

# The Quadripartite Classification of Triangles: A Unified Geometric and Numerical Field.

Jean-Yves Boulay

Jean-Yves BOULAY independent researcher – FRANCE – [jean-yvesboulay@orange.fr](mailto:jean-yvesboulay@orange.fr) ORCID: 0000-0001-5636-2375  
<https://www.researchgate.net/profile/Jean-Yves-Boulay>

*Abstract: Grounded in a novel mathematical framework, this study demonstrates that any Euclidean triangle can be uniquely categorized into one of four canonical classes based on the intersection of isosceles and right-angled characteristics. We prove that these four classes form a complementary entanglement capable of saturating a rectangular space without voids. Furthermore, this geometric configuration is shown to be isomorphic with specific numerical assemblies in number theory, establishing a direct link between the fundamental sequence of whole numbers and the stability of geometric structures.*

**Keywords:** Quadripartite Classification, Novel Triangle Model, Euclidean Geometry, Number Theory, Ultimate Numbers, Active Zero.

## 1. Introduction

Euclidean geometry and number theory are the two ancient pillars of mathematics, yet they are often treated as distinct domains with little structural overlap. Classic definitions of triangles based on sides or angles create functional classifications but do not reveal the underlying mathematical "DNA" that governs their stability and fundamental organization. In parallel, number theory is inherently fragmented, notably with the necessary exclusion of zero (0) and one (1) from the fundamental definitions of primes and composites. This traditional boundary leaves the two absolute foundations of the number line outside of the principal arithmetic categories.

In this paper, we propose to bridge these fields by introducing a novel mathematical framework: a Unified Geometric and Numerical Field. This framework is grounded in a "Novel Triangle" model that partitions all Euclidean triangles into four canonical and entangled classes based on an intersection of isosceles and right-angled characteristics.

We demonstrate that this geometric system is not arbitrary but is, in fact, isomorphic with specific sequences in number theory. We move from higher matrix configurations ( $4 \times 6$ ,  $4 \times 4$ ,  $4 \times 5$ ) to a final, minimalist quantum space where number and form are perfectly unified.

This unity is only possible through the introduction of a new arithmetical classification: the **Ultimate Numbers**. This concept allows for a formal, compact definition that, unlike prime number theory, naturally integrates the singular and foundational whole numbers 0 and 1, creating a complete and absolute arithmetic basis. The convergence of this new theory of ultimacy with our quadripartite geometric model provides the final proof of the structural stability of the  $\{0, 1, 2, 3\}$  sequence, offering a new perspective on primordially.

## 2. Four canonical triangle classes

In Euclidean geometry, every triangle can be classified as isosceles or non-isosceles. Likewise, every triangle can be classified as right-angled or non-right-angled. Isosceles triangles and right triangles are classically defined as follows:

- An isosceles triangle is a triangle that has at least two sides of equal length;
- A right triangle is a triangle that has one angle equal to 90 degrees.

However, we propose the following alternative definitions for isosceles triangles and right triangles:

- An isosceles triangle is a triangle that has two identical angles;
- A right triangle is a triangle in which the sum of two angles is equal to the third.

Thus, any triangle can be isosceles or non-isosceles, right-angled or non-right-angled. Also, these two dual criteria can be combined, and then any triangle belongs solely to one of these four classes:

- pure scalene triangle:  
does not admit pair of identical angles and does not admit pair of angles whose the sum is equal to the 3<sup>rd</sup>;
- non-right isosceles triangle:  
admits two identical angles and does not admit pair of angles whose the sum is equal to the 3<sup>rd</sup>;

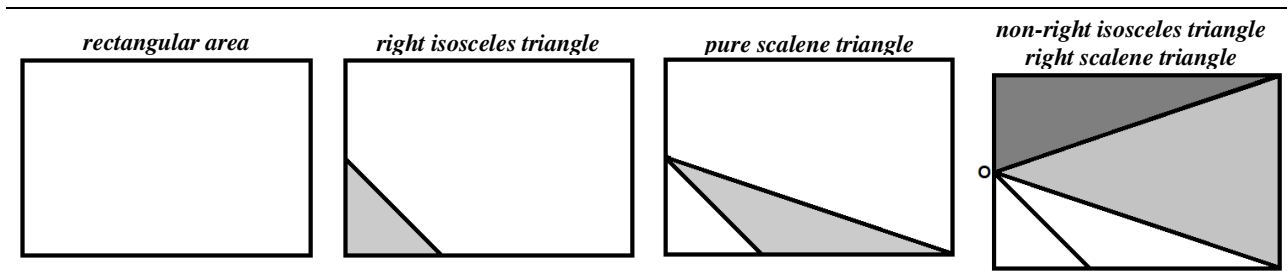
- right scalene triangle:  
does not admit pair of identical angles and admits 2 angles whose the sum is equal to the 3<sup>rd</sup>;
- right isosceles triangle:  
admits two identical angles and admits 2 angles whose the sum is equal to the 3<sup>rd</sup>.

Figure 1 illustrates the classification of any triangle into these four classes with intricate criteria.

	does not admit pair of angles which the sum is equal to the 3 <sup>rd</sup>		
does not admit pair of identical angles	<i>pure scalene triangle</i>	<i>non-right isosceles triangle</i>	admits two identical angles
	<i>right scalene triangle</i>	<i>right isosceles triangle</i>	
	admits 2 angles which the sum is equal to the 3 <sup>rd</sup>		

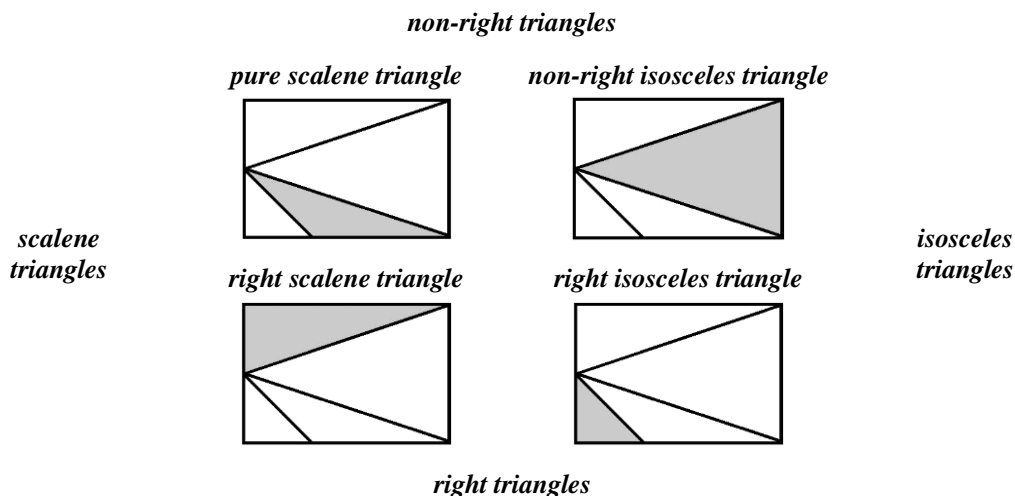
**Figure 1:** According to isosceles concept and right-angled concept, the four canonical classes of Euclidean triangles.

We propose to consider a real entanglement applying to these four classes of triangle where any triangle can be defined. As shown in Figures 2 and 3, it is indeed possible to represent these four canonical triangle classes in a single geometric space without voids, thus confirming their complementary entanglement.



**Figure 2:** Geometric construction of the four canonical classes of Euclidean triangles from a rectangular space.

Thus, as illustrated in Figure 2, within a rectangular space, it is possible to construct a right isosceles triangle followed by an adjacent pure scalene triangle. Subsequently, a non-right isosceles triangle is established adjacent to the latter, and the remaining space naturally completes the matrix as a right scalene triangle. In this unified configuration, all four triangles share a common vertex, designated as **point O**. This central pivot is necessarily located at the midpoint of the rectangle's vertical side, acting as the geometric origin from which the four canonical classes radiate and entangle.



**Figure 3:** According to isosceles concept and right-angled concept, geometric entanglement of the four canonical classes of Euclidean triangles.

Therefore, Figure 3 clearly demonstrates that, within this initial rectangular configuration (which may also be square), all four canonical triangle classes introduced earlier can be represented:

- pure scalene triangle;
- non-right isosceles triangle;
- right scalene triangle;
- right isosceles triangle.

Furthermore, with reference to the different types of triangular angles, the three other transcendental categories of triangles are also represented in this rectangular space of four complementary triangles:

- obtuse triangle → in the pure scalene triangle;
- acute triangle → in the non-right isosceles triangle;
- right triangle → in the right scalene or right isosceles triangle.

However, these latter angular characteristics (obtuse, acute) are not the focus of this paper, which restricts its study to the four previously introduced classes. Similarly, the **equilateral triangle**, while acknowledged here as a specific case of the **non-right isosceles triangle**, is not explicitly studied.

### 3. Four canonical triangle classes and number theory

#### 3.1 Matrix of 4 × 6 integer units

We will demonstrate here that the geometric dimensions of the four canonical triangle classes previously introduced are intimately linked to number theory. As Figure 4 clearly illustrates, the four classes of triangles can be represented within a closed rectangular space, a 4×6 matrix, whose total area equals the value of the first four whole numbers in each class:

- 2 → first prime number;
- 4 → first raised;
- 6 → first composite square free;
- 12 → first composite non square free.

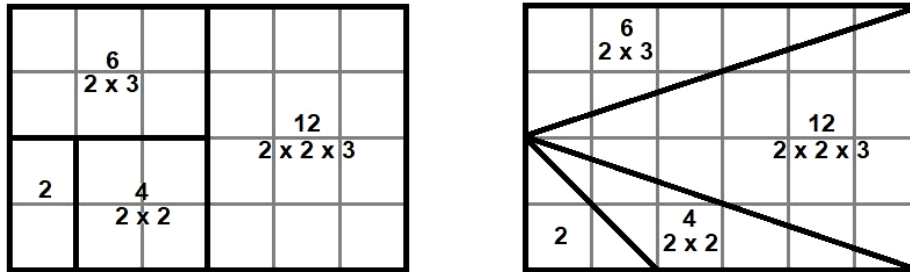


Figure 4: Respective geometric representation of the 4 number classes and the 4 triangle classes in identical areas.

Thus, it is possible to represent 4 triangles belonging to the four defined each with a respective area equal to 2, 4 (→ 2 × 2), 6 (→ 2 × 3) and 12 (→ 2 × 2 × 3). This is precisely the four values of the first active numbers of the four classes of whole numbers (with  $n \geq 2$ ) defined by their non-trivial divisors. These different areas therefore have **integer quantum values**, establishing a direct bridge between the discrete nature of number theory and the continuous appearance of geometric space.

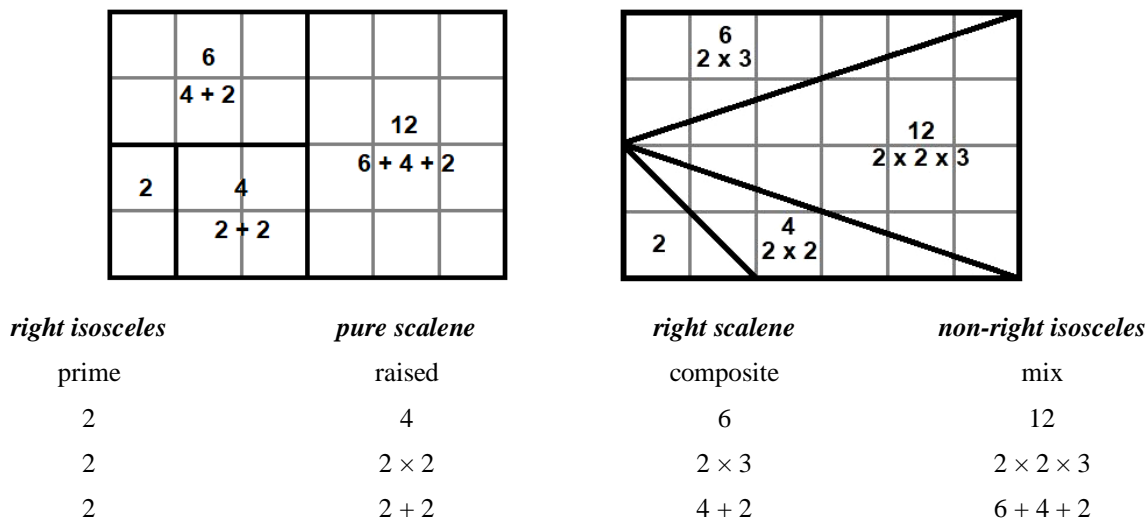
Like Figure 1 for triangle particularities, Figure 5 describes the intricate attributes of the four classes of whole numbers with the first ones of each of the four categories illustrated.

	does not admit different non trivial divisors		
does not admit identical non trivial divisors	2 <i>prime</i>	4 <i>raised</i>	admits identical non trivial divisors
	6 <i>pure composite</i>	12 <i>mixed composite</i>	
	admits different non trivial divisors		

Figure 5: The four canonical classes of whole numbers according to their constituent divisors. (See Figure 1 for comparison).

### 3.2 The Singularity of the "Four Pillars": 2, 4, 6, 12

Figure 6 illustrates an exceptional numerical structure where geometry, multiplication (factorization), and addition (partition) align in perfect harmony. This sequence—composed of the first representative of four distinct categories (*prime, raised, composite, mix*)—possesses a unique recurrence property that vanishes as soon as one attempts to generalize it.



**Figure 6:** Unique stability window linking the four numerical classes to the four fundamental triangle classes in a structural duality of the {2, 4, 6, 12} sequence: additive partition by the progression through harmonic accumulation ( $6=4+2$ ;  $12=6+4+2$ ) and factorization shows the geometric expansion of the base unit (2).

### 3.3 A Progression through Harmonic Accumulation

In this model, each new level is literally "carried" by the sum of the preceding foundations:

- 2 (Ultimate): The base unit, the first even prime number.
- 4 (Raised): The first square ( $2 \times 2$ ), equal to the previous unit doubled ( $2 + 2$ ).
- 6 (Composite): The first oblong number ( $2 \times 3$ ), which is also the sum of the two preceding numbers ( $4 + 2$ ).
- 12 (Mix): The first highly composite number ( $2 \times 2 \times 3$ ) which is the sum of the three preceding numbers ( $6 + 4 + 2$ ).

### 3.4 The Exceptionality of the First Cycle

This fluidity between the product and the sum is a property exclusive to the first four numbers of each class. If we apply the same logic to the next four numbers in these categories (3, 8, 10, 18), the balance collapses:

- The next "Composite" (10) is not the sum of 8 and 3 (11).
- The next "Mix" (18) is not the sum of 10, 8, and 3 (21).

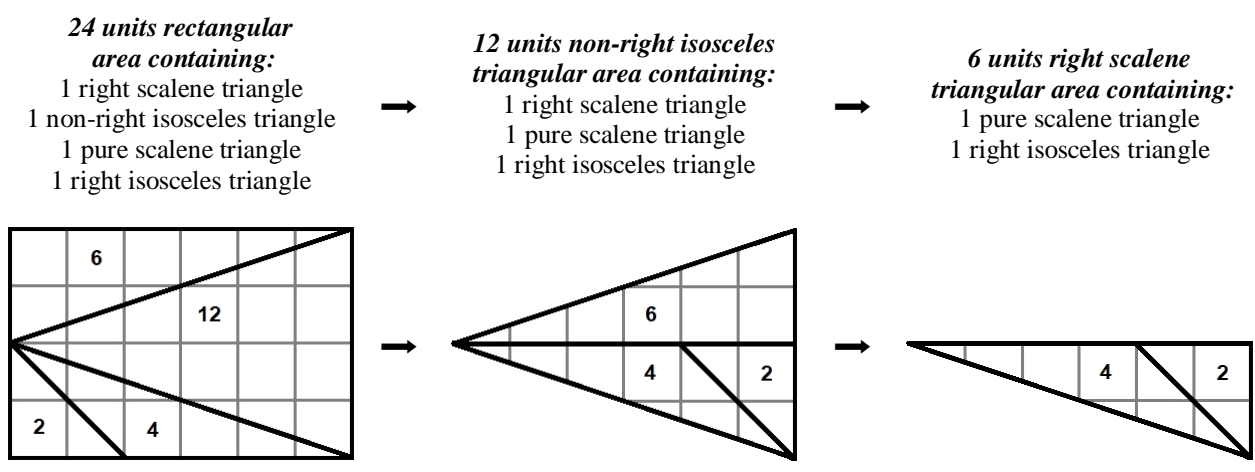
### 3.5 A Window of Structural Stability

The visual grid demonstrates that for the numbers 2, 4, 6, and 12, there is a rare coincidence between factor decomposition and additive partition. These four numbers form a "window of stability" where the number acts like a lung, capable of expanding through multiplication or fragmenting through addition while maintaining a coherent geometric structure.

Beyond this group, the complexity of prime numbers and the distribution of factors break this symmetry, making the sequence {2, 4, 6, 12} a fundamental and isolated signature within numerical architecture.

### 3.6 An intricate geometric structure

This numerical architecture also reveals itself geometrically in an intricate organization. Thus, as we have demonstrated, a rectangular area contains the four canonical classes of triangles. As illustrated in Figure 7, it turns out that within this rectangular space, the non-right isosceles area can contain the three other triangular classes: a right scalene triangle, a pure scalene and a right isosceles triangle. Finally, it appears that the right scalene area can contain a pure scalene triangle and a right isosceles triangle. This geometric hierarchy is mirrored numerically: this means that 24 contains the values 12, 6, 4 and 2, that 12 contains the values 6, 4 and 2 and that 6 contains 4 and 2.



**Figure 7:** Geometric inclusion of the four canonical classes of Euclidean triangles from a rectangular space to final triangular areas.

This demonstration, both numerical and geometric, made with the integer values 2, 4, 6 and 12, remains valid within any rectangular space systematically containing the four classes of triangles previously introduced.

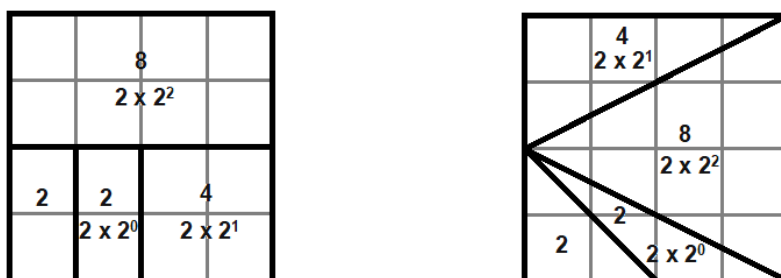
#### 4. Four canonical triangle classes in other matrices

The geometric representation of the four classes of triangles occupying complementary areas in integer values forming a rectangular zone can be done in multiple ways as we will show in different demonstrations.

##### 4.1 Matrix of $4 \times 4$ integer units

In a Matrix of  $4 \times 4$  integer units, it is possible to construct four triangles of each of the four classes defined previously. In this matrix, the areas are equal to 2, then 2, then 4, and finally 8. These different values correspond to 2, the first prime number, then to two times three progressive powers of this number, i.e., 2 then  $2 \times 2^0$ ,  $2 \times 2^1$  and  $2 \times 2^2$ .

##### 4.1.1 The $4 \times 4$ power matrix: geometric evolution of the number 2



**Figure 8:** Respective geometric representation of the number 2 and its first three powers and the 4 triangle classes in identical area values.

This Figure 8 illustrates the internal mechanics of a 16-unit matrix ( $4 \times 4$ ) demonstrating how the prime number 2 generates a sequence of areas that correspond to the four fundamental classes of triangles. It reveals a perfect transition from a linear prime value to exponential growth.

##### 4.1.2 The exponential ladder

The matrix is partitioned into four areas that follow the progressive powers of 2. This is not a simple addition, but a doubling maturation:

- Area 2 (2): The first prime number, acting as the seed.
- Area 2 ( $2 \times 2^0$ ): The first manifestation of the power, mirroring the seed.
- Area 4 ( $2 \times 2^1$ ): The doubling of the foundation.
- Area 8 ( $2 \times 2^2$ ): The final expansion, occupying exactly half of the  $4 \times 4$  matrix.

### 4.1.3 The four triangle classes and rectangular zones as equal areas

The brilliance of this model lies in the assertion that these areas (2, 2, 4, 8) are not just quantities, but spatial containers for the four classes of triangles:

- Each area allows for the construction of a specific triangle class (Equilateral, Isoceles, Right, and Scalene) while maintaining "identical area values" relative to their structural complexity.
- As the triangle's "freedom" increases (from the rigid Equilateral to the versatile Scalene), the matrix provides a proportionally larger "lung" (area) to accommodate its geometry.

### 4.1.4 Structural symmetry: the static vs. radiant view

The two diagrams offer a dual perspective on the same truth:

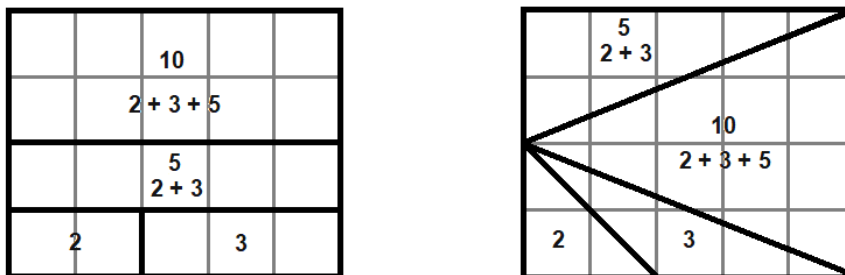
- The Modular View (Left): Shows the matrix as a stable construction of integer units. It emphasizes the hierarchy of the powers of 2.
- The Dynamic View (Right): Reinterprets these powers as radiant sectors. Here, the diagonal lines act as "vectors of expansion," showing how the number 2 "breathes" through the matrix to create different geometric depths.

### 4.1.5 The 16-unit completion

By using a  $4 \times 4$  grid (totalling 16 units), the model achieves a closed system. The sum of the areas ( $2 + 2 + 4 + 8 = 16$ ) perfectly fills the matrix. This suggests that the first prime number and its first three powers constitute a complete geometric set, capable of mapping the entire diversity of triangular forms within a single, unified stable structure.

### 4.2 Matrix of $4 \times 5$ integer units

In a Matrix of  $4 \times 5$  integer units, it is possible to construct four triangles of each of the four classes defined previously. In this matrix, the areas are equal to 2, then 3, then 5, and finally 10. These different values correspond respectively to the first prime number (2), then to the second prime (3), then the third prime (5) and finally 10, the sum of these first three prime number.



<i>right isosceles</i>	<i>pure scalene</i>	<i>right scalene</i>	<i>non-right isosceles</i>
first prime	second prime	third prime	Synthesis
2	3	5	10
2	3	3 + 2	5 + 3 + 2

**Figure 9:** respective geometric representation of the first three prime numbers then their sum and the 4 triangle classes in identical area values.

As shows in Figure 9, this is not just any sequence. 2 and 3 are the first two prime numbers, and 5, the third consecutive prime, is also the sum of the first two. 10, the fourth in the series, is logically the sum of the first three.

It turns out that this series of four values is constructed identically in this matrix as in the two previously introduced: the third and fourth values are the sum of the smaller values. Also, although the smallest value remains at an amplitude of 2, its addition to the second gives a value proportional to the size of the different matrices ( $2 + 2$  for a  $2 \times 4$  matrix,  $2 + 3$  for a  $2 \times 5$  matrix, etc.).

### 4.2.1 The prime synthesis: The 2-3-5-10 configuration

This figure demonstrates a fundamental additive synthesis using the first three consecutive prime numbers. Unlike the previous model which explored different classes of numbers, this configuration focuses on the **internal growth** of the prime sequence and its convergence into a decimal base.

#### 4.2.2 The cumulative prime ladder

The structure follows a strict additive progression where each step integrates the previous ones, creating a "staircase" of numerical density:

- **2 (First Prime):** The absolute foundation.
- **3 (Second Prime):** The first odd prime, which acts as the next structural layer.
- **5 (Third Prime):** The sum of the first two primes ( $2 + 3 = 5$ ). Here, the prime value and its additive history coincide perfectly.
- **10 (The Synthesis):** The sum of all three ( $2 + 3 + 5 = 10$ ). This represents the completion of the cycle, reaching the base of our decimal system.

#### 4.2.3 Geometric translation: from grid to pulse

The two diagrams illustrate the transition from **static partition** to **dynamic expansion**:

- **The Rectangular Grid (Left):** Shows the "stacking" of values. It visualizes the number 10 as a stable, layered construct where 2 and 3 support 5, which in turn supports the whole.
- **The Radiant Model (Right):** Reinterprets the same values through a focal point. This evokes the "breath" of the system, where the numbers 2, 3, and 5 radiate from a single origin to define the boundaries of the 10-unit space.

#### 4.2.4 The uniqueness of the 2-3-5 trio

This configuration is highly specific. In the infinite string of prime numbers, the trio  $\{2, 3, 5\}$  is the only sequence where the sum of the first two exactly equals the third.

- If we move to the next prime (7), the harmony breaks:  $3 + 5 \neq 7$ .
- This makes the **10** (the sum of 2, 3, and 5) a "Natural Fulcrum"—a point where the prime sequence and the decimal base meet through a perfect additive mirror.

#### 4.2.5 The "lung" of the decimal base

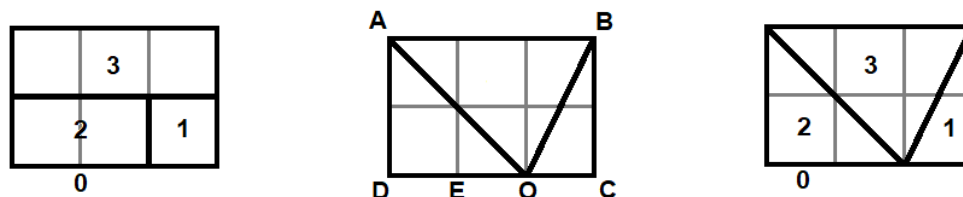
By visualizing 10 not just as a quantity, but as the combined "expiration" of 2, 3, and 5, this model treats the number as a living geometry. It serves as a bridge between the **indivisibility of primes** and the **structured stability of the decade**, reinforcing the idea that the first steps of mathematics are governed by a unique structural resonance that cannot be replicated further up the number line.

### 5 Matrix of $2 \times 3$ integer units

Having established the link between the four canonical triangle classes and the diverse natures of integer composition (Figure 4, 8 & 9), we must now demonstrate that this geometric entanglement is not dependent on high numerical values. This relationship is deeply embedded in the very first sequence of the number line.

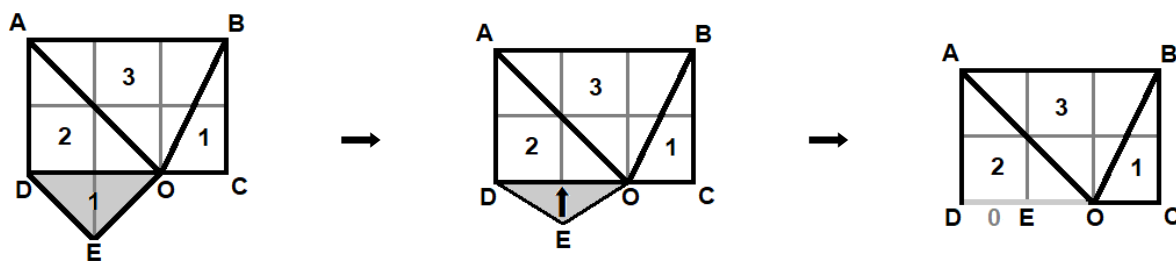
#### 5.1 The minimalist 0-1-2-3 structural grid

Having established the link between the four canonical triangle classes and the diverse natures of integer composition, we must now demonstrate that this geometric entanglement is not dependent on high numerical values. This relationship is deeply embedded in the very first sequence of the number line. Figure 10 presents a critical demonstration: the mapping of these four classes into a quantized space defined by a matrix of just six integer units. Within this minimalist grid, a continuous and complete geometry emerges, proving that the model's structural integrity is embedded in the very first elements of the number line.



**Figure 10:** Respective geometric representation of the first four whole numbers and the four triangle classes within identical areas of integer quantum values. The configuration reveals the presence of a **degenerate**, non-right isosceles triangle at **DEO** with an area of 0. (See Figure 11 for the dynamic explanation of this state).

The inclusion of the fourth class—the non-right isosceles triangle—within such a constrained matrix necessitates the concept of geometric degeneracy. To maintain the structural entanglement of all four classes within a total area of six units ( $0+1+2+3$ ), the non-right isosceles triangle must exist in a state of zero-area. As illustrated in Figure 11, this is achieved through a 'flattening' process: by reducing the altitude of the triangle at point E until it converges with the base segment DO, the triangle remains topologically present as a non-right isosceles form while its physical area nullifies. This 'limit state' allows the model to remain mathematically complete even at the origin of the number line.



**Figure 11:** Dynamic construction of a degenerate isosceles triangle (area = 0) in **DEO** by flattening the altitude at point E. This transition demonstrates how the fourth class is structurally preserved at the zero-point of the quantum sequence.

Building upon the established correlation between the four canonical triangle classes and their **corresponding integer quantum values** (Figure 10), we must now investigate the physical grounding of this model. The geometric entanglement of these classes does not require higher numbers; it is inherently coded within the most fundamental sequence of whole numbers.

Figure 10 presents a critical demonstration: the mapping of these four classes into a quantized space defined by a matrix of just **six integer units**. Within this minimalist grid, a continuous and complete geometry emerges, proving that the model's structural integrity is embedded in the very first elements of the number line.

The visual representation in Figure 10 reveals a profound insight into the relationship between number and form. By partitioning a rectangular field of total area 6 ( $0+1+2+3$ ), we observe a unified entanglement of the four triangle classes, each assigned an "**integer quantum value**" of area corresponding to the first four whole numbers.

The most striking originality of this manifestation lies in its completeness: all four canonical possible triangles are present and structurally stable.

- **Pure Scalene** (Non-Rectangle + Non-Isosceles): Manifested by the central triangle **ABO** with an area of 3.
- **Scalene Right** (Rectangle + Non-Isosceles): Manifested by the right-angled triangle **BCO** with an area of 1.
- **Right Isosceles** (Rectangle + Isosceles): Manifested by the right-angled triangle **ADO** with an area of 2.
- The final and crucial piece of the entanglement is the small area marked **0 (triangle DEO)**. To ensure a field without voids, the model requires the presence of the fourth canonical class: the **Non-Right Isosceles**.

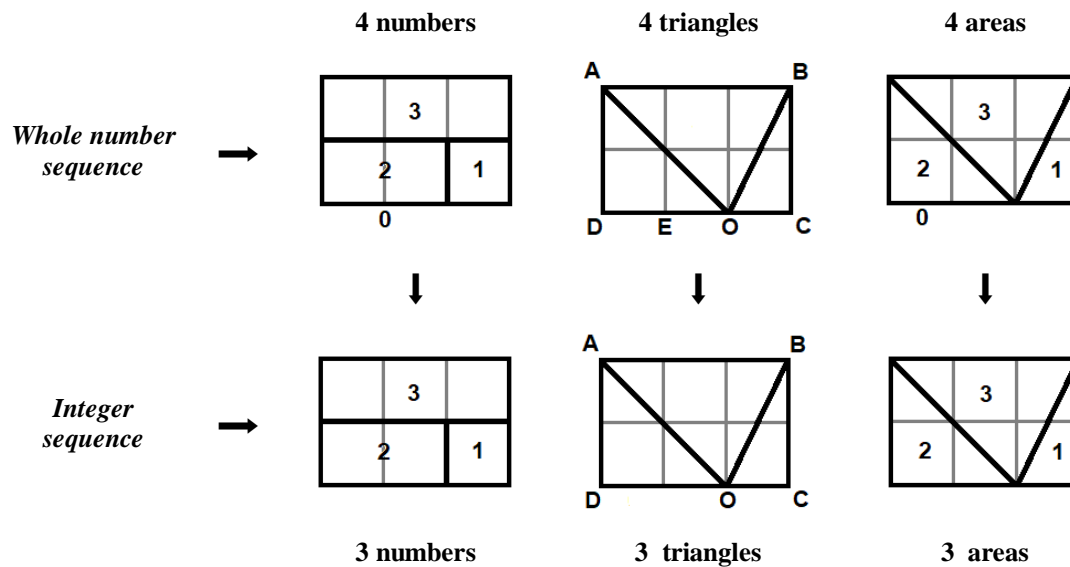
In this quantified matrix, this class is manifested precisely at area **0**, taking the form of a degenerate triangle. Far from being a mathematical void, this degenerate triangle **DEO** functions as an "**active non-manifested**" agent, completing the geometric field and anchoring the entire matrix of four possible triangles in the first whole number. This proves that the field of geometric possibility is fully defined from the very beginning of the whole number sequence. The triangle **DEO**, although degenerate due to its zero-area, maintains its logical identity within the matrix, acting as the critical link between the whole number sequence and geometric manifestation.

## 5.2 Integer sequence versus whole number sequence

The transition from the **integer sequence** (1, 2, 3) to the **whole number sequence** (0, 1, 2, 3) is not merely an arithmetic addition; it represents a fundamental shift in geometric topology.

As shown in Figure 12, the **integer sequence** creates a partial and fragmented geometry. With only three numbers, we can only manifest three triangle classes. This configuration leaves a structural 'void' or an incomplete matrix, where the non-right isosceles class is absent. In this state, the geometric field is disconnected from its source of total stability.

Conversely, the **whole number sequence** (0, 1, 2, 3) enables the 'Quadripartite' manifestation. The introduction of the zero-point (0) allows the fourth canonical class—the non-right isosceles—to emerge as a degenerate triangle at **DEO**. This 'active zero' is the bridge that closes the circuit of geometric possibilities. While the areas 1, 2, and 3 represent the *manifested* forms, the area 0 represents the *non-manifested* regulatory basis of the structure.



**Figure 12:** Respective geometric representation of the first four whole numbers and the four triangle classes and of the first three integer and only three triangle classes. **Degenerate**, non-right isosceles triangle at **DEO** with area **0**. (See Figure 10 and 11).

This comparison proves that the 'Novel Triangle' model requires the presence of the zero to reach structural integrity. Without the whole number sequence, the entanglement is broken; with it, the matrix becomes a unified, stable, and continuous field where number and form are perfectly isomorphic.

### 5.3 Intrication of the 0-1-2-3 sequence: the convergence of the fourth ultimate

The structural integrity of this model finds its final proof in the remarkable convergence of the sequence  $\{0, 1, 2, 3\}$ . This sequence acts as a triple-anchor for the theory:

1. **Arithmetical Primordality:** These are the first four **whole numbers**, the foundation of all numerical manifestation.
2. **The Frontier of Ultimacy:** They constitute the first four **Ultimate Numbers\***. As defined by their divisor structure, they represent a closed field of simplicity; a threshold that is breached as soon as we reach the number 4.
3. **Geometric Completeness:** They represent the four **minimalist areas** required to manifest the four canonical classes of triangles. Without the 'active zero', the entanglement is broken; with it, the matrix is complete.

\* (A formal definition of this new arithmetic classification is provided in Section 6).

The most profound property of this sequence lies in the unique nature of the number **3**. As the final element of this primordial set, 3 operates as a **unifying synthesis**:

- **Arithmetically:** It is the only natural number equal to the sum of all its predecessors ( $0 + 1 + 2 = 3$ ).
- **Geometrically:** Within the  $2 \times 3$  matrix, the area of the final class (Pure Scalene) is the exact sum of the three other areas ( $0 + 1 + 2$ ).

Thus, the number 3 does not merely follow 0, 1, and 2; it **contains** them. This 'Finality of 3' proves that the four canonical classes of triangles are not independent entities, but a single, unified geometric field. The model is therefore not just a classification, but an isomorphic map of the fundamental laws of structure and number.

## 6 The concept of ultimate number

In a previous paper (Boulay, 2023a), we introduced the concept of the ultimate numbers. We now recall this innovative concept.

Using a definition with inescapable characters and not subject to any ambiguity, we introduce here the concept of ultimacy that a whole number can possess. Recalling the notion of prime number, we then introduce a compact and absolute definition splitting the set  $\mathbb{N}_0$  into two primary sets: **ultimate** and **non-ultimate**. A development, taking the very first numbers as an example, clarifies this innovative concept which turns out to be of a strong simplicity.

## 6.1 Prime number definition

In mathematical literature the definition of primes is typically presented as follows:

*“Prime numbers are numbers greater than 1. They only have two factors, 1 and the number itself. This means these numbers cannot be divided by any number other than 1 and the number itself without leaving a remainder.”*

This is commonly supplemented by:

*“Numbers that have more than 2 factors are known as composite numbers.”*

These definitions supposed to describe and classify the whole numbers immediately present two great ambiguities since they are embarrassed by the two singular numbers that are zero (0) and one (1).

These definitions, while functional for traditional number theory, present two significant ambiguities when attempting to classify the set of whole numbers ( $\mathbb{N}_0$ ). They are fundamentally "embarrassed" by the two singular numbers—zero (0) and one (1)—which do not fit into either category (prime or composite). This exclusion reveals a structural gap in the traditional classification, as 0 and 1 are the very foundations of the number line, yet they remain outside the primary definitions of primality.

## 6.2 New definition approach

The conventional definition applied to determine whether a number is prime or not does not specify the character of inferiority of the divisors. This omission appears trivial at first glance. However, with regard to the particular numbers that are zero and one, this notion becomes essential.

Indeed, the number zero, the first of the whole numbers, has many divisors, but all of its positive divisors are strictly greater than itself. Likewise, the number one, the second whole number, admits no positive divisor strictly smaller than itself, since zero cannot be considered as a divisor.

This refined approach to the concept of divisibility of which explicitly introduces the notion of inferiority (and consequently superiority) of divisors, makes it possible to construct a classification in which all whole numbers can be consistently included in two unique sets. This classification is not based on primality but it is based on the number of proper divisors, which naturally includes 0 and 1 without exception.

## 6.3 Formal definitions

Considering the set of all whole numbers ( $\mathbb{N}_0 = \{0, 1, 2, 3, \dots\}$ ) and defining a divisor of a number as a non-negative integer in  $\mathbb{N}_0$ , this set can be partitioned into two disjoint subsets: ultimate numbers and non-ultimate numbers.

Ultimate number definition:

*An ultimate number has at most one divisor less than itself.*

Non-ultimate number definition:

*A non-ultimate number has at least two divisors less than itself.*

## 6.4 Conventional designations

As "primes" designates prime numbers, it is agreed that appellation "ultimates" designates ultimate numbers. Also it is agreed that appellation "non-ultimates" designates non-ultimate numbers. So, the concept introduced here is therefore called that of ultimity of whole numbers. Otherwise transcendent conventional appellations will be applied to these two different classes of whole number just defined, in particular with the development of Sophie Germain's number concept, which we are going to extend to all whole number.

## 6.5 Development

This new concept of ultimity, which may seem intriguing, is now explained in detail with, more concretely, the first of the whole numbers as representative examples, including the exotic numbers zero (0) and one (1).

### 6.5.1 Expanded definitions

Let  $n$  be a whole number (belonging to  $\mathbb{N}_0$ ),  $n$  is called Ultimate if it has **at most one divisor less than itself**.

Let  $n$  be a whole number (belonging to  $\mathbb{N}_0$ ),  $n$  is called non-Ultimate if it has **at least two divisors less than itself**.

## 6.5.2 Development

Below are listed, to illustration of definition, some of the first ultimate or non-ultimate numbers defined above, especially particular numbers zero (0) and one (1).

- 0 is Ultimate: although it admits an infinite number of divisors superior to it, **since it is the first whole number**, number 0 does not have any divisor less than itself;
- 1 is Ultimate: number 1 does not have any divisor (whole number) less than it;
- 2 is Ultimate: number 2 has only one divisor less than itself: 1;
- 4 is non-Ultimate: number 4 has 1 and 2 as divisors less than it, so more than one divisor;
- 6 is non-Ultimate: number 6 has numbers 1, 2 and 3 as divisors less than it, so more than one divisor;
- 7 is Ultimate: number 7 has only one divisor less than itself: 1;
- 12 is non-Ultimate: number 12 has numbers 1, 2, 3, 4 and 6 as divisors less than it, so more than one divisor.

Thus, by these previous definitions and demonstrations, the set of whole numbers ( $\mathbb{N}_0$ ) is organized just into these two entities:

- the set of ultimate numbers, which is the fusion of the prime numbers sequence with the number 0 and number 1;
- the set of non-ultimate numbers identifying to the non-prime numbers sequence, free of the numbers 0 and 1.

## 7 Discussions and conclusion

By the previous demonstrations, we can say that, in Euclidean geometry, every triangle can be categorized in four classes with entangled characteristics. Indeed, depending on whether a triangle is isosceles or not and is right or not, it belongs and can only belong to one of these four geometric classes:

- pure scalene triangle:  
does not admit pair of identical angles and does not admit pair of angles whose the sum is equal to the 3<sup>rd</sup>;
- non-right isosceles triangle:  
admits two identical angles and does not admit pair of angles whose the sum is equal to the 3<sup>rd</sup>;
- right scalene triangle:  
does not admit pair of identical angles and admits 2 angles whose the sum is equal to the 3<sup>rd</sup>;
- right isosceles triangle:  
admits two identical angles and admits 2 angles whose the sum is equal to the 3<sup>rd</sup>.

We have also demonstrated that it is possible to assemble these four geometric classes into a single rectangular area and have observed that it is possible to associate the value of the four different areas thus created with singular sequences of integers.

From this, we have proven concrete links between geometry and arithmetic fields by merging the concepts of two-dimensional space and number theory.

We have indeed been able to demonstrate that within rectangular configurations (which can also be square), the respective values of the areas of the four geometric classes of triangle that we have defined according to the isosceles and right-angled criteria are identical to sequences of particular numbers.

Configured in a rectangular area, these four classes of triangles can have sides of 2, 4, 6 and 12 respectively, corresponding exactly to the arithmetic values of the first active numbers of the four classes of whole numbers (with  $n \geq 2$ ) defined by their non-trivial divisors: prime, raised, composite free square and mixed composite. In another configuration, these four triangles are arranged in the arithmetic values 2, 2, 4, and 8, which are progressive powers of the first prime number. In another rectangular configuration, these triangles have sides of 2, 3, 5, and 10, values corresponding to the first three prime numbers and their sum. These different areas therefore have **integer quantum values**, establishing a direct bridge between the discrete nature of number theory and the continuous appearance of geometric space.

Finally, by introducing a degenerate triangle, a flattened non-right isosceles triangle of area 0, we were able to construct a configuration where the four classes of triangles have respective areas corresponding to the sequence  $\{0,1,2,3\}$  that is, very

precisely the first four whole numbers, which are simultaneously the first four numbers that we call ultimates by a novel definition unifying the primes and the exotic number 0 and 1.

This convergence, where the final class (Pure Scalene of area size 3) acts as a unifying synthesis containing all preceding values (both geometrically and arithmetically), constitutes the final proof of the structural stability of the field. The introduction of the active zero-point (0) is not a mere convenience; it is the regulatory basis and structural anchor that allows the 'Quadripartite' manifestation to exist without voids, bridging the non-manifested foundational rules with the manifested geometric forms. Beyond this primordial sequence  $\{0, 1, 2, 3\}$ , numbers become 'Non-ultimate,' signifying a transition from fundamental structural identity to complex composition. The isomorphism discovered between this minimalist matrix and the theory of ultimate numbers proves that these four canonical classes are not independent, but a single, closed, and unified geometric field, governed by the same primordial laws that order the beginning of the number line.

## References

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