A Paradoxical Increase in the Center of Mass Momentum of a Three Body System due to an Unknown External Impulse

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Abstract

Euler's first law states that only external impulses can increase the center of mass momentum of a multi-body system. However, after 30 test trials the mean value of known external friction impulses acting on a three-body system accounted for only ~ 8.2 per cent (standard deviation .037 and standard error .0068) of the increase in the final momentum of the system, leaving a ~ 91.8 per cent discrepancy. The three-body system consisted of two spheres, constrained to roll around quarter-circle barriers attached to a third body. As the spheres rounded the curves, centripetal contact forces acted on the spheres while equal and opposite centrifugal reactive contact forces acted on the inner walls of the curved barriers. These centrifugal reactive contact forces caused the system to accelerate, inducing an unknown external impulse that increased the

`orbital angular speed of the spheres with respect to our laboratory inertial frame. We derive a general impulse equation that demonstrates how the impulse history of centrifugal reactive contact forces couples to the increase in angular speed of the spheres due to this unknown external impulse and how this couples to the increase in momentum of the system. We suggest that more rigorous experiments in friction-free environments be conducted to see if the increase in momentum persists after all external friction impulses are removed which may shed light on the nature of the unknown external impulse.

1. Introduction

The investigation of this discrepancy as presented in this experiment as far as we know is unique and we are not aware of any prior experiments or papers that address this paradox in the existing literature. The motivation for this reported experiment arose from the results of other simple experiments.

In one experiment a sphere was constrained to travel around a quarter circle curve. The curve was attached to a platform that was free to move with one degree of freedom horizontal to the earth. There were two separate experimental tests with two separate conditions. In the first test the sphere entered the platform in the positive x-direction, rounded its quarter-circle barrier, and then exited the curve in the positive y-direction. In the first condition, the static case, the platform was held steady as the sphere rounded the curve. A velocity meter recorded the initial speed of the sphere as it entered the platform and another velocity meter attached to the platform measured the final speed as the sphere exited the quarter-circle barrier. The second meter showed that the speed of the sphere decreased due to friction effects as expected.

In the second condition, the accelerating case, we removed the constraint on the platform and allowed it to accelerate. The acceleration of the platform in the negative y-direction was due to the y-component of a centrifugal reactive contact force, opposite the centripetal contact force on the sphere, that acted on the curves in the negative y-direction. The velocity meter attached to the platform showed that the final speed of the sphere in the dynamic case increased over the speed in the static case, given the same initial speed of the sphere for both conditions.

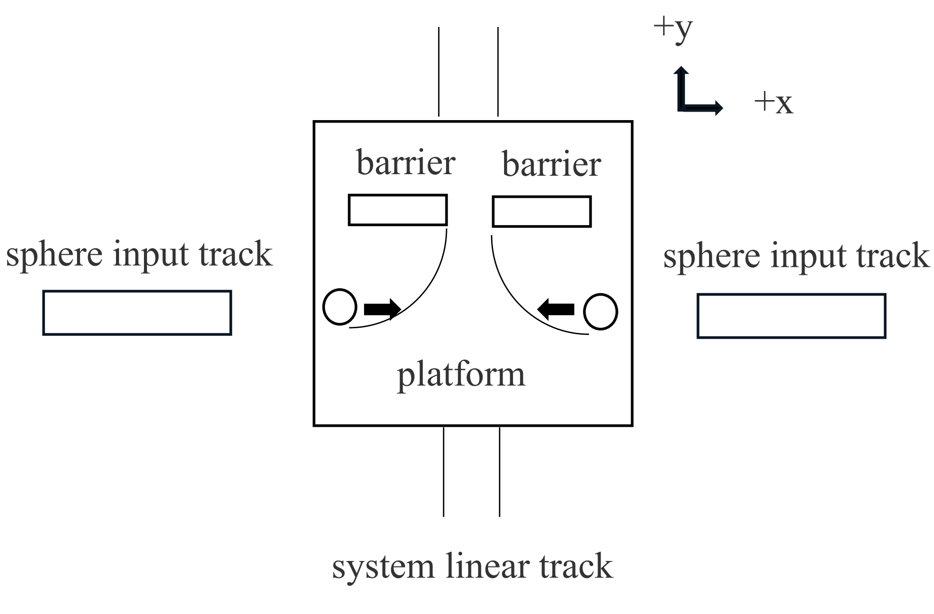
In the second experiment we allowed the sphere to enter the platform initially moving in the positive y-direction, rounding its curve, and exiting in the negative x-direction. For the dynamic case in this setup, the final speed of the sphere was even less than the speed for the static case for the same initial speed of the sphere as before. All of this suggested that acceleration of the system somehow induced a change in the orbital angular speed of the sphere.

In another experiment a rotating body (rotator) was inserted at one end into a spinning, bearing assembly that was attached to a platform that could slide along bearings attached to the platform within the grooves of a linear track. One Pasco accelerometer was attached to the platform and one to the rotator. The platform was initially butted against a barrier so that it could not move in the negative y-direction. A clockwise rotation was imparted to the rotator. When the rotator passed the 0 degrees point, the platform accelerated in the positive y-direction. The accelerometer reported the data via Blue Tooth to a Pasco program on a computer that plotted the graph of the acceleration. The data graphed a sinusoidal wave with zero acceleration for the platform at the zero degrees point, a maximum acceleration at the 90 degrees point, and zero acceleration at the 180 degrees point, followed by a maximum acceleration at the 270 degrees point and so on. When the rotator passed the 90 degrees and the 270 degrees point, the accelerometer on the rotator showed a bump up in orbital angular speed of the rotator. This agreed with the results of the first test, second condition of the above experiment with the sphere.

All of these experiments motivated us to perform experiments that answered a basic question. Since the same increase in angular speed is observed in both the accelerated frame and in our inertial frame is it possible that with respect to our inertial frame, could there be an increase in the center of mass momentum of a linearly-accelerated system relating to this frame invariant increase in angular speed?

2. Materials and Methods

Figure 1



In this experiment we attached a platform to the top of a Pasco Smart Cart which was free to travel on top of an aluminum linear track horizontal to the earth with one degree of freedom as shown above in Figure 1. Attached to the surface of the platform were two quarter-circle curves and two barriers. The platform, curves, barriers, and Pasco Smart Cart taken together constituted the mass designated as m_2. Two spheres of equal mass, designated by m_1, entered the surface of the platform simultaneously, rolling along the x-axis with equal initial velocities, designated by v_i. The two spheres were constrained to travel around quarter-circle barriers. One sphere rounded a curve on the left and traveled counter-clockwise from 270 degrees to 0 degrees and the other sphere rounded a curve on the right and traveled clockwise from 270 degrees to 180 degrees.

As the spheres rounded their curves, centrifugal reactive contact forces, opposite the equal and opposite centripetal contact forces on the spheres, acted on the curves and caused the platform-cart assembly to accelerate in the negative y-direction. Immediately, after rounding and existing the curves, both spheres collided with barriers in the positive y-direction, causing the spheres-platform system to acquire a post collision momentum in the positive y-direction. The time and velocity values of the system were communicated by Blue Tooth from the Pasco Smart Cart to a laptop.

It was crucial in this experiment that the two spheres entered the platform with the same speed and that they entered the surface of the platform simultaneously. This was necessary in order to cancel out any forces acting on the platform along the x-axis. We were only interested in forces that acted along the y-axis.

Each sphere had a m_1 mass of .5330 kg ± .0001 kg. The total m_2 mass was .4492 kg ± .0001 kg. A digital level with a resolution of .1 degrees was used to check if the linear track was level with respect to the horizontal x and y-axes. Each barriers the spheres collided into contained neodymium magnets to prevent bounce back after the collision of the spheres. The mass of each magnet was 14.7 ± .1 g.

The core of this experiment was to measure the impact of friction impulses on the system to see if they accounted for all of the final momentum of the system. When the platform rolled backwards in the negative y-direction as the spheres rounded their curves, external friction impulses acted on the system in the positive y-direction. The friction force for each trial was determined by dividing the post collision momentum of the system by the time it took the system to come to a stop. The friction impulse was calculated by multiplying the time the cart moved in the negative y-direction times this friction force.

It was not necessary to determine the rolling friction between the spheres and platform's surface and curved barriers since these were internal force pairs that would have no impact on the center of mass of the system. Air friction effects were also ignored since the speeds of the cart and spheres were small enough that any air friction drag effects would have negligible impact on the outcome.

Note, the initial speed of the spheres in this experiment had a mean average of ~ .66 m/s. This was not some kind of resonant speed that led to the reported discrepancy. In any replication of this experiment what is essential is the ratio of the mass of the spheres to the mass of the platform and the initial speed of the spheres. A higher ratio of the masses and a greater initial speed should lead to a smaller mean percentage contribution of the friction impulses.

A 50 Hz sample rate was used in the Pasco software for collecting the velocity and time data to reduce the signal to noise ratio. Higher rates would be too sensitive to vibrations and noise and would obscure the relevant information.

3. Results

Table 1. Data summary of 30 Test Trials

Initial Sphere Velocity m/s	Back Time s	Friction Force N	Friction Impulse N · s	Post Collision Velocity m/s	Post Collision Momentum Kg · m/s	Impulse to Momentum Ratio
0.65	0.154	0.0249	0.0038	0.047	0.071	0.05
0.65	0.116	0.0704	0.0082	0.053	0.080	0.10
0.65	0.171	0.0131	0.0022	0.025	0.038	0.06
0.66	0.148	0.0144	0.0021	0.028	0.042	0.05
0.65	0.141	0.0281	0.0040	0.033	0.050	0.08
0.65	0.192	0.0309	0.0059	0.051	0.077	0.08
0.66	0.264	0.0360	0.0095	0.034	0.052	0.18
0.65	0.137	0.0492	0.0067	0.077	0.12	0.06
0.65	0.186	0.0370	0.0069	0.044	0.067	0.10
0.65	0.132	0.0292	0.0039	0.032	0.048	0.08
0.65	0.135	0.0206	0.0028	0.035	0.053	0.05
0.66	0.155	0.0498	0.0077	0.047	0.071	0.11
0.66	0.164	0.0489	0.0080	0.06	0.091	0.088
0.66	0.2	0.0323	0.0065	0.046	0.070	0.09
0.67	0.212	0.0463	0.0098	0.063	0.095	0.10
0.65	0.136	0.0374	0.0051	0.039	0.059	0.09
0.66	0.195	0.0543	0.0106	0.033	0.050	0.21

0.67	0.195	0.0150	0.0029	0.023	0.035	0.08
0.67	0.132	0.0349	0.0046	0.052	0.08	0.06
0.65	0.186	0.0410	0.0076	0.059	0.089	0.09
0.66	0.133	0.0172	0.0023	0.036	0.055	0.04
0.66	0.19	0.0175	0.0033	0.045	0.068	0.05
0.65	0.149	0.0379	0.0056	0.037	0.06	0.10
0.65	0.198	0.0147	0.0029	0.028	0.042	0.07
0.65	0.152	0.0388	0.0059	0.042	0.06	0.09
0.66	0.127	0.0147	0.0019	0.032	0.048	0.04
0.66	0.2	0.0219	0.0044	0.028	0.04	0.10
0.66	0.19	0.0172	0.0033	0.037	0.056	0.06
0.66	0.21	0.0124	0.0026	0.027	0.041	0.06
0.73	0.137	0.0249	0.0034	0.051	0.08	0.04

Statistical summary of impulse to momentum ratio for 30 test trials

Mean .082

Std Dev .037

Std Error .0068

Constants Used in Table 1 Calculations

Total Mass (kg) 1.5152 ± 0.0002 kg

The back time was the time the platform moved backwards from its initial at rest state to the time it attained its maximum value. Post collision velocity was the maximum velocity of the platform right after the collision of the two spheres with their barriers.

4. Discussion

The intermediate mechanism of how the center of mass of the system acquires a net momentum is demonstrated by applying the general impulse equation. We begin its derivation with the centrifugal reactive contact force acting on the inside of the curved barriers. This force has a magnitude of $nkm_1r\omega^2$. In this expression n is the number of spheres, $k = \frac{m_2}{nm_1+m_2}$, m_1 is the mass of each sphere, m_2 is the mass of the platform, r is the radius of curvature of the path of the center of mass of each sphere , and ω is the orbital angular velocity of each sphere. We will assume the orbital angular speed ω of each sphere remains constant as they round their curves in this derivation and we ignore friction between the sphere and the curves and platform surface since these internal force pairs will have no impact on the center of mass of the system.

Finally, since we are only interested in the y-component of the impulse J, we include a sine expression factor in front of the force equation. The final form of the y-component of the reactive centrifugal force equation is given by:

$$f_y = sin(\omega t)\, nkm_1r\omega^2 \qquad (1)$$

The k arises because we assume the spheres and m_2 are rigid bodies, and therefore, have the same instantaneous y-component of acceleration.

We integrate the force with respect to time to determine the impulse J on m_2 with respect to the y-axis. This integral is expressed by:

$$J_y = \int_{\frac{\theta_i}{\omega}}^{\frac{\theta_f}{\omega}} n sin(\omega t) k m_1 r \omega^2 \, dt \quad (2)$$

Recognizing the indefinite integral of $\int \sin(\omega t)\, dt$ is equal to $-\frac{1}{\omega}\cos(\omega t) + C$ and applying this to equation (2) and after moving the constants out of the expression, we evaluate the above integral, giving us the general impulse equation:

$$J_y = n\, k\, m_1 r\omega\left[cos(\theta_i) - \cos(\theta_f)\right] \quad (3)$$

Alternatively, we can set $r\omega = v_i$, which gives us:

$$J_y = n\, k\, m_1 v_i \left[cos(\theta_i) - \cos(\theta_f)\right] \quad (3a)$$

For the special case where $\theta_i = 0$ degrees and $\theta_f = 90$ degrees or $\Theta_i = 90$ degrees and $\Theta_f = 0$ degrees , the above reduces to:

$$J_y = n\, k\, m_1 v_i \quad (3b)$$

We now apply the above impulse equations in hypothetical examples that only focuses on the impact the speed of a sphere has on the center of mass momentum of a system. We begin with a constant sphere velocity case to show that the momentum of the center of mass of the system does not change. To simplify, we use one sphere. The masses of m_1 and m_2 each is 1 kg. The initial speed of the sphere is 1 m/s. We simplify further by invoking Equation (3b) for $\theta_i = 270$ degrees and $\theta_f = 0$ degrees, which gives an impulse of - .5 N · s on m_2.

From this we determine the momentum of m_2 in the negative y-direction to be -.5 kg · m/s. The velocity is -.5 m/s.

The momentum of the sphere with respect to our lab frame is .5 kg · m/s. This is determined by applying the Galilean transformation equation, adding the velocity of m_2 to the velocity of m_1 with respect to the m_2 frame, which gives us the velocity of the sphere with respect to our laboratory frame and then multiplying by the mass of the sphere. Hence, the total momentum of the center of mass of the system is equal to -.5 kg · m/s plus .5 kg · m/s which is equal to 0 kg · m/s. This was the initial momentum of the system along the y-axis, hence, momentum is conserved.

This conservation of momentum is expected for the constant speed case as it follows from the principle of least action. In the absence of any external force the sphere should retain its constant speed along the curve and the momentum state of the total system should be unchanged as a consequence, all of which follows from Newton's first law of inertia which can be derived from the Euler-Lagrange equation.

We now show the case where a hypothetical external impulse acts on the sphere which increases the orbital angular speed of the sphere as it rounds the curve. We keep everything the same as above except the speed of the sphere increases by .1 m/s for every 5 degrees angular displacement from 270 degrees to 0 degrees. We apply the general impulse equation (3a) for each angular increment, then sum the incremental impulses. Table 2 below summarizes the results.

Table 2. Increasing Orbital Angular Speed Result

Sphere Initial Speed	Sphere Final Speed	Initial Angle	Final Angle	Incremental Impulse
M/S	M/S	Degrees	Degrees	N · S

1	1	270	275	-0.04358
1.1	1.1	275	280	-0.04757
1.2	1.2	280	285	-0.0511
1.3	1.3	285	290	-0.05408
1.4	1.4	290	295	-0.05642
1.5	1.5	295	300	-0.05804
1.6	1.6	300	305	-0.05886
1.7	1.7	305	310	-0.05883
1.8	1.8	310	315	-0.05789
1.9	1.9	315	320	-0.05599
2	2	320	325	-0.05311
2.1	2.1	325	330	-0.04922
2.2	2.2	330	335	-0.04431
2.3	2.3	335	340	-0.03839
2.4	2.4	340	345	-0.03148
2.5	2.5	345	350	-0.0236
2.6	2.6	350	355	-0.0148
2.7	2.7	355	0	-0.00514

This time the sum of the incremental impulses from the last column equals ~ -0.7464 N · s. From this as before we determine the momentum of m_2 in the negative y-direction to be ~ -.746 kg · m/s. The momentum of the sphere with respect to our lab frame is ~1.95 kg · m/s.

The sum of the two gives us a net increase in momentum of the center of mass equal to ~1.2 kg · m/s in the positive y-direction.

5. Conclusion

This experiment reported that after 30 test trials, the external friction impulses only accounted for a fraction of the total final momentum of the system. However, we have shown that the discrepancy might be resolved if we assume the increase in orbital angular speed of the spheres is due to an external impulse that acts on the spheres. The application of the general impulse equation showed how the impulse history of the centrifugal reactive contact force couples this increase in speed to an increase in the center of mass momentum of the system. As to the origin of this external impulse, we leave it as an open question and an unresolved paradox.

It is recommended that further experiments be conducted in a near zero friction environment on an air track or within a plane in a zero-g dive or by a CubeSat in space where the effects of external friction forces would be reduced significantly. Appendix A contains a set of equations that can be used for further testing and confirmation of the effect in near frictionless environment.

Appendix A

This appendix presents a set of five equations for predicting the motions of a more complicated system that could be used to confirm or disapprove that the paradox continues. These equations apply in idealized environments where external friction impulses or other external impulses are nearly eliminated such as in the environment of inter-stellar space. In this more elaborate system, the total mass m is the sum of the masses of the spheres, the platform, and all attachments to the platform. We need to define what a cycle N means. In this system two spheres are propelled by the release of two compressed springs, afterwards the spheres enter and round quarter-circle curves, then have elastic collisions with elastic barriers after existing the curves, bounce back, and then round the curves again in the opposite direction, returning to their initial positions, and finally resting against the two recompressed springs. This is defined as one cycle N. We denote a virtual external impulse J_{virtual}, defined by measuring the change in momentum of the system after one cycle.

The compressed springs which propel the spheres store elastic potential energy (epe) which is a scalar or invariant quantity that does not depend on the velocity of the system. Denoting the total number of springs as n and the energy per compressed spring as epe, the cumulative energy applied to the spheres after N cycles is:

$$E_{\text{cumulative}} = N\, n\, epe \qquad (4)$$

The cumulative final velocity of the system, with external impulse J_{virtual}, after N cycles is:

$$v_{f\ \text{system}} = \frac{N\, J_{\text{virtual}}}{m} \tag{5}$$

Substituting $v_{f\ \text{system}}$ into the formula for kinetic energy yields:

$$KE_{\text{final}} = \frac{(N\, J_{\text{virtual}})^2}{2m} \tag{6}$$

The cycle N at which KE_{final} equals $E_{\text{cumulative}}$ is determined by setting Equation (4) equal to Equation (6), then solving for N, giving us:

$$N = \frac{2\, m\, epe}{J_{virtual}^2} \tag{7}$$

For other engineering applications and insight, it is useful to graph both Equation (4) and Equation (6) to visually see their relationship with N as the x-axis and energy as the y-axis.

The value for m can easily be predetermined by direct measurement. The value for epe can be calculated knowing the spring constant and the degree of compression. The virtual impulse J_{virtual} is determined experimentally by activating a single cycle when the system is at rest and measuring the system's final velocity v_f, giving us the virtual impulse as:

$$J_{\text{virtual}} = m\, v_f \tag{8}$$

Declaration of AI Use

During the preparation of this work, the author used generative AI to assist with academic formatting.   The author reviewed and edited the content and takes full responsibility for the publication.

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Data Availability

Pasco data relating to the experiments is available for review upon request.