Explanation of time dilation in a GPS satellite without using the relativity theory

Jorma Jormakka
Vantaa, Finland
jorma.o.jormakka@gmail.com

Abstract: In relativity theory the advance of an atomic clock in a GPS satellite of some 38 microseconds in 24 hours is explained by time dilation because of velocity from the Special Relativity Theory and time dilation in a gravitational field from the General Relativity Theory. The article shows that the time advance of a GPS satellite can be explained without any relativity theory and the formulas are in the first order the same as in the relativity theory. It is also shown that the calculation of the GPS clock advance from the relativity theory is incorrectly made: in relativity theory there is the equivalence principle and because of it, the time dilation caused by acceleration due to the curved orbit must be included. When it is included, the clock advance is incorrect. It is also shown that Einstein's 1907 calculation of the gravitational redshift is incorrectly made, though the result is correct.

Keywords: Precession time dilation, gravitational time dilation, GPS satellite, gravitational redshift.

1. Introduction

Einstein predicted the gravitational redshift in 1907 and 1911, the first one was seven years before he published the General Relativity Theory. Gravitational redshift was measured in the Pound-Rebka experiment in 1960. Gravitational time dilation was used to explain and correct the advance of an atomic clock in GPS satellites in the 1970ies and measured in 1976, the relativistic calculation fit measurements well. Time dilation by velocity was used as an explanation of the longer half-time of fast muons, first measured in 1940, the Special Relativity Theory formula fits well. There are more experiments of time dilation, all seem to support the relativity theory, except for the Sagnac experiment, but this experiment has been explained, albeit in a way that may not sound so clear to a sceptic.

It might appear that time dilation of the relativity theory is well verified. The presented article shows that the explanations from the relativity theory in the case of the GPS satellite clock can be explained without any relativity theory. It is also shown that the relativistic calculation is incorrect because it omits the acceleration caused time dilation and if this time dilation is included, then the relativistic calculation gives a wrong result. Additionally, it is shown that Einstein's calculation of the gravitational redshift from the equivalence principle in 1907 is invalid. The equivalence principle is invalid.

My other papers show that basically everything in the relativity theory is wrong, see the references in [4]. It merely means that a theory cannot be verified by experiments, it can only be falsified.

2. Precession time dilation

The orbit of a satellite conserves the angular momentum. The mass must be taken as an apparent mass. The orbit of an GPS satellite is nearly circular and therefore the apparent mass is very closely Lorentz's transverse apparent mass $m = \gamma m_0$, also called the relativistic mass.

The Hamiltonian can be written as:

$$E_{total} = T + V = \sqrt{p^2c^2 + m_0^2c^4} - m_0c^2 + V \tag{1}$$

where T is the apparent/relativistic kinetic energy and V is the potential energy. The potential energy has the form

$$V = -\frac{g}{r} = -\frac{GM}{r} \tag{2}$$

and we include the rest energy of the mass into E

$$E = E_{total} - m_0c^2. \tag{3}$$

See [1] for the solution of the equation of (1). In [1] the angular momentum is denoted by $p_\theta = mr^2\dot{\theta}$, but here we write $L = p_\theta = mvr$ as L is a more conventional name for the angular momentum. Here v is the orbital velocity of m_0 and r is the radius of the (circular) orbit. The square of the velocity v is solved from

$$F = \frac{mv^2}{r} = \frac{GMm}{r^2} \qquad v^2 = \frac{GM}{r}. \tag{4}$$

The solution of the Hamiltonian equation is

$$r = \frac{a(1-\epsilon^2)}{1 + \epsilon\cos(\omega\theta)} \tag{5}$$

and the relevant equations coming from the solution are (see [1]):

$$\omega^2 = 1 - \frac{g^2}{L^2c^2} \tag{6}$$

and

$$\frac{E}{m_0c^2} = \left(\sqrt{1 - \frac{g^2}{L^2c^2}}\right)\left(\sqrt{1 - \epsilon^2\frac{g^2}{L^2c^2}}\right)^{-1}. \tag{7}$$

The solution has precession ω. We can express ω in a more transparent way for the gravitational force by inserting $g = GMm$ and v^2

$$\omega^2 = 1 - \frac{(GMm)^2}{(mvr)^2c^2} = 1 - \frac{GM}{rc^2} = 1 - \frac{v^2}{c^2}. \tag{8}$$

Thus,

$$\omega = \sqrt{1 - \frac{v^2}{c^2}} = \gamma^{-1}. \tag{9}$$

Without precession the orbital time would be

$$T_\infty = \frac{2\pi r}{v} = 2\pi\sqrt{\frac{r^3}{GM}} \tag{10}$$

but as there is precession, the orbital time is actually

$$T_{precession} = \frac{1}{\omega}T_\infty = \gamma T_\infty = \gamma\frac{2\pi r}{v} = \gamma 2\pi\sqrt{\frac{r^3}{GM}}. \tag{11}$$

We get exactly the same time dilation from precession that the relativity theory derives from the SRT time dilation because of speed. This precession time dilation for a GPS satellite at the altitude $r = 26,571 * 10^6 m$ is $7.171\mu s$ in 24 hours.

3. Gravitational time dilation

The gravitational time dilation is caused by time dilation in an atomic clock in the satellite. Let us assume that the satellite clock is a cesium-133 atomic clock as it is the most common. A cesium-133 atom has 55 electrons, 54 are on stable orbits and the outermost electron is on the sixth S-orbit. It has two hyperfine states, F3 and F4, which differ by the orientation of the spin of the electron. In the state F3 the spin of the electron is opposite to the spin of the nucleus and in the state F4 the spins are parallel. Magnets with magnetic spin parallel repulse each other and with opposite spins attract each other, therefore we can (correctly) guess that F3 has slightly smaller orbit and slightly lower energy level than F4.

When there is an external gravitational field with the center of the gravity very far from the atom (it is the gravitation field of the Earth), this gravitational field causes a gravitational potential that changes the energy levels of the atom. As the center of gravity is very far, this gravitational potential can be considered constant. It is easily inserted to the solution of (1), it is included simply by adding it to the energy parameter E. If the atomic clock is on the Earth, we add the gravitational potential with the radius equalling the radius R of the Earth:

$$E = E_{total} - m_0c^2 + m\frac{GM}{R}. \tag{12}$$

Let the satellite be at the distance r from the center of the Earth. Then the energy changes to

$$E = E_{total} - m_0c^2 + m\frac{GM}{r}. \tag{13}$$

The solution of (1) is still a precessing ellipse with the same ω and the small energy difference of F3 and F4 is not changed. However, the energy of the photon that is released in the transition F4 to F3 is different on the Earth and in the satellite.

The energy difference of F4 and F3 in the satellite corresponds to the energy difference of F4 and F3 on the Earth as

$$\Delta E = mGM\left(\frac{1}{R} - \frac{1}{r}\right) = \frac{R-r}{rR}GMm. \tag{14}$$

This energy difference changes the energy $h\nu$ of a photon that is emitted in the transition

$$h\nu_{observed} = h\nu - \Delta E. \tag{15}$$

The mass m in (14) is not the mass of the electron that circulates in states F3 and F4. It is the apparent mass of a photon. This is explained at the end of Section 4.

A photon does not have mass, but it has momentum and from momentum we get an apparent mass

$$p = \frac{1}{c}h\nu \qquad p = mv = mc \tag{16}$$

as $v = c$ locally for a photon. Then

$$m = \frac{hv}{c^2} \tag{17}$$

and inserting it to (14) gives

$$\nu_{observed} = \nu - \frac{\Delta E}{h} = \nu - \frac{R-r}{rR}GM\frac{\nu}{c^2}. \tag{18}$$

The observed frequency difference in the atomic clocks in the satellite and on the Earth is

$$\Delta\nu_{observed} = \Delta\nu\left(1 - \frac{R-r}{rR}\frac{GM}{c^2}\right). \tag{19}$$

This formula is the same as the gravitational time dilation formula from General Relativity. It gives $-45.748\mu s$ time dilation in 24 hours to the clock in the satellite. Combining the time dilations from precession and gravitation gives $-45.748 + 7.171 = 38.577\mu s$ in 24 hours as the time dilation difference between clocks on the Earth and in a GPS satellite. This time dilation maches very well with the measured time dilation of about $38\mu s$ in 24 hours, but more importantly, the formulas that were obtained without any relativity theory are exactly the same as come from relativity theory in the first order approximation, i.e.,

$$1 - \frac{GM}{c^2} \approx \sqrt{1 - \frac{2GM}{c^2}}. \tag{20}$$

4. Gravitational redshift

Let us consider mass m moving from lower gravitational potential at point A to higher gravitational potential at point B. Let the potential difference be $\Delta\Phi$.

Something must do the work $W = \Delta\Phi m$. If this mass is apparent mass of a photon and the photon at A has the energy $E_A = h\nu_s$, then at B it has the energy

$$E_B = E_A - W = h\nu_s - \Delta\Psi m = h\nu_s - \Delta\Phi\frac{h\nu_s}{c^2} = h\nu_s\left(1 - \frac{\Delta\Phi}{c^2}\right). \tag{21}$$

The gravitational redshift is

$$\nu_0 = \nu_s\left(1 - \frac{\Delta\Phi}{c^2}\right). \tag{22}$$

This result does not require relativity theory or equivalence principle. It only requires the apparent mass of a photon behaving as a mass in this situation.

Einstein explained the redshift with the equivalence principle in the following way. Assuming that B moves to the higher gravitational potential with constant acceleration a that equals the gravitational acceleration g, then the Doppler effect is

$$\nu_o = \nu_s\left(1 - \frac{\Delta v}{c}\right) = \nu_s\left(1 - \frac{gt}{c}\right). \tag{23}$$

Assuming that the speed of light is constant, $t = \frac{s}{c}$, then

$$\nu_o = \nu_s\left(1 - \frac{gs}{c^2}\right) = \nu_s\left(1 - \frac{\Delta\Phi}{c^2}\right). \tag{24}$$

The problem with this derivation is the equivalence principle. While it is true that in Newton's mechanics the inertial mass equals the gravitational mass e.g. in the equation

$$F = ma = \frac{GMm}{r^2}, \tag{25}$$

i.e., the first m is inertial mass and the second m is gravitational mass and they have the same value, it is not true that there is no way to differentiate between the situations where a mass m is in accelerating motion with acceleration g and where the mass is in a gravitational field with the field strength as g. It is enough to wait for a while. If the mass is accelerating, its speed grows and the redshift increases. The redshift stays constant in time if a mass stays in one place in a gravitational field. It is easy to see the difference, so equivalence can only apply to a time instant, but there are other objections, as the following one.

Another problem in the equivalence principle is in the gravitational time dilation of the GPS satellite. The satellite is in free fall on its orbit, therefore if the time dilation from acceleration equals time dilation from gravitation, we would expect that the time dilation of the satellite should be counted twice, or, assuming that time dilation has a direction, dilation from gravitation and acceleration should cancel. They do neither: in order to get the correct result, time dilation from acceleration must be ignored.

The relativity theory has the equivalence principle and therefore in that theory there is time dilation caused by acceleration and it is equal to time dilation caused by gravitation. In that theory is is necessary to calculate time dilation by acceleration in the GPS satellite case and then the resulting clock time advance is incorrect. The conclusion is that the equivalence principle is invalid in this situation.

There is time dilation because of acceleration if the mass actually is accelerating. It is the explanation why muons live longer when they are accelerated to a high speed: the reason is not SRT time dilation by velocity, the reason is time dilation by acceleration from the time when these muons were created to the time when they reached a high constant speed. But in the case of the GPS satellite, the mass of the satellite does not move in the radial direction, the radius remains at a constant value. Therefore the acceleration time dilation is not included.

Notice that time dilation because of acceleration is a misnomer: it is not time that is changed. The physical world is a three-dimensional world with movement and the rate of movement is described by an imagined time axis, but only the present moment really exists. This imagined property time is a scalar in all models where it is used. It is not three-dimensional and it only moves forward. It cannot be moving to a positive direction in one spatial direction and to a negative direction in the opposite spatial direction. Instead, redshift can do exactly this: the Doppler effect changes sign when the direction is changed to opposite. There is no time dilation of any kind, it is always a question of frequency shift that is often different in different direction. The equivalence principle is not a valid principle. It sometimes can give correct results, but only if it is understood in some special way in the given situation. Notice also that in the 1907 explanation of gravitational redshift by the equivalence principle, the point B is actually not moving and therefore acceleration caused time dilation does not happen.

After equation (14) I promised to explain why the mass m in (14) is not the electron mass and why it has to be taken as the photon apparent mass. Notice first that in order to get the correct result, it must be so. The reason is the following. The effect of the gravitational field gives a constant because the center of gravity is so far that the field is practically constant in the neighborhood of the atom. Consequently, the orbits in the atom (solutions of the Hamiltonian) are not changed, only the energy level E is changed by a constant that is a different constant at the distance R and r. Therefore m is not the electron mass: electron orbits are not affected. What is affected is the stage that changes the energy difference into a photon, i.e., emission of a photon. The mass that is in (14) is the mass of an imagined photon that is sent from the higher energy level at R to the lower energy level at r. As in gravitational redshift such a photon loses frequency and the lost energy equals the potential difference times the apparent mass of the photon. It is not necessary to physically send such a photon from the higher energy level to the lower energy level. The effect can be seen by comparing the frequencies of photons used in the atomic clock at these

two distances from the center of gravity.

5. Discussion and Conclusions

The relativistic kinetic energy formula is wrong. It is refuted by Bertozzi's measurements, see [2] and [3], but for circular orbits the apparent mass is apparent transverse mass and has the exact formula of relativistic mass.

The time dilation because of velocity in the Special Relativity Theory (SRT) is wrong. This is because the coordinate system that the Lorentz transform gives to the moving frame does not have independent coordinates. The concept of a coordinate system having independent coordinates is not the same as the concept that the coordinate vectors are not linearly independent. They are linearly independent. The condition that a coordinate system has independent coordinates means that the projection of a point to the jth coordinate axis gives the same number as in the jth place in the n-tuple $(x_1, ..., x_j, ..., x_n)$. If a coordinate system does not have independent coordinates, it is necessary to project the point explicitly to the coordinate axis in order to get the projection, not to use the value in the n-tuple identifying the point. When the time is projected to the t'-axis in the Lorentz transform, we see that the speed of light is not c in the moving inertial frame. As a consequence, basically all of SRT fails including the time dilation because of velocity.

Gravitational time dilation from General Relativity (GRT) is wrong. There is no valid solution to the Einstein equations that apply to the gravitational field of the Earth. The Schwarzschild metric is not a valid metric, see [4] and its references, and additionally it does not have locally constant speed of light. This error alone suffices to discard GRT. See also [5] and [6] for reasons why the space-time is not curved.

Electrons in an atom follow precessing elliptic orbits, they are not somewhere in an orbital. The false idea of orbitals came from Schrödinger's solution to the hydrogen atom where he did not impose the conservation law of angular momentum, but angular momentum is not only conserved, it is quantized and gives the principal quantum number, see [7] for Schrödinger's error.

As a conclusion, one can admit that Einstein did predict the gravitational redshift, though his argument by the equivalence principle fails, and he did define the energy and momentum of a photon, both useful concepts, and there is time dilation. But they are not as described by SRT or GRT, also, there is no space-time curvature. There are alternative explanations. There can always be alternative explanations, we have not found all of them, this is why one never can verify a theory by experiments.

6. References

[1] Jormakka, J., "A Look at Sommerfeld's Fine Structure Formula", ResearcGate, 2026.

[2] Jormakka, J., "Calculation of the longitudinal mass from Bertozzi's experiment", ResearchGate, 2025

[3] Jormakka, J., "Apparent momentum in Compton scattering, ResearchGate", 2025.

[4] Jormakka, J., "Why the Schwarzschild metric is not a metric and reviews from General Relativity and Gravitation", ResearcGate, 2026.

[5] Jormakka, J., "Gravity is a field in flat space, not geometry of space-time", ResearchGate", 2025.

[6] Jormakka, J., "Why the precession of Mercury cannot be explained by adding terms to the gravitational potential?", ResearcGate, 2026.

[7] Jormakka, J., "The error in Schrödinger's equation", ResearcGate, 2026.