An experiment proposal testing 'spooky action at a distance'

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Abstract:

The results of numerous Bell tests are viewed as evidence of quantum entanglement: the instant inner connection between totally separated entangled particles, which explains the 'spooky action at a distance', a term coiled by Einstein. The paper proposes a variation to Bell tests, aiming to prove or disprove spooky action from a distance. The main variation is an added pair of half-waveplates, with which we can change the relative polarization angle of the entangled photon pairs and assess its impact. The paper has derived the quantum prediction of Bell tests due to the changes in both the relative polarisation angle of the entangled photon pairs and in the relative angle of the polarisation measurement devices. If the quantum prediction is proven correct by experiments, the claim of instant inner connection between entangled particles or spooky action from a distance must be false.

Keywords: Bell test, quantum coherence, quantum entanglement, polarization angle, half-waveplate

1. Introduction

Since John Bell (1964)[1] proposed a test for a claim by Albert Einstein, his two assistants Boris Podolsky and Nathan Rosen (1935)[2] that the quantum theory is incomplete, numerous Bell tests (Aspect et al, 1982[3] , Tittel et al, 1998[4], Pan et al, 2000[5], Rowe et al 2001[6], Groblacher et al, 2007[7], Ansmann et al, 2009[8], Giustina et al, 2013[9], Hensen et al 2015[10], Shalm et al, 2015[11], Schmied et al, 2016[12], BIG Bell test, 2018[13], Shin et al, 2019[14], Thenabadu et al, 2020[15], Storz et al, 2023[16]) have been conducted and the results of tests reject the local hidden variable hypothesis and support the concept of quantum entanglement. The standard interpretation of Bell test results is that the measurement of quantum state of a particle at location A will affect the quantum state of the entangled counterpart at location B immediately, i.e. the 'spooky action at a distance' termed by Einstein.

However, how this spooky action works is questionable. While many take the claim as given and believe there is an inexplicable inner connection between entangled particles, some try to explicate the mechanism behind this instantaneous effect. One postulate is that information can be communicated between entangled particles at superluminal speed. This postulate is against the maximum speed of light established by Einstein's Relativity Theory. Moreover, if we interpret 'instantly' as 'in no time', the communication speed must be infinite. This is unphysical, thus the postulate is untenable. The other postulate is that there are superposition copies of entangled particles in each location, so no signalling over distance is required for the instantaneous effect. While the concept of superposition has been accepted in quantum mechanics and superposition of quantum states in a micro world is understandable, superposition of entangled particles over a vast distance (e.g. million light-years) can be accepted only for a spiritual world, not in a physical world.

Are the experimental results showing violating Bell theorem caused by spooky action at a distance? This paper intends to redesign the conventional Bell test to answer the question. While a conventional Bell test examines the results of measuring quantum state of particles, the redesigned Bell test keeps the settings of quantum state measurement unchanged but changes the setting of the quantum states of the particles before measurement. Consequently, any results of violating Bell theorem is due to the quantum-state phase setting, rather due to the spooky action of a change in setting for measuring the quantum states.

The remaining paper is organised as follows. Section 2 describes the traditional Aspect-style Bell test and provides an equivalent but simplified test. Section 3 shows the redesigned Bell tests for 2-channel and 4-channel coincidence detection. Based on quantum mechanism, Section 4 derives the expected results for the redesigned Bell test. Section 5 provides the key requirements for the experimental setup. Section 6 concludes.

2. Conventional Bell tests

The settings of Bell tests performed vary considerably. Here we describe the conventional bell test based on the setting of Aspect (1982), shown in panel (a) of Fig.1.

The light source S is a spontaneous parametric down-conversion source that generates photon pairs A and B , which have the same but random polarisation angle β. The flipping mirrors A and B are controlled by a random device so that they can randomly flip in or out of position. When the flipping mirrors are out of position, the photon pair A and B will arrive the polarization analysers A1 and B1, respectively. If they pass the polarisation analysers, they will be detected by detectors A1 and B1. On the other hand, if the flipping mirrors are in position, the photon pairs will be reflected upwards and reach polarisation analysers A2 and B2, possibly be detected by detector A2 and B2. The detected signals are sent to a 4-channel coincidence detector and the correlation rate of the coincidence can be calculated.

The coincidence rates from the past Bell tests show that the changes in polarisation of photon pairs at the analysers are correlated. This result serves as evidence of quantum entanglement: when photon A changes polarisation at the polarization measurement device, it will cause the polarisation of photon B to change, so the polarisation changes of 2 photons are coordinated and thus correlated.

Fig. 1. Illustration of a Bell test with 4-channel coincidence

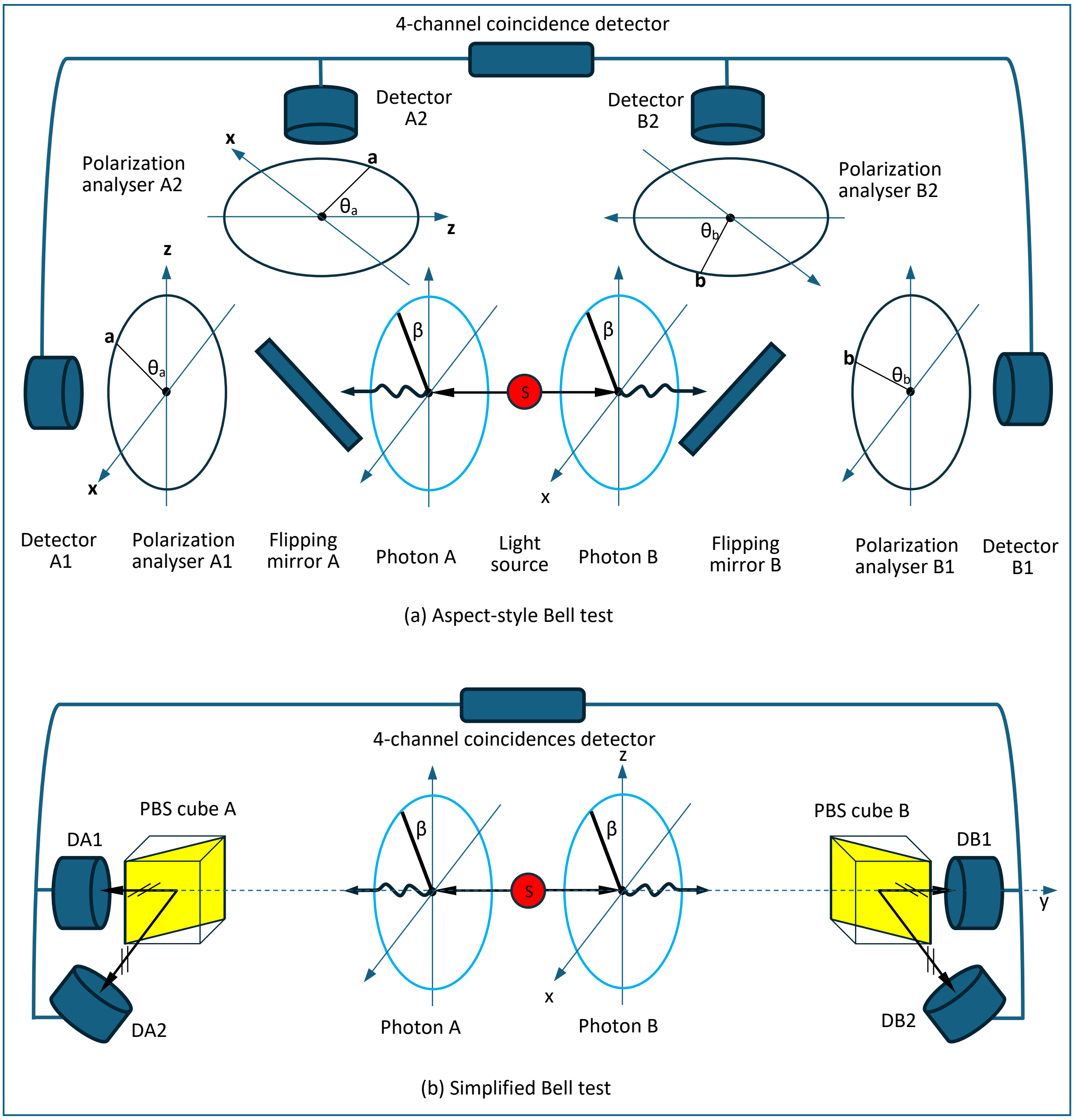


Fig. 2. Illustration of a Bell test with 4-channel coincidence

The setup in panel (a) involves a pair of flipping mirrors, and four polarisation analysers, and a random device. The function of these components can actually be replaced by a pair of polarisation beam splitter (PBS) cubes, as shown in panel (b). Each PBS cube serves as a pair of perpendicular polarisers: the photon with polarisation parallel to the split surface is reflected while the photon with orthogonal polarisation passes through. In wave theory of light, the PBS cube will split the polarized light to two orthogonal polarizations to be detected by the detectors behind the cube. However, since the parametric down conversion source emits a photon pair each time, only one photon arrives the PBS cube each time. The single photon cannot be divided so it has to make a probability-based choice of going through or being reflected. As such, the cube act as combined function of random flipping mirror and polarisation analysers. The 4 photon detectors DA1, DA2, DB1, and DB2 detect photons after the PBS, and the signals are sent to a 4-channel coincidence detector. By turning one of the PBS around the y-axis shown by the dashed line, we can change the relative angle θ of two PBSs (In Fig.1, the relative angle θ is zero). According to quantum mechanics, the correlation rate of the coincidence counts is:

$$P = \cos\theta \qquad (1)$$

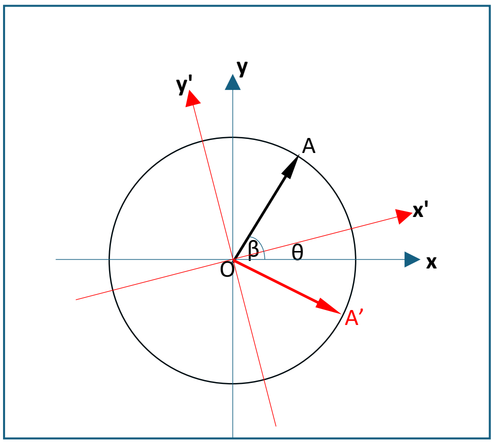
This coincidence rate gives an optimal angle of 22.5 degrees for a maximum violation of Bell theorem.

3. A variation to conventional Bell tests

In the above Bell test, we change the setting of devices for measuring the quantum state of the photon pairs (i.e. the setting of polarisation analysers in panel (a) or the relative angle of the PBS cubes in panel (b)) and assess its impact on coincidence rates, so the experimental results can be interpreted as the impact of measuring the quantum state of one photon on the quantum state of its entangled counterpart. To assess if the setting of measuring quantum state plays a non-replaceable role in the experimental results, we keep the settings of quantum state measurement unchanged but changes the setting of the quantum states of the particles through a pair of half-waveplates.

Waveplates are made out of a birefringent material (such as quartz or mica) with a carefully chosen crystal axes orientation and thickness. The index of refraction along the two perpendicular axes (fast and slow axes) is different for linearly polarized light, so wave plates retards one component of polarization with respect to its orthogonal component and thus leads to change in overall polarization. Since the function of a half-waveplate is crucial for the experimental setup, we briefly explain it with the aid of Fig.2.

Fig.2. polarization change around fast/slow axis of a half-waveplate



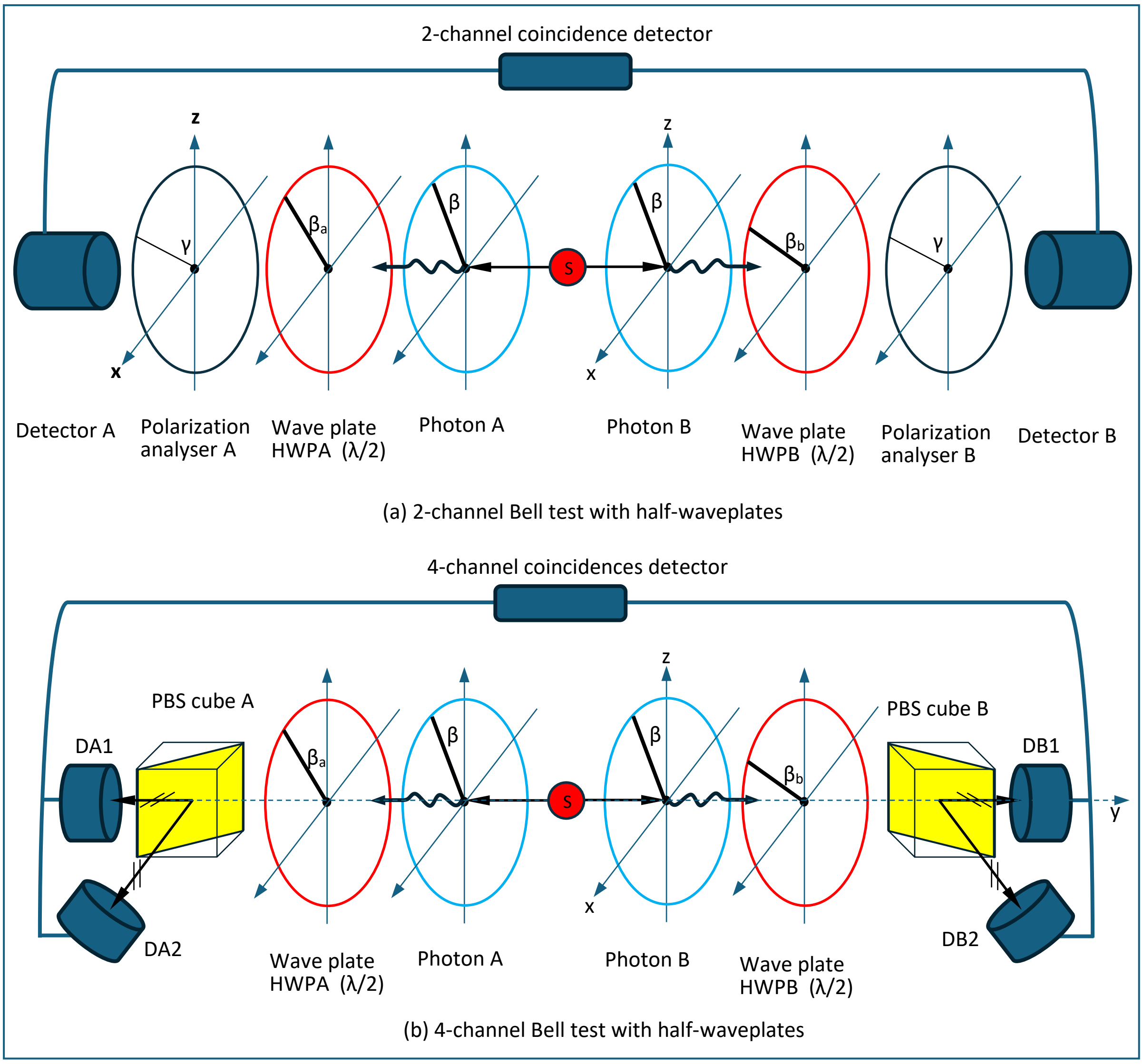
In a 2D system shown in Fig.2, the fast axis of the half-waveplate is x' axis, which forms an angle of θ with the x axis, and the slow axis is y' axis. The polarization of a photon is shown as OA, which has an angle of β with the x axis. After the half-waveplate, the polarization of the photon will rotate around the fast axis (i.e., the x' axis) by an angle of β – θ to OA'. As a result, the polarization angle of OA' is

$$\beta' = -(\beta - 2\theta) = 2\theta - \beta \qquad (2)$$

This rule also applies to the slow axis y'. For a special case where the polarization of the photon is on x axis, β=0, so the polarization after the half-waveplate is 2θ.

The variation to the Bell test is shown in Fig.3. Panel (a) shows the case of 2-channel coincidence. Compare with traditional 2-channel Bell test, it simply adds a pair of half waveplates HWPA and HWPB before the polarization analysers.

Fig. 3. A variation to Bell test



The light source emits a photon pair of the same but random polarization angle β (i.e. zero relative polarization). Given the fast axes of HWPA and HWPB of an angle β_a and β_b, respectively, the polarization angle of the photon pair after HWPA and HWPB will be

$\beta_a' = 2\beta_a - \beta$ (3)

$\beta_b' = 2\beta_b - \beta$ (4)

The new polarization angles are random (due to random β) and are not equal anymore, however, the relative polarization angle of the photon pair is:

$\beta'_{ab} = (2\beta_a - \beta) - (2\beta_b - \beta) = 2(\beta_a - \beta_b) = 2\beta_{ab}$

This is a fixed value determined by the orientations of the half-waveplates. In other words, using a pair of half-waveplates changes the relative polarization angle of the photon pair, but does not affect the coherence of the pair.

Since we are testing the impact of the relative polarization angle of the photon pair, we keep the setting of the polarization analysers unchanged. For simplification, we can set the two analysers to the same polarization angle.

Panel (b) of Fig.3 shows the case of 4-channel coincidence. The setting for the source and half-waveplates are the same as in the case of 2-channel coincidence, so the relative polarization angle of the photon pair is also $2(\beta_a - \beta_b) = 2\beta_{ab}$. For simplification, we can set the two PBS cubes perpendicular to y axis and make sure the splitting faces with the same angle with y axis, so the relative angle of the two PBS cubes is zero. By varying the angle of β_a or β_b, we can evaluate the impact of β_{ab}.

4. Quantum prediction for the modified Bell test

For a photon pair arriving at A and B from the source, we assume they have the same polarisation. In terms of a normalised wavefunction, it can be expressed as:

$$|\psi> = (|H_A>|H_B> + |V_A>|V_B>)/\sqrt{2} \quad (5)$$

Assuming the photon pair has a polarization angle of β and the half-waveplate angle is set an angle θ, based on eq. (2) we can calculate the photon polarization angle after the half-waveplate as $(2\theta - \beta)$. As such the horizontal direction H and vertical direction V will rotate by an angle $(2\theta - \beta)$ to $\widetilde{H}$ and $\widetilde{V}$, respectively. The new state $|\widetilde{H}>$ and $|\widetilde{V}>$ can be obtained by rotating the initial states |H> and |V> by an angle $(2\theta - \beta)$. Based on the 2D rotation matrix, we can obtain:

$$|\widetilde{H}> = |H> cos(2\theta - \beta) + |V> sin(2\theta - \beta) \quad (6)$$

$$|\widetilde{V}> = -|H> sin(2\theta - \beta) + |V> cos(2\theta - \beta) \quad (7)$$

In considering that θ=β_a for half-waveplate A and θ=β_b for half-waveplate B, the wavefunction for the photon pair after the half-waveplates becomes:

$$|\tilde{\psi}> = \frac{|\tilde{H}_A>|\tilde{H}_B>+|\tilde{V}_A>|\tilde{V}_B>}{\sqrt{2}} = \frac{1}{\sqrt{2}}\{[(|H_A> cos(2\beta_a-\beta)+|V_A> sin(2\beta_a-\beta)][(|H_B> cos(2\beta_b-\beta)+|V_B> sin(2\beta_b-\beta)] +[-|H_A> sin(2\beta_a-\beta)+|V_A> cos(2\beta_a-\beta)][-|H_B> sin(2\beta_b-\beta)+|V_B> cos(2\beta_b-\beta)]\} = \frac{1}{\sqrt{2}}[|H_A> H_B> cos2(\beta_a-\beta_b)+|V_A>|V_B> cos2(\beta_a-\beta_b)-|H_A> V_B> sin2(\beta_a-\beta_b)+|V_A> H_B> sin2(\beta_a-\beta_b)] \quad (8)$$

We consider two kinds of settings for the polarization measurement devices, i.e. the PBS cubes.

First, we consider the case shown in Fig.3(b), where the PBS cubes are perfectly place so that the photons passing through the cubes travels horizontally and photons reflected by the cubes travels vertically. For the photons passing through both the H direction at PBS cube A (to DA1) and the H direction at PBS cube B (to DB1), i.e. with +1 values at both A and B, the amplitude of the wave function can be calculated as:

$$\mathcal{L}(++) = (<H_A|<H_B|)|\tilde{\psi}> = \frac{cos2(\beta_a-\beta_b)}{\sqrt{2}} \quad (9)$$

Similarly, the amplitude of photons passing through V (recorded as -1) at both A and B is:

$$\mathcal{L}(--) = (<H_A|<H_B|)|\tilde{\psi}> = \frac{\cos 2(\beta_a-\beta_b)}{\sqrt{2}} \quad (10)$$

The amplitude for photons passing through H at A and V at B is:

$$\mathcal{L}(+-) = (<H_A|<V_B|)|\tilde{\psi}> = -\frac{\sin 2(\beta_a-\beta_b)}{\sqrt{2}} \quad (11)$$

The amplitude for photons passing through V at A and H at B is:

$$\mathcal{L}(-+) = (<V_A|<H_B|)|\tilde{\psi}> = \frac{\sin 2(\beta_a-\beta_b)}{\sqrt{2}} \quad (12)$$

The probability of photons passing through each combination of axes is the amplitude squared, so we have:

$$p(++) = p(--) = \frac{\cos^2 2(\beta_a-\beta_b)}{2} \quad (13)$$

$$p(+-) = p(-+) = \frac{\sin^2 2(\beta_a-\beta_b)}{2} \quad (14)$$

The expected values of photons passing through and being detected by DA1, DA2, DB1, and DB2, can be obtained as follows:

$$E(\beta_a, \beta_b) = (+1)(+1)p(++) + (-1)(-1)p(--) + (-1)(+1)p(-+) + (+1)(-1)p(+-)$$
$$= \cos^2 2(\beta_a - \beta_b) - \sin^2 2(\beta_a - \beta_b) = \cos 4(\beta_a - \beta_b) = cos4\beta_{ab} \quad (15)$$

Where β_{ab} is the relative angle of fast axes of the two half-waveplates.

The above expected value is verified by numerous Aspect-style 4-way coincidence Bell tests:

$$E(\beta_a, \beta_b) = \frac{N(++)+N(--)-N(+-)-N(-+)}{N(++)+N(--)+N(+-)+N(-+)} \quad (16)$$

If we change the rotation angle at A to θ_a' and/or change the rotation angle at B to θ_b' , we can calculate the expected value at each setting as:

$$E(\beta_a, \beta_{b'}) = cos4\beta_{ab'} \quad E(\beta_{a'}, \beta_b) = cos4\beta_{a'b} \quad E(\beta_{a'}, \beta_{b'}) = cos4\beta_{a'b'} \quad (17)$$

Following Clauser and Shimony (1978)[17] and Aspect et al (1981), we can obtain the optical angles at which the maximum violation of Bell theorem: $\beta_{ab} = \frac{\pi}{16}, or\ \frac{3\pi}{16}$.

Second, we consider the case where the PBS cubes are so placed that the photons passing through the cubes travels horizontally but photons reflected by the cubes travels with a deviation angle of θ_a from the vertical direction at cube A and a deviation angle of θ_b at cube B. In this case, the measurement operation at A and B can be described as:

$$< \breve{H}_A| =< H_A|cos\theta_a +< V_A|sin\theta_a \quad (18)$$

$$< \breve{H}_B| =< H_B|cos\theta_b +< V_B|sin\theta_b \quad (19)$$

$$< \breve{V}_A| = -< H_A|sin\theta_a +< V_A|cos\theta_a \quad (20)$$

$$< \breve{V}_B| = -< H_B|sin\theta_b +< V_B|cos\theta_b \quad (21)$$

The photons passing through $\breve{H}$ have values of +1 at both A and B, and the amplitude of the wave function can be calculated as:

$$\mathcal{L}'(++) = \left(< \breve{H}_A|< \breve{H}_B|\right)|\tilde{\psi} > = (< H_A|cos\theta_a +< V_A|sin\theta_a)(< H_B|cos\theta_b +< V_B|sin\theta_b)|\psi >$$
$$= \{(< H_A|< H_B|cos\theta_a cos\theta_b +< V_A|< V_B|sin\theta_a sin\theta_b +< H_A| < V_B|cos\theta_a sin\theta_b +$$
$$< H_B| < V|cos\theta_b sin\theta_a)\}\frac{1}{\sqrt{2}}[|H_A > H_B > cos2(\beta_a - \beta_b) + |V_A > |V_B$$
$$> cos2(\beta_a - \beta_b) - |H_A > V_B > sin2(\beta_a - \beta_b) + |V_A > H_B > sin2(\beta_a - \beta_b)]$$
$$= \frac{\cos(\theta_a - \theta_b)\, cos2(\beta_a - \beta_b) + \sin(\theta_a - \theta_b)\, sin2(\beta_a - \beta_b)}{\sqrt{2}}$$
$$= \frac{\cos\,[(\theta_a - \theta_b) - 2(\beta_a - \beta_b)]}{\sqrt{2}} = \frac{cos(\theta_{ab} - 2\beta_{ab})}{\sqrt{2}} \quad (22)$$

Similarly, the amplitude of photons passing through $\breve{V}$ at both A and B is:

$$\mathcal{L}'(--) = \left(<\breve{V}_A\middle|<\breve{V}_B\middle|\right)|\tilde{\psi}>= \frac{cos(\theta_{ab}-2\beta_{ab})}{\sqrt{2}} \quad (23)$$

The amplitude for photons passing through $\breve{H}$ at A and $\breve{V}$ at B is:

$$\mathcal{L}(+-) = \left(<\breve{H}_A\middle|<\breve{V}_B\middle|\right)|\tilde{\psi}>= \frac{sin(\theta_{ab}-2\beta_{ab})}{\sqrt{2}} \quad (24)$$

The amplitude for photons passing through $\breve{V}$ at A and $\breve{H}$ at B is:

$$\mathcal{L}(-+) = \left(<\breve{V}_A\middle|<\breve{H}_B\middle|\right)|\tilde{\psi}>= \frac{-sin(\theta_{ab}-2\beta_{ab})}{\sqrt{2}} \quad (25)$$

The probability of photons passing through each combination of axes is the amplitude squared, so we have:

$$p(++) = p(--) = \frac{\cos^2(\theta_{ab}-2\beta_{ab})}{2} \quad (26)$$

$$p(+-) = p(-+) = \frac{\sin^2(\theta_{ab}-2\beta_{ab})}{2} \quad (27)$$

The expected values of photons passing through all combinations of axes of setting (a,b), i.e. rotating polarisation analyser at A by θ_a and the analyser at B by θ_b, can be obtained as follows:

$$E(a,b) = (+1)(+1)p(++) + (-1)(-1)p(--) + (-1)(+1)p(-+) + (+1)(-1)p(+-)$$
$$= \cos^2(\theta_{ab}-2\beta_{ab}) - \sin^2(\theta_{ab}-2\beta_{ab}) = \cos 2(\theta_{ab}-2\beta_{ab}) = \cos 2(\theta_{ab}-\beta'_{ab}) \quad (28)$$

Where θ_{ab} is the relative angle of polarization measurement devices, β_{ab} the relative angle of the fast axes of half-waveplates, β'_{ab} the relative angle of the entangled photon pair after the half-waveplates or just before the polarization measurement devices – the PBS cubes.

With the same approach as before, we can obtain the angle at which the maximum violation of Bell theorem:

$$\theta_{ab} - 2\beta_{ab} = \pm\frac{\pi}{8}, or\ \theta_{ab} - 2\beta_{ab} = \pm\frac{3\pi}{8} \quad (29)$$

5. Key elements in experimental setup

(1) Optical alignment for light source production

The laser alignment and calibration of the laser polarizer and quartz plate are crucial to obtain the entangled photon pair through parametric down-conversion photon production. The requirements for photon pair production are detailed in Dehlinger and Mitchell (2002)[18], Waseem et al (2020)[19] and Lahoz Sanz et al (2024)[20].

(2) Alignment of PBS cubes.

Proper positions of PBS cubes are important to make sure the passing through light and the reflected light are orthogonal. An appropriate adjustment is recommended to make sure the reflected light is vertical (i.e. the relative angle of the PBSs is 0). If the relative angle of the PBSs is θ, the predicted correlation rate is:

$P=\cos[\pm(\theta_{ab}-2\beta_{ab})]$, which gives an optimal angle between the fast axes of half-wavelength plates of ($\theta_{ab}/2 \pm 11.25$) degrees, corresponding to the maximum violating Bell test.

A series of experiments can be done at different values for θ_{ab} and β_{ab} so as to verify the above formula for maximum violation angle.

(3) No extra waveplates should be added.

We saw in section 2 that the use of a pair of half-waveplates can change the relative polarization angle of the photon pairs but maintain the coherence of the photon pairs. Use odd number of half-waveplates, however, can reduce or even destroy the coherence of the photon pairs. For example, given that the source emits a photon pair of a random polarization angle β, using one half-waveplate of an angle θ can change the polarization of one photon of the pair to $\beta' = 2\theta - \beta$, so the relative polarization angle of the photon pair becomes $2\theta - 2\beta$. No matter what the setting of half-waveplates θ is, the relative polarization angle $2\theta - 2\beta$ is random because β is random, implying that the coherence of the photon pair is destroyed, i.e. the photon pair are not entangled any more.

The use of quarter waveplates will cause circularly or elliptically polarized light and thus complicate the Bell test, so quarter waveplates should not be used in this experiment.

In fact, Lahoz Sanz et al (2024) performed a Bell test shown in Fig. 3(b). However, they used 3 half-waveplates and a pair of quarter wave plates. Their theoretical reasoning and mathematical derivation are correct but are critically hinged on the implicit assumption that the |H> and |V> basis of the photon pair is literally horizontal and vertical in their analysis. This assumption is apparently inappropriate given that the polarization angles of the photon pair are random. The use of 3 half-waveplates reduces the coherence of the photon pairs and the use of quarter waveplates further complicated the experiments, rendering unreliable experimental results.

(4) Optical filters to be used to allow only the down-conversion photon pairs to pass through.

Entangled photon pairs are produced by shining laser beam on two type-I BBO crystals, so the entangled photon pairs are embedded in the stream of photons in the laser beam. Since the down-conversion photon pairs have a different frequency from that of the laser beam, using filters is necessary to filter out the photons in the laser beam.

6. Conclusions

To test the claim of spooky action at a distance, we designed a new type of Bell test. If the new tests shown in Fig.3 lead to a result of violating Bell theorem with predicted optimal relative angle of half-waveplates, we can claim that the correlated coincidence result is due to the coherence setting for the photon pair because the violation of the Bell theorem is caused by the relative polarisation angles of the photon pair. Likewise, since the measurement setting has not changed during the experiment, the violation of Bell test is definitively not due to any change in the measurement of polarisation of photons.

In order to prove or disprove eq. (28) or eq.(29), we can conduct a series experiments by varying the relative angle of fast axes of two half-waveplates at each given value for the relative angle of the PBS cubes. If eq. (28) is confirmed by experiments, we can reject the claim that the measurement of polarisation for one photon causes the polarisation change of its entangled counterpart. The reasoning can be demonstrated with the aid of Fig.2(b).

We let PBS cube A in Fig.2(b) closer to the light source than cube B does. When photon A arrives at PBS cube A at a polarisation angle of β and subsequently changes polarisation angle, if the claimed inner connection between the photon pair would change the polarisation angle of entangled photon B by a fixed amount Δβ before it arrives cube B, photon B will enter the cube B at a polarisation angle of β+Δβ. The predicted correlation can be expressed by eq. (28) but with β'_{ab} being replaced by Δβ, namely:

$$E(a,b) = \cos 2(\theta_{ab} - \Delta\beta) \qquad (30)$$

In this case, we can still see a violation of the Bell theorem, but the optimal angles for violation are no longer π/8 and 3π/8; they will shift by the amount of angle Δβ.

If the interaction of photon A with PBS cube A changes the polarisation of photon B to a fixed angle, e.g. to a portion of θ_a, we have $\beta'_a=\beta$, $\beta'_b=f\theta_a$, $\beta'_{ab}=\beta - f\theta_a$, where β'_{ab} is the relative polarization angle of the photon pair). Plugging β'_{ab} into eq. (28), we have:

$$E(a,b) = \cos 2(\theta_{ab} - \beta + f\theta_a) \qquad (31)$$

As β is random for each photon pair, the above equation will produce a random result, meaning no nonlocal correlation. In other words, the Aspect-style of Bell test should be very sensitive to the asymmetric distance from the light source to the two polarization measurement devices. Unequal distances render zero nonlocal correlation.

Given that the distances from light source to two polarisation measurement devices are not critical for results of Bell tests – some researchers made the distance between two polarisers (and also between the light source and polarisers) extremely large so as to avoid communication loophole and still obtained excellent results of violation of Bell theorem, we can conclude that the change in polarization of one photon A at the PBS cube A does not change the polarization of photon B in any way (e.g., by a fixed amount or a variable amount) and thus the spook action hypothesis (superluminal signalling, superposition over large distance, or any mysterious inner connection between the entangled particles) is false.

References:

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[1] J. S. Bell, "On the Einstein-Podolsky-Rosen Paradox," Physics 1, 195 (1964)
[2] A. Einstein, B. Podolsky, and N. Rosen, "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?," Phys. Rev. 47, 777 (1935).
[3] A. Aspect, J. Dalibard, and G. Roger, "Experimental Test of Bell's Inequalities Using Time- Varying Analyzers," Phys. Rev. Lett. 49, 1804 (1982).
[4] Tittel W., Brendel J., Gisin B., Herzog T., Zbinden H., Gisin N. Experimental demonstration of quantum-correlations over more than 10 kilometers. Physical Review A, 1998, v.57(5), 3229–3232.
[5] Pan J.W., Bouwmeester D., Daniell M., Weinfurter H., Zeilinger A. Experimental test of quantum nonlocality in three-photon GHZ entanglement. Nature, 2000, v.403(6769), 515–519.
[6] Rowe M.A.,Kielpinski D., Meyer V., Sackett C.A., Itano W.M., Monroe C., Wineland D.J. Experimental violation of a Bell's inequality with efficient detection. Nature, 2001, v.409(6822), 791–794.
[7] Gr¨oblacher S., Paterek T., Kaltenbaek R., Brukner S., Zukowski M., Aspelmeyer M., Zeilinger A. An experimental test of non-local realism. Nature, 2007, v. 446(7138), 871–875.
[8] Ansmann M., Wang H., Bialczak R., Hofheinz M., Lucero E., Neeley M., O'ConnellA.D., SankD., WeidesM., WennerJ., ClelandA.N., Martinis J.M. Violation of Bell's inequality in Josephson phase qubits. Nature, 2009, v.461, 504–506.
[9] Giustina M., Mech A., Ramelow S., Wittmann B., Kofler J., Beyer J., Lita A., Calkins B., Gerrits T., Nam S.W., Ursin R., Zeilinger A. Bell violation using entangled photons without the fair-sampling assumption. Nature, 2013, v.497(7448), 227–230.
[10] B. Hensen et al., "Loophole-free Bell Inequality Violation Using Electron Spins Separated by 1.3 Kilometres," Nature 526, 682 (2015).
[11] L. K. Shalm et al., "Strong Loophole-Free Test of Local Realism," Phys. Rev. Lett. 115, 250402 (2015)
[12] Schmied R., Bancal J.-D., Allard B., Fadel M., Scarani V., Treutlein P., Sangouard N. Bell correlations in a Bose-Einstein condensate. Science, 2016, v.352(6284), 441–444.
[13] BIG Bell Test Collaboration. Challenging local realism with human choices. Nature, 2018, v.557(7704), 212–216.
[14] Shin D.K., Henson B.M., Hodgman S.S., Wasak T., Chwedenczuk J., Truscott A.G. Bell correlations between spatially separated pairs of atoms. Nature Communications, 2019, v.104447.
[15] Thenabadu M., Cheng G.-L., Pham T.L.H., Drummond L.V., RosalesZ´arate L., Reid M.D. Testing macroscopic local realism using local nonlinear dynamics and time settings. Physical Review A, 2020, v.102, 022202.
[16] Storz S, Schär J, Kulikov A, Magnard P, Kurpiers P, Lütolf J, Walter T, Copetudo A, Reuer K, Akin A, Besse JC, Gabureac M, Norris GJ, Rosario A, Martin F, Martinez J, Amaya W, Mitchell MW, Abellan C, Bancal JD, Sangouard N, Royer B, Blais A, Wallraff A. Loophole-free Bell inequality violation with superconducting circuits. Nature. 2023 May;617(7960):265-270. doi: 10.1038/s41586-023-05885-0. Epub 2023 May 10. PMID: 37165240; PMCID: PMC10172133.
[17] Clauser J.F., Shimony A. Bell's theorem: experimental tests and implications. Reports on Progress in Physics, 1978, v.41, 1881.
[18] Dehlinger D, Mitchell M. 2002; Entangled photon apparatus for the undergraduate laboratory, Am J Phys. 70:898-902

[19] Waseem MH, Anwar MS, et al. (2020) Quantum mechanics in the single photon laboratory. Chap. Quantum state tomography. IOP Publishing;
[20] Lahoz Sanz., Lidia Lozano Martín, AdriàBrúiCortés, Martí Duocastella, Jose M. Gomez and Bruno Juliá-Díaz, 2024, Undergraduate setup for measuring the Bell inequalities and performing quantum state tomography , EPJ Quantum Technology , 11(86), https://doi.org/10.1140/epjqt/s40507-024-00298-y