\partial}{\partial r}(rB_\phi)\right]\mathbf{e}_\theta + \\ + \frac{1}{r}\left[\frac{\partial}{\partial r}(rB_\theta) - \frac{\partial B_r}{\partial\theta}\right]\mathbf{e}_\phi = \frac{1}{\varepsilon_0 c^2}\gamma\rho_{0q}\mathbf{v} = \frac{\rho_{0q}\,\omega r\sin\theta}{\varepsilon_0 c^2\sqrt{1 - \frac{\omega^2 r^2 \sin^2\theta}{c^2}}}\mathbf{e}_\phi.\end{gathered} \tag{160}$$

Considering that the components of the magnetic field induction vector do not depend on the angle ϕ, three equations follow from (160):

$$\frac{\partial}{\partial\theta}(B_\phi \sin\theta) = 0, \qquad \frac{\partial}{\partial r}(rB_\phi) = 0.$$

$$\frac{1}{r}\left[\frac{\partial}{\partial r}\left(rB_{\theta}\right)-\frac{\partial B_r}{\partial \theta}\right]=\frac{\rho_{0q}\,\omega r\sin\theta}{\varepsilon_0 c^2\sqrt{1-\frac{\omega^2 r^2\sin^2\theta}{c^2}}}. \qquad (161)$$

There is no charged matter outside the ball, $\rho_{0q}=0$, and for the external magnetic field (161) is simplified:

$$\frac{\partial}{\partial r}\left(rB_{0\theta}\right)-\frac{\partial B_{0r}}{\partial \theta}=0. \qquad (162)$$

The corresponding components of the magnetic field induction inside the ball (154) and outside the ball (156) exactly satisfy equations (161-162).

One of Maxwell's equations for the magnetic field $\mathbf{B}$ is that the divergence of this field at any point in space is equal to zero. In spherical coordinates, it looks like this:

$$\nabla\cdot\mathbf{B}=\frac{1}{r^2}\frac{\partial\left(r^2B_r\right)}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial\left(B_{\theta}\sin\theta\right)}{\partial\theta}+\frac{1}{r\sin\theta}\frac{\partial B_{\phi}}{\partial\phi}=0. \qquad (163)$$

Equation (163) is satisfied both for the magnetic field induction components (154) inside the ball and for the magnetic field induction components (156) outside the ball.

In curved space-time, the equation for the electromagnetic field tensor $F_{\mu\nu}$ has the following form [32]:

$$\nabla^{\sigma}\nabla_{\sigma}F_{\mu\nu}=\mu_0\nabla_{\mu}j_{\nu}-\mu_0\nabla_{\nu}j_{\mu}+F_{\nu\rho}R^{\rho}{}_{\mu}-F_{\mu\rho}R^{\rho}{}_{\nu}+R_{\mu\nu\lambda\eta}F^{\eta\lambda}, \qquad (164)$$

where $j_{\mu}=\rho_{0q}u_{\mu}$ is the charge four-current with covariant index; ρ_{0q} is the invariant charge density; u_{μ} is the four-velocity with covariant index; $R^{\rho}{}_{\mu}$ is the Ricci tensor with mixed indices; $R_{\mu\nu\lambda\eta}$ is the Riemann curvature tensor.

When deriving equation (164), it was assumed that, according to [38], for the signature $(+,-,-,-)$ of the metric for the curvature tensor and the Ricci tensor, the following relations are satisfied:

$$g^{\mu\eta} R_{\alpha\eta\beta\gamma} = R_{\alpha}{}^{\mu}{}_{\beta\gamma} = -\frac{\partial\Gamma^{\mu}_{\alpha\gamma}}{\partial x^{\beta}} + \frac{\partial\Gamma^{\mu}_{\alpha\beta}}{\partial x^{\gamma}} - \Gamma^{\mu}_{\sigma\beta}\Gamma^{\sigma}_{\gamma\alpha} + \Gamma^{\mu}_{\sigma\gamma}\Gamma^{\sigma}_{\beta\alpha}, \qquad R_{\alpha\gamma} = g^{\mu\eta} R_{\alpha\mu\eta\gamma}. \tag{165}$$

If we use the relations according to [39] for the curvature tensor and the Ricci tensor in the form

$$g^{\mu\eta} R_{\eta\alpha\beta\gamma} = R^{\mu}{}_{\alpha\beta\gamma} = \frac{\partial\Gamma^{\mu}_{\alpha\gamma}}{\partial x^{\beta}} - \frac{\partial\Gamma^{\mu}_{\alpha\beta}}{\partial x^{\gamma}} + \Gamma^{\mu}_{\sigma\beta}\Gamma^{\sigma}_{\gamma\alpha} - \Gamma^{\mu}_{\sigma\gamma}\Gamma^{\sigma}_{\beta\alpha}, \qquad R_{\mu\gamma} = g^{\alpha\eta} R_{\alpha\mu\eta\gamma}, \tag{166}$$

then the equation for the tensor $F_{\mu\nu}$ is written as:

$$\nabla^{\sigma}\nabla_{\sigma} F_{\mu\nu} = \mu_0 \nabla_{\mu} j_{\nu} - \mu_0 \nabla_{\nu} j_{\mu} - F_{\nu\rho} R^{\rho}{}_{\mu} + F_{\mu\rho} R^{\rho}{}_{\nu} + R_{\mu\nu\lambda\eta} F^{\eta\lambda}. \tag{167}$$

The components of the tensor $F_{\mu\nu}$ in the Cartesian frame of reference are expressed in terms of the components of the electric $\mathbf{E}$ and magnetic $\mathbf{B}$ fields as follows:

$$F_{\mu\nu} = \begin{pmatrix} 0 & \frac{E_x}{c} & \frac{E_y}{c} & \frac{E_z}{c} \\ -\frac{E_x}{c} & 0 & -B_z & B_y \\ -\frac{E_y}{c} & B_z & 0 & -B_x \\ -\frac{E_z}{c} & -B_y & B_x & 0 \end{pmatrix}. \tag{168}$$

In special relativity, space-time is not curved, so the tensors $R^{\rho}{}_{\mu}$ and $R_{\mu\nu\lambda\eta}$ are zeroed. In this case, the covariant derivatives ∇_{μ} are transformed into partial derivatives $\partial_{\mu} = \frac{\partial}{\partial x^{\mu}}$ with respect to four-dimensional coordinates x^{μ}. The four-velocity with covariant index becomes equal to $u_{\mu} = (\gamma c, -\gamma \mathbf{v})$, where γ is the Lorentz factor, $\mathbf{v}$ is the velocity of a small element of charge. Accordingly, the charge four-current with covariant index will be equal to

$$j_{\mu} = \rho_{0q} u_{\mu} = \rho_{0q} (\gamma c, -\gamma \mathbf{v}) = (\gamma c \rho_{0q}, -\mathbf{j}), \tag{169}$$

where $\mathbf{j} = \gamma \rho_{0q} \mathbf{v}$ is the current density.

All this, taking into account (168-169), leads to a simplification of equations (164) and (167):

$$\partial^{\sigma} \partial_{\sigma} F_{\mu\nu} = \mu_0 \partial_{\mu} j_{\nu} - \mu_0 \partial_{\nu} j_{\mu}. \qquad (170)$$

In (170), the operator $\partial^{\sigma} \partial_{\sigma} = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \Delta$ is the d'Alembert operator, and the operator Δ is the Laplacian. With this in mind, it follows from (170):

$$\Delta F_{\mu\nu} - \frac{1}{c^2} \frac{\partial^2 F_{\mu\nu}}{\partial t^2} = \mu_0 \partial_{\nu} j_{\mu} - \mu_0 \partial_{\mu} j_{\nu}. \qquad (171)$$

Substituting (168) into (171), taking into account the relation $j_{\mu} = \rho_{0q} \left(\gamma c, -\gamma \mathbf{v} \right)$, gives two inhomogeneous wave equations, for electric field strength $\mathbf{E}$ and for magnetic field induction $\mathbf{B}$:

$$\Delta \mathbf{E} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \mathbf{E} = \frac{\rho_{0q}}{\varepsilon_0} \nabla \gamma + \mu_0 \rho_{0q} \frac{\partial \left(\gamma \mathbf{v} \right)}{\partial t}.$$

$$\Delta \mathbf{B} - \frac{1}{c^2} \frac{\partial^2 \mathbf{B}}{\partial t^2} = -\mu_0 \nabla \times \mathbf{j} = -\mu_0 \rho_{0q} \nabla \times \left(\gamma \mathbf{v} \right). \qquad (172)$$

In the case under consideration, the rotation of the charged body is constant, so that the magnetic field $\mathbf{B}$ and the Lorentz factor γ of the moving charges of the body are independent of time. However, inside the body, γ depends on the coordinates of the rotating charges, which coincide with the coordinates of the observation point at which the magnetic field is being searched. Therefore, the rotor operator also acts on the Lorentz factor γ. Then the inhomogeneous Laplace equation for the magnetic field follows from (172).:

$$\Delta \mathbf{B} = -\mu_0 \rho_{0q} \nabla \times \left(\gamma \mathbf{v} \right). \qquad (173)$$

Equation (173) is written in vector form and is therefore valid in any coordinate system.

In spherical coordinates, the Laplace vector operator $\Delta\mathbf{B}$ is expressed by formula (3), where instead of the spherical components of the vector $\mathbf{b}$, the spherical components of the magnetic field vector $\mathbf{B}$ should be substituted. To calculate the curl $\nabla\times(\gamma\mathbf{v})$, we use (6), where instead of $\mathbf{A}$ we substitute the product $\gamma\mathbf{v}$:

$$\nabla\times(\gamma\mathbf{v})=\frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\gamma v_\phi\sin\theta\right)-\frac{\partial\left(\gamma v_\theta\right)}{\partial\phi}\right]\mathbf{e}_r+\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial\left(\gamma v_r\right)}{\partial\phi}-\frac{\partial}{\partial r}\left(r\gamma v_\phi\right)\right]\mathbf{e}_\theta+$$
$$+\frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\gamma v_\theta\right)-\frac{\partial\left(\gamma v_r\right)}{\partial\theta}\right]\mathbf{e}_\phi. \tag{174}$$

Inside a rotating charged body, the velocity $\mathbf{v}$ has spherical components according to (79) in the form $\mathbf{v}=\left(0,0,v_\phi\right)=\omega r\sin\theta\,\mathbf{e}_\phi$, so that the components of velocity v_r and v_θ in (174) are zero. All this, taking into account (174), gives in (173) the following:

$$\Delta\mathbf{B}_i=\left(\Delta B_{ir}-\frac{2B_{ir}}{r^2}-\frac{2}{r^2\sin\theta}\frac{\partial\left(B_{i\theta}\sin\theta\right)}{\partial\theta}-\frac{2}{r^2\sin\theta}\frac{\partial B_{i\phi}}{\partial\phi}\right)\mathbf{e}_r+$$
$$+\left(\Delta B_{i\theta}-\frac{B_{i\theta}}{r^2\sin^2\theta}+\frac{2}{r^2}\frac{\partial B_{ir}}{\partial\theta}-\frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial B_{i\phi}}{\partial\phi}\right)\mathbf{e}_\theta+$$
$$+\left(\Delta B_{i\phi}-\frac{B_{i\phi}}{r^2\sin^2\theta}+\frac{2}{r^2\sin\theta}\frac{\partial B_{ir}}{\partial\phi}+\frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial B_{i\theta}}{\partial\phi}\right)\mathbf{e}_\phi=$$
$$=-\mu_0\rho_{0q}\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\gamma v_\phi\sin\theta\right)\mathbf{e}_r+\mu_0\rho_{0q}\frac{1}{r}\frac{\partial}{\partial r}\left(r\gamma v_\phi\right)\mathbf{e}_\theta. \tag{175}$$

Outside the body, where there is no current density $\mathbf{j}$ and the charge density ρ_{0q} is zero, according to (173), the following equation is obtained:

$$\Delta \mathbf{B}_o = \left(\Delta B_{or} - \frac{2B_{or}}{r^2} - \frac{2}{r^2 \sin\theta} \frac{\partial \left(B_{o\theta} \sin\theta \right)}{\partial \theta} - \frac{2}{r^2 \sin\theta} \frac{\partial B_{o\phi}}{\partial \phi} \right) \mathbf{e}_r +$$

$$+ \left(\Delta B_{o\theta} - \frac{B_{o\theta}}{r^2 \sin^2\theta} + \frac{2}{r^2} \frac{\partial B_{or}}{\partial \theta} - \frac{2\cos\theta}{r^2 \sin^2\theta} \frac{\partial B_{o\phi}}{\partial \phi} \right) \mathbf{e}_\theta + \qquad (176)$$

$$+ \left(\Delta B_{o\phi} - \frac{B_{o\phi}}{r^2 \sin^2\theta} + \frac{2}{r^2 \sin\theta} \frac{\partial B_{or}}{\partial \phi} + \frac{2\cos\theta}{r^2 \sin^2\theta} \frac{\partial B_{o\theta}}{\partial \phi} \right) \mathbf{e}_\phi = 0.$$

In (175-176) it can be taken into account that for a uniformly rotating axisymmetric charged body none of the components of the magnetic field depends on the angle ϕ. Substituting into (175) the expression for the velocity component $v_\phi = \omega r \sin\theta$ and the expression for the Lorentz factor $\gamma = \frac{1}{\sqrt{1 - \frac{\omega^2 r^2 \sin^2\theta}{c^2}}}$, we arrive at three equations for the components of internal magnetic field:

$$\Delta B_{ir} - \frac{2B_{ir}}{r^2} - \frac{2}{r^2 \sin\theta} \frac{\partial \left(B_{i\theta} \sin\theta \right)}{\partial \theta} = -\mu_0 \rho_{0q} \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta} \left(\frac{\omega r \sin^2\theta}{\sqrt{1 - \frac{\omega^2 r^2 \sin^2\theta}{c^2}}} \right).$$

$$\Delta B_{i\theta} - \frac{B_{i\theta}}{r^2 \sin^2\theta} + \frac{2}{r^2} \frac{\partial B_{ir}}{\partial \theta} = \mu_0 \rho_{0q} \frac{1}{r} \frac{\partial}{\partial r} \left(\frac{\omega r^2 \sin\theta}{\sqrt{1 - \frac{\omega^2 r^2 \sin^2\theta}{c^2}}} \right).$$

$$\Delta B_{i\phi} - \frac{B_{i\phi}}{r^2 \sin^2\theta} = 0. \qquad (177)$$

The right-hand side of the equations (177) contains the magnetic vacuum permeability $\mu_0 = \frac{1}{c^2 \varepsilon_0}$.

It is easy to verify that, taking into account the expression for the Laplacian (5) in spherical coordinates, the components of the external magnetic field (156) satisfy the vector equation (176), and the components of the internal magnetic field (154) satisfy the equations (177).

5. Conclusions

Using the harmonic method to find solutions to equations for magnetic field of a rotating uniformly charged ball rotating around its axis leads to solutions (153-156). These solutions yield formulas for the components of the vector potential and the magnetic induction vector, expressed in spherical coordinates.

In the course of the solution, the fact was used that the vector potential of an axisymmetric rotating body does not depend on the angle ϕ of rotation. The use of symmetry considerations when replacing z with $-z$ in solutions makes it possible to exclude some of the undefined constant coefficients and consider them equal to zero, which simplifies solutions. This is also due to the fact that at infinity the potentials and fields should be zeroed, and in the center of the rotating body they should not give infinity. Some of the coefficients can also be excluded based on the condition that the external and internal components of the vector potential and the magnetic field on the surface of the body must be equal to each other.

With this in mind, we calculated the components of the vector potential and the magnetic field inside and outside a uniformly charged ball, rotating with a constant angular velocity, with an accuracy of terms containing c^6 in the denominator. The calculation method allows us to explicitly find the components of fields with any given accuracy, it is only necessary to use a sufficient number of vector spherical polynomials $V_n(\theta)$ (28), $W_n(\theta)$ (47) and $Z_n(\theta)$ (55). In this case, the polynomials $V_n(\theta)$ are expressed through the product $\sin\theta$ by the associated Legendre polynomials $P_n^1(\cos\theta)$, the polynomials $W_n(\theta)$ are expressed through the product $\cos\theta$ by $P_n^1(\cos\theta)$, and the polynomials $Z_n(\theta)$ are equal to $P_n^1(\cos\theta)$ up to a constant coefficient.

The feature we found is that the polynomials $V_0(\theta)$, $W_0(\theta)$ and $Z_0(\theta)$ of degree 0 have values that are not expressed through the polynomial $P_0^1(\cos\theta)$. However, the polynomial $Z_0(\theta)$ for uniformly rotating axisymmetric bodies turns out to be necessary, since it is present in the component $A_{i\phi}$ of the vector potential inside the body, as indicated by the presence of $\sin\theta$ in the denominator of the first terms in (153). This suggests that when calculating the vector potential, it is insufficient to use the associated Legendre polynomials $P_n^1(\cos\theta)$ and their products with $\sin\theta$ and $\cos\theta$. Instead, it is more appropriate to take into account the spherical polynomials $V_n(\theta)$, $W_n(\theta)$ and $Z_n(\theta)$ as specific, independent vector polynomials when solving problems with vector functions in curved coordinates.

It should be noted the limitations that were used in the presented method. The main assumptions include the constancy of angular rotation of the ball around its axis and uniform distribution of electric charge over the entire volume of the ball. In order to simplify calculations, all possible solutions were limited to solutions in which the components A_r, A_θ, A_ϕ of the vector potential do not depend on the angle ϕ of the observation point and are symmetrical with respect to the axis of rotation and the center of the ball.

The calculations also assumed that the magnetic permeability of the substance of the ball itself and the magnetic permeability of the medium surrounding the ball are the same as in vacuum, that is $\mu = 1$. If necessary, the formulas obtained for the vector potential and the magnetic field can be easily adjusted to take into account the values of the magnetic permeability and magnetic susceptibility of the substance of the ball and the medium around it.

Declarations

Availability of data and materials

All data generated or analysed during this study are included in this published article.

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