# The Logic of Pi's Irrationality

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**Abstract**

The point on a unit circle that is associated with the arc $\pi/2$ is $(0, 1)$. We prove the bidirectional: every ordered pair of positive real numbers $(a, b)$ corresponds to a point on the circumference of a circle of radius $\sqrt{a^2 + b^2}$ with an associated radius with an arc length less than $\pi/2$. If $\pi/2$ is rational this gives a contradiction.

## Proof

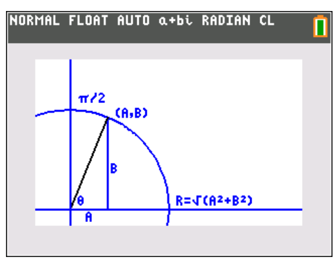


Figure 1: The ordered pair $(A, B)$ resides on a circle of radius $R$.

**Theorem 1.** *$\pi$ is irrational.*

*Proof.* As shown in Figure 1, every pair of positive reals, not necessarily integers or rational numbers can be placed as an ordered pair $(A, B)$ on a circle of radius $R = \sqrt{A^2 + B^2}$. A radius with the defined slope $B/A$ is created. The radian measure associated with this radius is less than $\pi/2$. As all circles are similar these slopes and radian measures will be the same for all circles.

Assume $\pi/2$ is the rational number $B/A$, this corresponds to the ordered pair $(A, B)$ and forces $\pi/2 < \pi/2$, a contradiction. $\square$

# References


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