Scattering of the spinless particles

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Abstract

The problem of scattering of the spinless by the spinless particles is considered under the condition that no other particles are created during the interaction. The matrix element of the scattering is defined and it is extracted from the general action of the current-current form. The differential cross section is then determined and the limiting formulas of it for the very high energy and very low energy is derived.

1 Introduction

The problem of scattering of the spin zero particles is considered under the condition that no other particles are created during the interaction. The matrix element of the scattering is defined and it is extracted from the general action of the current-current form. The differential cross section is then determined and the limiting formulas of it for the very high energy and very low energy is derived.

The causal situation which corresponds to the scattering of spin-less particles involves the act of emission the spin-less particles by the emission source and then their initial spatial separation where can be considered as non-interacting. Later, after sufficient time lapse the particles approach each other, interact, separate and are absorbed as the free particles by the detection source.

2 The source theory

Source theory, being the new quantum theory of fields (Schwinger, 1969; 1970; 1973; 1989; Dittrich, 1978), is the theoretical construction that uses quantum-mechanical particle language. Initially it was constructed for a description of the particle physics situations occurring in high-energy physics experiments. However, it was found that the original formulation simplifies the calculations in the electrodynamics and gravity where the interactions are mediated by the photon or graviton, respectively.

The basic formula in source theory $\langle 0_+|0_-\rangle = \exp[(i/\hbar)W]$ is of the physical meaning of the vacuum to vacuum amplitude (Schwinger, 1970; Schwinger et al., 1976), where the minus and plus signs on the vacuum symbol are causal labels, referring to any time before and after the space-time region where sources are manipulated. The exponential form with W being the action of fields, is introduced with regard to the existence of the physically independent experimental arrangements, which has a simple consequence that the associated probability amplitudes multiply and corresponding W expressions add (Schwinger, 1969; 1970; 1973; 1989).

3 Scattering of spin 0 particles

We shall treat the problem of scattering of the spin-less particles under condition that during interaction no other particles are created.

The question arises, from what action term can the information be obtained about such process. It may be easy to see that such process is involved in the action (Schwinger, 1970):

$$W = \frac{1}{2}\int (dx)(dx')j_c^\mu \Delta_+(x-x')j_{\mu c}(x'), \tag{1}$$

where $j_c^\mu(x)$ denotes the conserved current and the form of the action is the same as for the electromagnetic field with replacing J^μ by j_c^μ.

Since the causal situation is such that interaction occurs far from the emission and absorption sources, which is a part of the arrangement of the scattering experiment, the f^μ term in j_c^μ, causally tied to the source can be ignored and we have:

$$W_{40} = \frac{1}{2}\int (dx)(dx')j^\mu D_+(x-x')j_\mu(x'), \tag{2}$$

where we put for $j^\mu(x)$ the current for spin-less particles

$$j^\mu(x) = \partial^\mu\left(\varphi(x)\right) iq\varphi(x). \tag{3}$$

The total field $\varphi(x)$ between sources is the superposition of the in- and out-field, or

$$\varphi(x) = \varphi_1(x) + \varphi_2(x). \tag{4}$$

After inserting of eq. (4) into eq. (3) we get for the current density:

$$j^\mu(x) = i\partial^\mu\varphi_1(x)eq\varphi_1(x) + i\partial^\mu\varphi_2(x)eq\varphi_2(x) \quad +$$

$$i\partial^\mu \varphi_1(x) eq \varphi_2(x) - i\varphi_1(x) eq \partial^\mu \varphi_2(x). \tag{5}$$

Or,

$$j^\mu(x) = j^\mu_{11}(x) + j^\mu_{22}(x) + j^\mu_{12}(x). \tag{6}$$

Now, if we insert eq. (6) into eq. (2) we get, after retaining only contributions describing the considered scattering process:

$$\begin{aligned} W_{40} \quad \to \quad & \frac{1}{2}\int (dx)(dx') j^\mu_{12}(x) D_+(x-x') j^\mu_{12}(x) \quad + \\ & \int (dx)(dx') j^\mu_{11}(x) D_+(x-x') j^\mu_{22}(x). \end{aligned} \tag{7}$$

Putting

$$\varphi_1(x) = \sum_{pq} iK^*_{1pq} \varphi^*_{pq}(x), \tag{8}$$

$$\varphi_2(x) = \sum_{pq} \varphi_{pq}(x) iK_{2pq}, \tag{9}$$

where

$$\varphi_{pq} = (d\omega_p)^{1/2}\, \varphi_q e^{ipx} \tag{10}$$

is the field associated with the specific particle labeled by pq, which enters the interaction region after its creation by source K_{2pq}. Similarly φ^*_{pq} corresponds to K^*_{1pq}.

The two complex charge eigenvectors are

$$\varphi^*_+ = \frac{1}{\sqrt{2}}(1, -i) \tag{11}$$

$$\varphi^*_- = \frac{1}{\sqrt{2}}(1, i) \tag{12}$$

and they fulfill relations

$$\varphi^*_q \varphi_{q'} = \delta_{qq'} \tag{13}$$

$$\sum_q \varphi^*_q \varphi_q = 1 \tag{14}$$

$$\varphi^*_{q'} q = \varphi^*_{q'} q'; \quad q\varphi_{q'} = q'\varphi_{q'}; \quad \varphi^*_q = \varphi_{-q}, \tag{15}$$

where q is the obligate charge matrix

$$q = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}. \tag{16}$$

Using the above appropriate relations, we get for the current $j_{12}^{\mu}(x)$ the following formula

$$j_{12}^{\mu}(x) = \sum_{p_1 p_2 q} iK^*_{1p_1q} eq \left(p_1^{\mu} + p_2^{\mu}\right) \quad \times$$

$$\left(d\omega_{p_1} d\omega_{p_2}\right)^{1/2} e^{i(p_2-p_1)x} iK_{2p_2q}. \tag{17}$$

When only contributions from the incident particles of the same charge are considered, then,

$$W_{40} = \frac{1}{2}e^2 \sum_{p_1 p_2 g} iK^*_{1p_1q} iK^*_{1p_1'q} \left(d\omega_{p_1} d\omega_{p_1'} d\omega_{p_2} d\omega_{p_2'}\right)^{1/2} (p_1 + p_2)(p_1' + p_2') \quad \times$$

$$\left[\int (dx)(dx') e^{i(p_2-p_1)x} D_+(x - x') e^{i(p_2'-p_1')x'}\right] iK_{2p_2q} iK_{2p_2'q} \tag{18}$$

and the mathematical object in [...] can be evaluated as

$$[...] = \frac{1}{(p_1 - p_2)^2}(2\pi)^4 \delta(p_2 + p_2' - p_1 - p_1'), \tag{19}$$

which produces the momentum conservation by means of δ-function.

Now, we can pick out the T-matrix element from eq. (18). It corresponds to the term between $iK^*_{1p_1q} iK^*_{1p_1'q}$ and $iK_{2p_2q} iK_{2p_2'q}$. But this matrix element must be extended in order to be in agreement with Bose-Einstein statistics, or in other words, it must be symetrized in transformation of the incident particles

$$p_1 \quad \longleftrightarrow \quad p_1', \tag{20}$$

$$p_2 \quad \longleftrightarrow \quad p_2'. \tag{21}$$

The resulting matrix element is after symmetrization as follows:

$$\langle 1_{p_1q} 1_{p_1'q} | T | 1_{p_2q} 1_{p_2'q} \rangle \quad = \quad \left(d\omega_{p_1} d\omega_{p_1'} d\omega_{p_2} d\omega_{p_2'}\right)^{1/2} \quad \times$$

$$e^2 \left[\frac{(p_1 + p_2)(p_1' + p_2')}{(p_1 - p_2)^2} + \frac{(p_1 + p_2')(p_1' + p_2)}{(p_1 - p_2')^2}\right], \tag{22}$$

which is explicitly symmetrical in p_2, p_2' and also has the required p_1, p_1' symmetry since from the conservation law follows

$$p_1' - p_2 = -(p_1 - p_2'); \quad p_1' - p_2' = -(p_1 - p_2). \tag{23}$$

Knowledge of the T-matrix enables to calculate the cross section

$$d\sigma = \frac{d\omega_{p_1} d\omega_{p_1'} (2\pi)^4 \delta(p_1 + p_1' - p_2 - p_2') |\langle 1_{p_1 q} 1_{p_1' q} | T | 1_{p_2 q} 1_{p_2' q} \rangle|^2}{F}, \tag{24}$$

where F is the flux defined as an invariant measure of the relative flux of the incident particles, or,

$$F = d\omega_{p_a} d\omega_{p_b} 2 \left[M^2 - (m_a + m_b)^2 \right]^{1/2} \left[M^2 - (m_a - m_b)^2 \right]^{1/2} \tag{25}$$

for particles of masses m_a and m_b. In case that $m_a = m_b = m$, F is equal to

$$F = 2M(M^2 - 4m^2)^{1/2} d\omega_{p_2} d\omega_{p_2'}. \tag{26}$$

After inserting of eqs. (22) into (24) we get with regard to (26) the following differential cross section

$$d\sigma = \int d\omega_{p_1} d\omega_{p_1'} (2\pi)^4 \delta(p_1 + p_1' - p_2 - p_2' \frac{(4\pi\alpha)^2}{2M(M^2 - 4m^2)^{1/2}} \quad \times$$

$$\left[\frac{(p_1 + p_2)(p_1' + p_2')}{(p_1 - p_2)^2} + \frac{(p_1 + p_2')(p_1' + p_2)}{(p_1 - p_2')^2} \right]^{1/2}, \tag{27}$$

or, after some modification

$$\frac{d\sigma}{d\Omega} = \frac{1}{4} \frac{\alpha^2}{M^2} \left[\frac{(p_1 + p_2)(p_1' + p_2')}{(p_1 - p_2)^2} + \frac{(p_1 + p_2')(p_1' + p_2)}{(p_1 - p_2')^2} \right]^2 \quad =$$

$$\frac{\alpha^2}{M^2} \left[\frac{M^2 - 2m^2}{M^2 - 4m^2} \left(\frac{1}{\sin^2(\Theta/2)} + \frac{1}{\cos^2(\Theta/2)}, \right) - 1 \right]^2, \tag{28}$$

where $d\Omega$ is the element of the space angle, Θ is the deflection angle between outgoing particles and the full equivalence of the angles Θ and $\pi - \Theta$ in the last formula expresses the validity of the indistinguishably of the Bose-Einstein particles.

In derivation of the last formula we have used the reduction to the rest frame where particle energies equal to M/2, or,

$$-\frac{(p_1 + p_2)(p_1' + p_2')}{(p_1 - p_2)^2} = \frac{2M^2 - 4m^2}{(p_1 - p_2)^2} - 1 = 2\frac{M^2 - 2m^2}{M^2 - 4m^2} \cdot \frac{1}{\sin^2(\Theta/2)} - 1. \tag{29}$$

4 The limiting cases.

We consider here two limiting cases: the very high and the very low energy limits.
$\mathbf{M \gg 2m}$:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{M^2} \left(\frac{1}{\sin^2(\Theta/2)} + \frac{1}{\cos^2(\Theta/2)} - 1 \right)^2. \tag{30}$$

$\mathbf{M \approx 2m}$:

$$\frac{d\sigma}{d\Omega} = \frac{1}{16}\frac{\alpha^2}{(M-2m)^2}\left(\frac{1}{\sin^2(\Theta/2)} + \frac{1}{\cos^2(\Theta/2)}\right)^2. \tag{31}$$

For small scattering angles we get the Rutherford differential cross section:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\Theta\approx 0} = \left(\frac{\alpha}{2\mu v^2}\right)^2 \frac{1}{\sin^4\frac{\Theta}{2}}, \tag{32}$$

where

$$\frac{1}{2}\mu v^2 = M - 2m \tag{33}$$

is the relative kinetic energy of the particles.

5 Discussion

The probability of a transition per unit four-dimensional volume divided by invariant flux, defines a differential cross section. It is differential since the final particles are specified within small ranges of momenta, as limited by momentum conservation. Integrations over these differential elements supply various differential cross sections of lesser degrees of specification, leading finally to a total cross section, although the latter may not exist if very slight deflections carry a disproportionate weight.

We have used the symbol $d\sigma$ generally for all differential cross sections, relying on the explicitly stated differentials to indicate its precise nature. Energy-momentum conservation in a two-particle scattering process fixes the energies of the scattered particles and leaves free only two parameters that give the direction of the line along which both particles move, in the rest frame of the total momentum. We may as well integrate immediately over the distributions of those variables that assume precise values (Schwinger, 1970).

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